Notes in ECEN 5448

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Missed the first part

Talking about state space rep of a simple system.
State space matrices are F=A, G=B, H=C, and J=D.
J will be zero for any strictly proper transfer function.

you lazy slut, read the fucking slides cause you aren't paying attention.

Real notess

a zero of a discrete-time system is a value of z such that the system output y=0 even if the initial state $x(t_0)$ and the forcing input u are nonzero.

$$x(k+1) = \Phi x(k) + \Gamma u(k)$$
$$y(k) = Hx(k) + Ju(k) \rightarrow$$
$$(zI - \Phi)X(z) - \Gamma U(z) = 0$$
$$HX(z) + JU(z) = 0$$
$$\begin{bmatrix} zI - \Phi & \Gamma \\ H & J \end{bmatrix} \begin{bmatrix} X(z) \\ U(z) \end{bmatrix} = 0$$

first matrix is $(n+1) \times (n+1)$, second matrix is $(n+1) \times 1$.

Discrete Time Systems and Dynamic Response

talking about signal convergence.

$$e_4(k) = r^k \cos(k\theta)1(k)$$
$$E_f(z) = \frac{1}{2} \sum_{k=0}^{\infty} (r^k e^{jk\theta} z^{-k}) + r^k e^{-jk\theta} z^{-k}$$
$$= \frac{1}{2} \left(\frac{1}{1 - re^{j\theta} z^{-1}} + \frac{1}{1 - re^{-j\theta} z^{-1}} \right)$$

ROC $|re^{j\theta} z^{-1}| < 1 \rightarrow |rz^{-1}| < 1$.

$$E_4(z) = \frac{1}{2} \left(\frac{z}{z - re^{j\theta}} + \frac{z}{z - re^{-j\theta}} \right)$$

after some work, you get:

$$E_4(z) = \frac{z(z - r\cos(\theta))}{z^2 - 2r(\cos\theta)z + r^2}, \quad |z| > |r|$$

zeros at $0, r\cos\theta$.
poles at $\dfrac{2r\cos(\theta) \pm \sqrt{4r^2\cos^2\theta - 4r^2}}{2} = r\cos(\theta) \pm jr\sin(\theta) = re^{\pm j\theta}$
closer poles are to the origin, faster the decay is.

figure 4.24 is not a smith chart.

Correspondence of s plane poles and zeros with z plane poles and zeros.

$$Y(s) = \frac{s+a}{(s+a)^2 + b^2} = \frac{s+a}{s^2 + 2\zeta\omega s + \omega^2}$$

$$y(t) = e^{-at}\cos(bt)1(t)$$

poles: $s_1, s_2 = -a \pm jb = -\zeta\omega \pm j\omega\sqrt{1-\zeta^2}$

$$y(k) = y(t)\Big|_{t=kT} = e^{-aKT}\cos(bkT)1(kT)$$

poles: $z = e^{(-a\pm jb)T} = e^{s_1 T}, e^{s_2 T}$
In general, if a continuous signal $y(t)$ has Laplcae transform $Y(s)$ with poles $s_1, s_2, \ldots$, then the sampled (discrete) signal $y(kT)$ has z-transform $Y(z)$ with poles $z_1, z_2, \ldots$ where:

$$z_i = e^{s_i T}$$

$$s_1, s_2 = -\zeta\omega T + j\omega\sqrt{1-\zeta^2}$$

only intersted in that till the phase is $\pm\frac{\pi}{T}$ because of aliasing.
this all gives log spirals, which are the graph in her notes. Again, they only go to phase=π because aliasing produces the rest of the spirtal.