Homework 2

1. Alice attends a small college in which each class meets only once a week. She is deciding between 30 non-overlapping classes. There are 6 classes to choose from for each day of the week, Monday through Friday. Trusting in the benevolence of randomness, Alice decides to register for 7 randomly selected classes out of the 30, with all choices equally likely. What is the probability that she will have classes every day, Monday through Friday? (This problem can be done either directly using the naive definition of probability, or using inclusion-exclusion.)

2. Is it possible that an event is independent of itself? If so, when?

3. Give an example of 3 events A, B, C which are pairwise independent but not independent.
   Hint: find an example where whether C occurs is completely determined if we know whether A occurred and whether B occurred, but completely undetermined if we know only one of these things.

4. A bag contains one marble which is either green or blue, with equal probabilities. A green marble is put in the bag (so there are 2 marbles now), and then a random marble is taken out. The marble taken out is green. What is the probability that the remaining marble is also green?

5. Let G be the event that a certain individual is guilty of a certain robbery. In gathering evidence, it is learned that an event E1 occurred, and a little later it is also learned that another event E2 also occurred.

   (a) Is it possible that individually, these pieces of evidence increase the chance of guilt (so $P(G|E1) > P(G)$ and $P(G|E2) > P(G)$), but together they decrease the chance of guilt (so $P(G|E1, E2) < P(G)$)?

   (b) Show that the probability of guilt given the evidence is the same regardless of whether we update our probabilities all at once, or in two steps (after getting the first piece of evidence, and again after getting the second piece of evidence). That is, we can either update all at once (computing $P(G|E1, E2)$ in one step), or we can first update based on E1, so that our new probability function is $Pnew(A) = P(A|E1)$, and then update based on E2 by computing $Pnew(G|E2)$.

6. You are going to play 2 games of chess with an opponent whom you have never played against before (for the sake of this problem). Your opponent is equally likely to be a beginner, intermediate, or a master. Depending on which, your chances of winning an individual game are 90%, 50%, or 30%, respectively.

   (a) What is your probability of winning the first game?

   (b) Congratulations: you won the first game! Given this information, what is the probability that you will also win the second game (assume that, given the skill level of your opponent, the outcomes of the games are independent)? (c) Explain the distinction between assuming that the outcomes of the games are independent and assuming that they are conditionally independent given the opponent's skill level. Which of these assumptions seems more reasonable, and why?

7. A card player is dealt a 13 card hand from a well-shuffled, standard deck of cards. What is the probability that the hand is void in at least one suit ("void in a suit" means having no cards of that suit)?

8. A family has two children. Assume that birth month is independent of gender, with boys and girls equally likely and all months equally likely, and assume that the elder child's characteristics are independent of the younger child's characteristics).

   (a) Find the probability that both are girls, given that the elder child is a girl who was born in March.

   (b) Find the probability that both are girls, given that at least one is a girl who was born in March.

9. A fair coin is flipped 3 times. The toss results are recorded on separate slips of paper
   (writing "H" if Heads and "T" if Tails), and the 3 slips of paper are thrown into a hat.

   (a) Find the probability that all 3 tosses landed Heads, given that at least 2 were Heads.

(b) Two of the slips of paper are randomly drawn from the hat, and both show the
letter H. Given this information, what is the probability that all 3 tosses landed Heads?

10. The Jack of Spades (with cider), Jack of Hearts (with tarts), Queen of Spades (with a
wink), and Queen of Hearts (without tarts) are taken from a deck of cards. These four
cards are shuffled, and then two are dealt. Note: Literary references to cider, tarts, and
winks do not need to be considered when solving this problem.
(a) Find the probability that both of these two cards are queens, given that the first
card dealt is a queen.
(b) Find the probability that both are queens, given that at least one is a queen.
(c) Find the probability that both are queens, given that one is the Queen of Hearts.