Sampled-Data and Digital Control Systems

- Course overview
- Background knowledge
- Analog-to-Digital converters
  - Digital signals
- Approximation of differential equations using difference equations
- Digital-to-Analog converters
  - Delay

I assume you have a background in introductory feedback control (at the level of ECEN 4138/5138).

You should know:

- Laplace transforms
- Block diagram analysis
- P, I, D control
- Lead and lag compensation
- Stability
- Root locus
- Frequency response: Bode and Nyquist plots
- Introductory state-space representations
  - Relationship to transfer functions
  - State-feedback controllers

In practice, controllers are often implemented digitally:

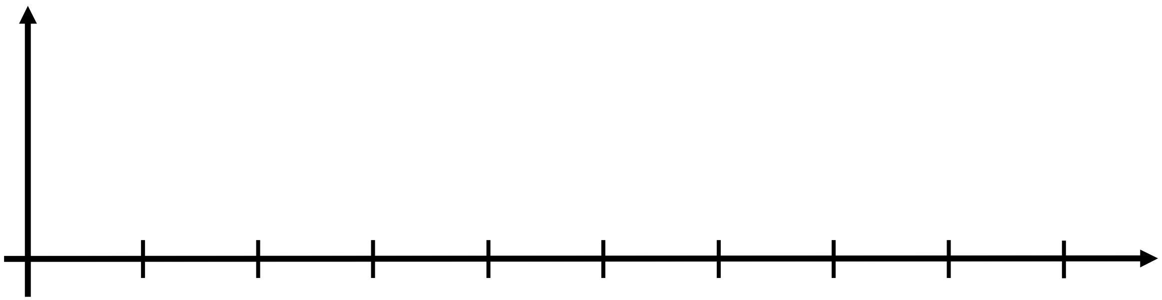
"Good" performance usually means . . .

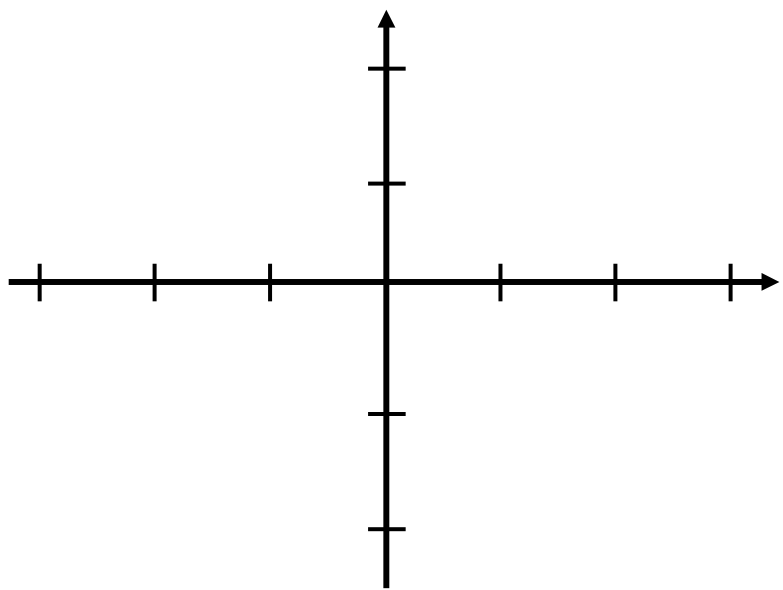
- **Output follows or tracks reference "well" despite . . .**
  - **Disturbances and sensor noise**
  - **Modeling errors**
  - **Parameter variations**

- **Feedback systems are more robust than open-loop systems.**

Analog-to-Digital (A/D) Converters

- **Convert a continuous physical variable (usually a voltage) to a stream of numbers**





- A discrete signal can only change at discrete times.
- A sampled-data system is a system having both discrete and continuous signals.
- A/D converters not only provide a discrete signal, but also a quantized signal --- that is, the signal must be stored in a finite number of bits.
  - Quantization is a nonlinear function.
- A signal that is both discrete and quantized is a digital signal.
  - Digital computers process digital signals.

- **Digital controller analysis and design: take into account effects of sampling period T and the quantization size q.**

- **If both T and q are extremely small . . .**
  - **the digital signals may be considered nearly continuous,**
  - **and continuous methods of analysis and design can be used, and then converted to the digital domain.**

- **In this course, we will try to gain an understanding of the effects of**
  - **Sample rates (fast and slow)**
  - **Quantization (large and small word sizes)**

- **Why not just always make sure the sampling rate is fast and the quantization size is small --- and then just design $D(s)$ and approximate it with $D(z)$?**

In this course, we will largely treat the problem of varying T and q separately.

- ~13 weeks
  - Consider only the effect of T, assuming $q = 0$.
  - Assume linearity and time-invariance
    (what is LTI?)

- ~1 week (depending on student interest and avail time)
  - Effects of $q \neq 0$
    - Some discussion on the effects of quantization will be made in conjunction with earlier lectures

In more detail . . .

Chapters 3 & 4

- ~13 weeks, $q = 0$
  - Discrete systems (linear, constant)
    - Z-transform of discrete signals
    - "pulse" transfer functions
  - Sampled-data systems
    - Discrete transfer functions of continuous systems that are sampled
      - System representations
        - Transform methods
        - State-space methods
  - Dynamic response of discrete systems

In more detail (continued) . . .

Chapter 5
- Inter-sample ripple
- Fourier analysis
- Aliasing, sampling theorem

Chapter 6
- Digital filters

Chapter 7
- Design of feedback controllers
  - Transform methods
    - Root locus
    - Frequency response

Chapter 8
  - State-space methods

In more detail (continued) . . .

Depending on student interest and available time:

Chapter 10

- ~1 week
  - Effects of $q \neq 0$

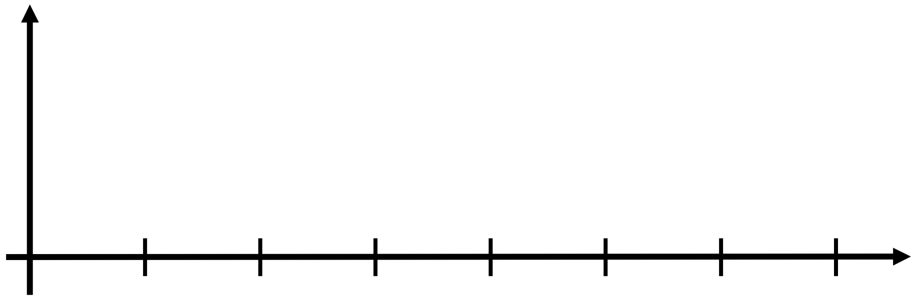
- Will use MATLAB/Simulink in homeworks and labs/projects

Course-Level Learning Goals

- Analyze sampled-data systems to determine stability and predict responses
- Design digital controllers and analyze their performance using both frequency-domain (root locus, Bode, Nyquist) and time-domain (state-space methods)
- Design digital controllers to meet certain (speed and precision) performance specifications

Digitization

- How can we approximate a differential equation using a difference equation?



- Euler's Forward Rectangular Rule (or Euler's Method):

$$\dot{x}(kT) \cong \frac{x\big((k+1)T\big) - x(kT)}{T}$$

- You will explore Euler's Backward Rectangular Method in HW 1 . . .

Example (3.1 of text)

- Suppose

$$D(s) = \frac{U(s)}{E(s)} = K_0 \frac{s+a}{s+b}, \qquad a > 0, \quad b > 0$$

  - General form of a lead or lag compensator

- How can we implement this compensator using a difference equation?
  - First, what is the differential equation that $D(s)$ represents?

Example (continued) . . .

- **Apply Euler's Method**

$$\dot{u} + bu = K_0\left(\dot{e} + ae\right)$$

- **Rework to write $u(k+1)$ (the new "control") as a function of the past control $u(k)$ and the "current" and past "errors" $e(k+1)$ and $e(k)$.**

Comments on Example

- Best to rearrange calculations so that $u(k+1)$ is output to plant as quickly as possible after "current" y and r are "read".
  - See example in Table 3.1 of text.

- Implementing $D(s)$ in this way will work well (meaning digital implementation leads to essentially the same performance as $D(s)$)
  - if the sample rate is $\geq 30 \times$ BW of the system (where the bandwidth is of the closed-loop continuous system with $D(s)$ . . .)

- **As the sample rate decreases below 30 x BW, the closed-loop control performance degrades**
  - **More overshoot**
  - **Appears to be less damped**
  - **Longer settling times**

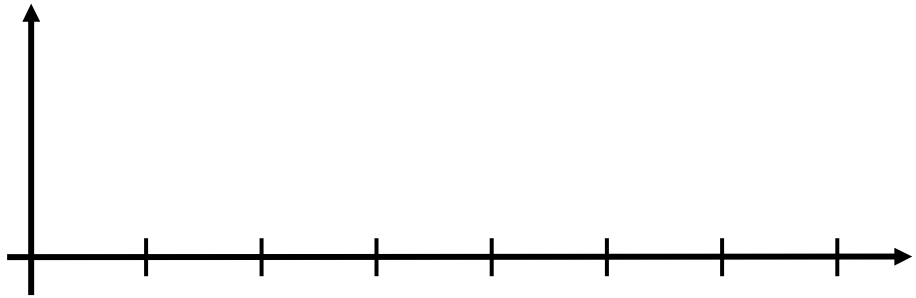
- **In this class, we will spend some time discussing "good" methods of converting $D(s)$ for discrete-time implementation if a continuous $D(s)$ design is already available.**

- **We will also discuss methods of <u>directly</u> designing a discrete controller.**

Digital-to-Analog (D/A) Converters

- **Single most important impact of implementing a control system digitally is the <u>delay</u> associated with the D/A.**

- **Each value of $u(kT)$ is typically held constant until the next value is available from the computer (called a Zero-Order Hold or ZOH).**
  - **The continuous value of $u(t)$ consists of steps --- that on average lag $u(kT)$ by $T/2$.**



- **If we simply include a $T/2$ delay in a continuous-time analysis of a digital system ➔ good agreement results . . .**

- **Performance of discrete-time implementation of $D(s)$ with sample time T can be approximated by this system.**

- **Delay in any feedback system degrades damping and ultimately stability of the system.**

- **Could plot a <u>locus</u> of roots as a function of T to understand the behavior of the discretized implementation of $D(s)$ on system performance . . .**

- **Alternatively, the delay effect can be analyzed using frequency response techniques.**