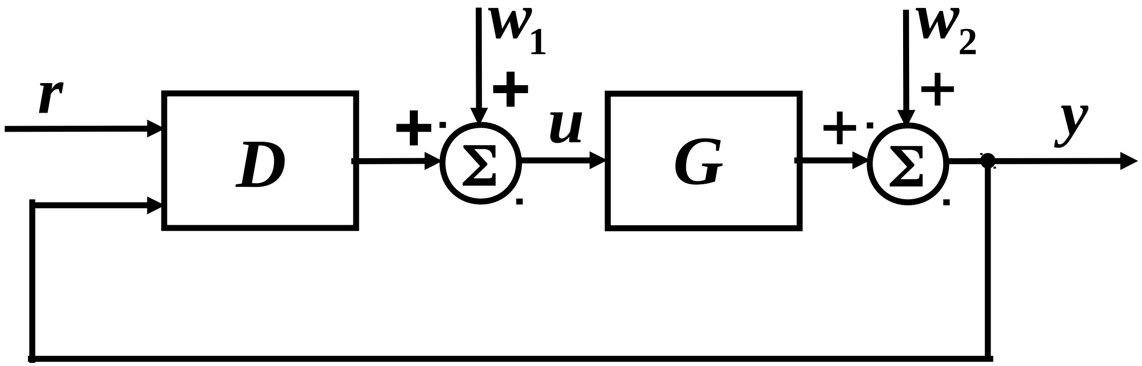
Integral Control and Disturbance Estimation

- Integral control
- Bias Estimation (or internal model) control
  - Constant and sinusoidal disturbances
- Comparison of compensator design methods

Integral Control



Plant:

$$\mathbf{x}(k+1) = \Phi \mathbf{x}(k) + \Gamma u(k)$$
$$y(k) = \mathbf{H}\mathbf{x}(k)$$

To implement integral control in this state-space system, we need to augment the state of the plant with x_I:

$$x_I(k+1) = x_I(k) + y(k) - r(k) = x_I(k) + \mathbf{H}\mathbf{x}(k) - r(k)$$

Transfer Function from E to X_I

$$(z-1)X_I(z) = E(z) \Longrightarrow \frac{X_I(z)}{E(z)} = \frac{1}{z-1}$$ pole at $z = 1$

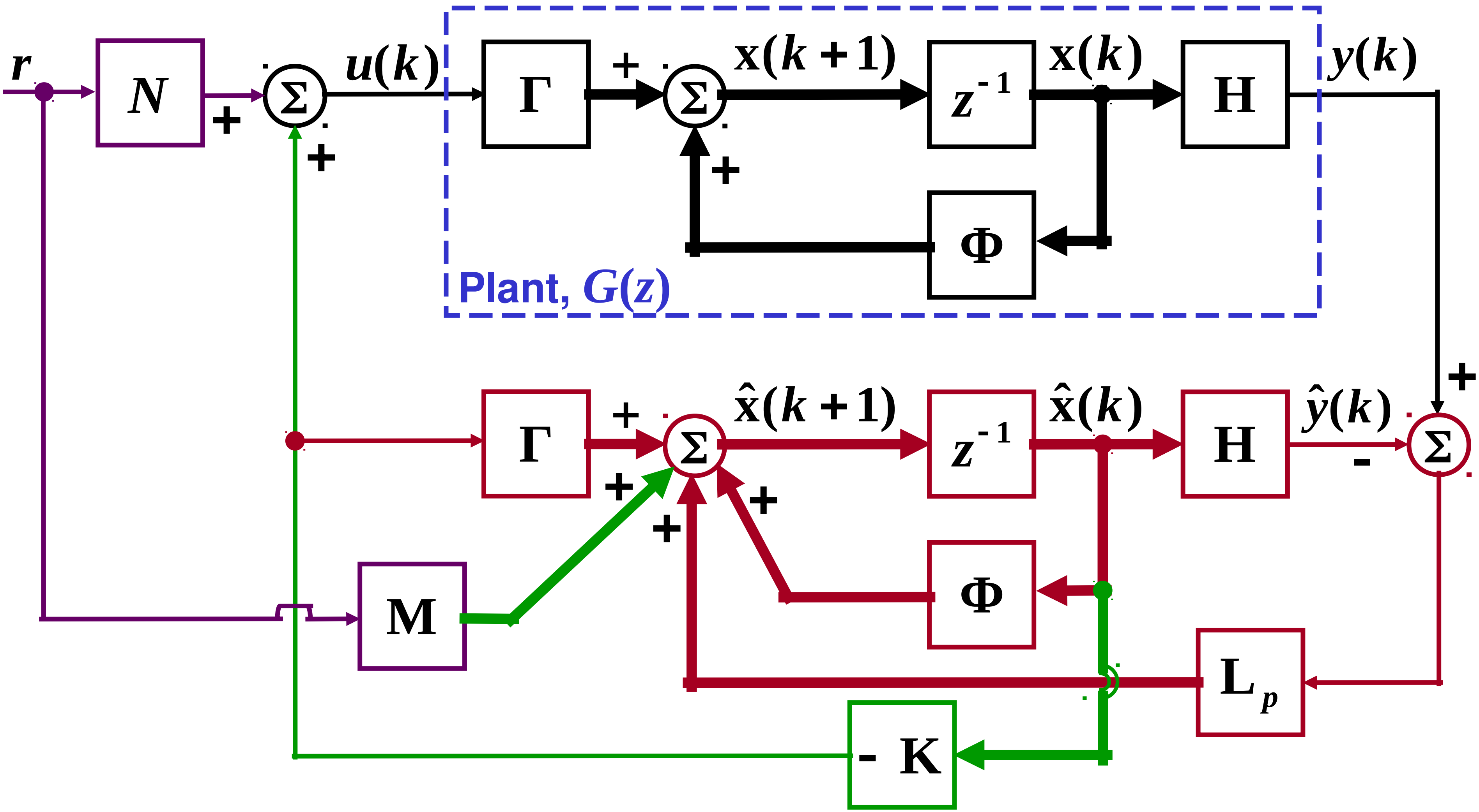
Augmented system:

$$\begin{bmatrix} x_I(k+1) \\ \mathbf{x}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & \mathbf{H} \\ \mathbf{0} & \Phi \end{bmatrix} \begin{bmatrix} x_I(k) \\ \mathbf{x}(k) \end{bmatrix} + \begin{bmatrix} 0 \\ \Gamma \end{bmatrix} u(k) + \begin{bmatrix} -1 \\ \mathbf{0} \end{bmatrix} r(k)$$

Control law:

$$u(k) = -\begin{bmatrix} K_I & \mathbf{K} \end{bmatrix} \begin{bmatrix} x_I(k) \\ \mathbf{x}(k) \end{bmatrix} + Nr(k)$$

Block Diagram



Overall System Transfer Function

In general, with integral control, the overall system transfer function is

$$\frac{Y(z)}{R(z)} = \eta \frac{\gamma(z) b(z)}{\alpha_e(z) \alpha_{c_I}(z)}$$

If $\mathbf{M} = \Gamma N$,

$$\frac{Y(z)}{R(z)} = \eta \frac{b(z)}{\alpha_{c_I}(z)}$$

Integral control can improve steady-state performance of the system. But it may cause worse transient performance.

Bias Estimation (or Internal Model) Control

Plant model: $\mathrm{x}(k+1) = \Phi \mathrm{x}(k) + \Gamma u(k) + \Gamma_1 w(k)$

The idea is to augment the model of the system to include a model of disturbance or reference, and then cancel it out.

To approximate integral control (to lead to zero error due to a step or constant disturbance), the model to incorporate is:

$$\dot{w} = 0$$

To follow or reject a sinusoid signal, augment system with a more complex model:

$$\ddot{w} = -\omega_0^2 w \qquad \omega_0 : \text{frequency of oscillation}$$

Generally, the disturbance can be modeled as:

$$\dot{\mathbf{x}}_d = \mathbf{F}_d \mathbf{x}_d$$

$$w = \mathbf{H}_d \mathbf{x}_d$$

What are $\mathbf{F}_d$ and $\mathbf{H}_d$ for a constant disturbance?

What are $\mathbf{F}_d$ and $\mathbf{H}_d$ for a sinusoidal disturbance?

The discrete model of the disturbance is:

$$\mathbf{x}_d(k+1) = \Phi_d \mathbf{x}_d(k)$$
$$w(k) = \mathbf{H}_d \mathbf{x}_d(k)$$

where $\Phi_d = e^{\mathbf{F}_d T}$

Augmented plant model:

$$\begin{bmatrix} \mathbf{x}(k+1) \\ \mathbf{x}_d(k+1) \end{bmatrix} = \begin{bmatrix} \Phi & \Gamma_1 \mathbf{H}_d \\ 0 & \Phi_d \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{x}_d(k) \end{bmatrix} + \begin{bmatrix} \Gamma \\ 0 \end{bmatrix} u(k)$$

$$y = \begin{bmatrix} \mathbf{H} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{x}_d(k) \end{bmatrix}$$

Then, we can design the compensator using state-space concepts:

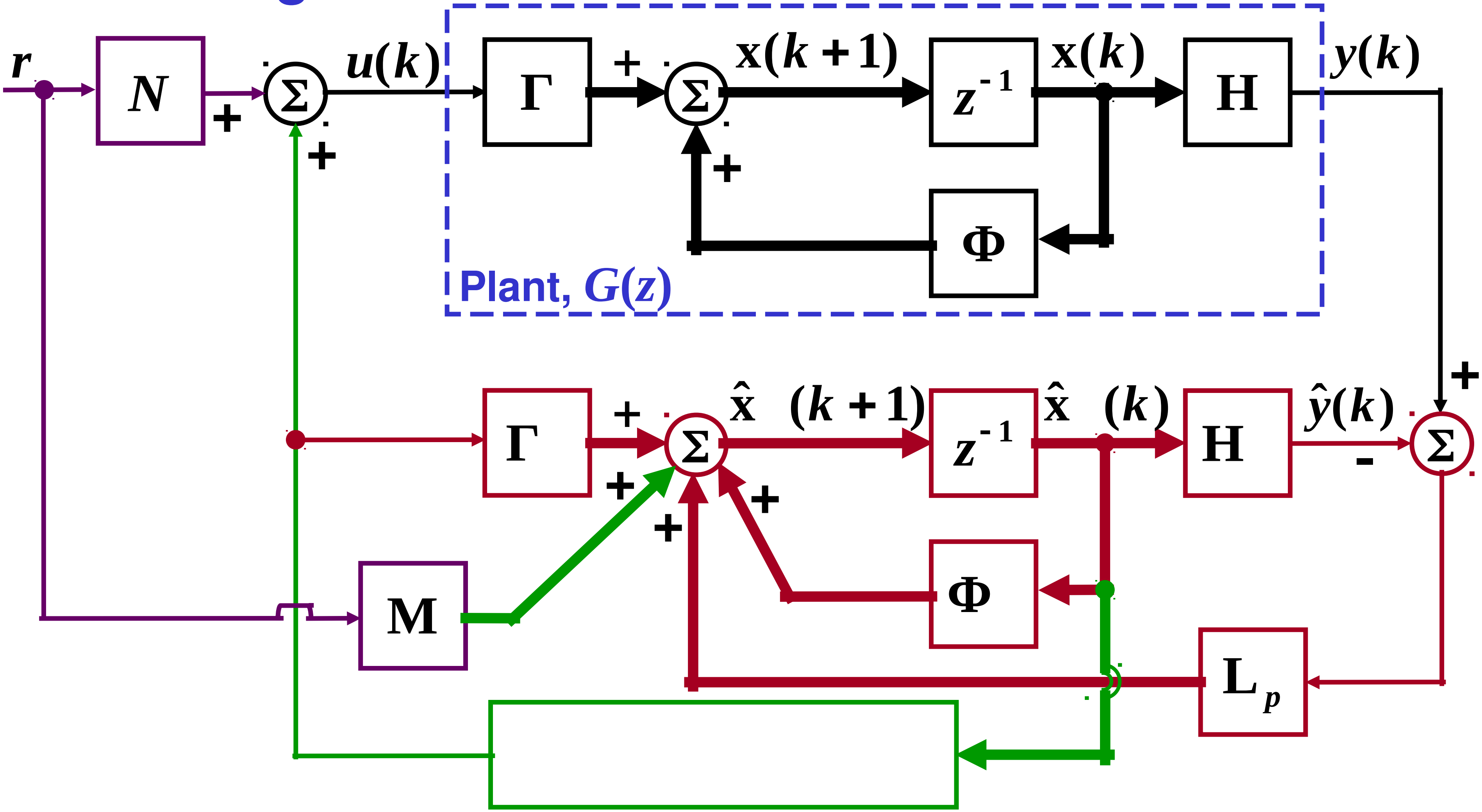
State-feedback design:

- Design $\mathbf{K}$ for the n states $\mathbf{x}(k)$.
- Additional states $\mathbf{x}_d$ are uncontrollable.

Estimator design:

- Design $\mathbf{L}_p$ for augmented state vector.

Block Diagram



Compensator Design Methods

Root Locus: Closed-loop poles constrained to move on a locus.

Bode: Design the compensator based on BW, PM, GM

- Often without even computing/knowing closed-loop poles

State-Space: Matrix calculations to place closed-loop poles (and zeros) at desired locations

- More general and powerful without specifying compensator poles and zeros
- More complex