Homework 6

1. Raindrops are falling at an average rate of 20 drops per square inch per minute. What would be a reasonable distribution to use for the number of raindrops hitting a particular region measuring 5 $inches^2$ in t minutes? Why? Using your chosen distribution, compute the probability that the region has no rain drops in a given 3 second time interval.

2. Harvard Law School courses often have assigned seating to facilitate the "Socratic method." Suppose that there are 100 first year Harvard Law students, and each takes two courses: Torts and Contracts. Both are held in the same lecture hall (which has 100 seats), and the seating is uniformly random and independent for the two courses.

   (a) Find the probability that no one has the same seat for both courses (exactly; you should leave your answer as a sum).

   (b) Find a simple but accurate approximation to the probability that no one has the same seat for both courses.

   (c) Find a simple but accurate approximation to the probability that at least two students have the same seat for both courses.

3. Explain why if X $\sim$ Bin(n, p), then n - X $\sim$ Bin(n, 1 - p).

4. There are 100 passengers lined up to board an airplane with 100 seats (with each seat assigned to one of the passengers). The first passenger in line crazily decides to sit in a randomly chosen seat (with all seats equally likely). Each subsequent passenger takes his or her assigned seat if available, and otherwise sits in a random available seat. What is the probability that the last passenger in line gets to sit in his or her assigned seat? (This is another common interview problem, and a beautiful example of the power of symmetry.)

   Hint: call the seat assigned to the jth passenger in line "Seat j" (regardless of whether the airline calls it seat 23A or whatever). What are the possibilities for which seats are available to the last passenger in line, and what is the probability of each of these possibilities?

5. A stick is broken into two pieces, at a uniformly random chosen break point. Find the CDF and average of the length of the longer piece.

6. Let U be a Uniform r.v. on the interval (-1, 1) (be careful about minus signs).

   (a) Compute E(U), Var(U), and E(U^4).

   (b) Find the CDF and PDF of U^2. Is the distribution of U^2 Uniform on (0, 1)?

7. For $X \sim \text{Pois}(\lambda)$, find $E(X!)$ (the average factorial of X), if it is finite