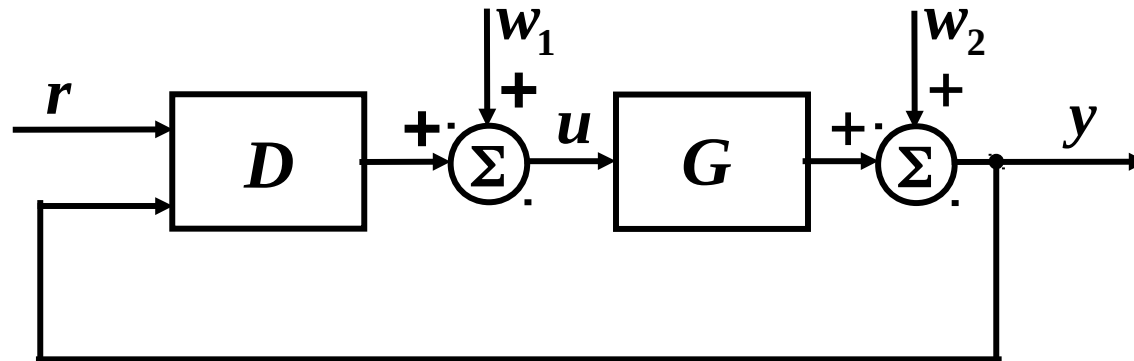


Integral Control and Disturbance Estimation

- **Integral control**
- **Bias Estimation (or internal model) control**
 - **Constant and sinusoidal disturbances**
- **Comparison of compensator design methods**

Integral Control



Plant:

$$\mathbf{x}(k+1) = \Phi \mathbf{x}(k) + \Gamma u(k)$$
$$y(k) = \mathbf{H} \mathbf{x}(k)$$

To implement integral control in this state-space system, we need to augment the state of the plant with x_I :

$$x_I(k+1) = x_I(k) + y(k) - r(k) = x_I(k) + \mathbf{H} \mathbf{x}(k) - r(k)$$

Transfer Function from E to X_I

$$(z - 1)X_I(z) = E(z) \implies \frac{X_I(z)}{E(z)} = \frac{1}{z - 1} \quad \text{pole at } z = 1$$

Augmented system:

$$\begin{bmatrix} x_I(k+1) \\ \mathbf{x}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & \mathbf{H} \\ 0 & \Phi \end{bmatrix} \begin{bmatrix} x_I(k) \\ \mathbf{x}(k) \end{bmatrix} + \begin{bmatrix} 0 \\ \Gamma \end{bmatrix} u(k) + \begin{bmatrix} -1 \\ 0 \end{bmatrix} r(k)$$

Control law:

$$u(k) = - \begin{bmatrix} K_I & \mathbf{K} \end{bmatrix} \begin{bmatrix} x_I(k) \\ \mathbf{x}(k) \end{bmatrix} + Nr(k)$$

The diagram illustrates a control system with a Kalman filter. The top section, labeled "Plant, $G(z)$ ", shows the plant dynamics. The input $u(k)$ is processed by block N and summed with a feedback signal to produce the control signal $u(k)$. This signal is then processed by block Γ and summed with a prediction to produce the next state $x(k+1)$. The state $x(k+1)$ is passed through a delay block z^{-1} to become $x(k)$, which is then processed by block H to produce the output $y(k)$. The state $x(k)$ is also passed through a transition block Φ to produce the next prediction. The bottom section shows the Kalman filter. The state estimate $\hat{x}(k)$ is passed through a delay block z^{-1} to become $\hat{x}(k+1)$, which is then processed by block H to produce the predicted output $\hat{y}(k)$. The predicted output $\hat{y}(k)$ is compared with the actual output $y(k)$ to produce the error signal. The error signal is used to calculate the Kalman gain K and to update the state estimate $\hat{x}(k+1)$. The error signal is also used to calculate the Kalman gain K . The Kalman gain K is used to update the state estimate. The error signal is also used to calculate the Kalman gain K . The Kalman gain K is used to update the state estimate.

Overall System Transfer Function

In general, with integral control, the overall system transfer function is

$$\frac{Y(z)}{R(z)} = \eta \frac{\gamma(z)b(z)}{\alpha_e(z)\alpha_{c_I}(z)}$$

If $\mathbf{M} = \mathbf{\Gamma}N$,

$$\frac{Y(z)}{R(z)} = \eta \frac{b(z)}{\alpha_{c_I}(z)}$$

Integral control can improve steady-state performance of the system. But it may cause worse transient performance.

Bias Estimation (or Internal Model) Control

Plant model: $\mathbf{x}(k+1) = \Phi\mathbf{x}(k) + \Gamma u(k) + \Gamma_1 w(k)$

The idea is to augment the model of the system to include a model of disturbance or reference, and then cancel it out.

To approximate integral control (to lead to zero error due to a step or constant disturbance), the model to incorporate is:

$$\dot{w} = 0$$

To follow or reject a sinusoid signal, augment system with a more complex model:

$$\ddot{w} = -\omega_0^2 w \quad \omega_0 : \text{frequency of oscillation}$$

Generally, the disturbance can be modeled as:

$$\dot{\mathbf{x}}_d = \mathbf{F}_d \mathbf{x}_d$$

$$\mathbf{w} = \mathbf{H}_d \mathbf{x}_d$$

What are \mathbf{F}_d and \mathbf{H}_d for a constant disturbance?

What are \mathbf{F}_d and \mathbf{H}_d for a sinusoidal disturbance?

The discrete model of the disturbance is:

$$\begin{aligned} \mathbf{x}_d(k+1) &= \Phi_d \mathbf{x}_d(k) \\ w(k) &= \mathbf{H}_d \mathbf{x}_d(k) \end{aligned} \quad \text{where} \quad \Phi_d = e^{\mathbf{F}_d T}$$

Augmented plant model:

$$\begin{bmatrix} \mathbf{x}(k+1) \\ \mathbf{x}_d(k+1) \end{bmatrix} = \begin{bmatrix} \Phi & \Gamma_1 \mathbf{H}_d \\ \mathbf{0} & \Phi_d \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{x}_d(k) \end{bmatrix} + \begin{bmatrix} \Gamma \\ \mathbf{0} \end{bmatrix} u(k)$$

$$y = \begin{bmatrix} \mathbf{H} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{x}_d(k) \end{bmatrix}$$

Then, we can design the compensator using state-space concepts:

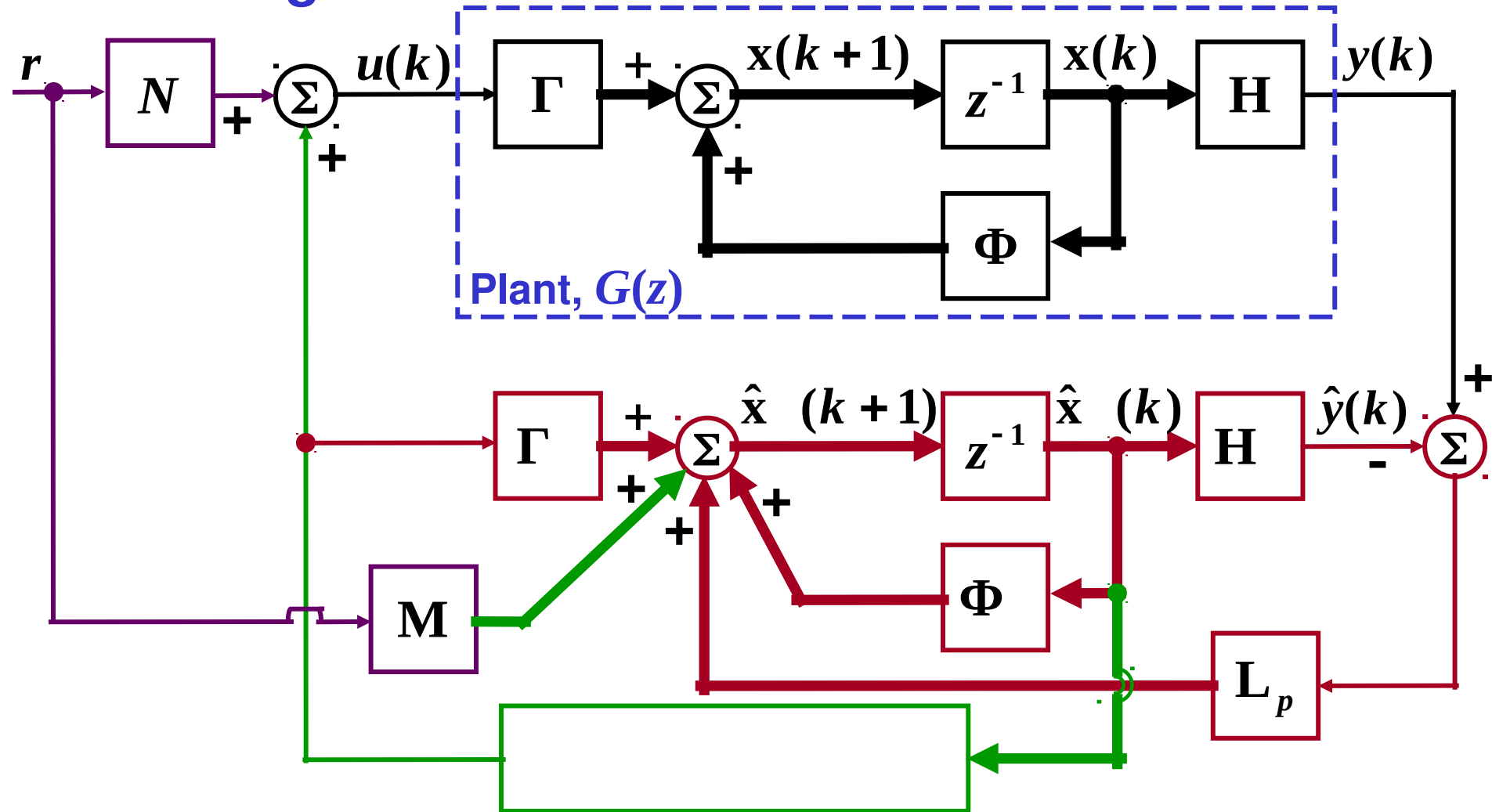
State-feedback design:

- **Design K for the n states $x(k)$.**
- **Additional states x_d are uncontrollable.**

Estimator design:

- **Design L_p for augmented state vector.**

Block Diagram



Compensator Design Methods

Root Locus: Closed-loop poles constrained to move on a locus.

Bode: Design the compensator based on BW, PM, GM

- Often without even computing/knowing closed-loop poles

State-Space: Matrix calculations to place closed-loop poles (and zeros) at desired locations

- More general and powerful without specifying compensator poles and zeros
- More complex