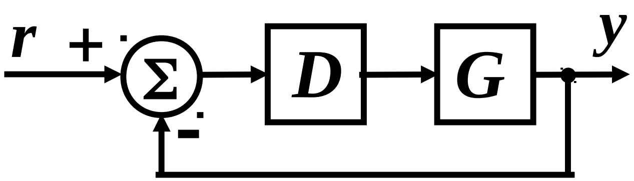
Ragazzini Method and Robust Control

- Direct design method of Ragazzini
  - Causality, stability, and steady-state constraints
- Robust Control
  - Sensitivity function
  - Stability robustness

Direct Design Method of Ragazzini



$$H(z) = \frac{DG}{1 + DG}$$

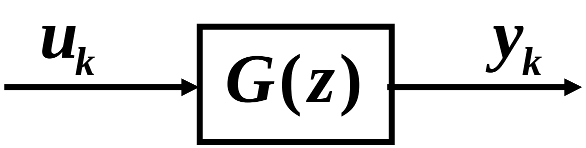
$$\boxed{D(z) = \frac{1}{G(z)} \frac{H(z)}{1 - H(z)}}$$

There must be some necessary constraints on $H(z)$ (and hence $D(z)$) so that $D(z)$ is implementable.

$D(z)$ causal: $D(z)$ must be well behaved as $z \to \infty$

If $G(z)$ has zeros at ∞ $\Longrightarrow$

Constraint 1: $H(z)$ must have a zero at infinity of the same order as the zero of $G(z)$ at infinity.



Also want the C.L. system to be stable

$$H(z) = \frac{DG}{1+DG} \Longrightarrow 1 + DG = 0$$

Define: $D(z) = \dfrac{c(z)}{d(z)}, \quad G(z) = \dfrac{b(z)}{a(z)}$

Suppose $D(z)$ cancels a pole or zero of $G(z)$:

$G(z)$ has a pole at $z = \alpha$ that is cancelled by $D(z)$.

$$G(z) = \frac{b(z)}{a(z)} = \frac{b(z)}{(z-\alpha)\bar{a}(z)}, \qquad D(z) = \frac{c(z)}{d(z)} = \frac{(z-\alpha)\bar{c}(z)}{d(z)}$$

$$D(z) = \frac{1}{G(z)} \frac{H(z)}{1 - H(z)}$$

If $D(z)$ should not cancel a pole or zero of $G(z)$:

If $D(z)$ does not cancel pole:

$\Longrightarrow$

If $D(z)$ does not cancel zero:

$\Longrightarrow$

Constraint 2: **$1 - H(z)$ must have roots at all poles of $G(z)$ on or outside unit circle.**

Constraint 3: **$H(z)$ must have zeros at all zeros of $G(z)$ on or outside unit circle.**

Generally also want to meet some steady-state requirements

$H(z)$ is the overall transfer function.

$$E(z) = R(z) - Y(z)$$

Steady-state error to a unit step:

$$e_{ss} = \lim_{z \to 1} (z-1)E(z)$$

$$\boxed{H(1) \geq 1 - e_{ss_{\max}}}$$ Type 0

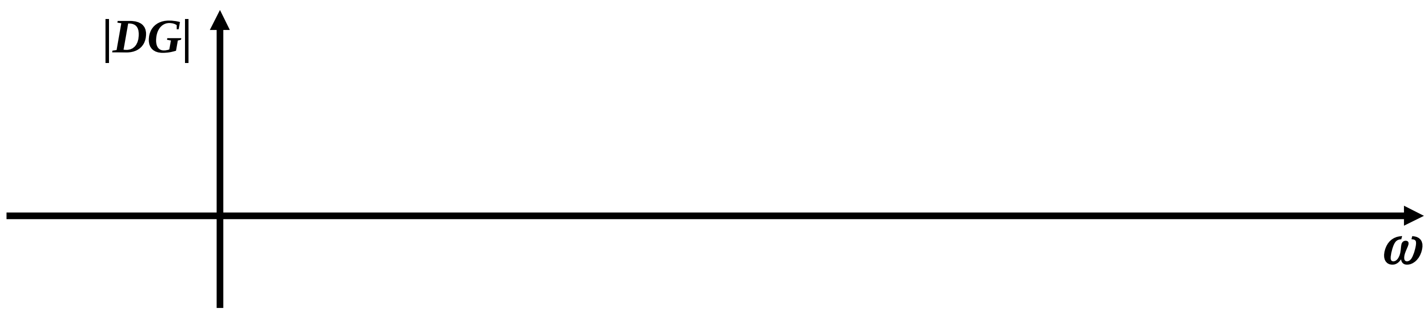
If $H(1) \neq 1 \Longrightarrow$

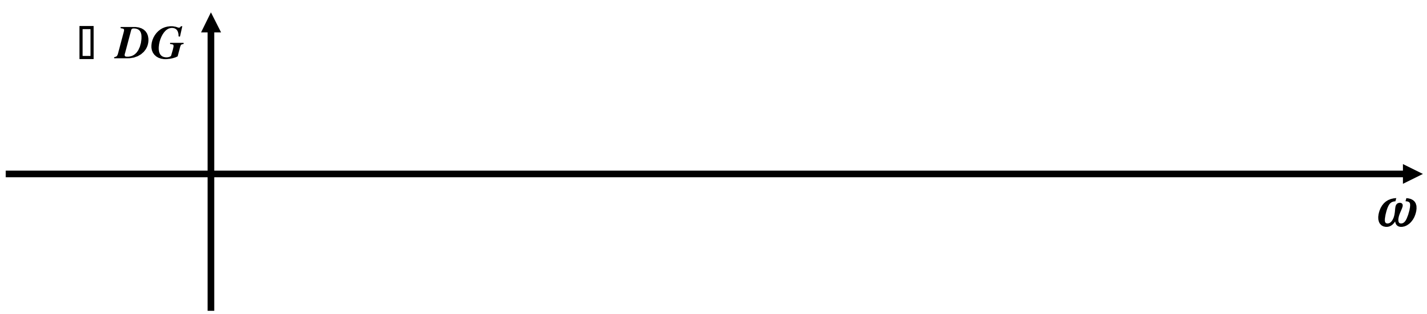
If want C.L. system to be Type 1 $\Longrightarrow$

$$e_{ss} = \lim_{z \to 1} \frac{T(1 - H(z))}{z-1}$$ $\Longrightarrow$ L'Hopital's: $\boxed{e_{ss} = \lim_{z \to 1} T\left[-\frac{dH}{dz}\right] < e_{ss_{\max}}}$ Type 1

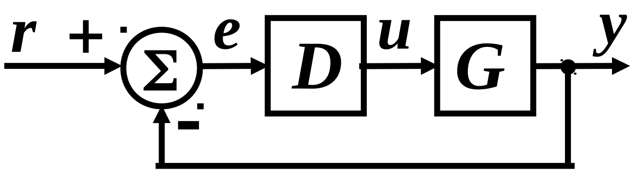
Robust Control

- **Discussion here applies equally well to continuous-time and discrete-time systems.**
- GM, PM, **and** ω_c **are <u>point</u> specifications.**
- **A more accurate "margin" can be given in terms of the <u>sensitivity function</u>, and this function can lead to specifications for the open-loop gain over a larger range of** ω**.**





Sensitivity Function



$$\boxed{E = \frac{1}{1+DG} R = SR}$$

$$S_\infty = \max_\omega |S|$$

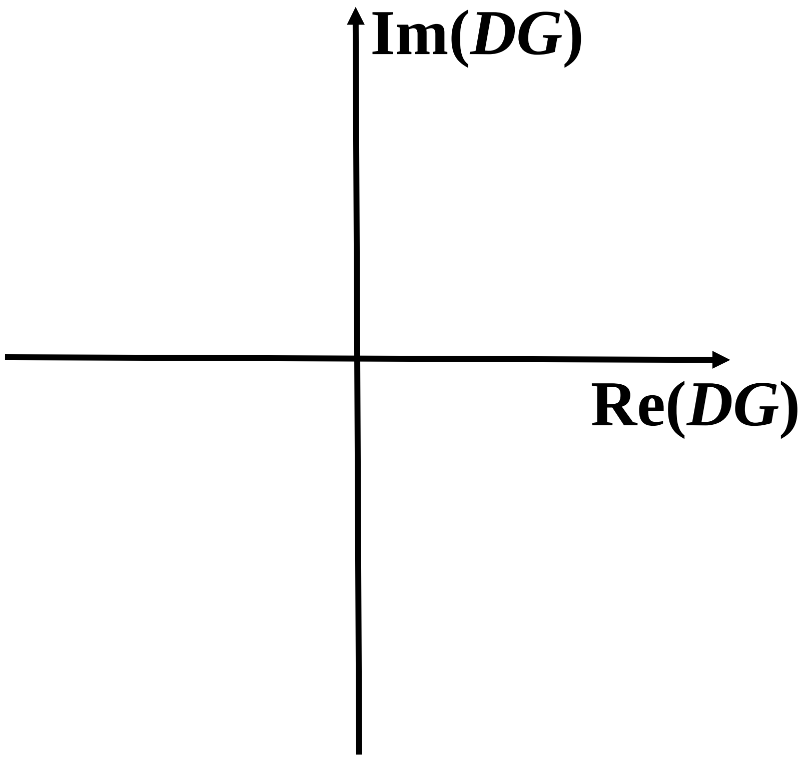
$$E(z) = \frac{1}{1 + D(z)G(z)} R(z), \quad E(s) = \frac{1}{1 + D(s)G(s)} R(s)$$

See Figure 7.23 of text.

Vector Gain Margin (VGM) is the worst case gain margin.

$$\frac{1}{VGM} + \frac{1}{S_\infty} = 1$$

$$VGM = \frac{S_\infty}{S_\infty - 1}$$



We can express more complete frequency domain design specifications than any of these margins (PM, GM, VGM) if we first give frequency descriptions for the external reference (and disturbance) signals:

- Generally, we'd like the system to track inputs (or reject disturbances) with frequency content up to a certain frequency.

  $$|E| = |SR| \le e_b$$

- To normalize problem without defining spectrum of R and the error bound e_b each time, define a real function of frequency:

  $$W_1(\omega) = \frac{|R|}{e_b}$$

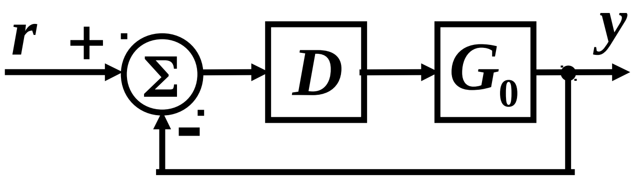
  $$S = \frac{1}{1 + DG} \approx \frac{1}{DG}$$

  want $|S|$ small

$$\boxed{|DG| \ge W_1(\omega)}$$

Stability Robustness

Usually it is expected that the control design works for a <u>range</u> of plants about the model used in designing the controller.



$$G(j\omega) = G_0(j\omega)[1 + w_2(\omega)\Delta(j\omega)]$$

w_2 : Magnitude function representing size of possible changes in plant; satisfies:

$$w_2(\omega) \le W_2(\omega)$$

$\Delta(j\omega)$: Represents uncertainty in phase; satisfies

$$|\Delta(j\omega)| \le 1$$

Stability robustness means the control design for G_0 will lead to a stable system for all $G = G_0\left(1 + w_2\Delta\right)$ for all $w_2(\omega) \leq W_2(\omega)$ and $\left|\Delta(j\omega)\right| \leq 1$.

In particular: $$1 + D(j\omega)G_0(j\omega) \neq 0, \quad \forall\omega$$

If there is stability robustness:

$$1 + D(j\omega)G(j\omega) \neq 0, \quad \forall\omega$$

$$\boxed{\left(1 + D(j\omega)G_0(j\omega)\right)\left(1 + Tw_2\Delta\right) \neq 0, \quad \forall\omega}$$

Complementary Sensitivity Function

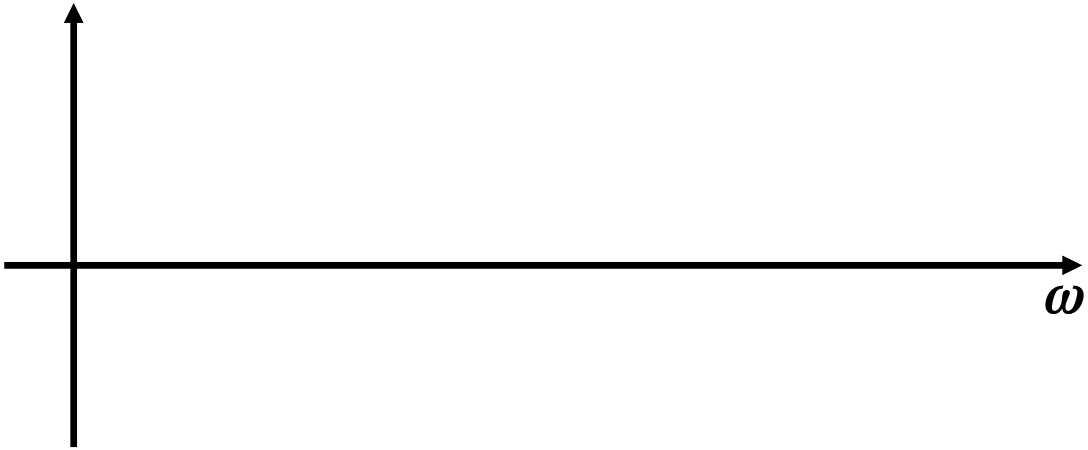
$$T = \frac{DG_0}{1 + DG_0}$$

$$\boxed{\left(1 + D(j\omega)G_0(j\omega)\right)\left(1 + Tw_2\Delta\right) \neq 0, \quad \forall \omega}$$

$1 + Tw_2\Delta \neq 0$ **is satisfied if** $\left|Tw_2\Delta\right| < 1$

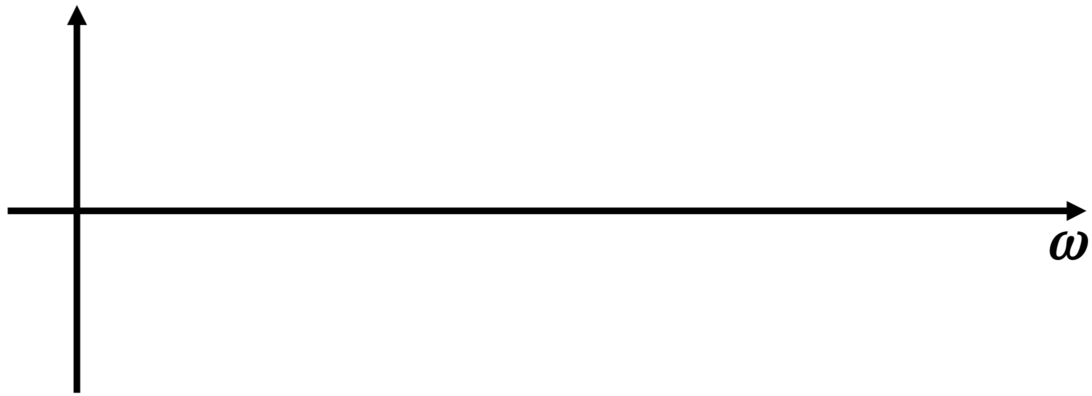
At high frequencies where there is usually more model uncertainty and W_2 is large, want DG_0 to be small.

$$T = \frac{DG_0}{1 + DG_0} \approx DG_0$$



Sensitivity Integral

$$S = \frac{1}{1 + DG}$$



For continuous-time systems:

$$\int_0^\infty \ln|S|\, d\omega = \pi \sum \mathrm{Re}\{p_i\}$$

p_i are the unstable (RHP) poles of DG

Assumes:
- DG roll-off at high frequencies is at a slope faster than -1
- all zeros of DG are minimum phase (LHP)

For discrete-time systems:

$$\int_0^\pi \ln\left(S(e^{j\phi})\right) d\phi = \pi \sum \ln\{r_i\}$$

r_i are the magnitudes of the unstable (outside U.C.) poles of DG