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Math 472
Exam 2

1. Given: $S = 250{,}000$, $x = 30$, $n = 30$, $i = 0.05$

a) Initial(Gross): $0.4P^G + 200$
Renewal(Gross): $(0.05P^G + 50)(\ddot{a}_{[30]:30\urcorner} - 1)$

(with the $100 expense)

$$\implies {}_oV^G = 250{,}100A_{[30]} + 0.4P^G + 200 + (0.05P^G + 50)(\ddot{a}_{[30]:30\urcorner} - 1) - P^G\ddot{a}_{[30]:30\urcorner} = 0$$

$$\implies P^G = \frac{1000(5002A_{[30]} + \ddot{a}_{[30]:30\urcorner} + 3)}{19\ddot{a}_{[30]:30\urcorner} - 7} = \$1356.08181203$$

$\implies A_{[30]} =$ in the table in the back

$$\ddot{a}_{[30]:30\urcorner} = \ddot{a}_{[30]} - v^{30}{}_{30}p_{[30]}\ddot{a}_{60} = 16.0422991470$$

(without the $100 expense)

$${}_oV^n = 250{,}000A_{[30]} - P_{net}\ddot{a}_{[30]:30\urcorner} = 0 \implies P_{net} = \frac{250{,}000A_{[30]}}{\ddot{a}_{[30]:30\urcorner}}$$

$$P_{net} = \$1198.86182297$$

b) Objective: Find ${}_1V^n$

$${}_1V^n = 250{,}000A_{[30]+1} - P_{net}\ddot{a}_{[30]+1:29\urcorner}$$

$$A_{[30]+1} = vq_{[30]+1} + \sum_{k=1}^{89} v^{k+1}{}_kp_{31} \times q_{31+k} = 0.080526176009$$

$$\ddot{a}_{[30]+1:29\urcorner} = \sum_{t=0}^{28} v^t{}_tp_{[30]+1} = 1 + \sum_{t=1}^{28} v^t{}_tp_{31} = 15.7989283795$$

$$\implies {}_1V^n = \$1190.81192435$$

c) Objective: Find ${}_1V^G$

$$\begin{aligned}
{}_1V^G &= 250{,}100A_{[30]+1} + 0.05P^G\ddot{a}_{[30]+1:29\urcorner} - 0.05P^G + 50\ddot{a}_{[30]+1:29\urcorner} - 50 - P^G\ddot{a}_{[30]+1:29\urcorner} \\
&= 250{,}100A_{[30]+1} + 0.05P^G\ddot{a}_{[30]+1:29\urcorner} + 50\ddot{a}_{[30]+1:29\urcorner} - P^G\ddot{a}_{[30]+1:29\urcorner} \\
&= \$576.135585193
\end{aligned}$$

d) Objective: Find ${}_{10}V^n$

$${}_{10}V^n = 250{,}000A_{40} - P_{net}\ddot{a}_{40:20\urcorner}$$

$$\ddot{a}_{40:20\urcorner} = \ddot{a}_{40} - v^{20}{}_{20}p_{40}\ddot{a}_{60} = 12.9937457615$$

$$\implies {}_{10}V^n = \$14{,}687.2942692$$

e) Objective: Find ${}_{10}V^G$

$$\begin{aligned}
{}_{10}V^G &= 250{,}100A_{40} + 0.05P^G\ddot{a}_{40:20\urcorner} + 50\ddot{a}_{40:20\urcorner} - P^G\ddot{a}_{40:20\urcorner} \\
&= 250{,}100A_{40} + (0.05P^G + 50 - P^G)\ddot{a}_{40:20\urcorner} \\
&= 250{,}100A_{40} + (50 - 0.95P^G)\ddot{a}_{40:20\urcorner} \\
&= \$14{,}187.2401056
\end{aligned}$$

f) Objective: Find AS_1

The formula is

$$AS_t = \frac{(1+i)^t(P-E) - Sq_{[x]+t-1}}{p_{[x]+t-1}} \implies AS_1 = \frac{(1.05)^1(0.6P^G - 200) - 250{,}000q_{[30]}}{p_{[30]}}$$

$$AS_1 = \$577.099127728$$

g) Objective: Find AS_1(not the same as in part f)

$$AS_1 = \frac{(1.075)(0.6P^G - 200) - 250{,}000(0.001)}{1 - 0.001} = \$410.082851611$$

h) Objective: Find the Surplus

$$\text{Surplus} = (1.075)(0.6P^G - 200) - 250{,}000(0.001) - {}_1V^G(1 - 0.001) = \$ - 165.886680851$$

2. Given:

$$\begin{aligned} x &= 45 \\ n &= 20 \\ \text{Initial: } & 0.4P + 100 \\ \text{Renewal: } & 0.05P + 25 \end{aligned}$$

a) Objective: Find P

$${}_oV = 50{,}000v^{20}{}_{20}p_{[45]}\ddot{a}_{65} - P\ddot{a}_{[45]:20\urcorner} + P(IA)_{[45]:20\urcorner}^{\ 1} + 0.4P + 100 + 0.05P + 25 = 0$$

Rearrange and solve for P, we get

$$P = \frac{125 + 50{,}000v^{20}{}_{20}p_{[45]}\ddot{a}_{65}}{\ddot{a}_{[45]:20\urcorner} - (IA)_{[45]:20\urcorner}^{\ 1} - 0.45}$$

$$\ddot{a}_{65} = 13.550 \ ; \ \ddot{a}_{[45]:20\urcorner} = 12.9410842501 \ ; \ (IA)_{[45]:20\urcorner}^{\ 1} = \sum_{k=0}^{19} v^{k+1}(k+1)_k p_{[45]} q_{[45]+k}$$

$$\implies (IA)_{[45]:20\urcorner}^{\ 1} = 0.291492991148$$

$$\implies P = \$20{,}002.37404$$

b) Objective: Find ${}_1V$

$${}_1V = 50{,}000v^{19}{}_{19}p_{[45]+1}\ddot{a}_{65} - P\ddot{a}_{[45]+1:19\urcorner} + P(IA)_{[45]+1:19\urcorner}^{\ 1}$$

Compute the missing values:

$$\ddot{a}_{[45]+1:19\urcorner} = \sum_{t=0}^{18} v^t{}_t p_{[45]+1} = 1 + \sum_{t=1}^{18} v^t{}_t p_{[45]+1} = 12.5457549282$$

$$(IA)^{1}_{[45]+1:19\urcorner} = \sum_{k=0}^{18} v^{k+1}(k+1)_k p_{[45]+1} q_{[45]+1+k}$$

$$= v q_{[45]+1} + \sum_{k=1}^{18} v^{k+1}(k+1)_k p_{46} q_{46+k}$$

$$= 0.281327860564$$

$$\implies {}_1V = \$10{,}941.8374963$$

c) Objective: Find AS_1

$$AS_1 = \frac{(1.06)(0.6P - 100 - 100) - 50{,}000(0.002)}{1 - 0.002} = \$12{,}434.3786467$$

d) Objective: Find the Surplus

$$\text{Surplus} = (1.06)(0.6P - 200) - 50{,}000(0.002) - {}_1V(1 - 0.002) = \$1{,}489.55606813$$

3. Given:

$x = 40$
$n = 20$
$S = 250{,}000$
$i = 0.05$

Initial: $0.4P^G + 150$

Renewal: $0.05P^G + 50$

a) Objective: Find P^G

$${}_oV^G = (S + 200)A_{[40]:20\urcorner} + 0.4P^G + 150 + 0.05P^G + 50 - P^G \ddot{a}_{[40]:20\urcorner} = 0$$

Rearrange and solve for P^G,

$$P^G = \frac{200 + 250{,}200A_{[40]:20\urcorner}}{\ddot{a}_{[40]:20\urcorner} - 0.45}$$

Compute the missing values:

$$A_{[40]:20\urcorner} = 1 - d\ddot{a}_{[40]:20\urcorner} = 0.381182312720$$

$$\ddot{a}_{[40]:20\urcorner} = \ddot{a}_{[40]} - v^{20}{}_{20}p_{[40]}\ddot{a}_{60} = 12.9951714329$$

$$\implies P^G = \$7{,}618.21511599$$

b) Objective: Find ${}_1V^G$

$$\begin{aligned} {}_1V^G &= 250{,}200A_{[40]+1:19\urcorner} - P^G\ddot{a}_{[40]+1:19\urcorner} \\ &= 250{,}200\left[1 - d\ddot{a}_{[40]+1:19\urcorner}\right] - P^G\ddot{a}_{[40]+1:19\urcorner} \end{aligned}$$

Compute the missing value:

$$\ddot{a}_{[40]+1:19\urcorner} = \sum_{t=0}^{18} v^t{}_t p_{[40]+1} = 1 + \sum_{t=1}^{18} v^t{}_t p_{41} = 12.5997917217$$

$$\implies {}_1V^G = \$4{,}094.55773443$$

c) Objective: Find ${}_{10}V^G$

$${}_{10}V^G = 250{,}200A_{50:10\urcorner} - P^G\ddot{a}_{50:10\urcorner}$$

Compute the missing values:

$$\ddot{a}_{50:10\urcorner} = \ddot{a}_{50} - v^{10}{}_{10}p_{50}\ddot{a}_{60} = 8.05551277557$$

$$A_{50:10\urcorner} = 1 - d\ddot{a}_{50:10\urcorner} = \text{Compute yourself!!!}$$

$$\implies {}_{10}V^G = \$92{,}855.6900229$$

d) Objective: Find AS_1

$$AS_1 = \frac{(1.07)(0.6P^G - 150 - 100) - 250{,}200(0.0015)}{1 - 0.0015} = \$4{,}254.47581819$$

e) Objective: Find the Surplus

$$\begin{aligned}\text{Surplus} &= (1.07)(0.6P^G - 250) - 250{,}200(0.0015) - {}_1V^G(1 - 0.0015)\\ &= \$159.678206637\end{aligned}$$