Notes in APPM 4650
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1 solution to matrix

$$Ax = b$$

A is invertible

$$x_1 = \frac{b_1 - \sum_{j=2}^{n} a_{ij}x_j}{a_{11}}$$
$$x_2 = \frac{b_2 - \sum_{j\neq 2}^{n} a_{ij}x_j}{a_{22}}$$
$$x_n = \frac{b_n - \sum_{j\neq n}^{n} a_{ij}x_j}{a_{nn}}$$

use an initial guess x_0. Correct with the above equations. Do this and redo until you converge.
This relies on doing preprocessing to make the diagnol of A large relative to everything else in the matrix.
Then a good initial guess is $x_i = \frac{b_i}{a_{ii}}$.
Little better if you do:

$$x_i^* = \frac{b_i - \sum_{j=1}^{i-1} a_{ij}x_j^* - \sum_{j=i+1}^{n} a_{ij}x_j}{a_{ii}}$$

Guass-Seidel.
looking for how to stop:
residual $r = b - Ax$, watch $|r| < \epsilon$.

$$x_i^* = x_i + \frac{r_i}{a_{ii}}$$