Homework 4

1. (a) In the World Series of baseball, two teams (call them A and B) play a sequence of games against each other, and the first team to win four games wins the series. Let p be the probability that A wins an individual game, and assume that the games are independent. What is the probability that team A wins the series?

   (b) Give a clear intuitive explanation of whether the answer to (a) depends on whether the teams always play 7 games (and whoever wins the majority wins the series), or the teams stop playing more games as soon as one team has won 4 games (as is actually the case in practice: once the match is decided, the two teams do not keep playing more games).

2. A sequence of n independent experiments is performed. Each experiment is a success with probability p and a failure with probability q = 1 p. Show that conditional on the number of successes, all possibilities for the list of outcomes of the experiment are equally likely (of course, we only consider lists of outcomes where the number of successes is consistent with the information being conditioned on).

3. (a) Consider the following 7-door version of the Monty Hall problem. There are 7 doors, behind one of which there is a car (which you want), and behind the rest of which there are goats (which you don't want). Initially, all possibilities are equally likely for where the car is. You choose a door. Monty Hall then opens 3 goat doors, and offers you the option of switching to any of the remaining 3 doors.
   Assume that Monty Hall knows which door has the car, will always open 3 goat doors and offer the option of switching, and that Monty chooses with equal probabilities from all his choices of which goat doors to open. Should you switch? What is your probability of success if you switch to one of the remaining 3 doors?

   (b) Generalize the above to a Monty Hall problem where there are $n \geq 3$ doors, of which Monty opens m goat doors, with $1 \leq m \leq n\ 2$.

4. Players A and B take turns in answering trivia questions, starting with player A answering the first question. Each time A answers a question, she has probability p1 of getting it right. Each time B plays, he has probability p2 of getting it right.

   (a) If A answers m questions, what is the PMF of the number of questions she gets right?

   (b) If A answers m times and B answers n times, what is the PMF of the total number of questions they get right (you can leave your answer as a sum)? Describe exactly when/whether this is a Binomial distribution.

   (c) Suppose that the first player to answer correctly wins the game (with no predetermined maximum number of questions that can be asked). Find the probability that A wins the game.

5. Calvin and Hobbes play a match consisting of a series of games, where Calvin has probability p of winning each game (independently). They play with a “win by two” rule: the first player to win two games more than his opponent wins the match. Find the probability that Calvin wins the match (in terms of p), in two different ways:

   (a) by conditioning, using the law of total probability.

   (b) by interpreting the problem as a gambler’s ruin problem.