

Let R denote the radius of the sphere, and r, H be the radius and height of each spherical cap respectively. Since the cylindrical hole has length 6, it follows that $H = R - 3$ and $r = \sqrt{R^2 - 9}$. Then,

$$\frac{4}{3}\pi R^3 - \left[2 \left(\frac{\pi H}{6} (3r^2 + H^2) \right) + 6\pi(R^2 - 9) \right] \quad (1)$$

$$= \frac{4}{3}\pi R^3 - \pi \left[\frac{(R-3)}{3} (3(R^2 - 9) + (R-3)^2) + 6(R-3)(R+3) \right] \quad (2)$$

$$= \frac{4}{3}\pi R^3 - \frac{\pi}{3} (R-3) \left[3(R^2 - 9) + (R-3)^2 + 18(R+3) \right] \quad (3)$$

$$= \frac{4}{3}\pi R^3 - \frac{4\pi}{3} (R-3) [R^2 + 3R + 9] \quad (4)$$

$$= \frac{4}{3}\pi R^3 - \frac{4\pi}{3} (R^3 - 3^3) \quad (5)$$

$$= \frac{4}{3}\pi (R^3 - R^3 + 3^3) \quad (6)$$

$$= 36\pi. \quad (7)$$