Homework 8

1. Fred wants to sell his car, after moving back to Blissville (where he is happy with the bus system). He decides to sell it to the first person to offer at least $15,000 for it. Assume that the offers are independent Exponential random variables with mean $10,000.

   (a) Find the expected number of offers Fred will have.

   (b) Find the expected amount of money that Fred gets for the car

2. Find E(X^3) for X ~ Expo(λ), using LOTUS and the fact that E(X)=1/λ and Var(X)=1/λ^2, and integration by parts at most once.

3. Let X_1,..., X_n be independent, with X_j ~ Expo(λ_j). (They are i.i.d. if all the λ_j 's are equal, but we are not assuming that.) Let M = min(X_1,...,X_n). Show that M ~ Expo(λ_1 + ⋯ + λ_n), and interpret this intuitively.

4. A post office has 2 clerks. Alice enters the post office while 2 other customers, Bob and Claire, are being served by the 2 clerks. She is next in line. Assume that the time a clerk spends serving a customer has the Exponential(λ) distribution.

   (a) What is the probability that Alice is the last of the 3 customers to be done being served Hint: no integrals are needed.

   (b) What is the expected total time that Alice needs to spend at the post office?

5. Find E(X^3) for X ~ Expo(λ) using the MGF of X (see also Problem 2).

6. If X has MGF M(t), what is the MGF of -X? What is the MGF of a + bX, where a and b are constants?