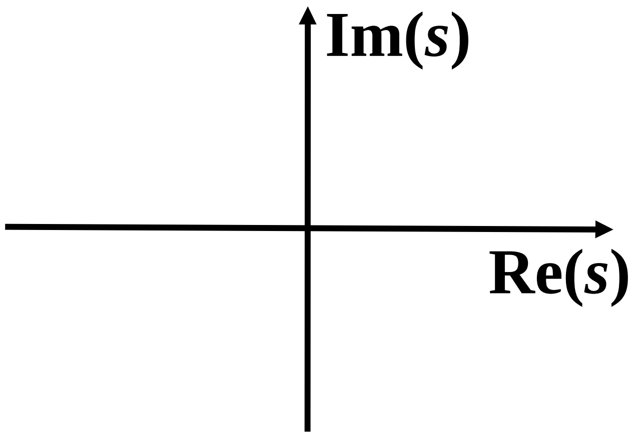
Nyquist Stability Criterion

- Continuous-time systems (review)
- Discrete-time systems

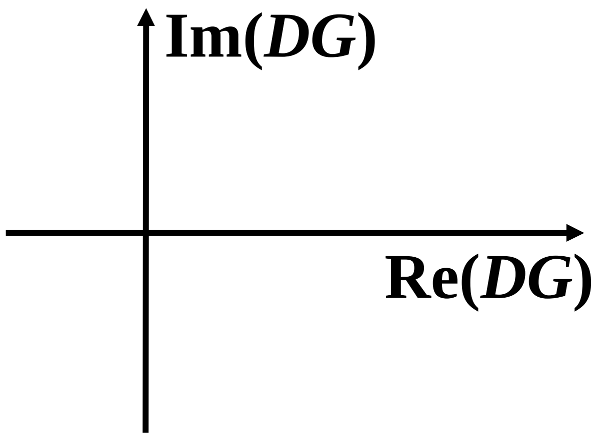
Nyquist Stability Criterion

Full proof way of determining stability. Based on complex analysis.

Characteristic equation: $1 + D(s)G(s) = 0$

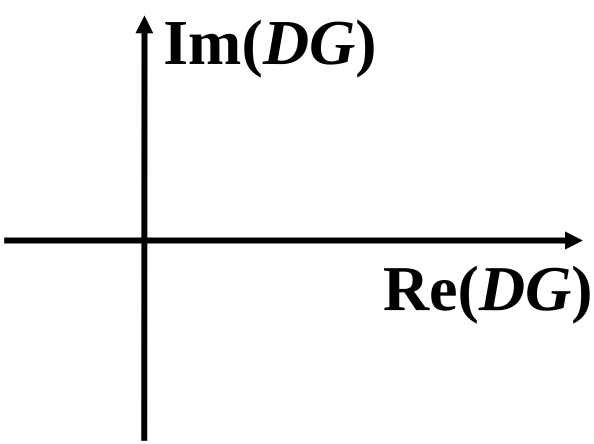


Evaluating $D(s)G(s)$ along contour yields trajectory of magnitudes and phases.



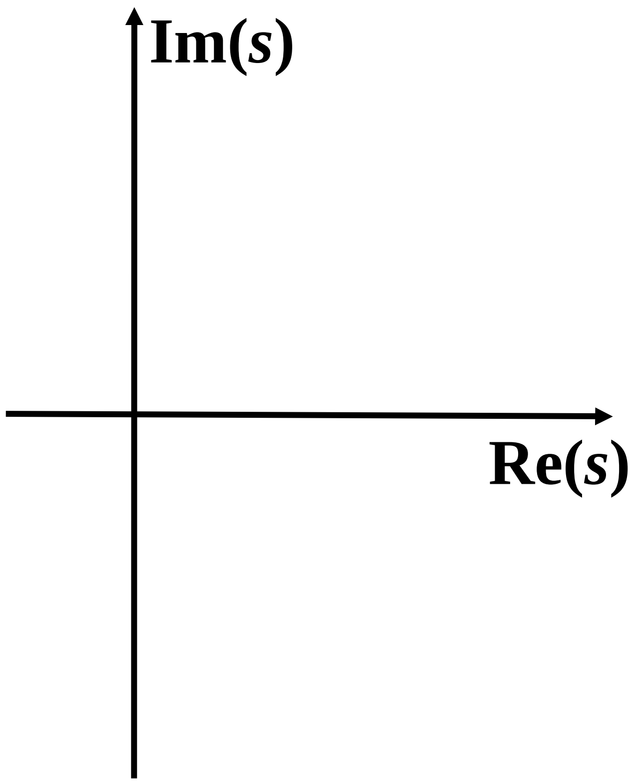
If no singularities (poles or zeros) of $D(s)G(s)$ are inside the contour, then evaluation will have no encirclements of the origin.

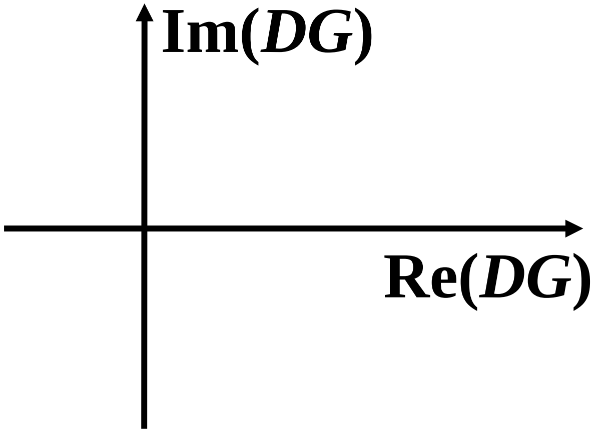
If there is a zero (and no poles) of $D(s)G(s)$ inside a CW contour, then evaluation of $D(s)G(s)$ around CW contour will lead to a CW encirlement of the origin.



- **If there are n_z zeros (and no poles) of $D(s)G(s)$ inside a CW contour, then evaluation of $D(s)G(s)$ around CW contour will lead to n_z CW encirlements of the origin.**
- **Similarly, if there are n_p poles (and no zeros) of $D(s)G(s)$ inside a CW contour, then evaluation of $D(s)G(s)$ around CW contour will lead to n_p CCW encirlements of the origin.**
- **If $D(s)G(s)$ has both poles and zeros inside a CW contour, let m_{net} be the net number of zeros inside the CW contour.**
    - **$m_{\text{net}} > 0$ indicates that there are <u>more</u> zeros than poles inside the contour.**
    - **$m_{\text{net}} < 0$ indicates that there are <u>fewer</u> zeros than poles inside the contour.**
    - **Evaluation of $D(s)G(s)$ around CW contour will lead to m_{net} CW encirclements of the origin. (If $m_{\text{net}} < 0$, encirclements are in the CCW direction.)**

- Want to know if roots of $1+D(s)G(s) = 0$ (closed-loop poles) are inside the RHP.





1: from Bode plots (DG)

2: complex conjugate of 1

3: $|D(s)G(s)|$ usually is close to zero

  - Evaluate $1+D(s)G(s)$ around this contour, then determine the number of encirclements of the origin.
  - Equivalently, evaluate $D(s)G(s)$ around this contour, then determine the number of encirclements of the -1 point.

Nyquist Stability Criterion

$$N = Z - P$$

$$Z = N + P$$

N: number of net CW encirclements of -1

Z: number of zeros of $1+D(s)G(s)$ in RHP =
number of C. L. poles of system in RHP

P: number poles of $1+D(s)G(s)$ in RHP
= number of poles of $D(s)G(s)$ in RHP =
number of O.L. poles in RHP

Often, $P = 0$ (O.L. stable) $\Longrightarrow$ then, $Z = N$.

Suppose $P = 2,\ N = -2$

$\Longrightarrow \quad Z =$

$Z \geq 0$

If $Z = 0$**, then C.L. system is stable.**

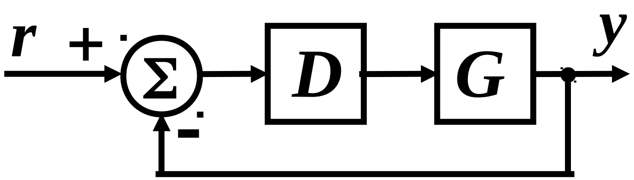
If $Z > 0$**, then C.L. system is unstable.**

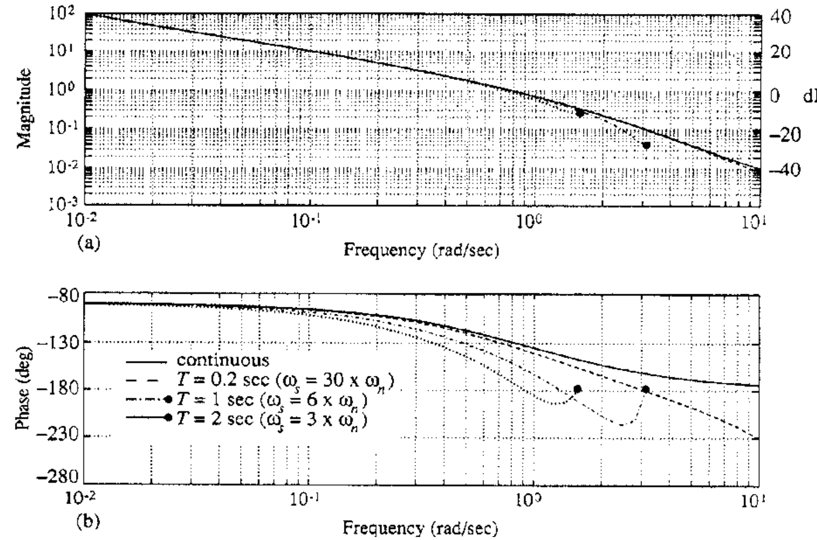
$P \geq 0$

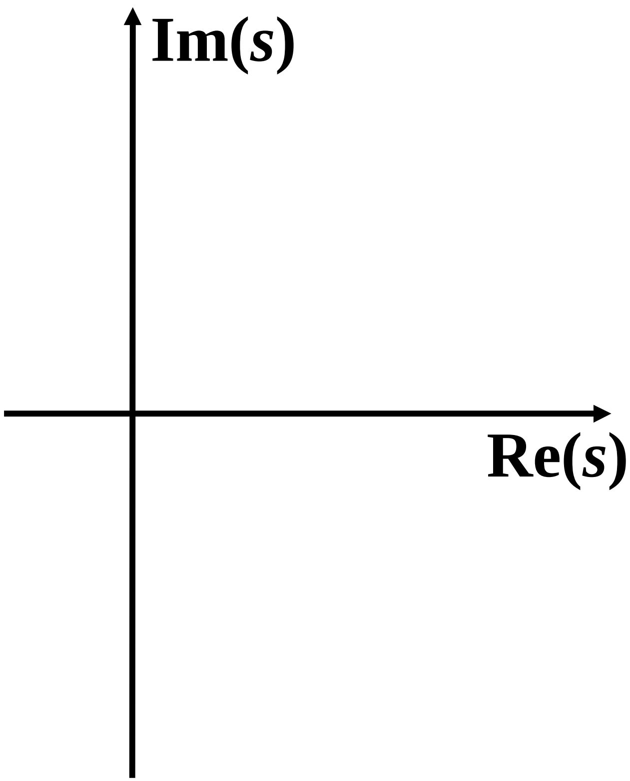
N **can be positive or negative.**

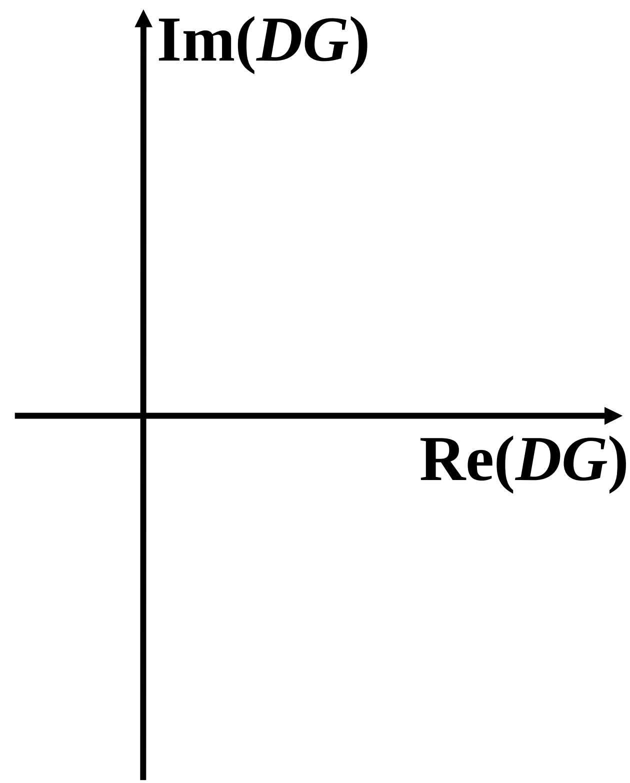
Example

$$G(s) = \frac{1}{s(s+1)} \qquad D(s) = K = 1$$

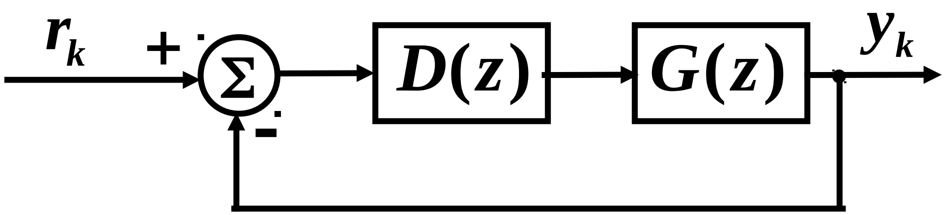








Nyquist for Discrete-Time Systems



$$H_{CL} = \frac{DG}{1+DG} = \frac{D(z)G(z)}{1+D(z)G(z)}$$

has poles outside the unit circle.

$\Longleftrightarrow$ $1+D(z)G(z)$ **has zeros outside the unit circle.**

Nyquist Stability Criterion for Discrete-Time Systems

$$Z = N + P$$

N: number of net CW encirclements of -1

Z: number of zeros of $1+D(z)G(z)$ outside U.C.

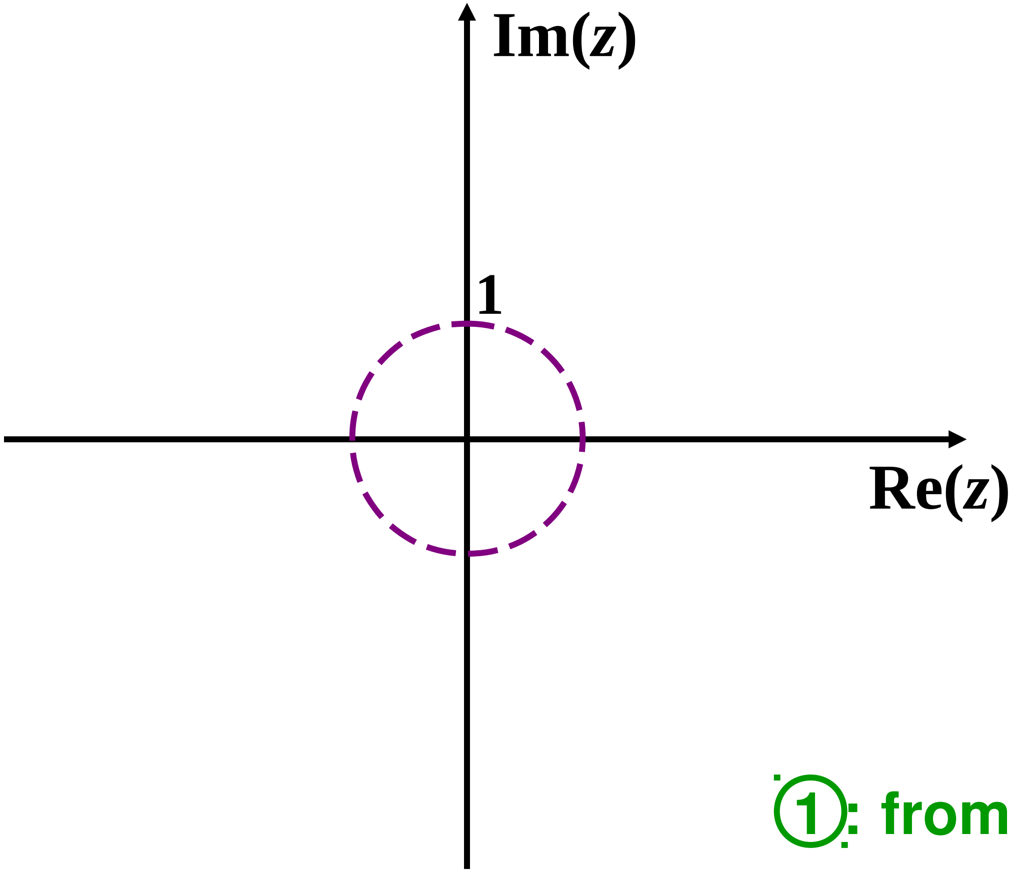
= number of C. L. poles of system outside U.C.

P: number poles of $1+D(z)G(z)$ outside U.C.

= number of poles of $D(z)G(z)$ outside U.C.

= number of O.L. poles outside U.C.

$$P \geq 0, \qquad Z \geq 0$$



1: from Bode plot

2: complex conjugate of 1

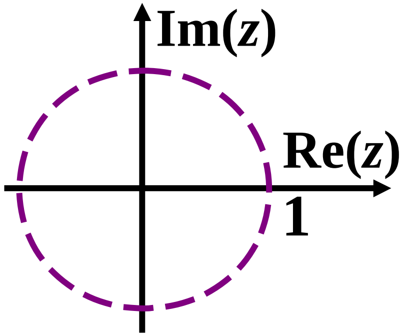
3: $|D(z)G(z)|$ usually is close to zero

4: need to evaluate

5: complex conjugate of 4

Alternative Nyquist Stability Criterion for Discrete-Time Systems

- Awkward to consider the unstable region of the z-plane <u>outside</u> the unit circle.
- How about considering the stable region <u>inside</u> the unit circle?



Let n be the number of zeros of $1+D(z)G(z) = 0$ (total number of closed-loop poles (stable or unstable)).

$$Z = P - N$$

N: number of net CCW encirclements of -1

= number of stable zeros – number of stable poles

$= (n - Z) - (n - P) = P - Z$

Z: number of zeros of $1+D(z)G(z)$ outside U.C.

P: number poles of $1+ D(z)G(z)$ outside U.C.

Discrete-Time Example

$$G(z) = 1.135\frac{z+0.523}{(z-1)(z-0.135)}, \quad T = 2 \text{ sec}$$

$$D = K = 1 \qquad\qquad DG = G$$