Sample Rate Selection

- Trade-offs
- Absolute lower bound
- Smoothness
- Delay
- Disturbances
- Quantization
- Anti-aliasing filters

Other Topics of Interest . . .

Sample Rate Selection Trade Offs

	Advantages	Disadvantages
Faster Sampling	• better performance (faster response, lower overshoot, . . .)	• cost
Slower Sampling	• less sensitive to word length limitations • less costly	• lower performance

- For mass produced systems: general rule of thumb is to design to sample as slow as possible such that performance specifications can be met.
- For one-of-a-kind or few-of-a-kind systems (e.g., satellites): sample faster to simplify control design because labor costs are significant portions of overall costs.

Absolute Lower Bound

The absolute lower bound on possible sample rates is dictated by the sampling theorem: $\boxed{\omega_s > 2\omega_b}$

If $\omega_s < 2\omega_b$, then sampling will cause aliasing of signals near bandwidth frequency ω_b to lower frequencies.

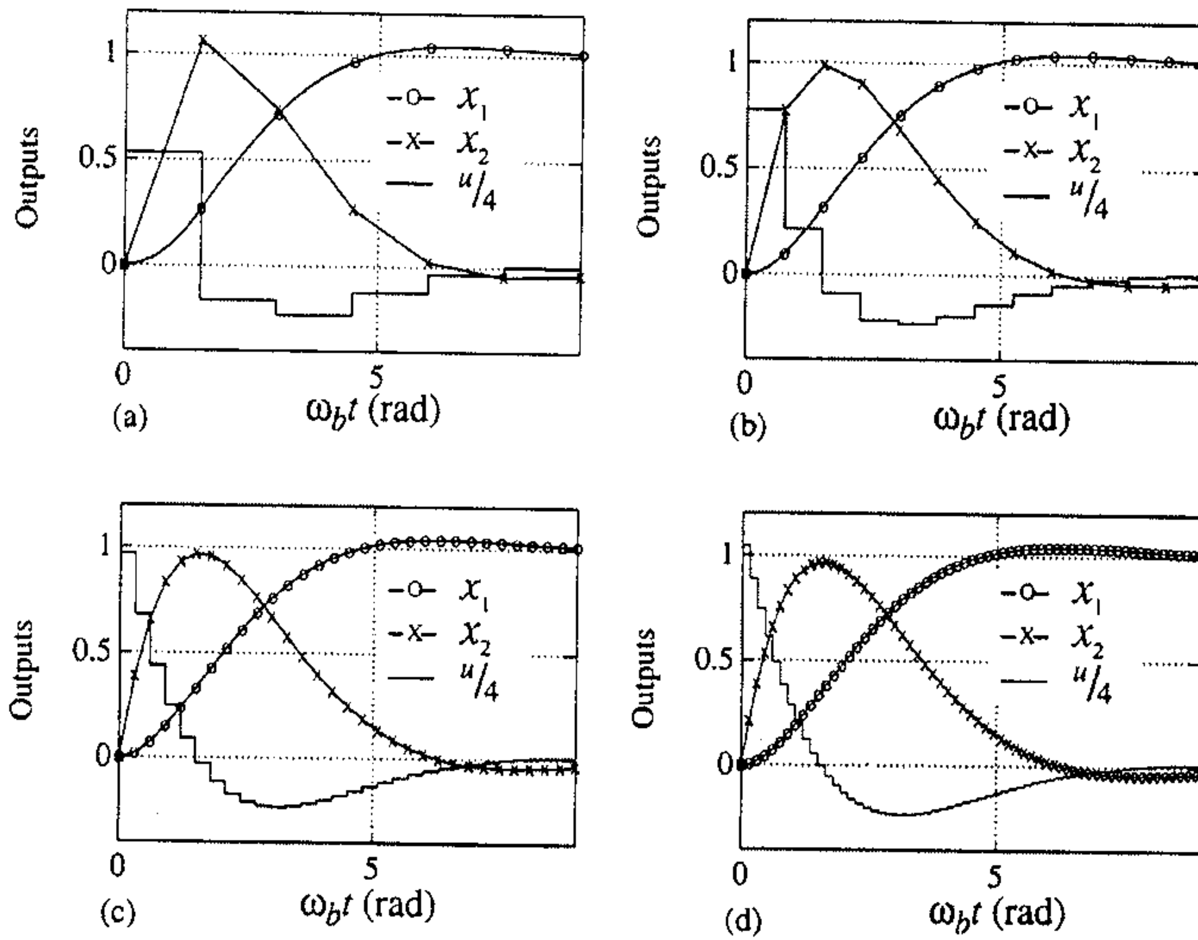
In state-space designs where overall closed-loop poles are those of controller and estimator poles, estimator poles usually are faster than controller poles. So ω_b is dictated largely by controller poles.

Typically: $$20 \le \frac{\omega_s}{\omega_b} \le 40$$

If ratio lower than 20, it may cause unsmooth responses and slower rise times.

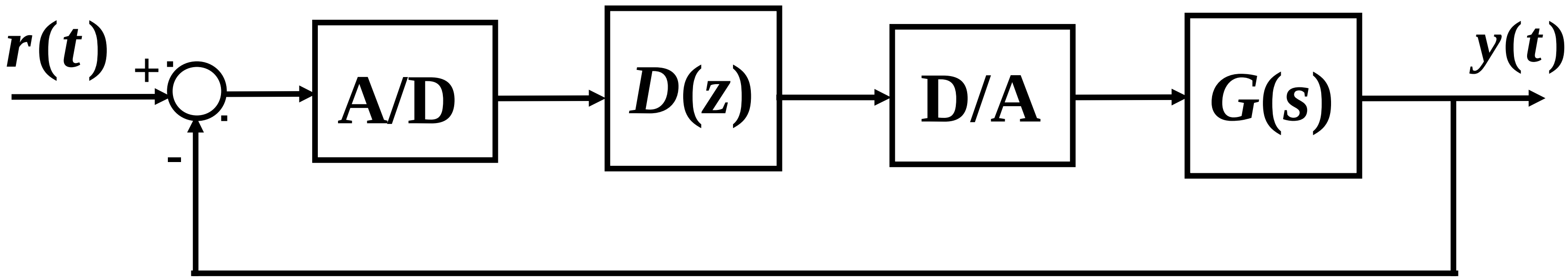
Sample rate selection affects smoothness of responses:

Figure 11.2
Double integrator step response for the sampling multiple with ω_s/ω_b equal to (a) 4, (b) 8, (c) 20, and (d) 40



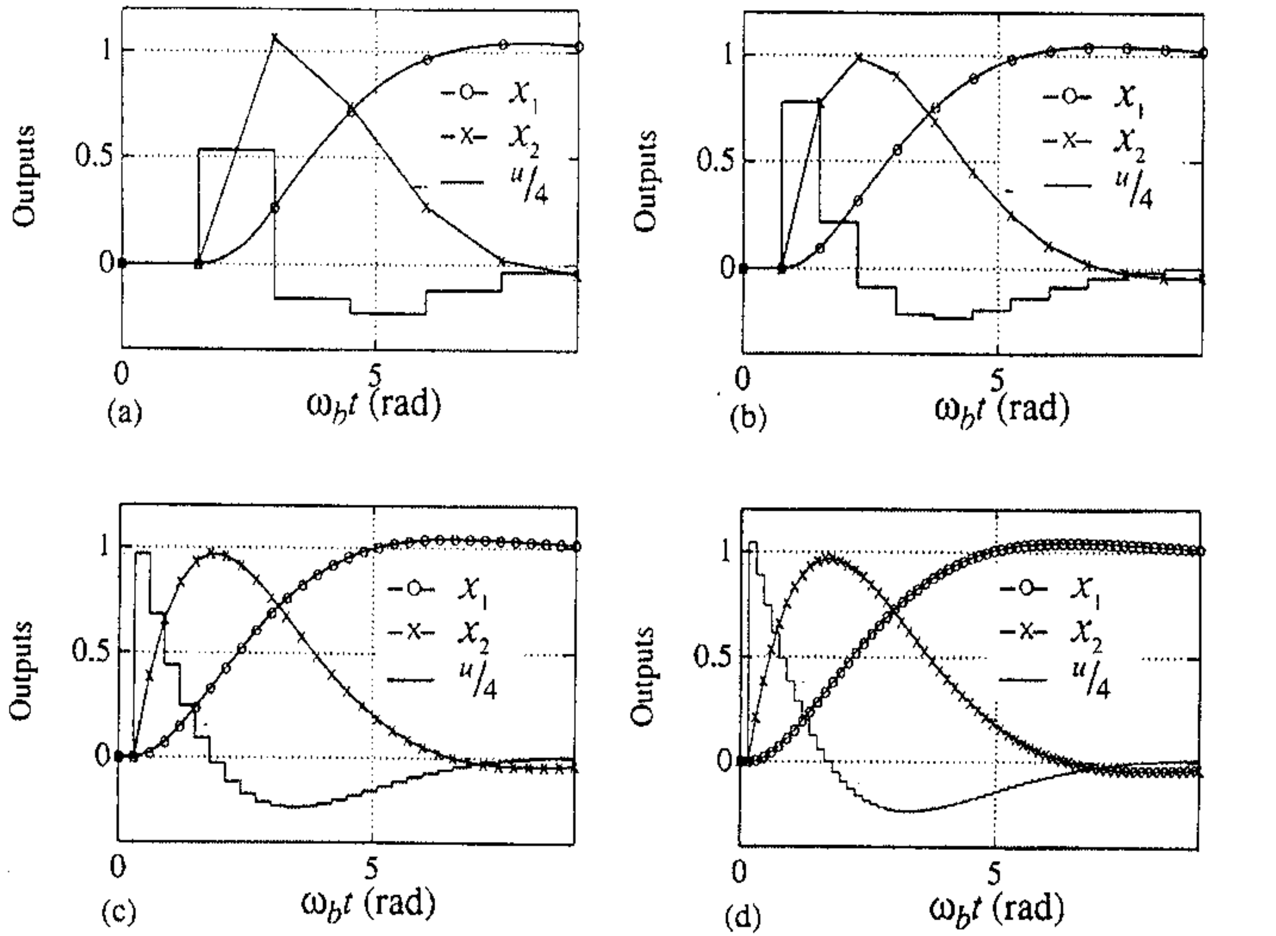
For reasonably smooth responses, need $\omega_s/\omega_b \geq 20$.

Reducing the sampling rate can also cause a delay up to one period in the response of the system.



Example 11.2 of Text

Figure 11.3
Double integrator step response with worst case phasing between command input and the sampler with ω_s/ω_b equal to (a) 4, (b) 8, (c) 20, (d) 40



Effects of ω_s and Disturbances

Figure 11.4
Block diagrams of the systems for disturbance analysis

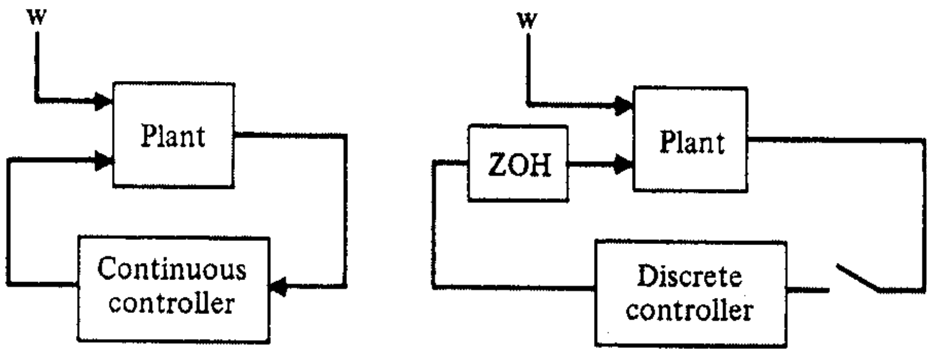
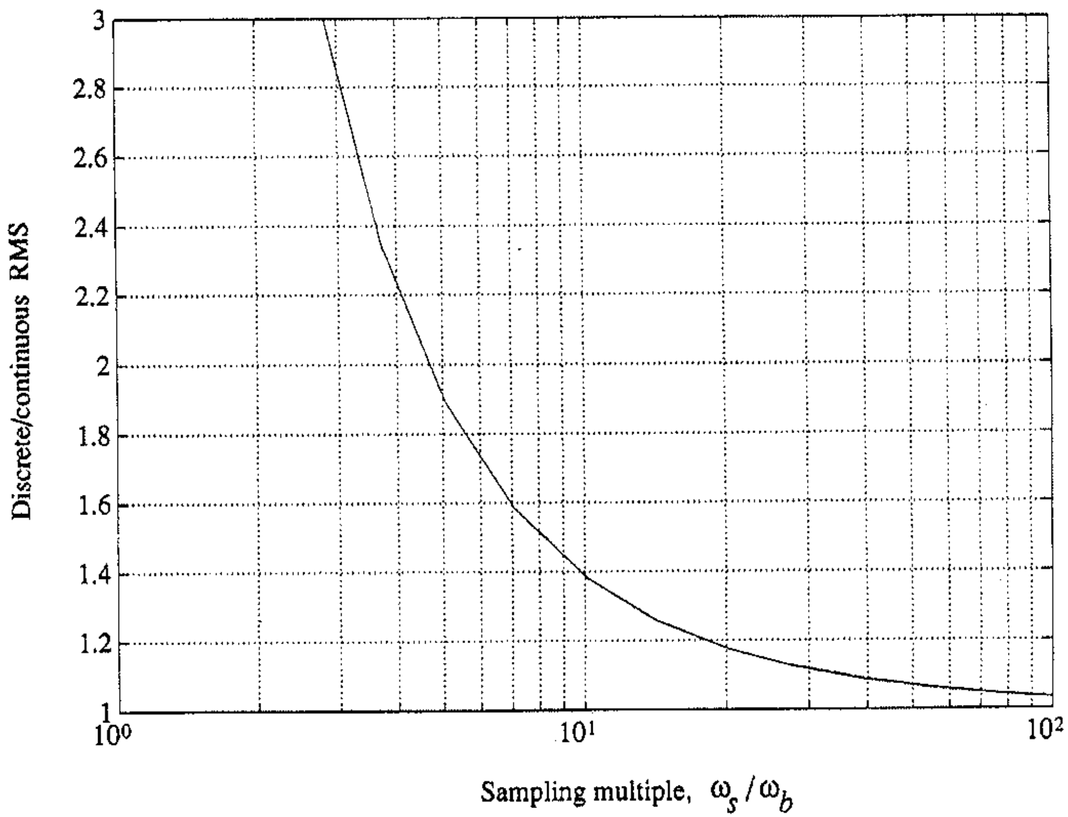
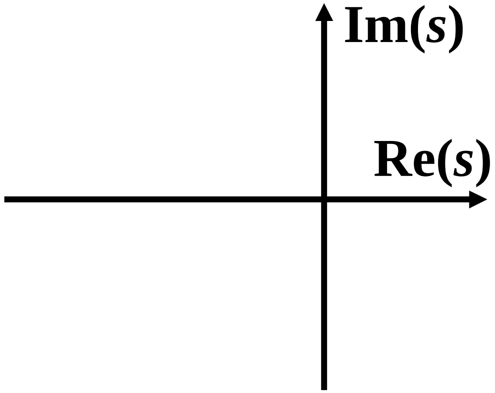


Figure 11.5
Discrete controller degradation versus sample rate for full state feedback and driven by a white disturbance, Example 11.3



Sample Rate Effects and Quantization

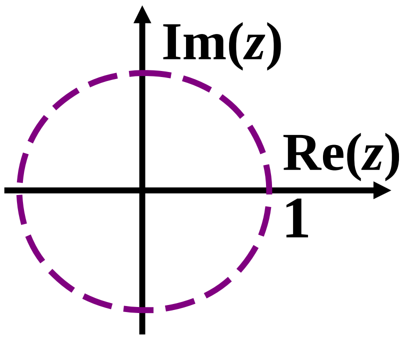
For slower ω_s , poles generally are more spread out.



$$s = -1, \ -2$$

$$T = 1 \sec$$

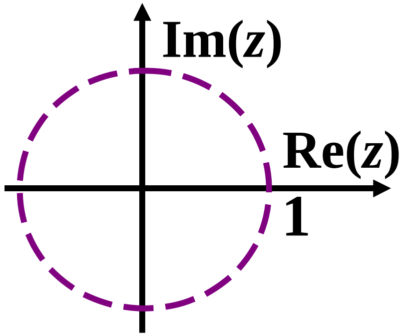
$$z = e^{sT} = 0.368, \ 0.135$$



For faster ω_s , poles move to the right and are more bunched together (near $z = 1$).

$$T = 0.01 \sec$$

$$z = e^{sT} = 0.99, \ 0.98$$



Some Rules of Thumb on Sample Rate Effects from Quantization

Generally, if the word size is ≥ 16 bits, sampling faster does not cause problems. ω_s is not affected by word size in both cascade and parallel realizations.

If the word size is ≤ 8 bits, sampling faster usually causes problems. Increasing ω_s increases the variance in states.

If $8 \text{ bits} < \text{word size} < 16 \text{ bits}$, the effects depend on pole locations and the implementation (cascade, parallel, direct realization). Cascade and parallel implementations of lower-order transfer functions are always better numerically than direct realizations of a higher-order controller.

Measurement Noise and Anti-Aliasing Filters

Anti-aliasing filters are generally needed to remove high-frequency noise in sensor measurements.

Anti-aliasing filter is a low-pass filter:

$$G_p(s) = \frac{\omega_p}{s + \omega_p}$$

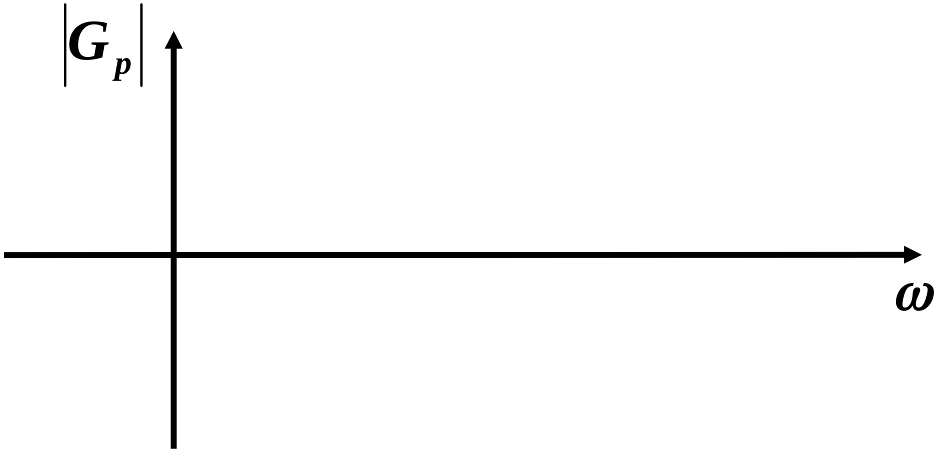
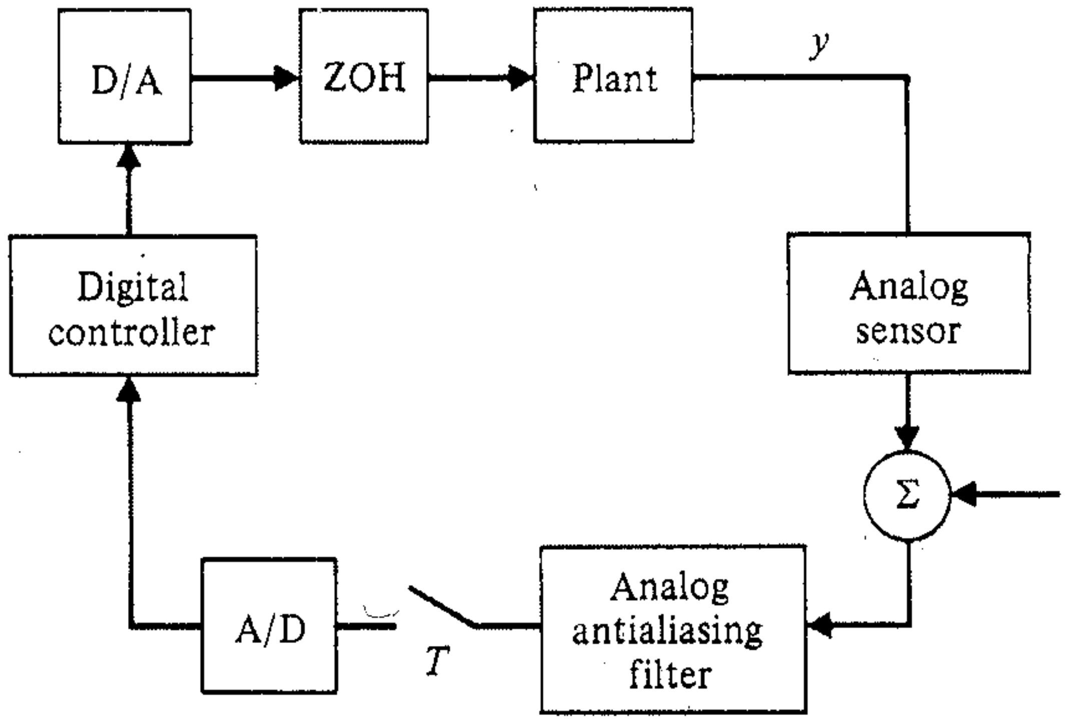


Figure 11.11
Block diagram showing the location of the antialiasing filter



Effect of Anti-Aliasing Filter on Sample Rate Selection

Example 11.6 of Text

Figure 11.12
Demonstration of the effects of an antialiasing filter or prefilter, Example 11.6. (a) Signal plus noise; (b) samples of (a) at $\omega_s = 28\ Hz$; (c) signal in (a) passed through antialiasing filter; (d) sampling of signal in (c)

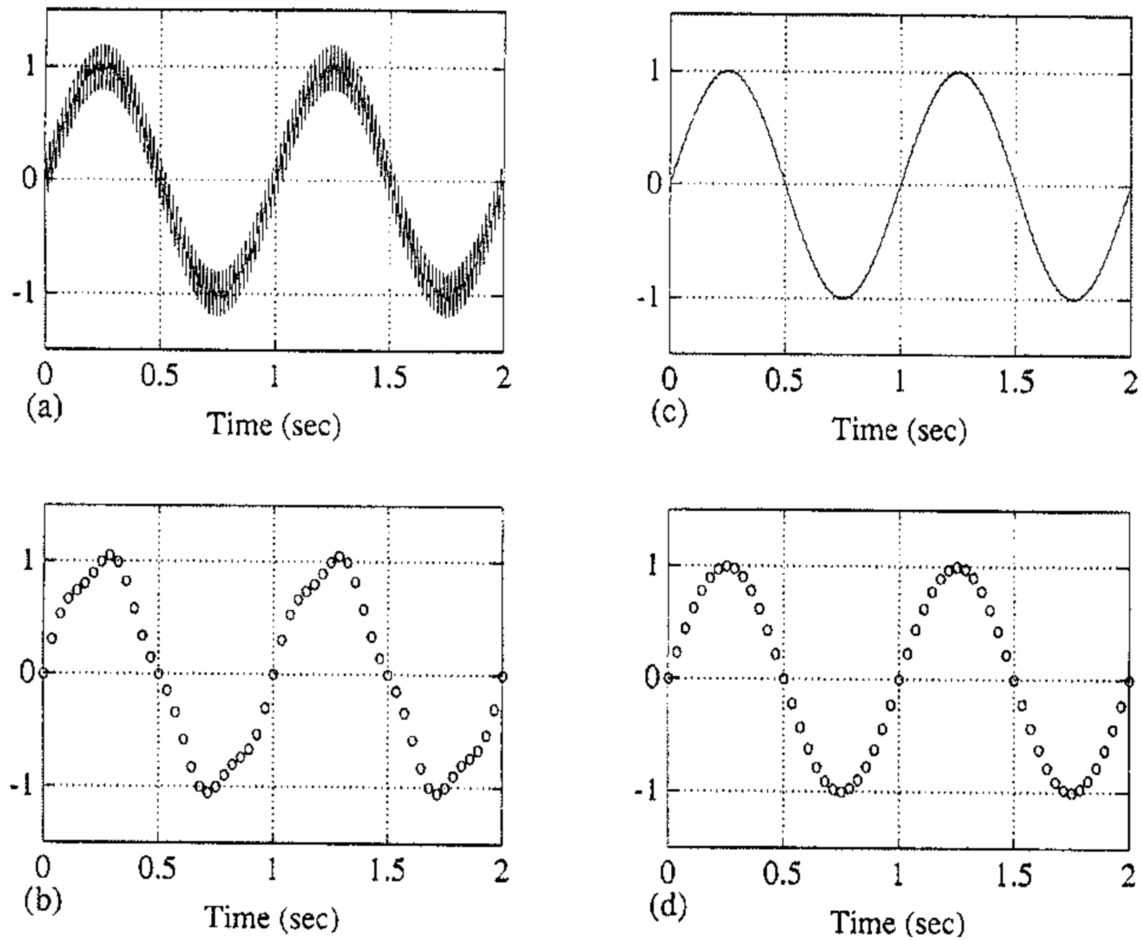
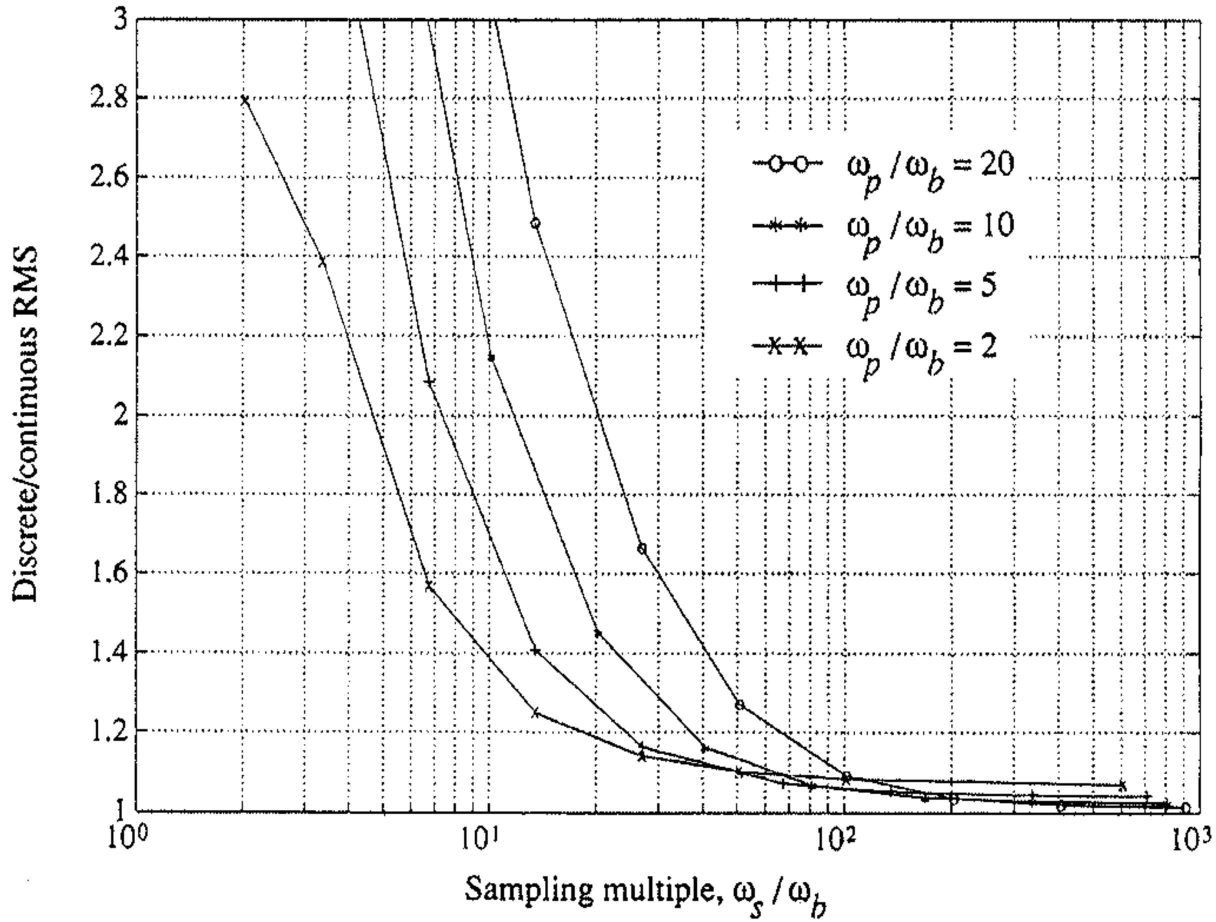


Figure 11.13
Root mean square response of Example 11.7 to white sensor noise showing effects of prefiltering, ω_p, and sampling, ω_s



In general, as $\omega_s \uparrow$ and/or $\frac{\omega_p}{\omega_b} \downarrow$ (but > 2 or so)

$\Longrightarrow$ less sensitivity of system to sensor noise

Other Sample Rate Selection Topics

- Effects of resonances (Section 11.3)
- Sensitivity to parameter variations (Section 11.4)
- Multi-rate sampling (Section 11.6)

Other Control System Topics of Interest (1/3)

- Robust Control (we briefly discussed at the level of Section 7.4.2, ECEN 50x8 covers in detail)
  - *Robust and optimal control*, K Zhou, JC Doyle, and K Glover, Prentice Hall, 1996
  - *Essentials of robust control*, K Zhou and JC Doyle, Prentice Hall, 1998
- Controllability, Observability, and other linear systems analysis concepts (ECEN 5448 covers in detail)
  - *Linear Systems*, T Kailath, Prentice Hall, 1980
  - *Linear System Theory and Design*, CT Chen, Oxford University Press, 1999

Other Control System Topics of Interest (2/3)

- **Optimal control (Chapter 9 introduces, ECEN 5358 covers in detail)**
  - ***Calculus of Variations and Optimal Control Theory: A Concise Introduction*, D Liberzon, Princeton University Press, 2012**

- **Multi-variable control (Chapter 9 introduces, ECEN 5418 covers in detail)**
  - ***Linear Systems*, T Kailath, Prentice Hall, 1980**
  - ***Linear Systems Theory*, JP Hespanha, Princeton University Press, 2009**

Other Control System Topics of Interest (3/3)

- System identification (Chapter 12 introduces, ASEN 6xxx covers in detail)
  - *System Identification: Theory for the User*, L Ljung, Prentice Hall, 2nd Edition, 1999

- Nonlinear systems and control (Chapter 13 introduces, ECEN 7438 covers in detail)
  - *Nonlinear Systems*, HK Khalil, Prentice Hall, 3rd Edition, 2002
  - *Applied Nonlinear Control*, JE Slotine and W Li, Prentice Hall, 1991