Notes in APPM 4650
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1 Project Continued

$F+0x$, $T\Big|_{t=0} = T_0$ and at $T=T_0$
Cons of E: $\rho c_p \frac{\partial T}{\partial t} = A_f e^{-\frac{E}{RT}} + A(T-T_0)$ A_f exponential term is heat generation, other term on RHS is heat loss.
Non-dimensional problem.

$$\frac{d\theta}{d\sigma} = \delta e^{\theta} - \theta$$

Initial condition $\theta\Big|_{\mathfrak{T}=0} = 0$.
Theta as a function of $\mathfrak{T}$ is highly exponential.

$$\delta \propto \frac{1}{H}$$

$$\delta < \frac{1}{e}$$

fizzle

$$\delta > \frac{1}{e}$$

explotion.

$\delta = 1, \delta = \frac{1}{5}$
integrate $\frac{\delta\theta}{\delta\sigma} = \delta e^{\theta} - \theta$
with too small of step size you will step right over the interesting part of the problem.
for the explosion case, you want to solve:

$$\frac{d\sigma}{d\theta} = \frac{1}{\delta e^{\theta} - \theta}$$

asymptotic value for this guy above and the original in the fizzle case.

$$0 = \delta e^{\theta_f} - \theta_f$$

can solve this with root finding.

RK4 for the differential equations.
fizzle problem

σ	θ
0	0
0.1	$\ldots$
0.2	$\ldots$
$\vdots$	$\vdots$
$\ldots$	θ_f
$\ldots$	θ_f

explotion problem

θ	σ
0	0
$\vdots$	$\vdots$
$\ldots$	$\sigma_{\text{explosion}}$

early solution:

$$\frac{d\theta}{d\sigma} = \delta e^{\theta} - \theta$$

I.C. $\theta\Big|_{\sigma=0} = 0.$
taylor series of e^{θ}.
diff eq$\approx$

$$\delta(1 + \theta + \frac{\theta^2}{2} + \ldots) - \theta$$
$$\approx \delta + \theta(\delta - 1)$$

$$\theta\Big|_{\theta=\sigma} = 0$$
$$\frac{d\theta}{d\sigma} \approx \delta + \theta(\delta - 1)$$
$$\frac{d\theta}{\delta + \theta(\delta - 1)} = d\sigma$$

$$\frac{d\theta}{\theta + \frac{\delta}{\delta-1}} = (\delta - 1)d\sigma$$

$$\ln(\theta + frac\delta\delta - 1) = (\delta - 1)\sigma + c$$

$$\ln\left(\frac{\delta}{\delta - 1}\right) = c$$

$$\ln\left(\theta + \frac{\delta}{\delta - 1}\right) = (\delta - 1)\sigma + \ln\left(\frac{\delta}{\delta - 1}\right)$$

$$\ln\left(\frac{\theta + \frac{\delta}{\delta-1}}{\frac{\delta}{\delta-1}}\right) = (\delta - 1)\sigma$$

$$\theta = \frac{\delta}{\delta - 1}e^{(\delta-1)\sigma} - \frac{\delta}{\delta - 1}$$

or $0 = \dots$

Clearly this is the first project. Start plotting these at the beginning of the problem next to the actual solutions.
θ_f is the late solution for the fizzle, and we have early solutions for the fizzle and explosion. Need late solution for the explosion.

late solution to $\frac{d\theta}{d\sigma} = \delta e^{\theta} - \theta$.

$$\frac{d\theta}{d\sigma} = \delta e^{\theta}$$

.

$$e^{-\theta} d\theta = \delta d\sigma$$

$$-e^{-\theta} \approx \delta\sigma + c$$

$$\theta \to \infty$$

$$0 = \delta\sigma_{\text{explosion}} + c$$

$$c = -\delta\sigma_{\text{explosion}}$$

$$-e^{-\theta} = \delta\sigma - \delta\sigma_{\text{explosion}}$$

$$-e^{-\theta} = \delta(\sigma - \sigma_{\text{explosion}}$$