Notes in APPM 4650
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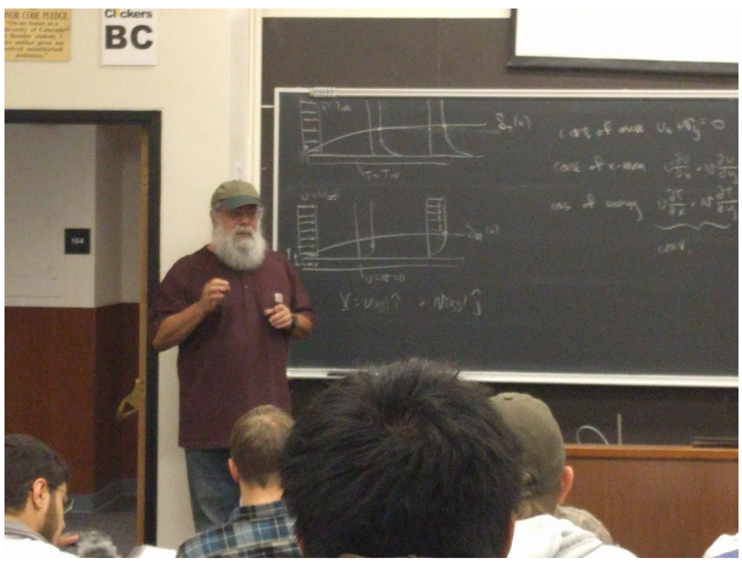
1 2nd Project



conservation of mass $u_x + \mathfrak{v}_y = 0$.
conservation of x-momentum $u\frac{du}{dx} + \mathfrak{v}\frac{du}{dy} = \nu\frac{d^2u}{dy^2}$
conservation of energy $u\frac{d\tau}{dx} + \mathfrak{v}\frac{dT}{dy} = \alpha\frac{d^2T}{dy^2}$
first 2 terms are convection and last is diffusion.
$v = u(x,y)\hat{i} + \mathfrak{v}(x,y)\hat{j}$ u is horizontal and v is vertical.
$T(x,y)$ need this and $u(x,y)$ and $\mathfrak{v}(x,y)$

highest order x derivative of u is 1, $x = 0$ $u = u_\infty$
highest order y derivative of u is 2, $y = 0$ $u = 0$, $y \to \infty$ $u \to u_\infty$

highest order v derivative with respect to x is 0
highest order v derivative with respect to y is 1, $y = 0$ $v = 0$

highest order T derivative with respect to x is 1, $x = 0$ $T = T_\infty$
highest order T derivative with respect to y is 2, $y = 0$ $T = T_\infty$
and $y \to \infty$ $T = T_\infty$

$$\eta = \frac{y}{\sqrt{\frac{\nu x}{u_\infty}}}$$

$$F(\eta) = f(\eta)$$

$$u = u_\infty F'(\eta)$$

$$\mathfrak{v} = \frac{1}{2}\sqrt{\frac{\nu u_\infty}{x}}(\eta F'(\eta) - F(\eta))$$

$$G = \frac{T - T_\infty}{T_\omega - T_\infty} = G(\eta, Pr)$$

Pr is the Prendel number $Pr = \frac{\nu}{\alpha}$.

these transform to:

$$f''' + \frac{1}{2} f f'' = 0$$

$$g'' + \frac{Pr}{2} f g' = 0$$

Leading edge $\eta \to \infty$ $F'(\infty) = 1$ $G(\infty) = 0$
Wall $\eta = 0$ $F'(0) = 0$ $G(0) = 1$ $f(0) = 0$
For shield $\eta \to \infty$

Our job:
use RK-4

$$u_1 = f$$

$$f' = u_1' = u_2$$

$$f'' = u_2' = u_3$$

$$f''' = u_3' = -\frac{1}{2} f f'' = \frac{-1}{2} u_1 u_3$$

$$u_4 = g$$

$$g' = u_4' = u_5$$

$$g'' = u_5' = \frac{-Pr}{2} u_1 u_5$$

η	u_1	u_2	u_3	u_4	u_5
0	0	0	$\cdot$	1	$\cdot$
$\cdot$	$\cdot$	1	$\cdot$	0	$\cdot$

so this is actually a boundary value problem because we don't have all the initial conditions.