Notes in APPM 4650
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1 Project Continued

Cone of energy...

$$\frac{d\theta}{d\sigma} = \delta e^{\theta} - \theta$$

$\delta < \frac{1}{e}$ fizzle.
$\delta > \frac{1}{e}$ explosion
$\delta = \frac{1}{5}$ and $\delta = 1$.
for fizzle solutions your just going to integrate the ode with RK4. This should asymptote to θ_f.
Late solution for fizzle will be when $\theta_f = \delta e^{\theta_f}$.
Early solution for fizzle will be from $\frac{d\theta}{d\sigma} = \delta + (\delta - 1)\theta$.

$$\frac{d\theta}{\delta + (\delta - 1)\theta} = d\sigma$$

$$\frac{d\theta}{\theta + (\frac{\delta}{\delta - 1})} = (\delta - 1)d\sigma$$

$$\theta = \frac{\delta}{\delta - 1}(e^{(\delta - 1)\sigma} - 1)$$

This early solution works for both fizzle and explosion.
for explosion, solving for σ is uesful.

$$\sigma = \left(\frac{1}{\delta - 1}\right) \ln \left[\frac{\theta + \dfrac{\delta}{\delta - 1}}{\dfrac{\delta}{\delta - 1}}\right]$$

Late solution for the explosion:

$$\frac{dy}{dx} = \frac{1}{\delta e^{x} - x}$$

$y = \sigma$, $x = \theta$.

$$r = \frac{\ln\left(\frac{\theta + \frac{\delta}{\delta-1}}{\frac{\delta}{\delta-1}}\right)}{\delta - 1}$$

$$\lim_{\delta \to 1} \sigma =$$

should give ou sigma vs theta at the beginning..

on explosion:

$$\frac{d\theta}{d\sigma} \approx \delta e^{\theta}$$

$$e^{-\theta} d\theta = \delta d\sigma$$

$$-e^{-\theta} = \sigma\delta + c$$

$$\theta\Big|_{\sigma \to \sigma_{expl}} \to \infty$$

$$c = -\delta\sigma_{expl}$$

$$\sigma\delta = \delta\sigma_{expl} - e^{-\theta}$$

$$\sigma = \sigma_{expl} - \frac{1}{\delta} e^{-\theta}$$

late explosion approximation.
σ_{expl} is unkown though.

$$\frac{d\sigma}{d\theta} = \frac{1}{\delta e^{\theta} - \theta}$$

$$\int_{\theta=0}^{\infty} \frac{d\sigma}{d\theta} d\theta = \int_{\theta=0}^{\infty} \frac{d\theta}{\delta e^{\theta} - \theta}$$

$$0 = \theta_{expl} - \sigma\Big|_{\theta=0}$$

$$\theta_{expl} = int_{\theta=0}^{\infty} \frac{d\theta}{\delta e^{\theta} - \theta}$$

Simpson's something for solution.
by the time θ is about 10 your good.
jacobian