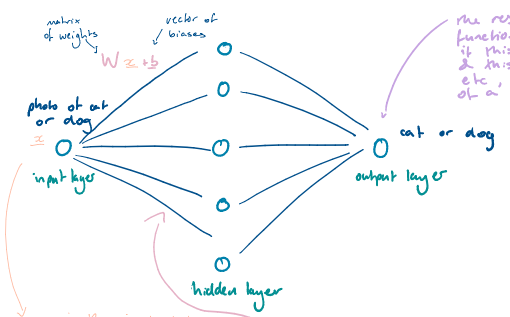
22 August 2022 10:56

Neural Networks



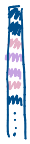
the result of the activation function tells use e.g. if this pixel is this colour & this pixel is this colour... etc, then this is a pic. of a dog or a cat.

x is the input which is the pixel values in a photo

so, if I have a photo

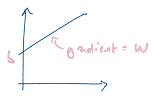
each pixel has a corresponding value

its chopped up so its a stacked column



$\Rightarrow x = \begin{bmatrix} 255 \\ 255 \\ 123 \\ 148 \\ 148 \\ \vdots \end{bmatrix}$

the weights & biases are applied to the input - a linear function



- gives a function y
- an activation function is applied to y

$\Rightarrow a_1(y_1)$, e.g. the relu fn. which makes -ve 0

- so now we have the values

SINDy algorithm

- the weights are the ξ values

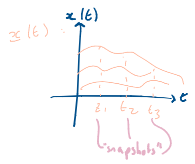
connection between NN & SINDy-PI

SINDy ← the original algorithm

dynamical system of the form

$$\frac{d}{dt}x(t) = f(x(t))$$

$$\Rightarrow \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \vdots \\ \dot{x}_n(t) \end{bmatrix} = \begin{bmatrix} f_1(x_1(t)) & \cdots & f_1(x_n(t)) \\ \vdots & & \\ f_n(x_1(t)) & \cdots & f_n(x_n(t)) \end{bmatrix}$$



the library of candidate functions

$$\Theta(x) = [\theta_1(x) \quad \theta_2(x) \quad \cdots] \quad \text{e.g. } [1, x, x^2, \sin(x), \cos(x) \ldots]$$

so we have

$$\frac{d}{dt}x_k(t) = f_k(x(t)) \approx \Theta(x)\xi_k$$

sparse vector

EXAMPLE

system: $\begin{bmatrix} \dot{c}(t) \\ \dot{o}(t) \\ \dot{CO_2}(t) \end{bmatrix} = \begin{bmatrix} -0.1\, c(t)o(t) \\ -0.2\, c(t)o(t) \\ 0.1\, c(t)o(t) \end{bmatrix}$

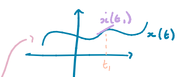
this is what we're trying to find

library: $\Theta(x) = [c(t), o(t), co_2(t), c(t)o(t)]$

$$X = \begin{bmatrix} c(t_1) & o(t_1) & CO_2(t_1) \\ c(t_2) & O(t_2) & CO_2(t_2) \end{bmatrix}$$

volumes of C, O, & CO_2 measured at snapshots (instances in time)

— refered to as trajectories



you can take the slope of a point on the line

$$\dot{X} = \begin{bmatrix} \dot{c}(t_1) & \dot{o}(t_1) & \dot{CO_2}(t_1) \\ \dot{c}(t_2) & \dot{O}(t_2) & \dot{CO_2}(t_2) \end{bmatrix}$$

time derivate of each trajectory
— measured
— or estimated

$$\dot{X} \approx \Theta(X)\Xi$$

$$\Rightarrow \begin{bmatrix} \dot{c}(t_1) & \dot{o}(t_1) & \dot{CO_2}(t_1) \\ \dot{c}(t_2) & \dot{O}(t_2) & \dot{CO_2}(t_2) \end{bmatrix} \approx \begin{bmatrix} c(t_1) & \cdots & o(t_1)CO_2(t_1) & c(t)o(t_1) \\ c(t_2) & \cdots & o(t_2)CO_2(t_2) & c(t_2)O(t_2) \end{bmatrix} \begin{bmatrix} \xi_{11} & \xi_{12} & \xi_{13} \\ \vdots & \vdots & \vdots \\ \xi_{41} & \xi_{42} & \xi_{43} \end{bmatrix}$$

associated w/ carbon; assoc. w/ oxygen; assoc. w/ CO_2

$$\frac{d}{dt}c(t) = -0.1\, c(t)o(t)$$

$$\frac{d}{dt}c(t) \approx \xi_{11}c(t) + \xi_{21}o(t) + \xi_{31}co_2(t) + \xi_{41}c(t)o(t)$$

$\therefore$ we would get out that $\xi_{41} = -0.1$ & the rest are 0.

implicit-SINDy

we now have the form

$$\Theta(x, \dot{x})\Xi = 0$$

dependant on derivative too

EXAMPLE

$$\Rightarrow \Theta = [c, o, \dot{c}, \dot{o}, \dot{co_2}, c \times O, c\dot{c}]$$

$$\begin{bmatrix} c(t_1) & \cdots & \dot{c}(t_1) & \cdots \\ c(t_2) & \cdots & \dot{c}(t_2) & \cdots \end{bmatrix} \begin{bmatrix} \xi_{11} & \cdots & \xi_{13} \\ \vdots & \ddots & \vdots \\ \xi_{81} & \cdots & \xi_{83} \end{bmatrix} \approx \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot{c}(t_1) \\ \dot{c}(t_2) \end{bmatrix} = \begin{bmatrix} c(t_1) & \cdots \\ c(t_2) & \cdots \end{bmatrix} \begin{bmatrix} \xi_{14} \\ \vdots \\ \xi_{18} \end{bmatrix}$$

$\Theta(x, \dot{x} \mid \theta_j(x, \dot{x}))$

"given"