A Proofs

A.1 Proposition 1

Let us consider a symmetric Bayesian Nash equilibrium, where a firm bids a price if its cost is smaller than some $\hat{c} \in (0, \bar{c})$ and a bribe equal to zero, but bribes a positive quantity if its cost is larger than $\hat{c}$ and demands a price equal to r. With symmetric and linear price and bribe bids:

$$(p(c_i), b(c_i)) = \begin{cases} (ac_i + k, 0) & if \quad c_i < \hat{c} \\ (r, m(r - c_i) + d) & if \quad c_i \geq \hat{c} \end{cases}, \text{ with } a, k, m, d > 0 \tag{3}$$

For a firm with $c_i < \hat{c}$, the probability of winning given a price bid p_i and conditional on meeting an honest PO, which happens with probability α, is equal to:

$$\begin{aligned} Pr(\text{win}|p_i) &= Pr(\text{win} \cap \text{firm } j \text{ bribes}|p_i) + Pr\left(\text{win} \cap \text{firm } j \text{ bids } p_j|p_i\right) \\ &= Pr(c_j \geq \hat{c}) + Pr\left(p_i < p_j = ac_j + k \wedge c_j < \hat{c}|p_i\right) \\ &= 1 - F(\hat{c}) + Pr\left(c_j > \frac{p_i - k}{a} \wedge c_j < \hat{c}|p_i\right) \\ &= 1 - F(\hat{c}) + Pr\left(\frac{p_i - k}{a} < c_j < \hat{c}|p_i\right) \\ &= 1 - F(\hat{c}) + \left(F(\hat{c}) - F\left(\frac{p_i - k}{a}\right)\right) \\ &= 1 - F\left(\frac{p_i - k}{a}\right) \end{aligned}$$

Therefore, the firm solves the problem below. Since this equals the problem with an always honest competition, its price bid is equal to $p(c_i) = \frac{c_i + \bar{c}}{2}$ and its expected utility is $u_i^H = \alpha\frac{(\bar{c} - c_i)^2}{2\bar{c}}$.

$$\max_{p_i}(p_i - c_i)\alpha\left(1 - F\left(\frac{p_i - k}{a}\right)\right) = (p_i - c_i)\alpha\left(1 - \frac{p_i - k}{a\bar{c}}\right)$$

For a firm with cost $c_i > \hat{c}$, the probability of winning given a bribe and conditional on meeting a dishonest PO, which happens with probability $1 - \alpha$, is equal to:

$$\begin{aligned} Pr(\text{win}|b_i) &= Pr(\text{win} \cap \text{firm } j \text{ bribes}|b_i) + Pr\left(\text{win} \cap \text{firm } j \text{ bids } p_j|b_i\right) \\ &= Pr(b_i > b_j = m(r - c_j) + d \wedge c_j \geq \hat{c}|b_i) + Pr(c_j < \hat{c}) \\ &= Pr(c_j > r - \frac{b_i - k}{a} \wedge c_j \geq \hat{c}|b_i) + F(\hat{c}) \\ &= Pr\left(c_j > \max\left\{r - \frac{b_i - k}{a}, \hat{c}\right\}\right) + F(\hat{c}) \\ \text{if } \frac{ar - b_i + k}{a} \geq \hat{c} \rightarrow &= 1 - F\left(\frac{ar - b_i + k}{a}\right) + F(\hat{c}) \end{aligned}$$

Therefore, the firm with costs $c_i \geq \hat{c}$ solves the following problem:

$$\begin{aligned} &\max_{b_i}(1 - \alpha)(r - c_i - b_i)\left(1 + F(\hat{c}) - F\left(\frac{ar - b_i + k}{a}\right)\right) - \alpha M \\ &= (1 - \alpha)(r - c_i - b_i)\frac{(a\bar{c} + a\hat{c} - ar + b_i + k)}{a\bar{c}} - \alpha M \end{aligned}$$

The first order condition is that $r - c_i - b_i = a\bar{c} + a\hat{c} - ar + b_i + k$ and therefore, based on the proposed linear strategy, the firms bribes $b(c_i) = \frac{r-c_i}{2} + \frac{r-\bar{c}-\hat{c}}{2} = r - \frac{c_i+\bar{c}+\hat{c}}{2}$. Note that this bribe is smaller than the case where all firms bribe. Also, its expected utility is equal to $u_i^C = (1-\alpha)\frac{(\bar{c}+\hat{c}-c_i)^2}{2\bar{c}} - \alpha M$ and note that for this bribe, the inequality $\frac{ar-b_i+k}{a} = c_i \geq \hat{c}$ holds.

In order to find the value $\hat{c}$, a firm with cost $c_i = \hat{c}$ must be indifferent between bribing and competing honestly. Therefore, $\bar{c}$ results from the following equation:

$$\alpha\frac{(\bar{c}-\hat{c})^2}{2\bar{c}} = (1-\alpha)\frac{\bar{c}^2}{2\bar{c}} - \alpha M$$

$$\alpha\hat{c}^2 - 2\alpha\bar{c}\hat{c} + (2\alpha-1)\bar{c}^2 + 2\alpha\bar{c}M = 0 \tag{4}$$

This results in a quadratic equation with up to two real solutions, one less than $\bar{c}$ and one larger than $\bar{c}$, which are $\hat{c}_{1,2} = \bar{c}\left(1 \pm \sqrt{\frac{(1-\alpha)}{\alpha} - \frac{2M}{\bar{c}}}\right)$. Therefore, $\hat{c} = \bar{c}\left(1 - \sqrt{\frac{(1-\alpha)}{\alpha} - \frac{2M}{\bar{c}}}\right)$

In order to have a $\hat{c}$ in the set $(0, \bar{c})$, it must be that the root of Equation 4 is in that set (condition i). Additionally, the square root in $\hat{c}$ must be real valued, and therefore this term must be larger than zero (condition ii). Also, since the firms expected revenue is decreasing in its costs, it must be the case that for a firm with costs $c_i = \bar{c}$, the utility of being corrupt must be at least 0 (the utility of competing honestly, in condition iii). Finally, the bribes must be non negative, and it is sufficient that the bribe of a firm with cost $\bar{c}$ is at least zero (condition iv). Thus, these conditions are:

(i) $1 - \sqrt{\frac{(1-\alpha)}{\alpha} - \frac{2M}{\bar{c}}} > 0 \iff (1-2\alpha)\bar{c} < 2\alpha M \iff \alpha > \frac{\bar{c}}{2M+2\bar{c}}$.

(ii) $\frac{(1-\alpha)}{\alpha} - \frac{2M}{\bar{c}} > 0 \iff (1-\alpha)\bar{c} > 2\alpha M \iff \alpha < \frac{\bar{c}}{2M+\bar{c}}$.

(iii) $\frac{(1-\alpha)\hat{c}^2}{2\bar{c}} - \alpha M \geq 0 \iff \alpha \leq \frac{\hat{c}^2}{2\bar{c}M+\hat{c}^2} \underbrace{<}_{\hat{c}\leq\bar{c}} \frac{\bar{c}}{2M+\bar{c}}$

(iv) $r - \frac{2\bar{c}+\hat{c}}{2} \geq 0 \iff r \geq \frac{2\bar{c}+\hat{c}}{2} = \bar{c} + \frac{\hat{c}}{2} > \bar{c}$

Finally, we must check that a low cost firm does not want to bribe and that a high cost firm does not want to compete honestly. If a firm with low costs $c_i \leq \hat{c}$ decided to bribe, it will surely win the bribery game with a bribe slightly less than $b(\hat{c}) = r - \frac{2\hat{c}+\bar{c}}{2}$ with an utility equal to $u_i^{deviate} = (1-\alpha)\left(\hat{c} - c_i + \frac{\bar{c}}{2}\right) - \alpha M$. Therefore, its expected profits can be at most equal to this value, which is less than the utility of being honest $u_i^H = \frac{\alpha(\bar{c}-c_i)^2}{2\bar{c}}$. This is because both values are equal to the same value at $c_i \to \hat{c}$ (because of the definition of $\hat{c}$), but since $\frac{\partial u_i^H}{\partial c_i} < \frac{\partial u_i^{deviate}}{\partial c_i}$, the firm has no incentive to deviate. This last inequality holds because of conditions i and ii. Similarly, if a firm with high costs $c_i \geq \hat{c}$ decides to compete honestly, it would get at most an utility equal to $\frac{\alpha(\bar{c}-c_i)^2}{2\bar{c}}$, which is less than $(1-\alpha)\frac{(\bar{c}+\hat{c}-c_i)^2}{2\bar{c}} - \alpha M$ because of the listed conditions. Also, note that if $M = 0$, then all conditions hold if $\alpha \in (\frac{1}{2}, 1)$ ∎

A.2 Proposition 2

In the separating equilibrium, $\mu(N) = 1$ and $\mu(S) = 0$, this is, the PO that burns money is not honest and the one that does not consume conspicuously is honest. If that is the case, from a standard procurement auction and from Beck and Maher (1986) we know that firms should bid $p(c|S) = r$ and $b(c|S) = r - \frac{c+\bar{c}}{2}$ if they observe S, but should bid $p(c|N) = \frac{c+\bar{c}}{2}$ and $b(c|N) = 0$ if not.

Given the firms' strategies and beliefs, we must check that the PO effectively wants to signal out its type. If the corrupt PO burns money, it loses $\psi > 0$ but has an expected revenue from bribes equal to $r - \frac{2\bar{c}}{3} > 0$. Thus, if $0 < \psi < r - \frac{2\bar{c}}{3}$, then the PO is strictly better by signaling and prefers to do so.

Note that the firms have no incentives to deviate: if they decided not to bribe given $s = S$, then they will not be granted the project for sure. Also, if they do not see this signal and decided to bribe, they would pay the penalty M for sure. Additionally, the PO has no incentives to deviate: if the corrupt PO decided not to consume conspicuously, it would get no bribe for sure. If the honest PO decided to burn money, it would only lose $\psi > 0$.

Finally, the firm with the lowest cost wins the procurement auction: with probability α by bidding the lowest price and with probability $1 - \alpha$ by submitting the largest bribe. Therefore, the expected price with an honest PO is $\frac{2\bar{c}}{3}$ and with a dishonest PO is r. ∎

A.3 Proposition 3

1. Given the firms belief that $\mu(N) = \alpha$, let us consider the price and bribe strategies considered in Proposition 1, to bribe only if costs are above a mid value $\underline{c}$. In order for such a strategy to characterize an equilibrium, the same set of conditions are needed. In this case, the dishonest PO has a positive expected revenue and pays no cost. However, in order to ensure that the PO has no incentives to deviate, firms must not try to bribe a PO, so $b(c|S) = 0$. In that case, burning money yields no expected benefit and the PO would be strictly worse by trying to signal his type.

2. The dishonest PO has no incentives to select $s = S$, since it would lose $\psi > 0$ for sure and not get anything in return. This rules out the separating and pooling equilibria where the dishonest PO burns money. ∎

A.4 Proposition 5

Given a reserve price r and when facing an honest PO, following Krishna (2009), the optimal price bid for a firm is $p(c_i) = E[\min\{c_j, r\}|c_j > c_i]$. Since $Pr(c_j \geq x|c_j > c_i) = \frac{Pr(c_i < c_j \geq x)}{Pr(c_j > c_i)} = \frac{F(x) - F(c_i)}{1 - F(c_i)} = \frac{x - c_i}{\bar{c} - c_i}$ for the uniform distribution, this has a density function of $\frac{1}{\bar{c} - c_i}$. Therefore, $p(c_i) = \int_{c_i}^{r} \frac{c_j}{\bar{c} - c_i} dc_j + \int_{r}^{\bar{c}} \frac{r}{\bar{c} - c_i} dc_j = \frac{c_i + \bar{c}}{2} - \frac{(\bar{c} - r)^2}{2(\bar{c} - c_i)}$.

Since the government values the project in v, it will get an utility of $v - \min\{p_1, p_2\}$ when meeting a firm that has a cost lower than r. Therefore, the government expected utility is equal to $EU^H(r) = \int_0^r \left(v - \frac{c+\bar{c}}{2} + \frac{(\bar{c}-r)^2}{2(\bar{c}-c)}\right) \frac{2(\bar{c}-c)}{\bar{c}} dc$, which results in $EU^H(r) = \frac{\frac{4}{3}r^3 - r^2(v+2\bar{c}) + 2v\bar{c}r}{\bar{c}^2}$.

Taking the first order condition of $EU^H(r)$, the optimal reserve price r^H is such that:

$$
\begin{aligned}
0 &= 4r^2 - 2r(v + 2\bar{c}) + 2v\bar{c} \\
&= 2(r - \bar{c})(2r - v)
\end{aligned}
$$

Therefore, $r^H = \min\{\bar{c}, \frac{v}{2}\}$, which verifies the second order condition. This results also follows directly from Result 2 of Thomas (2005) which states that the optimal reserve price r^* is such that $v = r^* + \frac{F(r^*)}{f(r^*)}$ if $r^* \leq \bar{c}$ and $r^* = \bar{c}$ otherwise. ∎

A.5 Proposition 6

Given a maximum reserve price r, firm i will set the price of $p(c_i) = r$ if $c_i \leq r$ and select a bribe $b(c_i) = E\left[\max\{0; r - c_j\} | c_j > c_i\right] = \int_{c_i}^{r} \frac{r-c_j}{\bar{c}-c_i} dc_j = \frac{(r-c_i)^2}{2(\bar{c}-c_i)}$. However, the expected revenue of the government is not related to the bribe, and it is equal to $v - r$ times the probability that one of the firms has a cost less than r. This is, $EU^C = \int_0^r (v-r) \frac{2(\bar{c}-c)}{\bar{c}^2} dc$, which results iN $EU^C(r) = \frac{(v-r)(2\bar{c}r - r^2)}{\bar{c}^2} = \frac{r^3 - r^2(v+2\bar{c}) + 2v\bar{c}r}{\bar{c}^2}$.Taking the first order condition of $EU^C(r)$, the optimal reserve price r^C is such that $0 = 3r^2 - 2r(v + 2\bar{c}) + 2v\bar{c}$

Note that this is a quadratic equation and therefore has two roots, although it can only be applied if $r^C \leq \bar{c}$. When we evaluate it at $r = \bar{c}$ we obtain that the right hand side is equal to $-\bar{c}^2 < 0$, a negative value, and therefore the solution must be the smaller root of the quadratic equation. This is, $r^C = \frac{2(v+2\bar{c}) - \sqrt{4(v+2\bar{c})^2 - 24\bar{c}}}{6}$. Additionally, if we evaluate it at $r = \frac{v}{2}$, the right hand sind of the equation is equal to $-\frac{v}{4} < 0$, which again implies that r^C is smaller than $\frac{v}{2}$, proving that $r^C < r^H$. Also, note that if $v = \bar{c}$, then this is equal to $r^C = \left(1 - \frac{1}{\sqrt{3}}\right) \bar{c} < \frac{\bar{c}}{2}$ ∎

A.6 Proposition 7

Given a reserve price r, the dishonest PO chooses to "burn money", which reduces its utility in $\psi > 0$. After seeing this, a firm decides to bribe $b(c|S) = \frac{(r-c)^2}{2(\bar{c}-c)}$ if its cost is smaller than the reserve price and set a price $p(c|S) = r$. If $s = 0$, then it bids a price $p(c|N) = \frac{c+\bar{c}}{2} - \frac{(\bar{c}-r)^2}{2(\bar{c}-c)}$ with $b(c|N) = 0$. In this case, a firm has no incentive to deviate, since it would lose the procurement for sure if bidding honestly to a corrupt PO and it would pay a penalty if bribing an honest PO.

Note that the PO's expected revenue depends on this bribe. Since $b(c|S)$ is negatively related to the cost c, the PO is looking for the minimum value c which provides the largest bribe and therefore his expected revenue is equal to $\int_0^r \frac{(r-c)^2}{2(\bar{c}-c)} \frac{2(\bar{c}-c)}{\bar{c}^2} dc = \frac{r^3}{3\bar{c}}$. Therefore, given the firms belief, the dishonest PO effectively wants to signal his type if there is a positive expected benefit from it, this is as long as $\frac{r^3}{3\bar{c}} > \psi > 0$. In that case, neither the dishonest PO has incentive to deviate, because the signal provides a positive increase in his expected utility, nor the honest PO, because it would only burn money but would not get any of the benefit.

Finally, given the PO and firms strategies, the government has an expected utility equal to the revenue that it would get with each type of PO. Therefore, its expected utility is equal to $EU^S(r) = \alpha EU^H(r) + (1-\alpha) EU^C(r) = \frac{\alpha\bar{c}}{3} + \frac{r^3 - r^2(v+2\bar{c}) + 2v\bar{c}r}{\bar{c}^2}$. If $\alpha = 0$ then $EU^S(r) = EU^C(r)$ and if $\alpha = 0$ then $EU^S(r) = EU^H(r)$, and since this is a cubic equation, the solution must lie between the solutions $r^S(\alpha = 1) = r^H$ and $r^S(\alpha = 0) = r^C$. Therefore, $r^S \in \left(r^C, r^H\right)$. ∎

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