Differential Privacy

Privacy Preserving Information Access: Homework 2

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DIPARTIMENTO MATEMATICA

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Diabetes Dataset

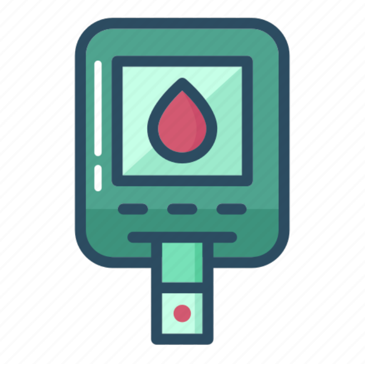
```
In [2]: diabetes_ds = pd.read_csv("diabetes.csv")

        # display dataset
        diabetes_ds
```

Out[2]:

	Pregnancies	Glucose	BloodPressure	SkinThickness	Insulin	BMI	DiabetesPedigreeFunction	Age	Outcome
0	6	148	72	35	0	33.6	0.627	50	1
1	1	85	66	29	0	26.6	0.351	31	0
2	8	183	64	0	0	23.3	0.672	32	1
3	1	89	66	23	94	28.1	0.167	21	0
4	0	137	40	35	168	43.1	2.288	33	1
...	...	...	...	...	...	...	...	...	...
763	10	101	76	48	180	32.9	0.171	63	0
764	2	122	70	27	0	36.8	0.340	27	0
765	5	121	72	23	112	26.2	0.245	30	0
766	1	126	60	0	0	30.1	0.349	47	1
767	1	93	70	31	0	30.4	0.315	23	0

Context

- Used to predict whether a patient has diabetes or not.
- The dataset contains the following information regarding patients:
- **Pregnancy: number of pregnancies**
- **BMI: body mass index (weight in kg/(height in m)^2)**
- **Age: years**
- **Glucose : glucose concentration**
- **BloodPressure: Diastolic blood pressure (mm Hg)**
- **output: diagnosis of diabetes (0 or 1)**
- **...**
- Identifiers have been removed (e.g., name, SSN), making this dataset "anonymized."



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Fake Data Generation

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Fake information was added to make the dataset more interesting: name, SSN, and job.

In [4]:
```
name_list = []
ssn_list = []
job_list = []


# use italian locale
fake = Faker('it_IT')

# generate fake data for all entries
for _ in range(768):
    name_list.append(fake.name_female())
    ssn_list.append(fake.ssn())
    job_list.append(fake.job())

# add new columns to csv file
diabetes_ds['Name'] = name_list
diabetes_ds['SSN'] = ssn_list
diabetes_ds['Job'] = job_list

diabetes_ds = diabetes_ds[['Name', 'SSN', 'Age', 'Job', 'Pregnancies',
                           'Glucose', 'BloodPressure', 'SkinThickness', 'Insulin',
                           'BMI', 'DiabetesPedigreeFunction', 'Outcome']]

# save the dataset
diabetes_ds.to_csv('output.csv')
```

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Dataset with PII

```
In [5]: # read new dataset
        diabetes_dataset = pd.read_csv("output.csv")
        diabetes_dataset.head()
```

Out[5]:

Name	SSN	Age	Job	Pregnancies	Glucose	BloodPressure	SkinThickness	Insulin	BMI	DiabetesPedigreeFunction	Outcome
Virginia Benigni	PVNLLT00E15I692J	50	Surveyor, planning and development	6	148	72	35	0	33.6	0.627	1
Sig.ra Ludovica Costanzi	SLTGDI16C14G909X	31	Editor, magazine features	1	85	66	29	0	26.6	0.351	0
Germana Schiavone	LZUGBL37H03C463U	32	Hospital doctor	8	183	64	0	0	23.3	0.672	1
Sig.ra Tiziana Alboni	MSTRNL72S25E539D	21	Programme researcher, broadcasting/film/video	1	89	66	23	94	28.1	0.167	0
Sig.ra Licia Malenchini	NTTVCR69A12I928V	33	Geographical information systems officer	0	137	40	35	168	43.1	2.288	1

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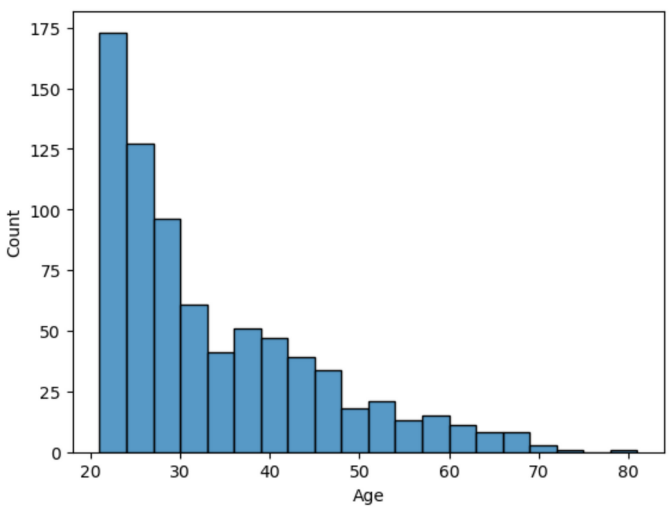
Histograms

Age:

Pregnancies:

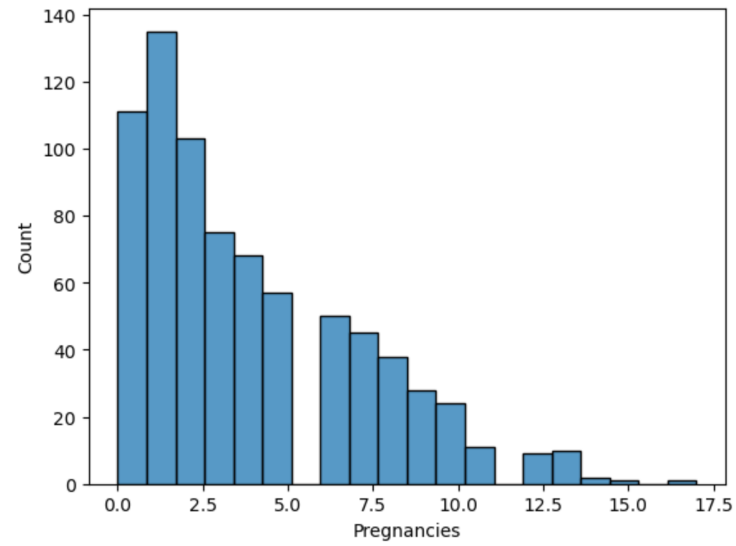
```
In [6]: sns.histplot(data=diabetes_dataset, x='Age', bins=20)
```

Out[6]: <AxesSubplot:xlabel='Age', ylabel='Count'>




```
In [7]: sns.histplot(data=diabetes_dataset, x='Pregnancies', bins=20)
```

Out[7]: <AxesSubplot:xlabel='Pregnancies', ylabel='Count'>



K-anonymity

Definition: A property of a dataset indicating its level of anonymity.

We say a dataset is k-anonymous when we have k-1 individuals with the same quasi-identifiers.

What is the best k value? Well, it depends...

```
In [9]: diabetes_ds = pd.read_csv("diabetes.csv")

        is_kanonymous(dataset=diabetes_ds, k_value=1)

Out[9]: True
```

For k-anonymity, two values **must be avoided**: k = 1 and k = n (table dimension).

```
In [14]: is_kanonymous(diabetes_ds, k_value=50)

Out[14]: False
```

We do not have 50-anonymity

Generalization

Suppression and **generalization** can be used to achieve k-anonymity.

In the following example, the generalization function is used to remove the second digit:

In [17]:
```
generalization_function(diabetes_ds, generalization_degree)
```

Out[17]:

	Pregnancies	Glucose	BloodPressure	SkinThickness	Insulin	BMI	DiabetesPedigreeFunction	Age	Outcome
0	0	140	70	30	0	30	0	50	0
1	0	80	60	20	0	20	0	30	0
2	0	180	60	0	0	20	0	30	0
3	0	80	60	20	90	20	0	20	0
4	0	130	40	30	160	40	0	30	0
...	...	...	...	...	...	...	...	...	...
763	10	100	70	40	180	30	0	60	0
764	0	120	70	20	0	30	0	20	0
765	0	120	70	20	110	20	0	30	0
766	0	120	60	0	0	30	0	40	0
767	0	90	70	30	0	30	0	20	0

768 rows × 9 columns

Value is completely lost!

Generalization (cont.)

Did we achieve the desired level of privacy?

```
In [19]: is_kanonymous(generalization_function(diabetes_ds, generalization_degree), 2)
Out[19]: False
```

Keep on generalizing until we reach it...

```
generalization_function(diabetes_ds, generalization_degree)
```

Out[20]:

	Pregnancies	Glucose	BloodPressure	SkinThickness	Insulin	BMI	DiabetesPedigreeFunction	Age	Outcome
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0
...	...	...	...	...	...	...	...	...	...
763	0	0	0	0	0	0	0	0	0
764	0	0	0	0	0	0	0	0	0
765	0	0	0	0	0	0	0	0	0
766	0	0	0	0	0	0	0	0	0
767	0	0	0	0	0	0	0	0	0

768 rows × 9 columns

```
In [21]: is_kanonymous(generalization_function(diabetes_ds, generalization_degree), 2)
Out[21]: True
```

We achieved it, but at what cost?

Distributional Queries

If we wanted to understand how many individuals in the dataset have a BMI over 50, we could do the following:

```
In [22]: diabetes_ds[diabetes_ds["Age"] >= 60].shape[0]
Out[22]: 32
```

Raw Results

How many individuals have been pregnant more than five times?

```
In [23]: diabetes_ds[diabetes_ds["Pregnancies"] > 5].shape[0]
Out[23]: 219
```

How can we protect patients?

Possible solution: Differential Privacy

DP: Laplace Mechanism

$$F(x) = \underbrace{f(x)}_{\text{Raw Results}} + \underbrace{\mathrm{Lap}\left(\frac{s}{\epsilon}\right)}_{\text{Added noise to achieve privacy}}$$

Raw Results

Added noise to achieve privacy

ε: Privacy Budget

S: Sensitivity

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DP: Laplace Mechanism (cont.)

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A query that looks for users with a BMI greater than or equal to 60.

```
In [26]: diabetes_ds[diabetes_ds["BMI"] >= 60].shape[0]
Out[26]: 1
```

Only one person

If the value for epsilon is **large**, we will not be able to protect the privacy of individuals, and **the value will be close to original value**:

```
In [27]: # Laplace mechanism definition:
         # F(x) = f(x) + Lap(s/eps)
         # s: sensitivity
         # eps: privacy budget

         sensitivity = 1
         eps = 0.8

         diabetes_ds[diabetes_ds["BMI"] >= 60].shape[0] + npr.laplace(scale=sensitivity/eps)
Out[27]: 0.325684438035028
```

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DP: Laplace Mechanism (cont.)

If the value for epsilon is **small**, we will **focus more on protecting the privacy of the user**, and **the returned value will be far from the original value**:

```
In [28]: sensitivity = 1
         eps = 0.001

         diabetes_ds[diabetes_ds["BMI"] >= 60].shape[0] + npr.laplace(scale=sensitivity/eps)

Out[28]: -220.89136497917102
```

Raw result + Laplace noise

Laplace Mechanism on Histograms

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We create a histogram based on blood pressure, with the number of individuals as the count (e.g., 57 individuals with blood pressure equal to 70).

In [33]:
```
# reset dataset

diab_ds = pd.read_csv("output.csv")
```

In [99]:
```
# choose variable glucose and count how many individuals have the same values
hist_diabetes = diab_ds["BloodPressure"].value_counts()

# display
hist_diabetes.to_frame().head(11)
```

Out[99]:

	BloodPressure
70	57
74	52
78	45
68	45
72	44
64	43
80	40
76	39
60	37
0	35
62	34

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Laplace Mechanism on Histograms (cont.)

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Laplace noise can be applied to histograms:

In [55]:
```
sensitivity = 1
eps = 0.03

lap_noise = lambda x: x + npr.laplace(scale=sensitivity/eps)

hist_noise_added = hist_diabetes.apply(lap_noise)

# noisy table
hist_noise_added.to_frame().head()
```

Out[55]:

	BloodPressure
70	30.907527
74	27.433998
78	48.338123
68	40.342932
72	59.949283

Values have been perturbed according to the privacy budget that we defined.

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Laplace noise can be applied to <u>contingency tables</u> as well. Using **pd.crosstab()**, multiple columns are joined to build the following table:

```
In [87]: contingency_table = pd.crosstab([diab_ds["Job"], diab_ds["Age"]], diab_ds['Outcome'])

         # display contingency table
         contingency_table
```

Out[87]:

Job	Age	Outcome 0	Outcome 1
Academic librarian	22	0	1
	36	0	1
	41	1	0
Accommodation manager	28	1	0
	70	0	1
...	...	...	...
Web designer	50	1	0
	60	0	1
Writer	21	1	0
	25	1	0
Youth worker	24	0	1

748 rows × 2 columns

The outcome determines whether or not a person has diabetes.(1 indicates diabetes; 0 indicates no diabetes).

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Laplace Mechanism on Contingency Tables (cont.)

Similar to what we encountered before, **if the privacy budget is large**, the final outcome **will not be drastically different from the original value**:

In [93]:
```
# define privacy budget which is large
eps = 50

noisy_contingency_table = contingency_table.apply(lap_noise)

# display
noisy_contingency_table
```

Out[93]:

	Outcome	0	1
Job	**Age**		
Academic librarian	**22**	0.008824	1.004068
	36	0.008824	1.004068
	41	1.008824	0.004068
Accommodation manager	**28**	1.008824	0.004068
	70	0.008824	1.004068
...	**...**	...	...
Web designer	**50**	1.008824	0.004068
	60	0.008824	1.004068
Writer	**21**	1.008824	0.004068
	25	1.008824	0.004068
Youth worker	**24**	0.008824	1.004068

748 rows × 2 columns

Laplace Mechanism on Contingency Tables (cont.)

If the privacy budget is small, the end result **will be distant from the original value**:

In [94]:
```python
# privacy budget chosen to be small
eps = 0.0005

noisy_contingency_table = contingency_table.apply(lap_noise)

# display
noisy_contingency_table
```

Out[94]:

	Outcome	0	1
Job	**Age**		
Academic librarian	**22**	-624.156473	1646.68496
	36	-624.156473	1646.68496
	41	-623.156473	1645.68496
Accommodation manager	**28**	-623.156473	1645.68496
	70	-624.156473	1646.68496
...	**...**	...	...
Web designer	**50**	-623.156473	1645.68496
	60	-624.156473	1646.68496
Writer	**21**	-623.156473	1645.68496
	25	-623.156473	1645.68496
Youth worker	**24**	-624.156473	1646.68496

748 rows × 2 columns

Final Remarks

- Resistant to record linkage attacks
- Bugs in implementation are possible (due to <u>incorrect sensitivity calculation</u> or <u>adding the incorrect amount of noise</u>).

- Not suitable for every scenario (e.g., medical data is sensitive; a small privacy budget is needed. Simultaneously, too much noise will make diagnosis difficult for doctors.)