Grad Project Equations in MCEN 5115: Rotating Inverted Pendulum

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Normal Inverted Pendulum

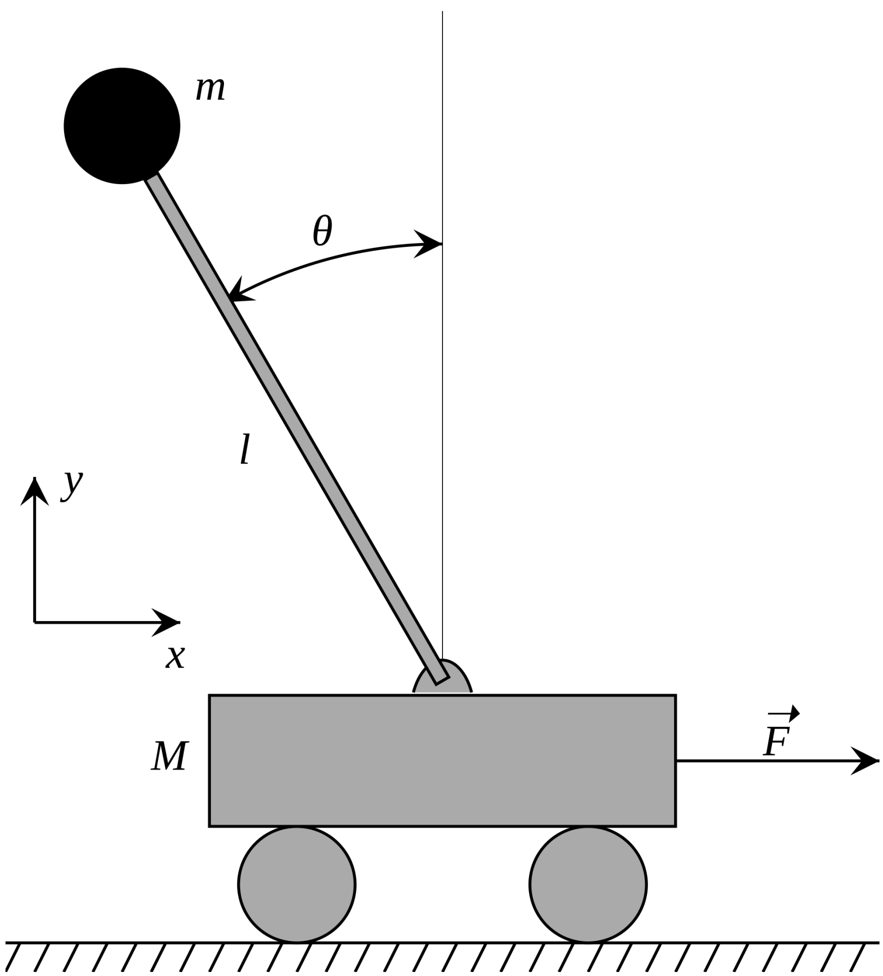

Figure 1: sup sup

Here we have the normal inverted pendulum equations for the image above:

$$(M+m)\ddot{x} - ml\ddot{\theta} + ml(\dot{\theta})^2\sin(\theta) = F$$
$$l\ddot{\theta} - g\sin(\theta) = \ddot{x}\cos(\theta)$$

The x position for us is actually ω because we are spinning. Also, the input force isn't F. What we have is a torque $\tau = ||r||||F|| \sin(\theta)$. Given that the θ here is 90 degrees or 270 degrees, force can be equated to $F = \frac{\tau}{r}$.
Torque is some function of the motor current and the motor properties that we can put in later. We also note that $x = \omega * r$ and thus $\ddot{x} = \ddot{\omega} r$.
Thus our equations become:

$$\frac{\tau}{r} = ml(\dot{\theta})^2 \sin(\theta) - ml\ddot{\theta} + (M+m)\ddot{\omega} r$$
$$\ddot{\omega} r \cos(\theta) = l\ddot{\theta} - g \sin(\theta)$$