Modifications of the Loss Random Variable

In order for a claim to be paid, d must be exceeded by a loss($X > d$), then the amount paid is $Y = X - d$ and the claim amount random variable Y is

$$Y = (X - d)_+ = \begin{cases} 0 \text{ for } X \leq d \\ X - d \text{ for } X > d \end{cases}$$

where $(X - d)_+$ is the "excess of X over d", if positive.

$Y = 0$ if $X \leq d$ so we have the $Discrete$ part of the distribution of Y as

$$p_y(0) = P(Y = 0) = P(X \leq d) = F_x(d)$$

$Continuous$ part of the distribution of Y is

$$f_Y(y) = f_X(d + y)$$

for $y > 0$, since $Y = y$ if $X = y + d$.

Moments of the random variable Y

If X is continuous,

$$E(Y^k) = E\left(X - d)_+^k \right) = \int_d^\infty (x - d)^k f_X(x)dx$$

If X is discrete,

$$E(Y^k) = E\left(X - d)_+^k \right) = \sum_{x>d} (x - d)^k p_X(x)$$

$$\begin{aligned} &Y \text{ is the random variable for the "amount paid per loss event"} \\ &Z \text{ is the random variable for the "amount paid per payment event"} \\ &E(Y) \text{ is the expected amount paid on a } loss \\ &E(Z) \text{ is the expected amount paid on a } payment \end{aligned}$$

When there is a payment($Y > 0$), Z is the same as Y and therefore

$$Z = Y|Y > 0$$

or

$$Z = Y|X > d$$

since $Y > 0$ and $X > d$ are equivalent events.

If X is continuous,

$$E(Z^k) = E(Y^k|Y > 0) = E\left((X - d)^k|X > d \right) = \frac{\int_d^\infty (x - d)^k f_X(x)dx}{1 - F_X(d)}$$

if X is discrete,

$$E(Z^k) = \frac{\sum_{x>d} (x - d)^k p_X(x)}{1 - F_X(d)}$$

$$E(Z^k) = \frac{E(Y^k)}{1 - F_X(d)}$$

The $E(Z)$ is also called the $Mean\ Excess\ Loss$.

In [ ]:

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