Homework 1

1. A round-robin tournament is being held with n tennis players; this means that every player will play against every other player exactly once.
   (a) How many possible outcomes are there for the tournament (the outcome lists out who won and who lost for each game)?

   (b) How many games are played in total?

2. (a) How many ways are there to split a dozen people into 3 teams, where one team has 2 people, and the other two teams have 5 people each?
   (b) How many ways are there to split a dozen people into 3 teams, where each team has 4 people?

3. How many paths are there from (0, 0) to (210, 211), where each step consists of going one unit up or one unit to the right, and the path has to go through (110, 111)?

4. To fulfill the requirements for a certain degree, a student can choose to take any 7 out of a list of 20 courses, with the constraint that at least 1 of the 7 courses must be a statistics course. Suppose that 5 of the 20 courses are statistics courses. How many choices are there for which 7 courses to take?

5. A certain casino uses 10 standard decks of cards mixed together into one big deck, which we will call a superdeck. Thus, the superdeck has 52 - 10 = 520 cards, with 10 copiesof each card. How many different 10-card hands can be dealt from the superdeck? The order of the cards does not matter, nor does it matter which of the original 10 decks the cards came from. Express your answer as a binomial coefficient.

   Hint: Bose-Einstein.

6. Four cards are face down on a table. You are told that two are red and two are black, and you need to guess which two are red and which two are black. You do this by pointing to the two cards you're guessing are red (and then implicitly you're guessing that the other two are black). Assume that all configurations are equally likely, and that you do not have psychic powers. Find the probability that exactly j of your guesses are correct,
   for j = 0, 1, 2, 3, 4.

7. An organization with 2n people consists of n married couples. A committee of size k is selected, with all possibilities equally likely. Find the probability that there are exactly j married couples within the committee.

8. There are n balls in a jar, labeled with the numbers 1, 2,…,n. A total of k balls are drawn, one by one with replacement, to obtain a sequence of numbers.
   (a) What is the probability that the sequence obtained is strictly increasing?
   (b) What is the probability that the sequence obtained is increasing (but not necessarily strictly increasing, i.e., there can be repetitions)?