

Homework 7

1. Let $Z \sim N(0, 1)$. Find $E(\phi(Z))$ without using LOTUS, where ϕ is the CDF of Z .
2. Let F be a CDF which is continuous and strictly increasing. The inverse function, F^{-1} , is known as the quantile function, and has many applications in statistics and econometrics. Find the area under the curve of the quantile function from 0 to 1, in terms of the mean μ of the distribution F .
Hint: Universality
3. Let $Z \sim N(0, 1)$ and let S be a “random sign” independent of Z , i.e., S is 1 with probability $1/2$ and -1 with probability $1/2$. Show that $SZ \sim N(0, 1)$.
4. Let $Z \sim N(0, 1)$. A measuring device is used to observe Z , but the device can only handle positive values, and gives a reading of 0 if $Z \leq 0$; this is an example of censored data. So assume that $X = ZI_{Z>0}$ is observed rather than Z , where $I_{Z>0}$ is the indicator of $Z > 0$. Find $E(X)$ and $\text{Var}(X)$.