Signaling corruption through conspicuous consumption*

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Abstract

We often see a link between conspicuous consumption of luxury goods and corruption from public officers. Since this raises public awareness about them, it can backfire and lead to an investigation that eventually finds them guilty. One plausible explanation to rationalize this behavior is that, by signaling their willingness to be corrupt, they can attract the pool of corrupt firms and get higher bribes. In this work, we consider a public procurement setting where the government delegates a supervisor to run the process. If the signaling cost is low enough, then there exists a separating equilibrium where the supervisor signals his type and obtains a higher bribe. Even when the government fixes a budget constraint or maximum price before assigning a supervisor, a signaling equilibrium can still exist, but with a lower reserve price than socially optimal. Therefore, even though the government can reduce the revenue that the supervisor gets, corruption and signaling can result in aggregate welfare loss.

Keywords: corruption, public procurement, signals.

JEL Codes: D44, D73, H57.

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1 Introduction

We frequently observe that (suspected) corrupt public officers are seen with luxury goods, which raises public awareness about them. For example, during his time at office as Argentina's Secretary of Transportation, Ricardo Jaime was repeatedly seen in a luxurious yacht and a private airplane (Alconada Mon, 2019), among other notorious goods and gifts. Jaime was in charge of allocating subsidies to transportation projects, some of which were poorly completed or not executed at all, and later on, he was found guilty of accepting bribes. Another Argentinian example is the former federal judge Norberto Oyarbide, who was seen with a notorious diamond ring valued at USD 250.000. In this case, he first claimed that he bought the ring but later on denied this version.[1] Another example, which illustrates that this is not only a local phenomenon, is the case of Equatorial Guinea's former Agriculture Minister Teodorin Obiang Jr., the son of the country's President.[2] Despite his formal 3.200 Euros wage, he was seen in Paris driving luxury cars such as a Ferrari or Porsche, among other goods.

Regardless of whether these public officers effectively bought the goods or not, it is surprising that they were *seen* using and consuming them.[3] After all, conspicuous consumption can backfire and lead to an investigation that eventually finds them guilty. To rationalize this behavior, there must be some gains from publicly consuming these goods. One explanation is that officers want to effectively signal their willingness to be corrupt and therefore attract a pool of more corrupt firms, raising their expected revenue from bribes. While some firms might be willing to bribe a public officer, there is uncertainty about the officer they are facing, and they would certainly not try to bribe the most honest supervisor. Consequently, with this signal, firms can have a better understanding of the type of agent that they are meeting and the actions that will let them win the procurement.

The idea that conspicuous consumption can act as a signalrelates to the work of Fabrizi and Lippert (2017), who, within a principal-agent model, characterize conditions for the existence of a signaling equilibrium. In their work, if the officer's bargaining power is high enough, there exists a separating equilibrium where a public officer consumes conspicuously or "burns money" and thus attracts higher bribes. However, they consider two types of supervisors, who distinguish from another according to the probability in which they are subjected to an audit. In their model, signaling lets a supervisor attract the *correct* amount of bribes. Therefore, the public officer is signaling his ability in corruption and not his willingness to be corrupt, which is the interest of this work.

In this article, we will show that, within the framework of public procurement, there exists a separating perfect Bayesian equilibrium in which a corrupt officer signals his type by buying a luxury good. To do so, we consider a procurement auction where the government delegates the allocation of the project to a supervisor, who is potentially corrupt. We find that, if the cost of conspicuous consumption is low enough, then there exists a separating equilibrium where firms learn about the supervisor's type and only try to bribe the corrupt one. Compared to the equilibrium without the signaling stage, the supervisor attracts higher bribes than when firms are uncertain about his type. As a result of this, conspicuous consumption lets the firms know who is running the procurement and

[1] Reported in *Perfil*, available here.

[2] Reported in *The Guardian*, available here. See Fabrizi and Lippert (2017) for a detailed discussion of his case.

[3] In the previous examples, both Ricardo Jaime and Teodorin Obiang were found guilty of accepting bribes and corruption, respectively.

what actions will let them win the auction.

Additionally, in a procurement auction the government can set a a maximum reserve price before assigning a supervisor. We find that, even with this budget constraint, a signaling equilibrium can still exist. Moreover, the optimal reserve price in this equilibrium is below the social optimum. This implies that, while the government can do a little better with a reserve price, corruption entails a social welfare loss and a higher probability of declaring the procurement void.

The article is organized as follows: in Section 2, we review the related literature regarding procurement, corruption mechanisms and signaling equilibria in corruption. In Section 3, we build our model and solve it, considering the case with and without conspicuous consumption and adding the possibility that the government sets a reserve price. Finally, in Section 4, we have some concluding remarks.

2 Related literature

Corruption can take many forms and economic theory has focused on a wide range of models to analyze corruption in public procurement.[4] In this work, we are interested in the collusion between a government officer and the bidders.

A majority of the literature has focused on public procurement using scoring rules that include price and quality bids, which started with the seminal work of Che (1993). Under a scoring scheme, an agent might bribe the inspector to: misrepresent the *ex-post* effective quality (Burguet, 2017; Celentani and J. J. Ganuza, 2002), manipulate the assessment of quality in the bid (Burguet and Che, 2004), or provide private information about the competition's bids to one firm so it can resubmit a more aggressive price bid (Compte, Lambert-Mogiliansky, and Verdier, 2005), among others. In these works, corruption typically entails that the government pays a higher price and gets a lower quality delivered, thus indicating inefficiency and welfare loss. However, for this to happen the corrupt politician must have enough manipulation power (Burguet and Che, 2004).

In this work, we will refrain from the scoring rule literature and consider the simpler framework of the public procurement of a single and homogeneous good, or a reverse auction. If a corrupt supervisor runs such procurement and assigns it to the firm with the largest bribe bid, under a predetermined price, Beck and Maher (1986) show that there is an isomorphism between bribery and competitive bidding. They show that there is a symmetric equilibrium, such that the firm with the lowest cost bids the largest bribe, and Lien (1986) shows that this equilibrium is unique. In this model, corruption entails no efficiency problem, only a redistribution between the supervisor and the government. However, if there is a collusive agreement between the supervisor and one firm, such that the former provides private information of the other bids, and all firms are aware of this informational advantage, Arozamena and Weinschelbaum (2009) show that corruption can change bidding behavior in a first price auction. In this model, there is a positive probability that the firm with the lowest cost does not win the auction, distorting the allocation of the project.

Since the government can usually do better by fixing a budget constraint or maximum reserve price, this raises the question of whether the government can discourage corruption, and reduce the potential incentives to signal that might emerge. Thomas

[4] For a more detailed discussion of the microeconomics of corruption in recent years, see Burguet, J.-J. Ganuza, and Montalvo (2016).

(2005) finds that in an infinitely repeated game where collusion between bidders can be sustained, setting a reserve price can shrink the set of discount factors for which collusion can be sustained, and by doing so it could deter collusion. In this work, we will analyze if this budget constraint can discourage corruption and signal in a one period model.

This article is also related to the signaling literature and luxury goods or conspicuous consumption as an informative signal about an agent's type (Di Tella and Weinschelbaum, 2008; Fabrizi and Lippert, 2017). It is closely related to the work of Fabrizi and Lippert (2017), who consider a principal-agent model where public servants distinguish from another by the probability in which they are subjected to an audit. With imperfect information about this audit probability, firms can not be certain about the value of the relationship and thus are uncertain about how much to bribe. If the officer's bargaining power is high enough, there can be a separating equilibrium where a public officer consumes conspicuously or "burns money" and thus attracts higher bribes. However, even though they characterize a signaling equilibrium, this public servant is actually signaling his ability to succeed at corruption and not its corruptness. This is because the firms here are certain that they should bribe the supervisor, but are uncertain about the correct level of bribery. In this work, we contribute by showing that a supervisor can directly signal his willingness to be corrupt through conspicuous consumption. By doing this, it can attract larger bribes from all firms, which are now certain of which supervisor would accept a bribe and who would never do so.

Finally, even though the motivation regarding conspicuous consumption and corruption comes from examples observed in reality, there is also a cross-country empirical correlation between the (perceived) corruption level and luxury car sales (Gokcekus and Suzuki, 2014), and that controlling corruption can reduce luxury spending (Tajaddini and Gholipour, 2018). In this work, we contribute by providing a micro foundation to rationalize this positive relationship.

The contribution of this paper is twofold. First, to the best of our knowledge it is one of the first to show that conspicuous consumption can perfectly signal a supervisor's corruptness within a standard public procurement framework. Second, we show that if government fixes a reserve price, a signaling equilibrium can still exist.

3 The model

Let us consider the case of $n = 2$ firms that can produce a good or a project. Each firm i independently draws its cost $c_i \sim U[0, \bar{c}]$, with a cumulative function $F(c_i)$ that is common knowledge. The government values it in $v \geq \bar{c}$, such that the production of this good is socially desirable, and it will be assigned following a procurement process. However, the government can not run this auction by itself and must assign a supervisor to conduct it, in addition to setting a budget constraint $r \geq 0$ to pay to the winning bidder.

This public servant or supervisor can be honest (H) with probability $\alpha \in (0, 1)$ or corrupt/dishonest (C) with probability $1 - \alpha$, and these probabilities are of common knowledge. We will assume that an honest supervisor will never assign the project to a firm that offers him a bribe and, similarly, a dishonest supervisor only assigns projects to firms that offer him strictly positive bribes. If a firm tries to bribe an honest supervisor, then that firm will pay a penalty equal to $M \geq 0$, which represents the monetary value of going to jail, paying a fine or even the opportunity cost of being excluded from any future procurement. Additionally, we will assume that the government cannot use a mechanism to discover the supervisor's type.

After being assigned by the government and before running the procurement auction, the supervisor can buy a luxury good or "burn money", following Fabrizi and Lippert (2017).[5] This will be through an action $s \in \{S, N\}$ visible to all, where $s = S$ represents the conspicuous consumption that costs $\psi > 0$ but provides no direct utility and $s = N$ refers to doing nothing. When the supervisor decides whether to burn money or not, the government has already set a maximum price r and assigned him to run the procurement.

After observing its own private cost c_i, each firm competes for the assignment of the project in the procurement. Each one must choose a non-negative amount b to bribe (potentially equal to 0) and a non-negative price p to bid. With an honest supervisor, all firms that submit a strictly positive bribe $b > 0$ will be reported and must pay the penalty. Because firms observe the signal s, they can use that information to calculate the posterior probability of meeting an honest supervisor $\mu(s)$ given s.

The timing of the more general model is as follows:

- $\mathbf{t = 0}$: The government assigns a supervisor to run the procurement process, given the budget constraint $r > 0$ for the project.
- $\mathbf{t = 1}$: Nature draws the costs $c_i \sim U[0, \bar{c}]$ of the firms and decides whether the supervisor is honest (with probability α) or not.
- $\mathbf{t = 2}$: The supervisor, knowing his type, decides whether to "burn money" (S) or not (N). If the action is $s = S$, it will cost him a given amount $\psi > 0$, which is the price of the luxury good.
- $\mathbf{t = 3}$: After observing s, firms submit a price and a bribe bid (potentially equal to zero).
- $\mathbf{t = 4}$: The honest supervisor grants the contract to the lowest price bid and reports firms that tried to bribe him. The dishonest supervisor assigns the project to the firm with the highest bribe.

Regarding these assumptions, even though the motivating fact is that conspicuous consumption can backfire and lead to an investigation, the model does not explicitly include an audit or conviction probability. This is because we want to show that there are *gains* from signaling corruption, given that signaling has costs. While conspicuous consumption can ignite an external audit, a supervisor can only be found guilty if there is evidence and proof that he accepted bribes. In countries with weak judicial institutions, such as the ones in the motivating examples, the probability of being convicted after consuming conspicuously will be small, and because of this, we will simplify the analysis by not explicitly incorporating this cost.

Moreover, we are considering the case of a single supervisor, that cannot communicate externally with the firms before or during the procurement and has no information about the firms. Thus, we rule out the possibility of favoritism and/or *ex-ante* agreement on information disclosure to some firm, in addition to the possibility of a network effect outside of the procurement process (where a supervisor already has a corrupt reputation). This is because, in order to build this reputation or to get to an agreement, the firms and the supervisor must communicate, but they are both uncertain about the type of agent that they are meeting. This is, a firm might be trying to get to an agreement with an honest supervisor, or a corrupt supervisor could try to build a reputation among

[5]The term of "burning money" refers to the fact that the agent derives no direct utility from the luxury good.

firms that would never bribe. Therefore, the model captures these information disclosure mechanisms through the price and bribe bidding process. Also, if a firm believes that the supervisor is more likely to be corrupt but is not perfectly certain, then there can still be gains from signaling (as it will become clear from Propositions 3 and 4).

Additionally, we are considering a static or one period model with a single supervisor, while a public servant usually works in government for many periods. This eliminates the reputation that the supervisor can build from successive procurement since eventually, all firms could try to bribe and thus learn about its type. However, the supervisor can also guarantee himself bribes for his entire lifespan by burning money once. Also, in the period following conspicuous consumption, the government could replace the supervisor. However, with a low discount factor, we expect that the one period gains from bribes can offset the infinite period discounted utility with firm uncertainty about its type. Therefore, the static model captures the relevant incentives that we aim to consider.

In order to solve the model and discuss the welfare implications of the set of equilibria, we will start from a standard procurement auction and add the additional stages. In section 3.1, we will discuss and characterize the equilibrium with a given reserve price $r \geq \bar{c}$ (such that a firm always wants to bid) without the signaling action. Then, in section 3.2 we will allow the supervisor to signal his type, given the reserve price $r \geq \bar{c}$, and characterize the set of equilibria. Finally, in section 3.3, we will add the previous stage of the game in which the government can set a reserve price that maximizes the expected utility and potentially set a price below the maximum cost $\bar{c}$.

3.1 Benchmark: procurement auction without signal

Let us first consider the case when the supervisor must conduct the procurement right after being assigned and is not allowed to "burn money" before the auction. This will be useful to characterize the bids and profits with and without corruption. Note again that, in this case, we are assuming that the maximum price r is at least $\bar{c}$, which implies that a firm can always bid a price that provides positive profits (or null with probability zero, only if $c_i = \bar{c} = r$).

If the supervisor was never corrupt (or $\alpha = 1$), then, for a given cost, firms will never try to bribe and will only compete in price. If a firm i with a cost c_i submits a price bid p_i and wins, it will get a profit equal to $p_i - c_i$ if the submitted price is lower than firm j's price p_j. Therefore, with an always honest supervisor, the firm i bids a price p_i that maximizes his expected profit:

$$\max_{p_i} (p_i - c_i) Pr(\text{win}|p_i) = \max_{p_i} (p_i - c_i) Pr(p_i < p_j) \tag{1}$$

This is the problem in a standard procurement or reverse auction, and it is a well-known result that with a symmetric and uniform distribution, the optimal price bid is a linear function of the cost. The following proposition characterizes this equilibrium:

Proposition 1 *With an honest supervisor, there exists a symmetric and linear Bayesian Nash equilibrium in which firms bid a bribe $b(c) = 0$ and a price bid $p(c) = \frac{c+\bar{c}}{2}$. In this equilibrium:*

i. The firm with the lowest cost wins the procurement.

ii. The expected profit of a firm is $u_i^H(c_i) = \frac{(\bar{c}-c_i)^2}{2}$ $\forall i$.

iii. The expected price that the government pays is $\frac{2\bar{c}}{3}$

Proof. Most of the proofs are relegated to the Appendix. This problem is similar to an independent private value auction with symmetric cost distribution. See Appendix A.1 for a formal proof. ■

Proposition 1 shows that the firm with the lowest cost has the largest expected utility and wins the auction. Given that the reserve price is larger than the maximum cost $\bar{c}$, all firms participate in the procurement and the expected paid price by the government equals the expected value of the minimum price bid, which is $\frac{2\bar{c}}{3}$ for the cost distribution. Therefore, the government has an expected utility equal to $v - \frac{2\bar{c}}{3} > 0$.

Then, if the supervisor was always corrupt (*i.e.*, $\alpha = 0$), the firm with the highest bribe wins the project and gets the asked price. Each firm i will submit a price p_i and a bribe b_i that maximizes its expected profit, subject to $p_i \leq r$. If the firm wins, then it will get a profit equal to its submitted price minus the cost and the bribe. Therefore, the firm resolves the following problem:

$$\max_{p_i \leq r, b_i} (p_i - c_i - b_i)Pr(b_i > b_j) \equiv \max_{b_i}(r - c_i - b_i)Pr(b_i > b_j) \tag{2}$$

Since the price does not affect the probability of winning and the expected profit is strictly increasing in p_i, the firm should select the highest price possible. Therefore, for all costs, the price bid will equal the budget constraint r. If we define the valuation of the firm as $r - c_i \equiv v_i \sim U[r - \bar{c}, r]$, then the firm's problem is a standard first price auction problem where the firm with the highest bribe bid wins. As a result of this, the firm with the largest valuation (or lowest cost) should win the auction.

Proposition 2 *With a corrupt supervisor, there exists a symmetric and linear Bayesian Nash equilibrium in which firms submit a price bid $p(c) = r$ and a bribe $b(c) = r - \frac{c+\bar{c}}{2}$. In this equilibrium:*

1. *The firm with the lowest cost wins the procurement.*
2. *The expected profit of a firm is $u_i^C(c_i) = \frac{(\bar{c}-c_i)^2}{2}$*
3. *The expected price that the government pays is r, and the expected bribe that the supervisor gets is $r - \frac{2\bar{c}}{3}$.*

Proposition 2 shows an interesting result: in this situation, corruption entails no distortion in the allocation of the project but a redistribution between the supervisor and the government. Compared to the equilibrium in Proposition 1, the supervisor now retains a share of the government's utility of the project. However, if we consider the welfare as the direct sum of individual utilities[6], then there is no welfare loss and the efficient assignment is attained even through this corruption procurement. The results of this proposition are precisely in line with Beck and Maher (1986) and Lien (1986), who show that this bribery game is isomorphic to the procurement process if $r \geq \bar{c}$.

Finally, both results are useful to characterize an equilibrium when the firm is uncertain about the type of supervisor that is running the procurement process (with $\alpha \in (0, 1)$). Because there is now a positive probability of meeting an honest supervisor, if a firm bribes it might be reported and forced to pay the penalty M. However, if a firm does not bribe, it will lose the procurement for sure when meeting a corrupt supervisor. Given the firm's uncertainty about the supervisor's type, the strategies in Propositions 1 and 2 of never bribing and always bribing, respectively, might not be the equilibrium strategies for $\alpha \in (0, 1)$.

[6]With a social welfare function that assigns a larger weight to the government's utility, then this result would imply a welfare loss under corruption. However, this is not the interest of this work.

To illustrate this, let us consider a firm i with high cost (or low gap $\bar{c} - c_i$). If firm i competes honestly in price with the other firm, its expected utility is at most equal to $\alpha\frac{(\bar{c}-c_i)^2}{2}$. However, if firm i decided to bribe the supervisor when the other firm competes in price, it will win the project for sure when meeting the corrupt supervisor. Since this happens with probability $1-\alpha$, it has an expected utility of $(1-\alpha)(r-b-c)-\alpha M$, which is larger than the previous utility when M is small and r is large enough. Therefore, when the cost is high enough, the utility of competing honestly is smaller and thus the incentives to bribe are higher. Also, if α is large enough, then a firm with low cost (and large gap $\bar{c} - c_i$) will never try to bribe.

Since the incentives to deviate in the previous example are different when a firm has a low or high cost, we can then expect a mid value $\hat{c} \in (0, \bar{c})$ to exist, such that a firm will bribe if and only if its cost is larger than $\hat{c}$, but will bid an honest price bid if its cost is lower than this mid value. Proposition 3 shows that there exists such a Bayesian Nash equilibrium. Moreover, when firms do not face a penalty for bribing an honest supervisor (with $M = 0$), it is sufficient that $\alpha \in (\frac{1}{2}, 1)$.

Proposition 3 *Given a reserve price $r > \bar{c}$ and a penalty $M \geq 0$, there exists a symmetric and linear Bayesian Nash equilibrium in which a firm decides to bribe only if their costs are larger than some $\hat{c} \in (0, \bar{c})$. For this equilibrium to exist, the probability α of meeting an honest supervisor must be in certain non-empty subset strictly contained in $(0, 1)$. In the simple case that $M = 0$, it is sufficient that $\alpha \in (\frac{1}{2}, 1)$.*

Proof. Given a mid value $\hat{c} \in (0, \bar{c})$, a firm i will bribe and bid a price of r when $c_i \geq \hat{c}$, but will not bribe when $c_i < \hat{c}$. Given the symmetric and linear strategy for a firm with cost lower than $\hat{c}$, the probability of winning equals α times the probability that firm j has a higher cost. Therefore, the price bid is the same as in Proposition 1.

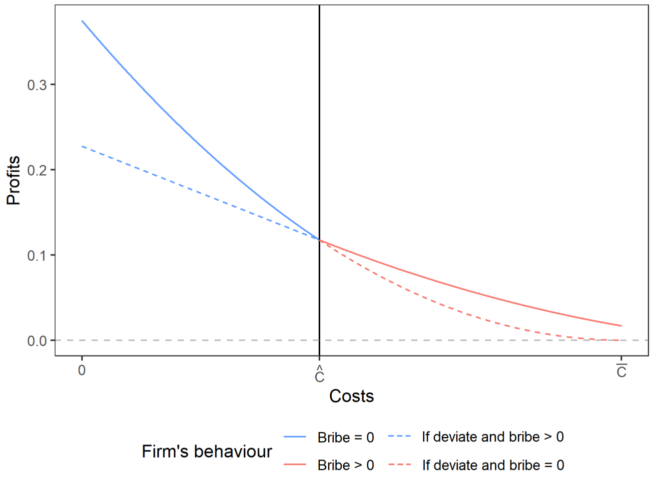
For a firm i with cost higher than $\hat{c}$, it will win if firm j has a higher cost but also if firm j has a cost lower than $\hat{c}$, conditional on meeting a corrupt supervisor. This increases the probability of winning and the expected utility if the penalty is small or nil. However, this reduces the bribe competition, since a firm bribe occasionally, and therefore reduces the bribe bid of firm i. Finally, given these strategies, a firm with costs $\hat{c}$ must be indifferent between bribing and competing honestly, which is how the mid value $\hat{c}$ is determined.

A set of conditions is needed to ensure that $\hat{c}$ is in the interval $(0, \bar{c})$, that the bribe bid is non negative, and that the utility of the firm that bribes is always non-negative for a given penalty $M \geq 0$. It can be shown that this set is non empty and also implies that no firm has incentives to deviate, because the expected utility functions are quadratic. In the simple case that $M = 0$, it is sufficient that $\alpha \in (\frac{1}{2}, 1)$.

See Appendix A.4 for a complete proof. ■

The results in Proposition 3 characterize an equilibrium where firms bribe sometimes. Unlike the previous case of perfect information regarding the supervisor's type (with α equal to 0 or 1), now a firm bribe only if its cost is large because the probability of meeting a firm with lower costs and losing the procurement results in a low expected utility of competing honestly. Figure 1 illustrates the expected utility of a firm as a function of its cost, and also the utility it can obtain by deviating (dashed line). As the Figure shows, in this equilibrium, a bribing firm is strictly better than without bribing.

Figure 1: Expected profits of a firm

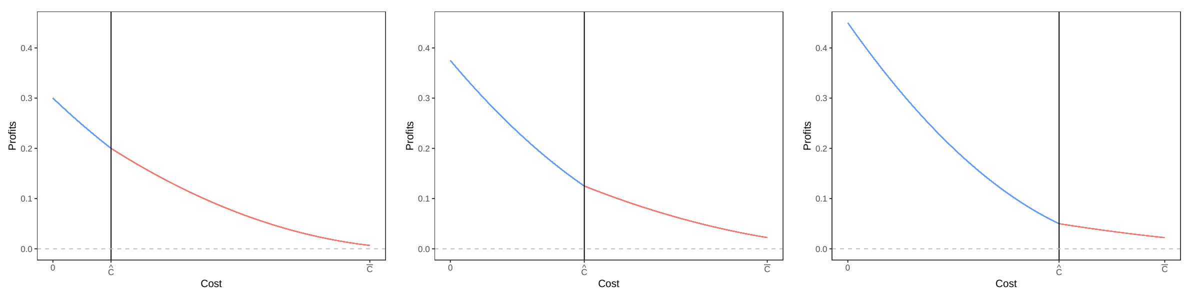


$\alpha = \frac{3}{4}$, $\bar{c} = 1$, $r = 2$ and $M = \frac{1}{100}$

Moreover, Proposition 3 shows that even if there is no direct punishment for trying to bribe an honest supervisor (if $M = 0$), a firm does not always bribe. Therefore, uncertainty about its willingness to be corrupt results in a firm only bribing sometimes. Figure 2 shows the expected utility of a firm when the probability α of meeting an honest supervisor increases. When the probability $1-\alpha$ of meeting a corrupt supervisor is smaller (larger α), the share of firms that bribe is smaller, as can be seen in the Figure. In this equilibrium, if $\alpha \in (0, 1)$ then a firm might or might not bribe with positive probability, which implies that any uncertainty regarding the supervisor's type can result in a firm bribing occasionally.

Figure 2: Expected profits of a firm, as function of α

(a) $\alpha = 0.6$ (b) $\alpha = 0.75$ (c) $\alpha = 0.9$



$\bar{c} = 1$, $r = 2$ and $M = 0$

An interesting result from Proposition 3 is that the bribe bids are strictly *smaller* than with an always dishonest supervisor, and therefore the expected bribe of a corrupt supervisor is lower. The intuition behind this result is that firms will try to bribe only sometimes, which reduces the bribery competition and the bribe bids. This results in the higher expected utility of the firm with high cost, because it can keep a larger share of the price. So, while bribery and corruption with perfect information increase bribe competition (Compte, Lambert-Mogiliansky, and Verdier, 2005), here the uncertainty reduces the bribery competition and the bribes.

However, unlike the case with perfect information about the supervisor's type, two inefficiencies arise in this equilibrium. First, if the firms bribe an honest supervisor or do not bribe a corrupt supervisor, which happens with positive probability, then the procurement will be declared void. Second, the firm with the highest cost can win the procurement by bribing while the other firm bids a price and loses. Given the equilibrium strategy in Proposition 3, with a dishonest supervisor, if a firms has a cost lower than $\hat{c}$ while the other firm's cost is larger than $\hat{c}$, then the latter will win the procurement process.

3.2 Procurement auction with conspicuous consumption

Now, since imperfect information results in lower bribes and the described inefficiencies, can the supervisor do any better? Let us introduce conspicuous consumption or the "burnt money" as a signal, a costly action that the supervisor makes before the procurement and that the firms can observe. Firms can incorporate this information to have a better understanding of the supervisor's type, to learn what action is going to let them win.

Given the signal $s \in \{S, N\}$, a firm considers that it is meeting an honest supervisor with probability $\mu(s)$, and given its cost c_i, firm i can bid a different price $p(c_i|s)$ and bribe $b(c_i|s)$ according to the observed action of the supervisor. With this additional action, there is a separating perfect Bayesian equilibrium where a corrupt supervisor signals his type. Proposition 4 characterizes the set of parameters for such an equilibrium to exist.

Proposition 4 *Given $r \geq \bar{c}$, if the cost of burning money is small enough ($\psi \leq r - \frac{2\bar{c}}{3}$), then there exists a separating equilibrium in which a corrupt supervisor signals that it is dishonest by consuming conspicuously. In this equilibrium:*

i. The firm with the lowest cost wins the procurement (by bribing or bidding a price).

ii. The government expected price is $\alpha\frac{2\bar{c}}{3} + (1-\alpha)r$.

Proof. Given the supervisor's action in the separating equilibrium, the firm has perfect information about the supervisor's type. Therefore, based on Proposition 2, a firm will bid a price equal to r and a bribe equal to $r - \frac{c+\bar{c}}{2}$ when $s = S$, and will not bribe when $s = 0$. The expected utility of a corrupt supervisor will be equal to $r - \frac{2\bar{c}}{3}$ minus the cost ψ.

Given the firm's strategies, consuming conspicuously must be the best response of a corrupt supervisor. Since the supervisor will not obtain any bribe if $s = 0$, it is sufficient that the expected utility $r - \frac{2\bar{c}}{3} - \psi$ is at least zero, which results in the condition $\psi \leq r - \frac{2\bar{c}}{3}$.

See Appendix A.4 for a complete proof. ■

In this signaling equilibrium, the corrupt supervisor burns money or buys a luxury good. By doing this, each firm updates its priors and can perfectly know who is running the procurement. Therefore, a firm will only try to bribe the supervisor with the luxury good and will compete honestly with a price bid if the supervisor does not consume conspicuously. For this equilibrium to exist, the cost of burning money must be small enough, such that the net profit of the bribes exceeds the cost, but positive such that the signal is costly and informative.

Additionally, in this equilibrium, the procurement will never be declared void and the firm with the lowest cost will always win the project. The government expected utility is equal to $v - \alpha\frac{2\bar{c}}{3} - (1-\alpha)r$ and the expected rent that the supervisor will get (weighted by the probability of being corrupt) is $(1-\alpha)(r - \frac{2\bar{c}}{3} - \psi)$. Therefore, the sum of expected utilities or total welfare is equal to $v - \frac{2\bar{c}}{3} - (1-\alpha)\psi$. This total welfare equals the government expected utility with an always honest supervisor or the maximum welfare attainable through a first price procurement auction, minus the cost of the signal. Therefore, while the signal eliminates the allocating distortion, it generates an efficiency problem through the welfare loss of burning money. This inefficiency is reduced when the probability of being corrupt or the signaling cost are small, but it is always positive.

Given the separating equilibrium that Proposition 4 characterizes, let us consider the other equilibria. As Proposition 5 states, there is a pooling equilibrium in which both an honest and dishonest supervisor do not burn money. In this equilibrium, the firms bid prices and bribes just like in Proposition 3. However, because the cost of the luxury good is strictly positive and the honest supervisor will never accept bribes nor assign the procurement to a corrupt firm, the honest supervisor will never choose $s = S$.

Proposition 5 *In this model:*

1. *There exists a pooling equilibrium with $\mu(N) = \alpha$, under the same set of conditions as in Proposition 3. In this equilibrium, if $s = N$ firms bid according to the strategy in Proposition 3. If $s = S$, then no one will bribe, and an out-of-equilibrium belief of $\mu(S) = 1$ is consistent with this strategy.*
2. *There is no pooling nor separating equilibrium where the honest supervisor burns money.*

Note that the existence of the pooling equilibrium in Proposition 5 implies a certain firm's behavior out of the equilibrium path. This equilibrium exists if firms do not bribe when $s = S$ (which, of course, does not happen in equilibrium), and it explains why a corrupt supervisor has no incentive to deviate from $s = N$.

If the signaling cost ψ is large enough and close to the expected revenue from bribes $r - \frac{2\bar{c}}{3}$, the benefit of signaling barely outweighs its costs and thus yields a very low return. Therefore, if ψ is large enough, the supervisor might be better by not signaling and getting bribes only from high cost firms than by signaling and getting bribes from all firms. Since both equilibrium are feasible if $\psi \in \left(0, r - \frac{2\bar{c}}{3}\right)$ and for the same set of parameters as in Proposition 3, it implies that the supervisor might be better in the separating equilibrium only if the signaling cost is low.

Moreover, because of the firms' symmetric cost distribution and the participant's risk aversion, the signaling equilibrium in Proposition 4 can be generalized to the case of a second-price auction (where the firm with the lowest price wins but obtains the second lowest price), an English auction and a Dutch auction. Under these settings, an analogous to a Revenue Equivalence Theorem holds, and the following Proposition formalizes it:

Proposition 6 *Given a reserve price $r \geq \bar{c}$, if the cost of burning money is small enough ($0 < \psi \leq r - \frac{2\bar{c}}{3}$), then there exists a separating equilibrium regardless of whether the honest supervisor runs a first-price, second-price, English or Dutch auction. In this equilibrium:*

i. *The firm with the lowest cost wins the procurement.*

ii. *The government expected price is $\alpha\frac{2\bar{c}}{3} + (1 - \alpha)r$.*

Proof. The result follows from the standard Revenue Equivalence Theorem. With probability α, the firms meet an honest supervisor who runs a first-price, second-price, English, or Dutch auction. Since firms draw their costs from an identical distribution, the Revenue Equivalence Theorem holds and the government expected revenue equals $v - \frac{2\bar{c}}{3}$ (or the expected price is $\frac{2\bar{c}}{3}$).

With probability $1 - \alpha$, the government meets a corrupt supervisor that perfectly signals his type, such that the expected price equals r. In this case, firms compete in a first bribe auction and the firm with the lowest cost wins. ■

3.3 Government optimal reserve price

Since the government can fix a budget constraint r that the supervisor has to respect, regardless of its type, we will now discuss the existence of the signaling equilibrium and the optimal reserve price r. Let us first consider the case of the government meeting an always honest supervisor ($\alpha = 1$) and an always corrupt supervisor ($\alpha = 0$). This will determine the optimal reserve price with an honest supervisor (r^H) and with an always corrupt supervisor (r^C), which will be useful as benchmarks.

With an honest supervisor and given a reserve price $r \leq \bar{c}$, following Krishna (2009)[7], the optimal price bid for a firm i with cost $c_i \leq r$ is to take the expected value of the minimum of cost c_j and r, given that the other firm has larger cost (or $c_j > c_i$). This results in a price bid of $p(c_i) = \frac{c_i+\bar{c}}{2} - \frac{(\bar{c}-r)^2}{2(\bar{c}-c_i)}$, as Proposition 7 states. Given the budget constraint, the government obtains a utility of $v - \min\{p_1, p_2\}$ only if at least one of the firms has a cost lower than r. The following Proposition characterizes this equilibrium.

Proposition 7 *If the government values the project in $v \geq \bar{c}$ and the supervisor is always honest, then:*

i. The optimal price bid for a firm with costs $c \leq r$ is $p(c) = \frac{c+\bar{c}}{2} - \frac{(\bar{c}-r)^2}{2(\bar{c}-c)}$, while a firm with cost $c > r$ does not bid.

ii. The optimal reserve price for the government is $r^H = \min\{\frac{v}{2}, \bar{c}\}$, with an expected utility of $EU^H(r) = \frac{\frac{4}{3}r^3 - r^2(v+2\bar{c}) + 2v\bar{c}r}{\bar{c}^2}$.

This implies that, if the government valuation is $v \leq 2\bar{c}$ such that $r^H = \frac{v}{2} < v$, then there is a positive probability that the procurement is declared void if both firms have costs above the maximum reserve price. This result is standard within the auction or procurement literature (Krishna, 2009; Thomas, 2005), and the intuition of this result is that the constraint lowers the price bids. Therefore, this increases the expected utility $v - \min\{p_1, p_2\}$ when one of the firms has a cost less than r, which offsets the loss of declaring the procurement void when meeting two high costs firms.

If the supervisor was always corrupt, this drastically modifies the government's problem because it will pay the reserve price for sure. In this case, the government expected utility equals $v - r$ times the probability that one of the firms has a cost less than r. Since the government will always pay r, there is a trade-off between setting a high price r and getting a low benefit $v - r$ with high probability or setting a low price with high benefit but low probability. As a result of this, the government should reduce the price compared to the case with an honest supervisor. Proposition 8 characterizes the government's optimal reserve price in this situation.

Proposition 8 *If the government values the project in $v \geq \bar{c}$ and the supervisor is always corrupt, then:*

i. The optimal bribe bid for a firm with costs $c \leq r$ is $b(c) = \frac{(r-c)^2}{2(\bar{c}-c)}$, while a firm with cost $c > r$ does not bid.

ii. The optimal reserve price for the government r^C depends on v and $\bar{c}$ and it is smaller than r^H. Also, the government expected utility is $EU^C(r) = \frac{r^3 - r^2(v+2\bar{c}) + 2v\bar{c}r}{\bar{c}^2}$

[7] The result for auctions is that, given two individuals with valuations v_i, v_j, the optimal bid in a first price auction if $v_i \geq r$ is $p(v_i) = E[\max\{v_j, r\} | v_j < v_i]$.

It is important to notice that, in this equilibrium, the expected utility of the government $EU^C(r)$ is smaller than with an honest supervisor $EU^H(r)$. Given a reserve price, the difference between these two utilities is exactly $\frac{r^3}{3\bar{c}}$, which is the expected revenue of the supervisor. However, with an always corrupt supervisor, the government should fix a lower reserve price r^H than with an honest supervisor r^H.

Finally, with imperfect information about the supervisor's type, a similar signaling equilibrium as in Proposition 4 exists. As in Proposition 4, if the signaling cost is positive but lower than the expected bribe revenue $\frac{r^3}{3\bar{c}}$, we can characterize a separating perfect Bayesian equilibrium where the supervisor signals his type. In this equilibrium, the firm with the lowest cost will win the procurement, either by bribery or by bidding a price.

Proposition 9 *Even if the government can select the reserve price before assigning the supervisor, then there exists a separating equilibrium where the supervisor signals his type with a sufficiently low cost* $0 < \psi < \frac{r^3}{3\bar{c}}$. *In this equilibrium, the government's optimal reserve price* r^S *is in the interval* (r^C, r^H).

Therefore, a separating equilibrium is still possible for a sufficiently low signaling cost and the government's optimal reserve price is below r^H but above r^C. However, in this equilibrium, the reserve price is not free and it implies two (related) inefficiencies. On the one hand, because the optimal price in this separating equilibrium is lower than with an honest supervisor (*i.e.*, $r^S < r^H$), there is a higher probability of declaring the procurement void. This is unwanted since the project is socially desirable given that $v \geq \bar{c}$. On the other hand, the total welfare before including the signaling cost is smaller with r^S than with r^H.

Regarding the latter claim, the corrupt supervisor expected revenue (before including the signaling cost) is $\frac{r^3}{3\bar{c}^2}$, while the government expected utility is:

$$\begin{aligned} EU^S(r) &= \alpha EU^H(r^S) + (1-\alpha)EU^C(r^S) \\ &= \frac{\alpha r^3}{3\bar{c}^2} + \frac{r^3 - r^2(v+2\bar{c}) + 2v\bar{c}r}{\bar{c}^2} \end{aligned}$$

The total expected welfare is the sum of the expected utilities (weighted by the likelihood of meeting the dishonest supervisor), which equals:

$$\begin{aligned} W(r) &= \frac{(1-\alpha)r^3}{3\bar{c}^2} + EU^S(r) \\ &= \frac{\frac{4}{3}r^3 - r^2(v+2\bar{c}) + 2v\bar{c}r}{\bar{c}^2} \end{aligned}$$

From Proposition 7 we know that the reserve price that maximizes this total welfare $W(r)$ is r^H, which implies that $W(r^S) < W(r^H)$. As a result of this, imperfect information results in a welfare loss before accounting for the signaling cost, even when the supervisor can perfectly signal its type to the firms. Therefore, allowing the government to set a reserve price does not eliminate the distortions and, while reserve price can increase the government expected utility, corruption and signaling result in a total welfare loss.

4 Concluding remarks

We motivated this work with some real-life examples of a connection between luxury goods or conspicuous consumption and corruption from former public servants. While the losses and consequences of raising public awareness are clear, we were interested in the potential gains from signalizing the type, such that this observed behavior can be rationalizable.

In our model, the supervisor has incentives to signal his type to attract higher bribes from all firms. The underlying mechanism is that the firms can understand better the actions that will let them win the procurement, and thus they can completely switch to competition in bribes instead of competing with price bids. As a result of this, it raises the supervisor expected revenue from bribes. Moreover, if the government tried to deter corruption by fixing a budget constraint, it could improve its expected utility but not completely eliminate corruption and conspicuous consumption. With a low signaling cost, the supervisor can still perfectly signal his type, which is a surprisingly weak condition.

Regarding the allocation of the procurement and the total welfare, in the signaling equilibrium, the supervisor never grants the procurement to a firm with the highest cost. Therefore, even though the government pays a higher price, the efficient firm provides the good, although the money that the corrupt supervisor burns in conspicuous consumption directly results in welfare loss. Moreover, with the budget constraint $r \leq \bar{c}$, the government should set a maximum price below the social optimum. As a result of this, there is a higher probability of declaring the procurement void than with an honest supervisor, which results in total welfare loss.

Therefore, in this model, signaling corruption never implies that the winning firm has a higher cost than its competitors, while other inefficiencies may arise. Since the literature suggests that there is a positive probability that the firm with the lowest cost does not win the model, resulting in allocation distortion, future research can extend the model to capture the firm's heterogeneity in the bribery technology. Overall, we conclude that conspicuous consumption can act as a strong signal about a public servant's type and honesty.

References

Alconada Mon, Hugo (2019). *La raíz (de todos los males): cómo el poder montó un sistema para la corrupción y la impunidad en la Argentina*. Planeta, pp. 102–103.

Arozamena, Leandro and Federico Weinschelbaum (2009). "The effect of corruption on bidding behavior in first-price auctions". In: *European Economic Review* 53.6, pp. 645–657.

Beck, Paul J. and Michael W. Maher (1986). "A comparison of bribery and bidding in thin markets". In: *Economics Letters* 20.1, pp. 1–5.

Burguet, Roberto (May 2017). "Procurement design with corruption". In: *American Economic Journal: Microeconomics* 9.2, pp. 315–341.

Burguet, Roberto and Yeon-Koo Che (2004). "Competitive procurement with corruption". In: *RAND Journal of Economics*, pp. 50–68.

Burguet, Roberto, Juan-José Ganuza, and Jose G Montalvo (2016). "The Microeconomics of Corruption. A Review of Thirty Years of Research". In:

Celentani, Marco and Juan José Ganuza (July 2002). "Corruption and competition in procurement". In: *European Economic Review* 46.7, pp. 1273–1303.

Che, Yeon-Koo (1993). "Design competition through multidimensional auctions". In: *The RAND Journal of Economics*, pp. 668–680.

Compte, Olivier, Ariane Lambert-Mogiliansky, and Thierry Verdier (2005). "Corruption and competition in procurement auctions". In: *Rand Journal of Economics*, pp. 1–15.

Di Tella, Rafael and Federico Weinschelbaum (2008). "Choosing agents and monitoring consumption: A note on wealth as a corruption-controlling device". In: *The Economic Journal* 118.532, pp. 1552–1571.

Fabrizi, Simona and Steffen Lippert (2017). "Corruption and the public display of wealth". In: *Journal of Public Economic Theory* 19.4, pp. 827–840.

Gokcekus, Omer and Yui Suzuki (Aug. 2014). "Is there a Corruption-effect on Conspicuous Consumption?" In: *Margin* 8.3, pp. 215–235.

Krishna, Vijay (2009). *Auction theory*. Academic press.

Lien, Da-Hsiang Donald (1986). "A note on competitive bribery games". In: *Economics Letters* 22.4, pp. 337–341.

Tajaddini, Reza and Hassan F. Gholipour (Nov. 2018). "Control of Corruption and Luxury Goods Consumption". In: *Kyklos* 71.4, pp. 613–641.

Thomas, Charles J. (2005). "Using reserve prices to deter collusion in procurement competition". In: *Journal of Industrial Economics* 53.3, pp. 301–326.

A Proofs

A.1 Proposition 1

With the linear symmetric Bayesian equilibrium strategy $p(c_i) = ac_i + k$, then each firm solves the following problem:

$$\begin{aligned}
\max_{p_i}(p_i - c_i)Pr(\text{win}|p_i) &= (p_i - c_i)Pr(p_i < p_j(c_j) = ac_j + k|p_i) \\
&= (p_i - c_i)Pr\left(c_j > \frac{p_i - k}{a}|p_i\right) \\
&= (p_i - c_i)\left(1 - \frac{p_i - k}{a\bar{c}}\right)
\end{aligned}$$

From the first order conditions and the symmetric bid, this results in a price bid equal to $p(c_i) = \frac{c_i}{2} + \frac{\bar{c}}{2}$. Since the bid is strictly increasing in the cost, the firm with the lowest cost will bid the lowest price and therefore wins, which proves the first item. Also, if we replace this price bid in the firm's expected utility, we get that it is equal to $u_i^H(c_i) = \frac{(\bar{c}-c_i)^2}{2\bar{c}}$, proving the second item.

Finally, the expected price that the government pays with the honest supervisor equals the expected value of the minimum bid. This is equal to $E\left[\min_{i=1,2}\{\frac{c_i+\bar{c}}{2}\}\right] = \frac{E[\min_{i=1,2}\{c_i\}]+\bar{c}}{2} = \frac{\frac{\bar{c}}{3}+\bar{c}}{2}$ and therefore equal to $\frac{2\bar{c}}{3}$. ∎

A.2 Proposition 2

Let us consider the case of a symmetric equilibrium bribe with $b(c_i) = m(r - c_i) + d$ with $m, d > 0$, which is linear in the firm's valuation. The firm i maximizes:

$$\begin{aligned}
\max_{b_i}(r - c_i - b_i)Pr(b_i > b_j = m(r - c_j) + d|b_i) &= (r - c_i - b_i)Pr(\frac{b_i - d}{m} > r - c_j|b_i) \\
&= (r - c_i - b_i)\frac{\frac{b_i-d}{m} - r + \bar{c}}{\bar{c}} \\
&= (r - c_i - b_i)\frac{b_i - d - mr + m\bar{c}}{m\bar{c}}
\end{aligned}$$

From the first order condition and the symmetric bid, this results in a bribe equal to $b(c_i) = \frac{r-c_i}{2} + \frac{r-\bar{c}}{2} = r - \frac{c_i+\bar{c}}{2}$, which is decreasing in the cost. Therefore, the firm with the lowest cost will submit the highest bribe and therefore wins the procurement. Also, the expected revenue for a firm with cost c_i is also equal to the revenue from the standard procurement $u_i^C(c_i) = \frac{(\bar{c}-c_i)^2}{2\bar{c}}$.

Finally, government always pays the price r and the supervisor's profit equals the expected value of the maximum bribe. Thus, $E\left[\max_{i=1,2}\{r - \frac{c_i+\bar{c}}{2}\}\right] = r - \frac{E[\min_{i=1,2}\{c_i\}]+\bar{c}}{2} = r - \frac{2\bar{c}}{3}$. ∎

A.3 Proposition 3

Let us consider a symmetric Bayesian Nash equilibrium, where a firm bids a price if its cost is smaller than some $\hat{c} \in (0, \bar{c})$ and a bribe equal to zero, but bribes a positive quantity if its cost is larger than $\hat{c}$ and demands a price equal to r. With symmetric and linear price and bribe bids:

$$(p(c_i), b(c_i)) = \begin{cases} (ac_i + k, 0) & if \quad c_i < \hat{c} \\ (r, m(r - c_i) + d) & if \quad c_i \geq \hat{c} \end{cases}, \text{ with } a, k, m, d > 0 \tag{3}$$

For a firm with $c_i < \hat{c}$, the probability of winning given a price bid p_i and conditional on meeting an honest supervisor, which happens with probability α, is equal to:

$$\begin{aligned}
Pr(\text{win}|p_i) &= Pr(\text{win} \cap \text{firm } j \text{ bribes}|p_i) + Pr\left(\text{win} \cap \text{firm } j \text{ bids } p_j|p_i\right) \\
&= Pr(c_j \geq \hat{c}) + Pr\left(p_i < p_j = ac_j + k \,\wedge\, c_j < \hat{c}|p_i\right) \\
&= 1 - F(\hat{c}) + Pr\left(c_j > \frac{p_i - k}{a} \,\wedge\, c_j < \hat{c}|p_i\right) \\
&= 1 - F(\hat{c}) + Pr\left(\frac{p_i - k}{a} < c_j < \hat{c}|p_i\right) \\
&= 1 - F(\hat{c}) + \left(F(\hat{c}) - F\left(\frac{p_i - k}{a}\right)\right) \\
&= 1 - F\left(\frac{p_i - k}{a}\right)
\end{aligned}$$

Therefore, the firm solves the problem below. Since this equals the problem with an always honest competition, its price bid is equal to $p(c_i) = \frac{c_i + \bar{c}}{2}$ and its expected utility is $u_i^H = \alpha \frac{(\bar{c} - c_i)^2}{2\bar{c}}$.

$$\max_{p_i} (p_i - c_i)\alpha\left(1 - F\left(\frac{p_i - k}{a}\right)\right) = (p_i - c_i)\alpha\left(1 - \frac{p_i - k}{a\bar{c}}\right)$$

For a firm with cost $c_i > \hat{c}$, the probability of winning given a bribe and conditional on meeting a dishonest supervisor, which happens with probability $1 - \alpha$, is equal to:

$$\begin{aligned}
Pr(\text{win}|b_i) &= Pr(\text{win} \cap \text{firm } j \text{ bribes}|b_i) + Pr\left(\text{win} \cap \text{firm } j \text{ bids } p_j|b_i\right) \\
&= Pr(b_i > b_j = m(r - c_j) + d \wedge c_j \geq \hat{c}|b_i) + Pr(c_j < \hat{c}) \\
&= Pr(c_j > r - \frac{b_i - k}{a} \wedge c_j \geq \hat{c}|b_i) + F(\hat{c}) \\
&= Pr\left(c_j > \max\left\{r - \frac{b_i - k}{a}, \hat{c}\right\}\right) + F(\hat{c}) \\
\text{if } \frac{ar - b_i + k}{a} \geq \hat{c} \rightarrow &= 1 - F\left(\frac{ar - b_i + k}{a}\right) + F(\hat{c})
\end{aligned}$$

Therefore, the firm with costs $c_i \geq \hat{c}$ solves the following problem:

$$\begin{aligned}
&\max_{b_i} (1 - \alpha)(r - c_i - b_i)\left(1 + F(\hat{c}) - F\left(\frac{ar - b_i + k}{a}\right)\right) - \alpha M \\
&= (1 - \alpha)(r - c_i - b_i)\frac{(a\bar{c} + a\hat{c} - ar + b_i + k)}{a\bar{c}} - \alpha M
\end{aligned}$$

The first order condition is that $r - c_i - b_i = a\bar{c} + a\hat{c} - ar + b_i + k$ and therefore, based on the proposed linear strategy, the firms bribes $b(c_i) = \frac{r - c_i}{2} + \frac{r - \bar{c} - \hat{c}}{2} = r - \frac{c_i + \bar{c} + \hat{c}}{2}$. Note that this bribe is smaller than the case where all firms bribe. Also, its expected utility is equal to $u_i^C = (1 - \alpha)\frac{(\bar{c} + \hat{c} - c_i)^2}{2\bar{c}} - \alpha M$ and note that for this bribe, the inequality $\frac{ar - b_i + k}{a} = c_i \geq \hat{c}$ holds.

In order to find the value $\hat{c}$, a firm with cost $c_i = \hat{c}$ must be indifferent between bribing and competing honestly. Therefore, $\bar{c}$ results from the following equation:

$$\alpha\frac{(\bar{c} - \hat{c})^2}{2\bar{c}} = (1 - \alpha)\frac{\bar{c}^2}{2\bar{c}} - \alpha M$$

$$\alpha\hat{c}^2 - 2\alpha\bar{c}\hat{c} + (2\alpha - 1)\bar{c}^2 + 2\alpha\bar{c}M = 0 \tag{4}$$

This results in a quadratic equation with up to two real solutions, one less than $\bar{c}$ and one larger than $\bar{c}$, which are $\hat{c}_{1,2} = \bar{c}\left(1 \pm \sqrt{\frac{(1-\alpha)}{\alpha} - \frac{2M}{\bar{c}}}\right)$. Therefore, $\hat{c} = \bar{c}\left(1 - \sqrt{\frac{(1-\alpha)}{\alpha} - \frac{2M}{\bar{c}}}\right)$

In order to have a $\hat{c}$ in the set $(0, \bar{c})$, it must be that the root of Equation 4 is in that set (condition i). Additionally, the square root in $\hat{c}$ must be real valued, and therefore this term must be larger than zero (condition ii). Also, since the firms expected revenue is decreasing in its costs, it must be the case that for a firm with costs $c_i = \bar{c}$, the utility of being corrupt must be at least 0 (the utility of competing honestly, in condition iii). Finally, the bribes must be non negative, and it is sufficient that the bribe of a firm with cost $\bar{c}$ is at least zero (condition iv). Thus, these conditions are:

(i) $1 - \sqrt{\frac{(1-\alpha)}{\alpha} - \frac{2M}{\bar{c}}} > 0 \iff (1-2\alpha)\bar{c} < 2\alpha M \iff \alpha > \frac{\bar{c}}{2M+2\bar{c}}$.

(ii) $\frac{(1-\alpha)}{\alpha} - \frac{2M}{\bar{c}} > 0 \iff (1-\alpha)\bar{c} > 2\alpha M \iff \alpha < \frac{\bar{c}}{2M+\bar{c}}$.

(iii) $\frac{(1-\alpha)\hat{c}^2}{2\bar{c}} - \alpha M \geq 0 \iff \alpha \leq \frac{\hat{c}^2}{2\bar{c}M+\hat{c}^2} \underbrace{<}_{\hat{c}\leq\bar{c}} \frac{\bar{c}}{2M+\bar{c}}$

(iv) $r - \frac{2\bar{c}+\hat{c}}{2} \geq 0 \iff r \geq \frac{2\bar{c}+\hat{c}}{2} = \bar{c} + \frac{\hat{c}}{2} > \bar{c}$

Finally, we must check that a low cost firm does not want to bribe and that a high cost firm does not want to compete honestly. If a firm with low costs $c_i \leq \hat{c}$ decided to bribe, it will surely win the bribery game with a bribe slightly less than $b(\hat{c}) = r - \frac{2\hat{c}+\bar{c}}{2}$ with an utility equal to $u_i^{deviate} = (1-\alpha)\left(\hat{c} - c_i + \frac{\bar{c}}{2}\right) - \alpha M$. Therefore, its expected profits can be at most equal to this value, which is less than the utility of being honest $u_i^H = \frac{\alpha(\bar{c}-c_i)^2}{2\bar{c}}$. This is because both values are equal to the same value at $c_i \to \hat{c}$ (because of the definition of $\hat{c}$), but since $\frac{\partial u_i^H}{\partial c_i} < \frac{\partial u_i^{deviate}}{\partial c_i}$, the firm has no incentive to deviate. This last inequality holds because of conditions i and ii. Similarly, if a firm with high costs $c_i \geq \hat{c}$ decides to compete honestly, it would get at most an utility equal to $\frac{\alpha(\bar{c}-c_i)^2}{2\bar{c}}$, which is less than $(1-\alpha)\frac{(\bar{c}+\hat{c}-c_i)^2}{2\bar{c}} - \alpha M$ because of the listed conditions. Also, note that if $M = 0$, then all conditions hold if $\alpha \in (\frac{1}{2}, 1)$ ∎

A.4 Proposition 4

In the separating equilibrium, $\mu(N) = 1$ and $\mu(S) = 0$, this is, the supervisor that burns money is not honest and the one that does not consume conspicuously is honest. If that is the case, from Propositions 1 and 2 we know that firms should bid $p(c|S) = r$ and $b(c|S) = r - \frac{c+\bar{c}}{2}$ if they observe S, but should bid $p(c|N) = \frac{c+\bar{c}}{2}$ and $b(c|N) = 0$ if not.

Given the firms' strategies and beliefs, we must check that the supervisor effectively wants to signal out its type. If the corrupt supervisor burns money, it loses $\psi > 0$ but has an expected revenue from bribes equal to $r - \frac{2\bar{c}}{3} > 0$. Thus, if $0 < \psi < r - \frac{2\bar{c}}{3}$, then the supervisor is strictly better by signaling and prefers to do so.

Note that the firms have no incentives to deviate: if they decided not to bribe given $s = S$, then they will not be granted the project for sure. Also, if they do not see this signal and decided to bribe, they would pay the penalty M for sure. Additionally, the supervisor has no incentives to deviate: if the corrupt supervisor decided not to consume conspicuously, it would get no bribe for sure. If the honest supervisor decided to burn money, it would only lose $\psi > 0$.

Finally, the firm with the lowest cost wins the procurement auction: with probability α by bidding the lowest price and with probability $1 - \alpha$ by submitting the largest bribe. Therefore, the expected price with an honest supervisor is $\frac{2\bar{c}}{3}$ and with a dishonest supervisor is r. ∎

A.5 Proposition 5

1. Given the firms belief that $\mu(N) = \alpha$, let us consider the price and bribe strategies considered in Proposition 3, to bribe only if costs are above a mid value $\underline{c}$. In order for such a strategy to characterize an equilibrium, the same set of conditions are needed.

In this case, the dishonest supervisor has a positive expected revenue and pays no cost. However, in order to ensure that the supervisor has no incentives to deviate, firms must not try to bribe a supervisor, so $b(c|S) = 0$. In that case, burning money yields no expected benefit and the supervisor would be strictly worse by trying to signal his type.

2. The dishonest supervisor has no incentives to select $s = S$, since it would lose $\psi > 0$ for sure and not get anything in return. This rules out the separating and pooling equilibria where the dishonest supervisor burns money. ∎

A.6 Proposition 7

Given a reserve price r and when facing an honest supervisor, following Krishna (2009), the optimal price bid for a firm is $p(c_i) = E[\min\{c_j, r\}|c_j > c_i]$. Since $Pr(c_j \geq x|c_j > c_i) = \frac{Pr(c_i<c_j\geq x)}{Pr(c_j>c_i)} = \frac{F(x)-F(c_i)}{1-F(c_i)} = \frac{x-c_i}{\bar{c}-c_i}$ for the uniform distribution, this has a density function of $\frac{1}{\bar{c}-c_i}$. Therefore, $p(c_i) = \int_{c_i}^{r} \frac{c_j}{\bar{c}-c_i} dc_j + \int_{r}^{\bar{c}} \frac{r}{\bar{c}-c_i} dc_j = \frac{c_i+\bar{c}}{2} - \frac{(\bar{c}-r)^2}{2(\bar{c}-c_i)}$.

Since the government values the project in v, it will get an utility of $v - \min\{p_1, p_2\}$ when meeting a firm that has a cost lower than r. Therefore, the government expected utility is equal to $EU^H(r) = \int_0^r \left(v - \frac{c+\bar{c}}{2} + \frac{(\bar{c}-r)^2}{2(\bar{c}-c)}\right) \frac{2(\bar{c}-c)}{\bar{c}} dc$, which results in $EU^H(r) = \frac{\frac{4}{3}r^3 - r^2(v+2\bar{c}) + 2v\bar{c}r}{\bar{c}^2}$.

Taking the first order condition of $EU^H(r)$, the optimal reserve price r^H is such that:

$$\begin{aligned} 0 &= 4r^2 - 2r(v + 2\bar{c}) + 2v\bar{c} \\ &= 2(r - \bar{c})(2r - v) \end{aligned}$$

Therefore, $r^H = \min\{\bar{c}, \frac{v}{2}\}$, which verifies the second order condition. This results also follows directly from Result 2 of Thomas (2005) which states that the optimal reserve price r^* is such that $v = r^* + \frac{F(r^*)}{f(r^*)}$ if $r^* \leq \bar{c}$ and $r^* = \bar{c}$ otherwise. ∎

A.7 Proposition 8

Given a maximum reserve price r, firm i will set the price of $p(c_i) = r$ if $c_i \leq r$ and select a bribe $b(c_i) = E\left[\max\{0; r - c_j\}|c_j > c_i\right] = \int_{c_i}^{r} \frac{r-c_j}{\bar{c}-c_i} dc_j = \frac{(r-c_i)^2}{2(\bar{c}-c_i)}$. However, the expected revenue of the government is not related to the bribe, and it is equal to $v - r$ times the probability that one of the firms has a cost less than r. This is, $EU^C = \int_0^r (v - r) \frac{2(\bar{c}-c)}{\bar{c}^2} dc$, which results iN $EU^C(r) = \frac{(v-r)(2\bar{c}r-r^2)}{\bar{c}^2} = \frac{r^3-r^2(v+2\bar{c})+2v\bar{c}r}{\bar{c}^2}$.Taking the first order condition of $EU^C(r)$, the optimal reserve price r^C is such that $0 = 3r^2 - 2r(v + 2\bar{c}) + 2v\bar{c}$

Note that this is a quadratic equation and therefore has two roots, although it can only be applied if $r^C \leq \bar{c}$. When we evaluate it at $r = \bar{c}$ we obtain that the right hand side is equal to $-\bar{c}^2 < 0$, a negative value, and therefore the solution must be the smaller root of the quadratic equation. This is, $r^C = \frac{2(v+2\bar{c})-\sqrt{4(v+2\bar{c})^2-24\bar{c}}}{6}$. Additionally, if we evaluate it at $r = \frac{v}{2}$, the right hand sind of the equation is equal to $-\frac{v}{4} < 0$, which again implies that r^C is smaller than $\frac{v}{2}$, proving that $r^C < r^H$. Also, note that if $v = \bar{c}$, then this is equal to $r^C = \left(1 - \frac{1}{\sqrt{3}}\right)\bar{c} < \frac{\bar{c}}{2}$ ∎

A.8 Proposition 9

Given a reserve price r, the dishonest supervisor chooses to "burn money", which reduces its utility in $\psi > 0$. After seeing this, a firm decides to bribe $b(c|S) = \frac{(r-c)^2}{2(\bar{c}-c)}$ if its cost is smaller than the reserve price and set a price $p(c|S) = r$. If $s = 0$, then it bids a price $p(c|N) = \frac{c+\bar{c}}{2} - \frac{(\bar{c}-r)^2}{2(\bar{c}-c)}$ with $b(c|N) = 0$. In this case, a firm has no incentive to deviate, since it would lose the procurement for sure if bidding honestly to a corrupt supervisor and it would pay a penalty if bribing an honest supervisor.

Note that the supervisor expected revenue depends on this bribe. Since $b(c|S)$ is negatively related to the cost c, the supervisor is looking for the minimum value c which provides the largest bribe and therefore his expected revenue is equal to $\int_0^r \frac{(r-c)^2}{2(\bar{c}-c)} \frac{2(\bar{c}-c)}{\bar{c}^2} dc = \frac{r^3}{3\bar{c}}$. Therefore, given the firms belief, the dishonest supervisor effectively wants to signal his type if there is a positive expected benefit from it, this is as long as $\frac{r^3}{3\bar{c}} > \psi > 0$. In that case, neither the dishonest supervisor has incentive to deviate, because the signal provides a positive increase in his expected utility, nor the honest supervisor, because it would only burn money but would not get any of the benefit.

Finally, given the supervisor and firms strategies, the government has an expected utility equal to the revenue that it would get with each type of supervisor. Therefore, its expected utility is equal to $EU^S(r) = \alpha EU^H(r) + (1-\alpha)EU^C(r) = \frac{\alpha\bar{c}}{3} + \frac{r^3 - r^2(v+2\bar{c}) + 2v\bar{c}r}{\bar{c}^2}$. If $\alpha = 0$ then $EU^S(r) = EU^C(r)$ and if $\alpha = 0$ then $EU^S(r) = EU^H(r)$, and since this is a cubic equation, the solution must lie between the solutions $r^S(\alpha = 1) = r^H$ and $r^S(\alpha = 0) = r^C$. Therefore, $r^S \in \left(r^C, r^H\right)$. ∎