Due Friday, December 11, 2015

1. Consider solving the system of equations

$$\begin{pmatrix} 4 & 3 & 0 \\ 3 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 24 \\ 30 \\ -24 \end{pmatrix}$$

using Gauss–Seidel with relaxation. Determine the optimum relaxation factor ω where $x_{i+1} = x_i + \omega \dfrac{r_i}{a_{ii}}$ to get the solution to within 6 decimal places.

2. Consider the matrix $\mathbf{A} = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 5 & 0 \\ -1 & 1 & -1 \end{pmatrix}$.

   (a) Calculate $\mathbf{A}^{-1}$ exactly by using the cofactor method.

   (b) Start with the initial approximation to $\mathbf{A}^{-1}$

$$\mathbf{x}_0 = \begin{pmatrix} 0.5 & -0.1 & 0.4 \\ 0 & 0.2 & 0 \\ -0.4 & 0.3 & -1.5 \end{pmatrix}$$

   and use the iterative method $\mathbf{x}_{i+1} = \mathbf{x}_i(2\mathbf{I} - \mathbf{A}\mathbf{x}_i)$ to calculate the next approximation $\mathbf{x}_1$.

   (c) Calculate the deviations of $\mathbf{x}_0$ and $\mathbf{x}_1$ from the true inverse matrix $\mathbf{A}^{-1}$.

3. Use the "power method" to find the dominant eigenvalue λ and the corresponding eigenvector $\mathbf{V}$ for the matrix

$$\mathbf{A} = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}.$$

   (a) Start the procedure with the initial vector $\mathbf{x}_0 = (1, 1, 1, 1)^{\mathrm{T}}$.

   (b) Now, repeat the calculations starting with $\mathbf{x}_0 = (1, 1, 5, 1)^{\mathrm{T}}$.

   (c) Comment on the results from parts (a) and (b).

4. Consider the matrix $\mathbf{A} = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix}$ and the initial vector $\mathbf{x}_0 = (1, 1, 1)^{\mathrm{T}}$.

   (a) Calculate the Rayleigh quotient and the error estimate using $\mathbf{x}_2$ and $\mathbf{x}_3$.

   (b) Use the power method to find λ_{max} and the corresponding $\mathbf{V}_{max}$.