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# The Log of Gravity: An Update to a Report on Seriousness

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Term Paper for Econ 282F: Advance Macroeconomic Theory

Fall 2011

## **Abstract**

This paper reproduces a portion of the results found in Silva and Tenreyro (2006), "The Log of Gravity". I have updated the Poisson Pseudo-Maximum Likelihood (PPML) regression method with 2010 data for GDP and GDP per capita, and have added several new free trade accords to the Trade Preference variable. Fourteen countries (about 10% of the original country list) have been selected from the 136 countries illustrated in Silva and Tenreyro (2006). Four differing regressions are performed to test the consistency of the author's results upon this condensed set.

While the results are inconclusive for the updated subset, it can be shown that PPML estimates were similar and the significant variables (with one exception) were consistent. Inversely, the OLS estimates were too noisy and lacked enough consistency for a definitive analysis. This leads to an inability to make considerable conclusions at this time; but the results lead me to believe that further data accumulation will lead to better and more robust estimates.

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# 1. Issues with the Gravity Equation

Over the past few decades, there has been an unparalleled increase in international trade and interdependence of countries' economies. This interdependency has encouraged economists to explore and better understand the underpinnings of international trade. In the pursuit of understanding, modern literature uses a model that is analogous to Newton's law of gravity, which coincidentally is named the gravity equation. Despite the heavy use of the gravity equation, there are biases associated with the model. In exemplum, the majority of scholarly research log transforms the gravity equation and subsequently runs an OLS regression. However in the presence of heteroskedasticity (which appears in the data more often than not) or when the bilateral trade between countries is zero, the results of the OLS regression become suspect.

For this paper I will mainly be focusing on Silva and Tenreyro's (2006) "The Log of Gravity". The authors propose that using the level form of the gravity equation is better since the model will be free of biases caused by truncating the sample due to zero bilateral trade pairs. Moreover, Silva and Tenreyro devise a pseudo-maximum likelihood method that accounts for the inherent heteroskedastic issues. I will first review their data, and I will then conduct the original analysis and the updated analysis respectively.

Modern trade literature tends to use a log form of the gravity equation as the model to be analyzed.

Trade volume is defined by:

$$T_{ij} = \alpha_0 Y_i^{\alpha_1} Y_j^{\alpha_2} D_{ij}^{\alpha_3} \exp(\alpha_4 S_i + \alpha_5 S_j + \alpha_6 \delta_{ij}) \varepsilon_{ij}$$

where  $Y$  is the GDP of country  $i$  and country  $j$ ,  $D$  is the distance between the two economies,  $S_i$  and  $S_j$  are the barriers of trade for country  $i$  and country  $j$ ,  $\varepsilon$  is the error term, and the  $\alpha$ 's are the parameters

that are to be estimated. Following the convention of modern literature, the log of the above gravity equation is performed to obtain the following, and a successive OLS regression is conducted.

$$\ln(T_{ij}) = \ln(\alpha_0) + \alpha_1 \ln(Y_i) + \alpha_2 \ln(Y_j) + \alpha_3 \ln(D_{ij}) + \alpha_4 S_i + \alpha_5 S_j + \alpha_6 \delta_{ij} + \ln(\varepsilon_{ij})$$

Silva and Tenreyro claim that the log-transformed gravity model is subject to four types of estimation bias. These biases could be the originators of the aforementioned controversy. The first source of bias comes from the heteroskedastic nature of the error term (which is the main focus of this paper). Since it is often the case that the level model suffers from heteroskedasticity problems, so then will the log transformed model. The second source of estimation bias is due to the nature of the log transformation. Country pairs with zero trade values will be dropped from the model, effectively truncating the sample. Another source of bias originates from omitted-variable bias. It is standard to assume that exchange rate volatility is exogenous; however, this is seldom true. For instance, two countries looking to increase their relative levels of trade could do so by reducing trade barriers or standardizing production regulations. The final source of bias is from measurement error. There is a large variation between what countries report and what countries actually do.

Silva and Tenreyro (2006) focus specifically on the first two sources of bias: heteroskedasticity and log transformed gravity equations. The authors propose a PPML (Poisson Pseudo-Maximum Likelihood) regression method that compensates for the heteroskedastic nature of the data.<sup>1</sup> To compensate for zero value bilateral trade pairs, the authors use the level form of the gravity equation rather than the log transformed gravity equation. Furthermore, Silva and Tenreyro found, in the robustness checks, that to

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<sup>1</sup> The validity and robustness of this method is left to the analysis of the original paper, Silva and Tenreyro (2006).

obtain consistent estimators using the PPML regression, the gravity equation must be in its level form; further perpetuating that the level form should be used.

## **2. Literature Review**

The Gravity Equation was first proposed by Tinbergen (1962) and was later expanded upon by Anderson and Van Wincoop (2003). This model was created to theorize bilateral trade in an international setting. The theory states that international trade between two countries is directly proportional to the relative size of the two countries' GDP and inversely proportional to the distance between them. Three papers which were found to support the Gravity Equation in theoretical foundations are Anderson (1979), Bergstrand (1985), and Anderson and Van Wincoop (2003). Anderson and Wincoop (2003) use cross-sectional data on monopolistic competition and propose a method to estimate gravity coefficients. They conclude that both the relative size of GDP and the distance between two countries are highly significant for bilateral trade. Further, they determine that trade costs directly influence bilateral trade volume.

There are problems associated with the Gravity Equation however. These issues were outlined in Silva and Tenreyro (2006) and later in Tenreyro (2007). Silva and Tenreyro (2006) state that in the presence of heteroskedasticity, the log transformed gravity model suffers from biased estimates. The log transformed model also breaks down in the presence of zero valued bilateral trade volume. The authors then propose a PPML regression method on the level form of the Gravity Equation. In Tenreyro (2007) four potential biases plague the Gravity Equation. These biases must be addressed simultaneously in order to avoid misleading results. It is here that Tenreyro puts forth a PPML-IV method regressed on the level form gravity model. This simultaneously accounts for the aforementioned zero valued trade volume, heteroskedasticity, endogeneity, and measurement error. The results of this paper are that

exchange rate volatility have only marginal effects upon trade volume, and that distance between a client country and an anchor country increases the probability that the client will peg its currency to the anchor.

### **3. Data**

#### **3.1 Original Data Set**

Silva and Tenreyro (2006) use cross-sectional data that covers 136 countries in 1990.<sup>2</sup> The variables of interest are trade, country GDP, country GDP per capita, the log of distance, country's remoteness, and openness. The authors also control the following variables using dummies: common borders, common language (first and secondary), colonial ties, if the country is landlocked, and if the country is part of a free trade agreement.

The data for trade, or bilateral exports, comes from Feenstra et al. (1997). The data on real GDP, GDP per capita, and population are found in the World Bank's (2002) World Development Indicators. The CIA *World Factbook* (2002) provides the data for the contiguity, common language, colonial ties, and access to water dummies. Andrew Gray (2001) computes the relative distance of countries. Wei (1996) computes remoteness as the log of GDP as a weighted average of distance to all other countries. Data on preferential trade statuses is arranged by Frankel (1997).

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<sup>2</sup> A list of countries is included in Appendix A1.

### 3.2 Updated Data Set

The updated data contains 14 randomly selected countries from Silva and Tenreyro's original sample of 136 countries.<sup>4</sup> Each of these 14 countries contains updated data for real GDP, real GDP per capita, and favorable trade agreements. The real GDP and real GDP per capita data utilized in my research is the 2010 statistics from the December update for the World Bank's (2011) *World Development Indicators*. The list of favorable trade agreements and their respective member nations were taken from their respective hosted websites<sup>6</sup>. All other time-invariant data were taken from Silva and Tenreyro's data set; these variables include: distance, contiguity, common language, colonial ties, landlocked, and remoteness.

The bilateral trade variable was also taken from the Silva and Tenreyro (2006) data set, but was updated to 2010 values in order to streamline the results. Bilateral trade was updated via multiplication of an inflation coefficient calculated by the average annual inflation over the last twenty years to account for the 1990 to 2010 update. The inflation coefficient was found through the Saint Louis Federal Reserve website (FRED).<sup>7</sup>

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<sup>4</sup> The 14 randomly chosen countries constitute approximately 10% of the original sample of countries. A listing of these countries can be found in Appendix A2.

<sup>6</sup> This updated data is also from 2010. A list can be found in Appendix A4.

<sup>7</sup> The FRED estimated the inflation coefficient over this time period to be 66.56% as calculated through the CPI.



## 4. Original Data Analysis

In this section I reproduce sections of Table 3 in Silva and Tenreyro (2006).<sup>8</sup> I ran two OLS regressions and two regressions that utilize the PPML method, accounting for heteroskedasticity. The first OLS regression emulates the current standard log transformed gravity equation; thus the dependant variable is  $\ln(trade)$ . This regression removes all bilateral trade pairs that have zero trade from the data, effectively truncating the sample.

The second regression is still the standard OLS log transformed regression, but compensates for the zero bilateral trade pairs. The value of 1 is added to all trade values and the log transformed dependent variable is  $\ln(1 + trade)$ . As a result of this transformation, no observations are dropped or removed from the data.

The third and fourth model uses the level form of the gravity equation utilizing the PPML regression. The difference is that regression three only uses values of trade that are greater than 0, while regression four uses the entire sample.

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<sup>8</sup> Refer to Table 3, Silva and Tenreyro (2006) "The Log of Gravity", *The Review of Economics and Statistics*, page 650.

Table 1 - reproduction of Silva and Tenreyro's results:

Estimator: Dependent Variable:	OLS $\ln(T_{ij})$	OLS $\ln(1 + T_{ij})$	PPML $T_{ij} > 0$	PPML $T_{ij}$
Log Exporter's GDP	0.938** (0.012)	1.128** (0.011)	0.721** (0.027)	0.732** (0.027)
Log Importer's GDP	0.798** (0.011)	0.866** (0.011)	0.732** (0.028)	0.741** (0.027)
Log Exporter's GDP per capita	0.207** (0.017)	0.277** (0.017)	0.154** (0.053)	0.157** (0.053)
Log Importer's GDP per capita	0.106** (0.017)	0.217** (0.017)	0.133** (0.044)	0.135** (0.045)
Log Distance	-1.166** (0.034)	-1.151** (0.037)	-0.776** (0.055)	-0.784** (0.055)
Contiguity Dummy	0.314* (0.143)	-0.241 (0.164)	0.202 (0.105)	0.193 (0.104)
Common-Language Dummy	0.678** (0.064)	0.742** (0.064)	0.751** (0.134)	0.746** (0.135)
Colonial-Tie Dummy	0.397** (0.068)	0.392** (0.068)	0.012 (0.150)	0.025 (0.150)
Landlocked- Exporter Dummy	-0.062 (0.065)	0.106* (0.060)	-0.872** (0.157)	-0.863** (0.157)
Landlocked- Importer Dummy	-0.665** (0.063)	-0.278** (0.060)	-0.703** (0.141)	-0.696** 0.141
Exporter's Remoteness	0.467** (0.078)	0.526** (0.089)	0.647** (0.135)	0.660** (0.134)
Importer's Remoteness	-0.205** (0.081)	-0.109 (0.089)	0.549** (0.120)	0.562** (0.119)
Free-Trade Agreement Dummy	0.491** (0.105)	1.289** (0.143)	0.179* (0.090)	0.181* (0.089)
Openness	-0.170** (0.049)	0.739** (0.048)	-0.139 (0.133)	-0.107 (0.131)
Observations	<b>9613</b>	<b>18360</b>	<b>9613</b>	<b>18360</b>
RESET test p-values	<b>0.000</b>	<b>0.000</b>	<b>0.941</b>	<b>0.332</b>

The replicated results generally equal the results from the original paper. The only discrepancies that appear are in the third or fourth significant digit; thus any difference becomes negligible.

For a full analysis refer to Silva and Tenreyro (2006). The main result to be noted from the above table is that it is heteroskedasticity, not truncation of the data, that is the cause of the apparent differences.

Notice that the estimated coefficients for both PPML regressions are nearly identical, with only minor

variation. When we compare this to both OLS regressions, we see significant differences in the estimates. A secondary result is that the OLS tends to have over-exaggerated the estimates of real GDP for imports and exports, log distance, and colonial ties. Finally, the RESET Test favors the PPML methods over the OLS method, illustrated in the test's p-values located at the bottom of Table 1.

## **5. Analysis with Updated Data**

This section follows the same format previously outlined but with the condensed 14 country subset 2010 data. The expected results are four-fold in response. First, are the PPML estimated coefficients similar in value? Second, are the estimates for the log of GDP for importer and exporter small for the PPML? Third, are the values for log of distance and colonial-ties over-exaggerated for the OLS regressions? Finally, fourth, does the RESET test favor the PPML method? The results of the updated regressions can be found in Table 2.

Table 2 - Results using the updated data subset.

Estimator: Dependent Variable:	OLS $\ln(T_{ij})$	OLS $\ln(1 + T_{ij})$	PPML $T_{ij} > 0$	PPML $T_{ij}$
Log Exporter's GDP	0.806** (0.148)	1.212** (0.144)	0.888** (0.166)	1.022** (0.216)
Log Importer's GDP	0.799** (0.139)	0.869** (0.144)	0.640** (0.082)	0.826** (0.133)
Log Exporter's GDP per capita	0.464* (0.207)	0.328 (0.208)	-0.016 (0.205)	-0.236 (0.251)
Log Importer's GDP per capita	0.227* (0.208)	0.561** (0.207)	0.378** (0.143)	0.107 (0.178)
Log Distance	-1.769** (0.507)	-0.041 (0.452)	-2.398** (0.413)	-1.943** (0.405)
Contiguity Dummy	(Omitted)	-0.774 (1.077)	(dropped)	(dropped)
Common-Language Dummy	-0.370 (0.709)	2.364** (0.842)	-0.815 (0.591)	-0.412 (0.644)
Colonial-Tie Dummy	0.334 (0.798)	-2.139** (0.834)	-0.206 (0.663)	-0.917 (0.644)
Landlocked- Exporter Dummy	0.949 (0.574)	0.838 (0.543)	-1.243 (0.730)	-0.923 (0.735)
Landlocked- Importer Dummy	-0.502 (0.590)	0.492 (0.543)	-0.671 (0.438)	-0.373 (0.432)
Exporter's Remoteness	3.481** (1.307)	0.807 (1.502)	3.219** (0.828)	3.018** (0.995)
Importer's Remoteness	0.163 (1.306)	-0.518 (1.501)	-0.092 (0.782)	-0.299 (0.995)
Free-Trade Agreement Dummy	-2.969 (1.494)	0.041 (0.971)	-5.365** (1.479)	-4.683** (0.845)
Openness	-0.333 (0.511)	0.119 (0.504)	-0.010 (0.417)	0.367 (0.572)
Observations	<b>72</b>	<b>182</b>	<b>72</b>	<b>172</b>
RESET test p-values	<b>0.259</b>	<b>0.000</b>	<b>0.005</b>	<b>0.140</b>

Unfortunately the answers to the above four questions are not as obvious as the results found in Silva and Tenreyro (2006). In comparison to the Silva and Tenreyro results, the PPML estimates are no longer as close in value, but neither are the values too divergent. The two estimates that vary the most are the colonial-ties dummy and the common language dummy. Diversely, reviewing the OLS regression estimates, the difference in values can vary wildly and dramatically.

When comparing the estimates of log GDP for imports and exports, between the OLS and PPML regression, it is no longer clear if one estimating method overvalues or undervalues the estimate. The same is true for the proximity variable and the colonial ties dummy.

Finally the RESET test also shows a degree of ambiguity. The RESET test favors the OLS regression with truncated trade values and the PPML regression using all trade values.<sup>9</sup> Conversely, the RESET test states that the OLS regression (with modified positive trade values) and the PPML method (using only positive trade values) can be improved using a linear fit variable.

There are three apparent problems that need to be addressed before the above results can be safely interpreted. The first problem is that the updated set only includes 10% of the original samples countries. In Silva and Tenreyro (2006) the data set contains 136 countries which calculates to  $(136*135)$  18,360 data points. The above data set only has 14 countries, or 186 bilateral trade data points. So decreasing the amount of countries to 14 reduces the sample size by about 18,180 data points. There is a vast amount of information missing that may be causing a bias to the results. A second problem is losing the common border dummy. Out of the 14 randomly chosen countries, only two share a border. Consequently, almost all of the contiguity dummy entries are zero. Thus any effect that sharing a border has, in this subset analysis, is lost. The final issue is that only GDP, GDP per capita, and preferential trade agreements have been updated to 2010 values. For the analysis to stand up to robustness checks bilateral trade levels and the openness values need to also be updated to 2010 values. I did change the bilateral trade values from 1990 dollars to 2010 dollars but this effect proved negligible in correcting the model.

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<sup>9</sup> Including zero bilateral trade values.

## 6. Conclusions

As it stands, the updated results are ambiguous at best. None of the four main Silva and Tenreyro (2006) results could be verified with this condensed subset. The PPML estimates for both models were not close enough in value to make the claim that it is heteroskedastic bias, not truncation, that fuels the differences. I could also not claim that the OLS models overstate the importance of real GDP, real GDP per capita, and the colonial-ties dummy. Finally, the RESET test was not consistent in determining which regression type is favored.

Despite these inconclusive results, much work needs to be done before any decisive results can be drawn. For further research and analysis, the updated 2010 data needs to be expanded to all 136 countries and bilateral trade data for 2010 needs to be tabulated. My updated information is only estimating lagged values of trade and the importance of this model is dubious. A richer, more robust model, must have all variables calculated in 2010 and then regressed using both OLS and PPML methods. It is at this point that true conclusions could be drawn.

## 7. Appendices

### Appendix A1 - Silva and Tenreyro (2006) country list

<b>Albania</b>	Denmark	Kenya	Romania
<b>Algeria</b>	Djibouti	Kiribati	Russian Federation
<b>Angola</b>	Dominican Rep.	Korea, Rep.	Rwanda
<b>Argentina</b>	Ecuador	Laos P. Dem. Rep.	Saudi Arabia
<b>Australia</b>	Egypt	Lebanon	Senegal
<b>Austria</b>	El Salvador	Madagascar	Seychelles
<b>Bahamas</b>	Eq. Guinea	Malawi	Sierra Leone
<b>Bahrain</b>	Ethiopia	Malaysia	Singapore
<b>Bangladesh</b>	Fiji	Maldives	Solomon Islands
<b>Barbados</b>	Finland	Mali	South Africa
<b>Belgium-Lux</b>	France	Malta	Spain
<b>Belize</b>	Gabon	Mauritania	Sri Lanka
<b>Benin</b>	Gambia	Mauritius	St. Kitts and Nevis
<b>Bhutan</b>	Germany	Mexico	Sudan
<b>Bolivia</b>	Ghana	Mongolia	Suriname
<b>Brazil</b>	Greece	Morocco	Sweden
<b>Brunei</b>	Guatemala	Mozambique	Switzerland
<b>Bulgaria</b>	Guinea	Nepal	Syrian Arab Rep.
<b>Burkina Faso</b>	Guinea-Bissau	Netherlands	Tanzania
<b>Burundi</b>	Guyana	New Caledonia	Thailand
<b>Cambodia</b>	Haiti	New Zealand	Togo
<b>Cameroon</b>	Honduras	Nicaragua	Trinidad and Tobago
<b>Canada</b>	Hong Kong	Niger	Tunisia
<b>Central Africa Rep.</b>	Hungary	Nigeria	Turkey
<b>Chad</b>	Iceland	Norway	Uganda
<b>Chile</b>	India	Oman	United Arab Em.
<b>China</b>	Indonesia	Pakistan	United Kingdom
<b>Colombia</b>	Iran	Panama	United States
<b>Comoros</b>	Ireland	Papua New guinea	Uruguay
<b>Congo Dem. Rep.</b>	Israel	Paraguay	Venezuela
<b>Congo Rep.</b>	Italy	Peru	Vietnam
<b>Costa Rica</b>	Jamaica	Philippines	Yemen
<b>Ivory Coast</b>	Japan	Poland	Zambia
<b>Cyprus</b>	Jordan	Portugal	Zimbabwe

## Appendix A2 - Updated subsample country list

Belize	Nigeria	Niger
Mali	Gabon	Dominican Rep.
Malawi	Pakistan	Madagascar
Japan	Honduras	Guinea
Spain	Burkina Faso	

## Appendix A3 - List of Preferential Trade Agreements

EEC/EC	CARICOM	CACM
Belgium	Bahamas	Costa Rica
Denmark	Barbados	El Salvador
France	Belize	Guatemala
Germany	Dominican Rep.	Honduras
Greece	Guyana	Nicaragua
Ireland	Haiti	
Italy	Jamaica	<b>Bilateral Agreements</b>
Luxembourg	Trinidad and Tobago	EC-Cyprus
Netherlands	St. Kitts and Nevis	EC-Malta
Portugal	Suriname	EC-Egypt
Spain		EC-Syria
United Kingdom	<b>SPARTECA</b>	EC-Algeria
	Australia	EC-Norway
<b>EFTA</b>	New Zealand	EC-Iceland
Iceland	Fiji	EC-Switzerland
Norway	Kiribati	Canada-United States
Switzerland	Papua New Guinea	Israel-United States
Liechtenstein	Solomon Islands	
<b>CER</b>	<b>PATCRA</b>	
Australia	Australia	
New Zealand	Papua New Guinea	



## **Appendix A4 - Updated List of Preferential Trade Agreements**

### **EFTA - European Free Trade Association**

Iceland	Norway
Liechtenstein	Switzerland

### **CER - Closer Economic Relations**

Australia	New Zealand
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### **CARICOM - Caribbean Community**

Antigua and Bermuda	Grenada	Saint Kitts and Nevis
Bahamas	Guyana	Saint Lucia
barbados	Haiti	Saint Vincent & Grenadines
Belize	Jamaica	Suriname
Dominican Rep.	Montserrat	Trinidad and Tobago

### **SPARTECA - South Pacific Regional Trade and Economic Co-operation Agreement**

Cook Isle.	Nauru	Tonga
Australia	New Zealand	Tuvalu
Fiji	Papua New Guinea	Vanuatu
Marshal Isle.	Samoa	Kiribati
Micronesia	Solomon Isle.	Niue

### **PATCRA - Papua New Guinea-Australia Trade and Commercial Agreement**

Australia	Papua New Guinea
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### **CACM - Central American Common Market**

Belize	Honduras
Costa Rica	Nicaragua
El Salvador	Panama
Guatamala	Dominican Rep.

### **EU - European Union**

Austria	Germany	Netherlands
Belgium	Greece	Poland
Bulgaria	Hungary	Portugal
Cyprus	Ireland	Romania
Czech Rep.	Italy	Slovakia

Denmark	Latvia	Slovenia
Estonia	Lithuania	Spain
Finland	Luxembourg	Sweden
France	Malta	United Kingdom

### **NAFTA - North American Free Trade Agreement**

Canada	Mexico	United States
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### **ASEAN - Association of Southeastern Asian Nations**

Brunei	Laos	Singapore
Cambodia	Malaysia	Thailand
Indonesia	Philippines	Vietnam

### **BRICS**

Brazil	India	South Africa
China	Russia	

### **ECOWAS - Economic Community of West African States**

Benin	Ghana	Niger
Burkina Faso	Guinea	Nigeria
Cape Verde	Guinea-Bissau	Senegal
Cote d'Ivoire	Liberia	Sierra Leone
Gambia	Mali	Togo

### **EAC - East African Community**

Burundi	Rwanda	Uganda
Kenya	Tanzania	

### **CEN-SAD - Community of Sahel-Saharan States**

Burkina Faso	Senegal	Liberia
Chad	Egypt	Ghana
Libya	Morocco	Sierra Leone
Mali	Nigeria	Comoros
Niger	Somalia	Guinea
Sudan	Tunisia	Kenya
Central African Rep.	Burundi	Mauritania
Eritrea	Togo	Sao Tome and Principe
Djibouti	Cote d'Ivoire	
Gambia	Guinea-Bissau	

### **COMESA - Common Market for Eastern and Southern Africa**

Burundi	Kenya	South Sudan
Comoros	Libya	Swaziland
DRC	Madagascar	Uganda
Djibouti	Malawi	Zambia
Egypt	Mauritius	Zimbabwe
Eritrea	Rwanda	
Ethiopia	Seychelles	

### **SADC - Southern African Development Community**

Angola	Mauritius	Swaziland
Botswana	Mozambique	Tanzania
DRC	Namibia	Zambia
Lesotho	Seychelles	Zimbabwe
Malawi	South Africa	

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