APPM 4650 Homework #4 Fall 2015

Due Friday, November 20, 2015

1. Approximate the integral $\int_0^1 x^2 e^{-x}\,dx$ using Gaussian quadrature and compare your results to the exact values of the integral.
   (a) Use $n = 3$.
2. Use the 4^{th} order Runge–Kutta method for systems to approximate the solution of the following system of first–order differential equations for $0 \le t \le 1$, and $h = 0.2$, and compare the results to the actual solution.

$$\begin{aligned} u_1' &= 3u_1 + 2u_2 - (2t^2+1)e^{2t} \\ u_2' &= 4u_1 + u_2 + (t^2+2t-4)e^{2t} \end{aligned}$$

   The initial conditions are $u_1(0) = 1$ and $u_2(0) = 1$. For comparison, the exact solutions are $u_1(t) = \frac{1}{3}e^{5t} - \frac{1}{3}e^{-t} + e^{2t}$ and $u_2(t) = \frac{1}{3}e^{5t} + \frac{2}{3}e^{-t} + t^2e^{2t}$.
3. Consider the initial value, ordinary differential equation, $y' = x + y$ with $y(0) = 0$. Find $y(x)$ for $0 \le x \le 0.5$ with a step size of $h = 0.1$ using the following methods:
   (a) 4^{th} order Runge-Kutta
   (b) Improved Euler
   (c) Predictor-corrector using the Adams-Bashforth 4-step for the predictor and the Adams-Moulton 3-step as the corrector

   To determine any seed values, use the fact that $y \approx \dfrac{x^2}{2}$ when $x << 1$.