Caltech Ducted Fan Model
ECEN 5008

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1 Notes on Dynamics for Final Project

m_x is inertia in the x direction.
m_z is inertia in the z direction
I_{yy} is rotational inertia
ρ_d is air density

Aerodynamic compoents for lift drage and moment: $C_L = C_{L\alpha}(\alpha - \alpha_0)$ where α is the angle of attack. $x(0) = \alpha_0$. $C_{L\alpha}$ is the lift force
$C_D = C_{D0} + \frac{1}{2}C_{D\alpha}(\alpha - \alpha_0)^2$
$C_M = C_{M\alpha}(\alpha - \alpha_0)$

$f_x = T\cos(\delta_\tau)$
$f_z = -T\sin(\delta_\tau)$
$\delta_\tau = K_{\delta_\tau}\delta_p$

θ is the full angle combination of angle of attack, α and the flight path angle γ.

$$\ddot{x} = \frac{1}{m_x}\left(\cos(\theta)f_x + \sin(\theta)f_z + \frac{1}{2}\rho_d SV(\dot{x}C_L(\alpha) - \dot{z}C_D(\alpha))\right)$$

$$\ddot{z} = \frac{1}{m_z}\left(\cos(\theta)f_z - \sin(\theta)f_x - \frac{1}{2}\rho_d SV(\dot{x}C_L(\alpha) + \dot{z}C_D(\alpha)) + m_z g\right)$$

$$\ddot{\theta} = \frac{1}{I_{yy}}\left(l_t f_z + \frac{1}{2}\rho_d V^2 C_M(\alpha)S\bar{c} - b_\theta\dot{\theta}\right)$$

$$V = \sqrt{\dot{x}^2 + \dot{z}^2}$$

$$T = 38.89V_m - 3.14$$

$$\begin{pmatrix}\dot{x}\\ \dot{z}\end{pmatrix} = R_y(\gamma) = \begin{pmatrix}\cos(\gamma) & \sin(\gamma)\\ -\sin(\gamma) & \cos(\gamma)\end{pmatrix}\begin{pmatrix}v\\ 0\end{pmatrix}$$

$$\begin{pmatrix}\ddot{x}\\ \ddot{z}\end{pmatrix} = \begin{pmatrix}\cos(\gamma) & \sin(\gamma)\\ -\sin(\gamma) & \cos(\gamma)\end{pmatrix}\begin{pmatrix}\dot{v}\\ -v\dot{\gamma}\end{pmatrix}$$

$$R_y(\gamma) = \begin{pmatrix}\cos(\gamma) & 0 & \sin(\gamma)\\ 0 & 1 & 0\\ -\sin(\gamma) & 0 & \cos(\gamma)\end{pmatrix}$$

basis vector:

$$\hat{e}_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

solve one of the above eqquations for accels.

$$\begin{pmatrix} \dot{v} \\ -v\dot{\gamma} \end{pmatrix} = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) \\ \sin(\gamma) & \cos(\gamma) \end{pmatrix} \begin{pmatrix} \ddot{x} \\ \ddot{z} \end{pmatrix}$$

$R_y(\gamma)$ is the 2 by 2 above.

$$= R_y(\alpha) R_y(-\theta) \begin{pmatrix} \ddot{x} \\ \ddot{z} \end{pmatrix}$$

which is the acceleration in the body frame is the first two terms, the first rotation matrix puts us in the local frame

$$\begin{pmatrix} m\dot{v} \\ -mv\dot{\gamma} \end{pmatrix} = \begin{pmatrix} -D \\ -L \end{pmatrix} + \begin{pmatrix} -\sin(\gamma) \\ \cos(\gamma) \end{pmatrix} mg + R_y(\alpha) \begin{pmatrix} f_x \\ f_z \end{pmatrix}$$

dynamics, with the problem that γ isn't one of our states so we need:

$$\gamma = \theta - \alpha$$

$$\dot{\gamma} = \omega - \dot{\alpha}$$

$$mv\dot{\alpha} = -L + mg\cos(\theta - \alpha) - f_x \sin(\alpha) + f_z \cos(\alpha) + mv\omega$$

$$J\dot{\omega} = l_\tau f_z + M_a$$

l_τ is the distance between the center of mass and force application. with M_a being the aerodynamic moment.