Signaling corruption through conspicuous consumption*

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Abstract

Repeatedly, we see a link between conspicuous consumption of luxury goods and corruption from a public officer. Since this raises public awareness about him, it can backfire and lead to an investigation that eventually finds him guilty. Thus, how can we rationalize this fact? One potential explanation is that there are gains from signaling their willingness to be corrupt, to attract the pool of corrupt firms and raise the expected revenue from bribes. In this work, we consider a public procurement setting, where the government delegates a supervisor to run the process. We find that if the signaling cost is low enough, then there exists a separating equilibrium where the supervisor signals his type and gets to keep a share of the firm's paid price. Also, since in a procurement auction the tender can increase his expected utility by setting a reserve price, we consider what happens if the government fixes a budget constraint or maximum price before assigning a supervisor. We find that a signaling equilibrium in such conditions can also exist, but with a lower reserve price than socially optimal. Therefore, even though the government can reduce the revenue that the supervisor gets, corruption and signaling can result in welfare loss.

Keywords: corruption, public procurement, signals.

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1 Introduction

We frequently observe that a (suspected) corrupt public officer is seen with luxury goods, which raises public awareness about him. For example, during his time at office as Argentina's Secretary of Transportation, Ricardo Jaime was repeatedly seen in a luxurious yacht and a private airplane (Alconada Mon, 2019), among other notorious goods and gifts. Jaime was in charge of allocating subsidies to transportation projects, some of which were poorly completed or not executed at all, and later on, he was found guilty of accepting bribes. Another Argentinian example is the former federal judge Norberto Oyarbide[1], who was seen with a notorious diamond ring. In this case, he first claimed that he bought the ring, valued at USD 250.000, but later on denied this version. A final example, which illustrates that this is not only a local phenomenon, is the case of Equatorial Guinea's former Agriculture Minister Teodorin Obiang Jr.[2], the son of the country's President. Despite his formal 3.200 Euros wage, he was seen in Paris driving luxury cars such as a Ferrari or Porsche, among other goods.

Regardless of whether these public officers effectively bought the goods or not[3], it is remarkable that they were *seen* using and consuming them. After all, conspicuous consumption can backfire and lead to an investigation that eventually finds them guilty. How is it possible that public officers repeatedly buy luxury goods? To rationalize this behavior, there must be some gains from consuming these goods. One potential explanation is that officers want to effectively signal their willingness to be corrupt and therefore attract a pool of more corrupt firms, raising their expected revenue from bribes. While some firms might be willing to bribe a public officer, they are not sure about the kind of counterpart they are facing, and they would certainly not try to bribe the most honest supervisor. Consequently, with this signal, firms might have a better understanding of who they are meeting and what kind of actions will let them win the procurement.

The idea that conspicuous consumption can act as a signal is related to the work of Fabrizi and Lippert (2017), who, within a principal-agent model, characterize conditions for the existence of a signaling equilibrium. In their work, if the officer's bargaining power is high enough, there can be a separating equilibrium where a public officer consumes conspicuously or "burns money" and thus attract higher bribes. However, they consider two types of supervisors that distinguish from another, according to the probability in which they are subjected to an audit, thus attracting the *correct* amount of bribes. Therefore, the public officer is signalizing his ability in corruption and not his willingness to be corrupt, which is the interest of this work.

In this article, we want to argue that, within the framework of public procurement, there exists a separating perfect Bayesian equilibrium in which a corrupt officer signals his type by buying a luxury good and being seen displaying it. To do so, we consider a public procurement where the government delegates the process to a supervisor, who is potentially corrupt. We find that, if the cost of the conspicuous consumption is positive but low enough, then there exists a separating equilibrium where the supervisor signals his type and all firms try to bribe him. Compared to the equilibrium without signal, the supervisor effectively attracts higher bribes compared to when firms are not perfectly informed about his type. As a result, it let the firms know who is running the procurement

[1]Reported in *Perfil*, available here.

[2]Reported in *The Guardian*, available here. See Fabrizi and Lippert (2017) for a detailed discussion of his case.

[3]In the previous examples, both Ricardo Jaime and Teodorin Obiang were found guilty of accepting bribes and corruption, respectively.

and what are the actions that will make them win the auction.

Additionally, since in a procurement auction the government can typically do better by setting a maximum reserve price, we consider whether a signaling equilibrium can exist in that case. We find that the government can do better by setting a budget constraint or maximum price before assigning a supervisor. However, in that case, there still exists a signaling equilibrium with a reserve price that is below the social optimum. This implies that, while the government can do a little better, corruption entails a social welfare loss and a higher probability of declaring the procurement void.

The article is organized as follows: in Section 2, we review the related literature regarding procurement, corruption mechanisms and signaling equilibria in corrupt auctions. In Section 3, we build our model and solve it, considering the case with and without conspicuous consumption and adding the possibility that the government sets a reserve price. In Section 4, we discuss the implications of the results and their scope, and, finally, in Section 5, we have some concluding remarks.

2 Related literature

Economic theory has focused on a wide range of models to capture the essence of corruption[4], in this case, the collusion between a government officer and a bidder. After all, corruption and bribery can take many forms and affect resource allocation and welfare through many mechanisms. While corruption usually entails distortions, it only does under certain conditions.

A vast majority of the literature has focused on the public procurement using scoring rules to consider both price and quality bids, which started with the seminal work of Che (1993). Under a scoring scheme, an agent might bribe the inspector to: misrepresent the *ex-post* effective quality (Burguet, 2017; Celentani and J. J. Ganuza, 2002), manipulate the assessment of quality in the bid (Burguet and Che, 2004), or provide private information about the competition's bids to one firm so it can resubmit a more aggressive price bid (Compte, Lambert-Mogiliansky, and Verdier, 2005), among others. In these works, corruption typically entails that the government pays a higher price and gets a lower quality delivered, thus indicating inefficiency and welfare loss. However, for this to happen the corrupt politician must have enough manipulation power (Burguet and Che, 2004).

However, we will refrain from the scoring rule literature and consider a simpler framework, but we will still keep in consideration the previous results and its welfare implications. Instead, we will consider the public procurement of a single and homogeneous good, or a reverse auction. In this literature, corruption might or might not imply allocation distortions, depending on the mechanism. On the one hand, if we consider a corrupt inspector running a procurement for a single good, which assigns the project to the supplier that submits the lowest price bid or the largest bribe with a predetermined price, Beck and Maher (1986) show that there is an isomorphism between bribery and competitive bidding. They show that there is a symmetric equilibrium, such that the lowest cost firm bids the largest bribe, and Lien (1986) shows that this equilibrium is unique. On the other hand, Arozamena and Weinschelbaum (2009) show that corruption can change bidding behavior in a first price auction if all firms are aware that there is a dishonest

[4] For a more detailed discussion of the microeconomics of corruption in recent years, see Burguet, J.-J. Ganuza, and Montalvo (2016).

bidder that has informational advantage. In that situation, there is a positive probability that the firm with the highest valuation (lowest cost) does not win the auction, distorting the allocation.

Thomas (2005) finds that, in an infinitely repeated game where collusion between bidders can be sustained, setting a reserve price can shrink the set of discount factor for which collusion can be sustained, and by doing so it could deter collusion. This raises the question about whether the government can do something similar to discourage the corruption between the public officer and bidders, and even whether it can eliminate the incentives to signal.

This article is also related to the signaling literature and luxury goods or conspicuous consumption as an informative signal about an agent's type (Di Tella and Weinschelbaum, 2008; Fabrizi and Lippert, 2017). It is more closely related to the work of Fabrizi and Lippert (2017), who consider a principal-agent model where public servants distinguish from another by the probability in which they are subjected to an audit. With imperfect information about this audit probability, firms can not be certain about the value of the relationship and thus are uncertain about how much to bribe. If the officer's bargaining power is high enough, there can be a separating equilibrium where a public officer consumes conspicuously or "burns money" and thus attract higher bribes. However, even though they characterize a signaling equilibrium, this public servant is actually signalizing his ability to succeed at corruption and not his willingness to be corrupt. This is because the firms here are certain that they should bribe the supervisor, but are uncertain about the correct level of bribery.

Finally, even though the motivation regarding conspicuous consumption and corruption comes from examples observed in reality, there is also cross-country empirical correlation between the (perceived) corruption level and luxury car sales (Gokcekus and Suzuki, 2014), and that controlling corruption can reduce luxury spending (Tajaddini and Gholipour, 2018).

3 The model

Let us consider the case of $n = 2$ firms that can produce a good or a project. Each company i independently draws its cost $c_i \sim U[0, \bar{c}]$, with identical cumulative function $F(c_i)$ that is common knowledge. The government values this project in $v \geq \bar{c}$, such that the production of this good is socially desirable, and it will be assigned following a procurement process. However, the government can not run this auction by itself and must assign a supervisor to conduct it, in addition to setting a maximum budget constraint $r \geq 0$ to pay to the winning bidder.

This public servant or supervisor can be honest (H) with probability $\alpha \in (0, 1)$ or corrupt/dishonest (C) with probability $1 - \alpha$, and these probabilities are of common knowledge. We will assume that an honest supervisor will never assign the project to a company that offers him a bribe and, similarly, a dishonest supervisor only assigns projects to firms that offer him positive bribes. If a firm tries to bribe an honest supervisor, then that firm will pay a penalty equal to $M \geq 0$, which represents the monetary value of going to jail, paying a fine or even the opportunity cost of being excluded from any future procurement. Additionally, we will assume that the government cannot use a mechanism to discover the supervisor's type.

After being assigned by the government and before running the procurement auction, the supervisor can buy a luxury good or "burn money", following Fabrizi and Lippert

(2017). This will be through an action $s \in \{S, 0\}$, where $s = S$ represents the conspicuous consumption that costs him $\psi > 0$ and $s = 0$ refers to doing nothing. When the supervisor decides whether to burn money or not, the government has already set a maximum price r and assigned him to run the procurement.

After seeing their private cost c_i, each firm competes for the assignment of the project in the procurement. Each one must choose a non-negative amount b to bribe (potentially equal to 0) and a non-negative price p to bid. With an honest supervisor, all firms that submit a positive bribe $b > 0$ will be reported and must pay the penalty. Because firms observe the signal s, they can use that information to calculate the posterior probability of meeting an honest supervisor $\mu(s)$ given s.

The timing of the more general model is as follows:

- $\mathbf{t = 0}$: The government assigns a supervisor to run the procurement process, given the budget constraint $r > 0$ for the project.
- $\mathbf{t = 1}$: Nature draws the costs $c_i \sim U[0, \bar{c}]$ of the firms and decides whether the supervisor is honest (with probability α) or not.
- $\mathbf{t = 2}$: The supervisor, knowing his type, decides whether to "burn money" (S) or not (0). If the action is $s = S$, it will cost him a given amount $\psi > 0$, which is the price of the luxury good.
- $\mathbf{t = 3}$: After observing s, firms submit a price and a bribe bid (potentially equal to zero).
- $\mathbf{t = 4}$: The honest supervisor grants the contract to the lowest price bid and reports firms that tried to bribe him. The dishonest supervisor assigns the project to the firm with the highest bribe.

In order to solve the model and discuss the welfare implications of the set of equilibria, we will start from a standard procurement auction and add the additional stages. In section 3.1, we will discuss and characterize the equilibrium with a given reserve price $r \geq \bar{c}$ (such that a firm always wants to bid) without the signaling action. Then, in section 3.2 we will allow the supervisor to signal his type, given the reserve price $r \geq \bar{c}$, and characterize the set of equilibria. Finally, in section 3.3, we will add the previous stage of the game in which the government can set a reserve price that maximize the expected utility, and potentially set a price below the maximum cost $\bar{c}$.

3.1 Benchmark: procurement auction without signal

Let us first consider the case when the supervisor must conduct the procurement right after being assigned and is not allowed to "burn money" before the auction. This will be useful to characterize the bids and profits with and without corruption. Note again that, in this case, we are assuming that the maximum price r is at least $\bar{c}$, which implies that a firm can always bid a price that provides positive profits (or null with probability zero, only if $c_i = \bar{c} = r$).

If the supervisor was never corrupt (or $\alpha = 1$), then for a given cost, firms will never try to bribe and would only compete in price. If a firm i with a cost c_i submits a price bid p_i and wins, it will get a profit equal to $p_i - c_i$. However, this only happens it the submitted price is lower than company j's price p_j. Therefore, with an always honest supervisor, the firm i submits a price p_i that maximizes his expected profit:

$$\max_{p_i}(p_i - c_i)Pr(\text{win}|p_i) = \max_{p_i}(p_i - c_i)Pr(p_i < p_j) \tag{1}$$

This is the problem in a standard procurement or reverse auction, and it is a well known result that with a symmetric and uniform distribution, the optimal price bid is a linear function of the cost. The following proposition characterizes this equilibrium:

Proposition 1 *With an honest supervisor, there exists a symmetric and linear Bayesian Nash equilibrium in which firms bid a bribe $b(c) = 0$ and a price bid $p(c) = \frac{c+\bar{c}}{2}$. In this equilibrium:*

i. The firm with the lowest cost wins the procurement.

ii. The expected profit of a firm is $u_i^H(c_i) = \frac{(\bar{c}-c_i)^2}{2}\ \forall\, i$.

iii. The expected price that the government pays is $\frac{2\bar{c}}{3}$

Proof. All proofs are relegated to the Appendix. ■

Proposition 1 shows that the firm with the lowest cost has the largest expected utility and wins the auction. In this situation, since the reserve price is larger than the maximum cost $\bar{c}$, all firms participate in the procurement and the expected price that the government pays is equal to $\frac{2\bar{c}}{3}$. Therefore, the government has an expected utility equal to $v - \frac{2\bar{c}}{3} > 0$.

Then, if the supervisor is always corrupt (*i.e.*, $\alpha = 0$), the firm with the highest bribe will win the project and get paid the bidded price. Firm i will submit a price p_i and a bribe b_i that maximizes its expected profit, subject to $p_i \leq r$. If the firm wins, then it will get a profit equal to their submitted price minus the cost and the bribe. Therefore, the firm resolves the following problem:

$$\max_{p_i \leq r, b_i}(p_i - c_i - b_i)Pr(b_i > b_j) \equiv \max_{b_i}(r - c_i - b_i)Pr(b_i > b_j) \tag{2}$$

Since the expected profit is strictly increasing in the price bid, which does not affect the probability of winning, the firm will optimally select the highest price possible, such that $p(c) = r$ for all possible costs. This transforms the problem into a standard first price auction problem, where each firm has a valuation $r - c_i \equiv v_i \sim U[r - \bar{c}, r]$ for the project, and the firm with the highest (bribe) bid wins. As a result, it is expected that the firm with the largest valuation (*i.e.*, the lowest cost) wins the project.

Proposition 2 *With a dishonest supervisor, there exists a symmetric and linear Bayesian Nash equilibrium in which firms submit a price bid $p(c) = r$ and a bribe $b(c) = r - \frac{c+\bar{c}}{2}$. In this equilibrium:*

1. The firm with the lowest cost wins the procurement.

2. The expected profit of a firm is $u_i^C(c_i) = \frac{(\bar{c}-c_i)^2}{2}$

3. The expected price that the government pays is r, and the expected bribe that the supervisor gets is $r - \frac{2\bar{c}}{3}$

Proposition 2 shows an interesting result: in this situation, corruption entails no distortion in the allocation of the project but a redistribution between the supervisor and the government. This is because, compared with the equilibrium in Proposition 1, now the supervisor retains a share of the government's utility of the project. However, if we consider the welfare as the direct sum of individual utilities[5], then there is no welfare loss and the efficient assignment is attained even through this corruption procurement. The results of this proposition are precisely in line with the results of Beck and Maher (1986) and Lien (1986), who show that this bribery game is isomorphic to the procurement process if $r \geq \bar{c}$.

Finally, both results are useful to characterize the equilibrium when the firm does not know what kind of a supervisor is running the procurement process. Note that if each firm has a positive probability of meeting an honest and dishonest supervisor, then there is a positive probability of being reported and paying the penalty M if they try to bribe. So, in this situation, how should a firm behave optimally when it has no idea who is on the other side of the table? Should this firm always bribe, never bribe or only within certain costs? Are the strategies described in Propositions 1 or 2 part of an equilibrium for a given α?

The answer to the latter question is that no, the strategies in Propositions 1 and 2 might not characterize an equilibrium for a given α. To illustrate this, let us consider the case of a firm with high cost (or low gap $\bar{c} - c$): if this company competes honestly in price with the other firm, its expected utility is at most equal to $\alpha\frac{(\bar{c}-c)^2}{2}$. However, by trying to bribe the supervisor with some $b > 0$, the firm wins the project for sure when it meets the corrupt supervisor, which happens with probability $1 - \alpha$, and can get an expected profit of $(1-\alpha)(r-b-c) - \alpha M$. For a sufficiently large r and a small M, this is a larger expected profit than by competing honestly. This implies that maybe firms will only try to bribe if their costs are too high, since in that case competing in prices will yield a small expected profit, and thus the strategies in Proposition 1 might not characterize the equilibrium. Also, for a sufficiently large α, companies with low cost (and therefore a large gap $\bar{c} - c$) have too much to lose if they try to bribe. They would prefer only to submit a price bid, which also implies that the strategies in Proposition 2 might not be part of the equilibrium.

In the previous examples, the incentives to deviate from the strategies depend on the cost, because low cost firms have potentially no incentive to bribe and high cost firms might not have incentives to compete honestly. We can then expect that there is a mid value $\underline{c} \in (0, \bar{c})$ such that a firm will try to bribe the supervisor if and only if its cost is larger than $\underline{c}$, but will bid an honest price bid if its cost is lower than this mid value. Proposition 3 shows that there exists such a Bayesian Nash equilibrium. In the simple case that the firms do not face a penalty for bribing an honest supervisor (with $M = 0$), then it is sufficient that $\alpha > \frac{1}{2}$.

Proposition 3 *Given a reserve price $r \geq \bar{c}$ and a penalty M, there exists a symmetric and linear Bayesian Nash equilibrium in which a firm decides to bribe only if their costs are larger than some $\underline{c} \in (0, \bar{c})$. For this equilibrium to exist, the probability α of meeting an honest supervisor must be in certain non-empty subset strictly contained in $(0, 1)$. In the simple case that $M = 0$, it is sufficient that $\alpha > \frac{1}{2}$.*

[5] With a social welfare function that assigns a larger weight to the government's utility, then this result would imply a welfare loss under corruption. However, this is not the interest of this work.

The results in Proposition 3 characterize an equilibrium in which firms will only bribe sometimes. Unlike the previous case of perfect information regarding who is running the procurement auction, now the firms optimally decide to bribe only if their costs are large, because the probability of meeting a firm with lower costs and losing the procurement results in a low expected profit of competing honestly. Even if there is no direct punishment from trying to bribe an honest supervisor (if $M = 0$), it is not true that firms will always try to bribe.

An interesting result from Proposition 3 is that the bribe bids are strictly *smaller* than with an always dishonest supervisor, and therefore the expected bribe of a corrupt public officer is lower. The intuition behind this result is that firms will try to bribe only sometimes, which reduces the bribery competition and the bribe bids. This results in a higher expected profit of the firms, because they can get to keep a larger share of the price. So, while bribery and corruption (with perfect information) increase bribe competition (Compte, Lambert-Mogiliansky, and Verdier, 2005), in this case the uncertainty reduces the bribery competition and the bribes.

However, unlike the case with perfect information, two relevant inefficiencies arise in this equilibrium. First, if the two firms try to bribe an honest supervisor or the two firms try to bid a price bid to a corrupt supervisor, the procurement will be declared void. As a result, there is a positive probability that the project is not assigned to any firm, unlike the case with full information. Second, the firm with the highest cost may win the procurement: with a dishonest supervisor, if one of the firms has a cost lower than $\underline{c}$ while the other has a cost higher than $\underline{c}$, then the latter will bribe and win the procurement process. Since this event can happen with positive probability, the imperfect information results in allocation distortion and efficiency loss.

3.2 Procurement auction with conspicuous consumption

Now, knowing that imperfect information results in lower bribes and some inefficiencies, can the supervisor do any better? Let us introduce conspicuous consumption or the "burnt money" as a signal, an action that the supervisor makes before the procurement and that the firms can observe. Each firm can incorporate this information to have a better understanding and infer better who is on the other side of the table, and what kind of actions are going to let him win.

Given the signal $s \in \{S, 0\}$, firms consider that they are meeting an honest supervisor with probability $\mu(s)$, and given the cost c_i, firm i can bid a different price $p(c_i|s)$ and bribe $b(c_i|s)$ according to the observed action of the supervisor. With this information, is there a separating perfect Bayesian equilibrium? Proposition 4 characterize the set of parameters for such an equilibrium to exist.

Proposition 4 *Given $r \geq \bar{c}$, if the cost of burning money $\psi > 0$ is small enough, then there exists a separating equilibrium in which a corrupt supervisor signals that it is dishonest. In this equilibrium:*

i. The firm with the lowest cost wins the procurement (by bribing or bidding a price).

ii. The government expected price is $\alpha\frac{2\bar{c}}{3} + (1-\alpha)r$.

As Proposition 4 shows, there exists a separating equilibrium in which a corrupt supervisor burns money or buys a luxury good. By doing so, the firms will perfectly know who is running the procurement and what kind of bid will make them win the auction. Here, they will only try to bribe the public officer that has a luxury good and they will compete honestly with a price bid if the supervisor does not consume conspicuously. However, for this equilibrium to exist, the cost of burning money must be small enough, such that the net profit of the bribes exceeds the cost of burning it, but big enough (positive) to be a costly and informative signal about the supervisor's type.

Additionally, in this equilibrium, the procurement will never be declared void and the firm with the lowest cost will always win the project. The government expected utility is equal to $v - \alpha\frac{2\bar{c}}{3} - (1-\alpha)r$ and the expected rent that the supervisor will get (weighted by the probability of being corrupt) is $(1-\alpha)(r - \frac{2\bar{c}}{3})$. Therefore, the sum of expected utilities or total welfare is equal to $v - \frac{2\bar{c}}{3}$, which is also the government expected utility with an always honest supervisor or the maximum welfare attainable through a first price procurement auction. In this equilibrium, there is only a distributive problem and not an efficiency problem.

Given the separating equilibrium that Proposition 4 characterizes, let us consider the other equilibria. As Proposition 5 states, there is a pooling equilibrium in which both an honest and dishonest supervisor do not burn money and the firms bid prices and bribes just like in Proposition 3. However, because the cost of the luxury good is strictly positive, the honest supervisor will never choose $s = S$. This is because the action is expensive but provides him no benefit, given that the honest supervisor will never take bribes nor assign the procurement to a corrupt firm. As a result of this, the honest supervisor will never decide to burn money.

Proposition 5 *In this model:*

1. *There exists a pooling equilibrium with $\mu(0) = \alpha$, under the same set of conditions as in Proposition 3. In this equilibrium, if $s = 0$ firms bid according to the strategy in Proposition 3, but if $s = S$ then no one will bribe a positive bid.*

2. *There is no pooling nor separating equilibrium in which the honest supervisor burns money.*

Note that the existence of the pooling equilibrium in Proposition 5 implies a certain firm's behavior out of the equilibrium path. This equilibrium exists if firms do not bribe when $s = S$ (which, of course, does not happen in equilibrium), which explains why a corrupt supervisor has no incentive to deviate from not consuming conspicuously.

Interestingly, if ψ is large enough and close to $r - \frac{2\bar{c}}{3}$, the expected return from signaling barely outweighs its costs and thus yields a very low return. Therefore, if ψ is large enough, the supervisor might be better by not signaling and getting low bribes only sometimes than by signaling and getting higher bribes all the time. Even for $\psi \in \left(0, r - \frac{2\bar{c}}{3}\right)$ and the same set of parameters as in Proposition 3, both the pooling and separating equilibrium described are feasible but, with a sufficiently large signaling cost ψ, the supervisor could be better in the pooling equilibrium.

3.3 Government optimal reserve price

As mentioned before, what if the government decided on the maximum reserve price before the procurement? How does this affect the project assignment and resources allocations? Since all the previous results hold when $r \geq \bar{c}$, can the government do better by setting a lower budget constraint? To answer this, let us first consider as another benchmark the situation when the government meets an always corrupt supervisor and also the case with an always honest supervisor. This will give us the optimal reserve price with an honest supervisor (r^H) and with an always corrupt supervisor (r^C).

With an honest supervisor and given a reserve price $r \leq \bar{c}$, following Krishna (2009)[6], the optimal price bid for a firm is $p(c_i) = E[\min\{c_j, r\}|c_j > c_i]$ if $c_i \leq r$. Since $Pr(c_j \geq x|c_j > c_i) = \frac{x-c_i}{\bar{c}-c_i}$ for the considered uniform distribution, it can be shown that the resulting optimal bid for firm i is $p(c_i) = \frac{c_i+\bar{c}}{2} - \frac{(\bar{c}-r)^2}{2(\bar{c}-c_i)}$, as Proposition 6 states. In this scenario, the government obtains a utility of $v - \min\{p_1, p_2\}$ only if at least one of the firms has a cost lower than r, so how should the government choose this price? The following proposition characterizes this equilibrium.

Proposition 6 *If the government values the project in $v \geq \bar{c}$ and the supervisor is always honest, then:*

i. The optimal price bid for a firm with costs $c \leq r$ is $p(c) = \frac{c+\bar{c}}{2} - \frac{(\bar{c}-r)^2}{2(\bar{c}-c)}$, while if $c > r$ then the firm does not submit a bid (or $p(c) = +\infty$).

ii. The optimal reserve price for the government is $r^H = \min\{\frac{v}{2}, \bar{c}\}$, with an expected utility of $EU^H(r) = \frac{\frac{4}{3}r^3 - r^2(v+2\bar{c}) + 2v\bar{c}r}{\bar{c}^2}$.

This implies that, if $v \leq 2\bar{c}$ and $r^H = \frac{v}{2}$, then there is a positive probability that the procurement is declared void if both firms have costs above the maximum reserve price. This is of course no surprise within the auction or procurement literature (Krishna, 2009; Thomas, 2005), but is interesting to discuss the mechanism behind these results. Even though there is a probability of not obtaining the good or project, the reserve price reduces the price bids of the firms. Therefore, this increases the expected utility $v - \min\{p_1, p_2\}$ when $\min\{c_1, c_2\} \leq r$, which offsets the loss of meeting two high costs firms.

Now, if the supervisor was always corrupt, this drastically modifies the government's problem because it will pay the reserve price for sure. In this case, the expected utility is not related to the bribes, and it is equal to $v - r$ times the probability that one of the firms has a cost less than r. Proposition 7 characterizes the government's optimal reserve price in this situation. The intuition behind this is that, since the government will always get to pay r, there is a trade-off between setting a high price r and getting a low benefit $v - r$ with high probability, or setting a low price with high benefit but low probability. As a result, it should reduce the price compared to the case with an honest supervisor.

Proposition 7 *If the government values the project in $v \geq \bar{c}$ and the supervisor is always corrupt, then:*

i. The optimal bribe bid for a firm with costs $c \leq r \leq \bar{c}$ is $b(c) = \frac{(r-c)^2}{2(\bar{c}-c)}$, while if $c > r$ then the firm does not submit a bid (or $p(c) = +\infty$).

ii. The optimal reserve price for the government r^C depends on v and $\bar{c}$ and it is smaller than r^H. In the case that $v = \bar{c}$, $r^C = \left(1 - \frac{1}{\sqrt{3}}\right)\bar{c}$. Also, the government expected utility is $EU^C(r) = \frac{r^3 - r^2(v+2\bar{c}) + 2v\bar{c}r}{\bar{c}^2}$

[6]The result for auctions is that, given two individuals with valuations v_1, v_2, the optimal bid in a first price auction if $v_i \geq r$ is $p(v_i) = E[\max\{v_j, r\}|v_j < v_i]$.

It is important to notice that, in this equilibrium, the expected utility of the government $EU^C(r)$ is smaller than with an honest supervisor. Given a reserve price, the difference between these two utilities is exactly $\frac{r^3}{3\bar{c}}$, which is precisely the expected revenue of the supervisor.

Finally, with imperfect information about the type of supervisor that will run the procurement after setting the reserve price r, a similar result as in Proposition 4 holds. Just like in this Proposition, if the cost of signaling is positive but low enough, we can characterize a separating perfect Bayesian equilibrium where the supervisor signals his type. In this equilibrium, the firm with the lowest cost will win the procurement, either by bribery or by bidding a price. However, the government can maximize its expected revenue by selecting a maximum reserve price below the maximum cost $\bar{c}$.

Proposition 8 *Even if the government can select the reserve price before assigning the supervisor, then there exists a separating equilibrium where the supervisor signals his type with a sufficiently low cost $\psi > 0$. In this equilibrium, the government's optimal reserve price r^S is in the interval (r^C, r^H).*

Therefore, even though the government can do better by selecting a lower price, a separating equilibrium is still possible for a sufficiently low signaling cost. Also, in this equilibrium, imposing a reserve price is not free and does not solve the distributive problems, since there are two (related) sources of inefficiencies. First, since the optimal reserve price in this separating equilibrium is lower than with an honest supervisor (*i.e.*, $r^S < r^H$), there is a higher probability of declaring the procurement void. This is of course unwanted, since the project is socially desirable given that $v \geq \bar{c}$. Second, there is an inefficiency not in the allocation of the procurement, that certainly goes to the firm with the lowest cost, but rather in the total welfare.

Regarding the latter claim, note that the expected revenue from the supervisor is $\frac{r^3}{3\bar{c}^2}$, while the expected utility of the government is $EU^S(r) = \frac{\alpha r^3}{3\bar{c}^2} + \frac{r^3 - r^2(v+2\bar{c}) + 2v\bar{c}r}{\bar{c}^2}$. Defining the total expected welfare as the sum of the expected utilities (weighted by the likelihood of meeting the dishonest supervisor), this equals $W(r) = \frac{(1-\alpha)r^3}{3\bar{c}^2} + EU^S(r) = \frac{\frac{4}{3}r^3 - r^2(v+2\bar{c}) + 2v\bar{c}r}{\bar{c}^2}$. From Proposition 6 we know that the reserve price that maximizes this total welfare $W(r)$ is r^H and therefore $W(r^S) < W(r^H)$. Therefore, imperfect information results in a welfare loss even when the supervisor perfectly signals his type to the firms. We can then conclude that allowing the government to set a reserve price does not eliminate the distortions. Even though the reserve price increases the expected utility of the government, corruption and signaling result in a total welfare loss.

4 Discussion

In this section, we will discuss the implications of the signaling equilibrium on total welfare and distribution. We will also limit the scope of the results and potential extensions that can be considered.

First of all, in our model, the supervisor has explicit incentives to signal his type to attract higher bribes rather than without the signal. The underlying mechanism behind this is that firms can learn better what are the actions that will let them win the procurement, and thus they can switch from competition in prices to competition in bribes, raising the supervisor expected revenue from bribes. While this increases the expected price the government pays, the most efficient firm still wins but only through a different action. Therefore, in this signaling equilibrium with reserve price $r \geq \bar{c}$, the corruption entails no welfare loss and no allocation distortion, only redistribution from the government to the corrupt supervisor. Also, a sufficient condition for the equilibrium to exist is that the signaling cost is low enough, which is a surprisingly weak condition.

However, if the government would try to implement a maximum reserve price, potentially below the maximum cost $\bar{c}$, it can increase his expected utility by extracting a lower price bid from firms that are meeting an honest supervisor. However, they might not fully deter corruption with this price, and the optimal reserve price to fix in the signaling equilibrium is below the socially optimum. Consequently, reserve prices can help the government to keep a higher level of utility, but it is not cost free and results in total welfare loss. More specifically, it implies a lower maximum price and therefore a higher probability of declaring the procurement void.

Regarding the model assumptions, it is worth mentioning that, even though our motivating fact was that conspicuous consumption can backfire and lead to an audit, we are technically not including that probability in our model. The reason behind this is that we want to show that there are *gains* from signaling corruption, given that we already know that signaling has costs. As Propositions 4 and 8 show, there are clear revenue gains from signaling the willingness to be corrupt. Also, even though the firms can perfectly update their posterior probability of meeting a corrupt supervisor, an external audit can only find the supervisor guilty if they have evidence and proofs that he accepted bribes. As long as there is a type II error in the judicial system, the probability of being convicted will be strictly less than one. Therefore, we expect to find a non empty set of parameters for the separating equilibrium to exist. Because of this, the model is still useful, but further research can go on this direction to characterize the equilibria with an ex-post audit.

Additionally, we are considering a static or one period model with a single supervisor, which is either corrupt or honest. If we extend the model to more periods, there is also a reputational value: the supervisor might guarantee bribes for his entire lifespan by burning money once. However, in this case, the government can also learn about the supervisor type, just like firms. He could then be replaced with a new supervisor in the following period. We conjecture that a semi-separating signaling equilibrium might exist, such that the government and firms cannot be certain about the supervisor type, but can have a better understanding every additional period.

It is also worth mentioning that we consider the case with a single supervisor, thus eliminating the potential competition among public officials to run the procurement. Since increasing the competitiveness of the economy might reduce the supervisors' incentives to be corrupt (Rose-Ackerman, 1975; Drugov, 2009), it is unclear whether a signaling equilibrium can emerge under these conditions. Further research could be extended in this direction, although we conjecture that, for stricter parameter conditions about the signaling cost, a dishonest supervisor might still attract bribes from high cost firms if he consumes conspicuously.

Finally, the equilibrium described in Theorem 4 implies no allocation inefficiency, only a redistributive problem between the supervisor and the government. This is, nonetheless, a surprising result given that corruption can entail a distortion in the allocation of the project to an inefficient firm that bribes a higher bid, as the literature suggests. In this way, how can we adjust the model so that it better represents this reality? One possibility is to incorporate a second dimension in which firms distinguish from each other, that captures the difference in the bribery technology among firms. This is because firms might have to incur in different costs to send a single dollar in bribes, either to hide that from the books or to send money without getting caught. In this scenario, we expect that corruption entails an inefficiency even with an always corrupt supervisor: a firm with high production cost but very low bribery costs might offer the best deal to the public officer running the procurement process.

5 Concluding remarks

We motivated this work with some real life examples of a connection between luxury goods or conspicuous consumption and corruption from former public servants. While the losses and consequences of raising public awareness are clear, we wanted to argue that there might be actual gains from signalizing the type, such that this behavior is rationalizable.

Overall, we discussed and characterized the conditions needed for a corrupt supervisor to explicitly signal his willingness to be corrupt. We showed that, without signals, firms might only bribe if their costs are large enough, but they could potentially try to bribe an honest supervisor. Here lies the informational advantage that the supervisor can exploit: by consuming conspicuously, firms can know what is the correct action that will let them win the procurement. As a result of this, it increases bribery competition between firms and raises the supervisor's expected revenue.

Additionally, if the government tried to deter corruption by fixing a maximum budget constraint, they could improve their expected utility but not eliminate it. With a low signaling cost, the supervisor can still perfectly signal his type, and the government should set a reserve price below the socially optimal to increase his expected utility. However, this is not free and it results in a higher probability of declaring the procurement void, even when the good is socially desirable.

In essence, conspicuous consumption can act as a strong signal about a public servant's type and honesty.

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A Proofs

A.1 Proposition 1

If we consider a linear symmetric Bayesian equilibrium with $p(c_i) = ac_i + k$, then each firm solves the following problem:

$$\begin{aligned}\max_{p_i}(p_i - c_i)Pr(\text{win}|p_i) &= (p_i - c_i)Pr(p_i < p_j(c_j) = ac_j + k|p_i)\\ &= (p_i - c_i)Pr(c_j > \frac{p_i - k}{a}|p_i)\\ &= (p_i - c_i)(1 - \frac{p_i - k}{a\bar{c}})\end{aligned}$$

From the first order conditions and the symmetric bid, this results in a price bid equal to $p(c_i) = \frac{c_i}{2} + \frac{\bar{c}}{2}$. Since the bid is strictly increasing in the cost, the firm with the lowest cost will bid the lowest price and therefore wins, which proves the first item. Also, if we replace this price bid in the firm's expected utility, we get that it is equal to $u_i^H(c_i) = \frac{(\bar{c}-c_i)^2}{2\bar{c}}$, proving the second item.

Finally, the expected price that the government pays with the honest supervisor equals the expected value of the minimum bid. This is equal to $E\left[\min_{i=1,2}\{\frac{c_i+\bar{c}}{2}\}\right] = \frac{E[\min_{i=1,2}\{c_i\}]+\bar{c}}{2} = \frac{\frac{\bar{c}}{3}+\bar{c}}{2}$ and therefore equal to $\frac{2\bar{c}}{3}$. ∎

A.2 Proposition 2

Let us consider the case of a symmetric equilibrium bribe with $b(c_i) = m(r - c_i) + d$ with $m, d > 0$, which is linear in the firm's valuation. The firm i maximizes:

$$\begin{aligned}\max_{b_i}(r - c_i - b_i)Pr(b_i > b_j = m(r - c_j) + d|b_i) &= (r - c_i - b_i)Pr(\frac{b_i - d}{m} > r - c_j|b_i)\\ &= (r - c_i - b_i)\frac{\frac{b_i-d}{m} - r + \bar{c}}{\bar{c}}\\ &= (r - c_i - b_i)\frac{b_i - d - mr + m\bar{c}}{m\bar{c}}\end{aligned}$$

From the first order condition and the symmetric bid, this results in a bribe equal to $b(c_i) = \frac{r-c_i}{2} + \frac{r-\bar{c}}{2} = r - \frac{c_i+\bar{c}}{2}$, which is decreasing in the cost. Therefore, the firm with the lowest cost will submit the highest bribe and therefore wins the procurement. Also, the expected revenue for a firm with costs c_i is also equal to the revenue from the standard procurement $u_i^C(c_i) = \frac{(\bar{c}-c_i)^2}{2\bar{c}}$.

Finally, the government always pays the price r and the supervisor gets a profit equal to the expected value of the maximum bribe. Thus, $E\left[\max_{i=1,2}\{r - \frac{c_i+\bar{c}}{2}\}\right] = r - \frac{E[\min_{i=1,2}\{c_i\}]+\bar{c}}{2} = r - \frac{2\bar{c}}{3}$. ∎

A.3 Proposition 3

Let us consider a symmetric Bayesian Nash equilibrium, in which a firm bids a price if their cost is smaller than some $\underline{c} \in (0, \bar{c})$ and a bribe equal to zero, but bribes a positive quantity if their cost is larger than $\underline{c}$ and demands a price equal to r. More specifically, let us consider the case of a symmetric linear price bid and a linear bribe, conditional on the cost:

$$(p(c_i), b(c_i)) = \begin{cases} (ac_i + k, 0) & if \quad c_i < \underline{c} \\ (r, m(r - c_i) + d) & if \quad c_i \geq \underline{c} \end{cases}, \text{ with } a, k, m, d > 0 \tag{3}$$

For an honest firm with $c_i < \underline{c}$, the probability of winning, given a price bid p_i and conditional on meeting an honest supervisor (that happens with probability α), is equal to:

$$\begin{aligned}
Pr(\text{win}|p_i) &= Pr(\text{win} \cap \text{firm } j \text{ bribes}|p_i) + Pr\left(\text{win} \cap \text{firm } j \text{ bids } p_j|p_i\right) \\
&= Pr(c_j \geq \underline{c}) + Pr\left(p_i < p_j = ac_j + k \wedge c_j < \underline{c}|p_i\right) \\
&= 1 - F(\underline{c}) + Pr\left(c_j > \frac{p_i - k}{a} \wedge c_j < \underline{c}|p_i\right) \\
&= 1 - F(\underline{c}) + Pr\left(\frac{p_i - k}{a} < c_j < \underline{c}|p_i\right) \\
&= 1 - F(\underline{c}) + \left(F(\underline{c}) - F\left(\frac{p_i - k}{a}\right)\right) \\
&= 1 - F\left(\frac{p_i - k}{a}\right)
\end{aligned}$$

Therefore, the firm with $c_i \leq \underline{c}$ maximizes the problem below. Since the problem for the low cost firm is the same as the honest competition, its price bid is equal to $p(c_i) = \frac{c_i + \bar{c}}{2}$ and its expected utility is $u_i^H = \alpha \frac{(\bar{c} - c_i)^2}{2\bar{c}}$.

$$\max_{p_i} (p_i - c_i)\alpha \left(1 - F\left(\frac{p_i - k}{a}\right)\right) = (p_i - c_i)\alpha \left(1 - \frac{p_i - k}{a\bar{c}}\right)$$

For a firm with cost $c_i > \underline{c}$, the probability of winning given a bribe and conditional on meeting a dishonest supervisor (that happens with probability $1 - \alpha$) is equal to:

$$\begin{aligned}
Pr(\text{win}|b_i) &= Pr(\text{win} \cap \text{firm } j \text{ bribes}|b_i) + Pr\left(\text{win} \cap \text{firm } j \text{ bids } p_j|b_i\right) \\
&= Pr(b_i > b_j = m(r - c_j) + d \wedge c_j \geq \underline{c}|b_i) + Pr(c_j < \underline{c}) \\
&= Pr(c_j > r - \frac{b_i - k}{a} \wedge c_j \geq \underline{c}|b_i) + F(\underline{c}) \\
&= Pr\left(c_j > \max\left\{r - \frac{b_i - k}{a}, \underline{c}\right\}\right) + F(\underline{c}) \\
\text{if } \frac{ar - b_i + k}{a} \geq \underline{c} \rightarrow &= 1 - F\left(\frac{ar - b_i + k}{a}\right) + F(\underline{c})
\end{aligned}$$

Therefore, the firm with costs $c_i \geq \underline{c}$ solves the following problem:

$$\begin{aligned}
&\max_{b_i} (1 - \alpha)(r - c_i - b_i)\left(1 + F(\underline{c}) - F\left(\frac{ar - b_i + k}{a}\right)\right) - \alpha M \\
&= (1 - \alpha)(r - c_i - b_i)\frac{(a\bar{c} + a\underline{c} - ar + b_i + k)}{a\bar{c}} - \alpha M
\end{aligned}$$

The first order condition from this problem is that $r - c_i - b_i = a\bar{c} + a\underline{c} - ar + b_i + k$ and therefore, based on the proposed linear strategy, the firms bribes $b(c_i) = \frac{r - c_i}{2} + \frac{r - \bar{c} - \underline{c}}{2} = r - \frac{c_i + \bar{c} + \underline{c}}{2}$. Note that this bribe is smaller than the case where all firms bribe. Also, its expected utility is equal to $u_i^C = (1 - \alpha)\frac{(\bar{c} + \underline{c} - c_i)^2}{2\bar{c}} - \alpha M$ and note that for this bribe, the inequality $\frac{ar - b_i + k}{a} = c_i \geq \underline{c}$ holds.

In order to find the value $\underline{c}$, it must be the case that someone with costs $c_i = \underline{c}$ must be indifferent between bribing and competing honestly. Therefore, $\bar{c}$ results from the following equation:

$$\begin{gathered}
\alpha \frac{(\bar{c} - \underline{c})^2}{2\bar{c}} = (1 - \alpha)\frac{\bar{c}^2}{2\bar{c}} - \alpha M \\
\alpha \underline{c}^2 - 2\alpha \bar{c}\underline{c} + (2\alpha - 1)\bar{c}^2 + 2\alpha \bar{c} M = 0
\end{gathered}$$

This results in a quadratic equation with up to two real solutions, one less than $\bar{c}$ and one larger than $\bar{c}$, which are $\underline{c}_{1,2} = \bar{c}\left(1 \pm \sqrt{\frac{(1-\alpha)}{\alpha} - \frac{2M}{\bar{c}}}\right)$. Therefore, $\underline{c} = \bar{c}\left(1 - \sqrt{\frac{(1-\alpha)}{\alpha} - \frac{2M}{\bar{c}}}\right)$

In order to have a $\underline{c}$ in the set $(0, \bar{c})$, it must be that the root of the last equation is in that set (condition i). Additionally, the root must be real, therefore the term in the square root must be larger than zero (condition ii). Finally, since the firms expected revenue is decreasing in its costs, it must be the case that for a firm with costs $c_i = \bar{c}$, the utility of being corrupt must be at least 0 (the utility of competing honestly, in condition iii). Thus, these conditions are:

(i) $1 - \sqrt{\frac{(1-\alpha)}{\alpha} - \frac{2M}{\bar{c}}} > 0 \iff (1-2\alpha)\bar{c} < 2\alpha M \iff \alpha > \frac{\bar{c}}{2M+2\bar{c}}$.

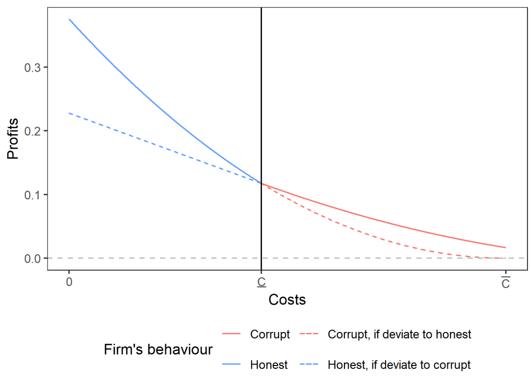
(ii) $\frac{(1-\alpha)}{\alpha} - \frac{2M}{\bar{c}} > 0 \iff (1-\alpha)\bar{c} > 2\alpha M \iff \alpha < \frac{\bar{c}}{2M+\bar{c}}$.

(iii) $\frac{(1-\alpha)\underline{c}^2}{2\bar{c}} - \alpha M \geq 0 \iff \alpha \leq \frac{\underline{c}^2}{2\bar{c}M+\underline{c}^2} \underbrace{<}_{\underline{c}\leq\bar{c}} \frac{\bar{c}}{2M+\bar{c}}$

Finally, we must check that a low cost firm does not want to bribe and that a high cost firm does not want to compete honestly. If a firm with low costs $c_i \leq \underline{c}$ decided to bribe, it will surely win the bribery game with a bribe slightly less than $b(\underline{c}) = r - \frac{2\underline{c}+\bar{c}}{2}$ with an utility equal to $u_i^{deviate} = (1-\alpha)\left(\underline{c} - c_i + \frac{\bar{c}}{2}\right) - \alpha M$. Therefore, its expected profits can be at most equal to this value, which is less than the utility of being honest $u_i^H = \frac{\alpha(\bar{c}-c_i)^2}{2\bar{c}}$. This is because both values are equal to the same value at $c_i \to \underline{c}$ (because of the definition of $\underline{c}$), but since $\frac{\partial u_i^H}{\partial c_i} < \frac{\partial u_i^{deviate}}{\partial c_i}$, the firm has no incentive to deviate. This last inequality holds because of conditions i and ii. Similarly, if a firm with high costs $c_i \geq \underline{c}$ decides to compete honestly, it would get at most an utility equal to $\frac{\alpha(\bar{c}-c_i)^2}{2\bar{c}}$, which is less than $(1-\alpha)\frac{(\bar{c}+\underline{c}-c_i)^2}{2\bar{c}} - \alpha M$ because of the listed conditions. Also, note that if $M = 0$, then all conditions hold if $\alpha \in (\frac{1}{2}, 1)$ ∎

The following figure illustrates the expected profit of a firm in a specific case, and also their incentives to deviate:

Figure A1: Expected profits of a firm



$\alpha = \frac{3}{4}$, $\bar{c} = 1$, $r = 2$ and $M = \frac{1}{100}$

A.4 Proposition 4

In the separating equilibrium, $\mu(0) = 1$ and $\mu(S) = 0$, this is, the supervisor that burns money is not honest and the one that does not consume conspicuously is honest. If that is the case, from Propositions 1 and 2 we know that firms should bid $p(c|S) = r$ and $b(c|S) = r - \frac{c+\bar{c}}{2}$ if they observe S, but should bid $p(c|0) = \frac{c+\bar{c}}{2}$ and $b(c|0) = 0$ if not.

Given the firms' strategies and beliefs, we must check that the supervisor effectively wants to signal out his type. If the corrupt supervisor burns money, he loses $\psi > 0$ but has an expected revenue from bribes equal to $r - \frac{2\bar{c}}{3} > 0$. Thus, if $0 < \psi < r - \frac{2\bar{c}}{3}$, then the supervisor is strictly better by signaling and prefers to do so.

Note that the firms have no incentives to deviate: if they decided not to bribe given $s = S$, then they will not be granted the project for sure. Also, if they do not see this signal and decided to bribe, they would pay the penalty M for sure. Additionally, the supervisor has no incentives to deviate: if the corrupt supervisor decided not to consume conspicuously, he would get no bribe for sure. If the honest supervisor decided to burn money, he would only lose $\psi > 0$, so it has no incentives to deviate.

Finally, the firm with the lowest cost wins the procurement auction: with probability α by bidding the lowest price and with probability $1 - \alpha$ by submitting the largest bribe. Therefore, the expected price with an honest supervisor is $\frac{2\bar{c}}{3}$ and with a dishonest supervisor is simply r, which proves the second item. ∎

A.5 Proposition 5

1. Given the firms belief that $\mu(0) = \alpha$, let us consider the price and bribe strategies considered in Proposition 3, to bribe only if costs are above a mid value $\underline{c}$. In order for such a strategy to characterize an equilibrium, the same set of conditions are needed. In this case, the dishonest supervisor has a positive expected revenue and pays no cost. However, in order to ensure that the supervisor has no incentives to deviate, firms must not try to bribe a supervisor, so $b(c|S) = 0$. In that case, burning money yields no expected benefit and the supervisor would be strictly worse by trying to signal his type.

2. The dishonest supervisor has no incentives to select $s = S$, since it would lose $\psi > 0$ for sure and not get anything in return. This rules out the separating and pooling equilibria where the dishonest supervisor burns money. ∎

A.6 Proposition 6

Given a reserve price r and when facing an honest supervisor, following Krishna (2009), the optimal price bid for a firm is $p(c_i) = E[\min\{c_j, r\}|c_j > c_i]$. Since $Pr(c_j \geq x|c_j > c_i) = \frac{Pr(c_i<c_j\geq x)}{Pr(c_j>c_i)} = \frac{F(x)-F(c_i)}{1-F(c_i)} = \frac{x-c_i}{\bar{c}-c_i}$ for the uniform distribution, this has a density function of $\frac{1}{\bar{c}-c_i}$. Therefore, $p(c_i) = \int_{c_i}^{r} \frac{c_j}{\bar{c}-c_i} dc_j + \int_{r}^{\bar{c}} \frac{r}{\bar{c}-c_i} dc_j = \frac{c_i+\bar{c}}{2} - \frac{(\bar{c}-r)^2}{2(\bar{c}-c_i)}$.

Since the government values the project in v, it will get an utility of $v - \min\{p_1, p_2\}$ when meeting a firm that has a cost lower than r. Therefore, the government expected utility is equal to $EU^H(r) = \int_0^r \left(v - \frac{c+\bar{c}}{2} + \frac{(\bar{c}-r)^2}{2(\bar{c}-c)}\right) \frac{2(\bar{c}-c)}{\bar{c}} dc$, which results in $EU^H(r) = \frac{\frac{4}{3}r^3 - r^2(v+2\bar{c}) + 2v\bar{c}r}{\bar{c}^2}$.

Taking the first order condition of $EU^H(r)$, the optimal reserve price r^H is such that:

$$\begin{aligned} 0 &= 4r^2 - 2r(v + 2\bar{c}) + 2v\bar{c} \\ &= 2(r - \bar{c})(2r - v) \end{aligned}$$

Therefore, $r^H = \min\{\bar{c}, \frac{v}{2}\}$, which verifies the second order condition. This results also follows directly from Result 2 of Thomas (2005) which states that the optimal reserve price r^* is such that $v = r^* + \frac{F(r^*)}{f(r^*)}$ if $r^* \leq \bar{c}$ and $r^* = \bar{c}$ otherwise. ∎

A.7 Proposition 7

Given a maximum reserve price r, firm i will set the price of $p(c_i) = r$ if $c_i \leq r$ and select a bribe $b(c_i) = E\left[\max\{0; r - c_j\}|c_j > c_i\right] = \int_{c_i}^{r} \frac{r-c_j}{\bar{c}-c_i} dc_j = \frac{(r-c_i)^2}{2(\bar{c}-c_i)}$. However, the expected revenue of the government is not related to the bribe, and it is equal to $v - r$ times the probability that one of the firms has a cost less than r. This is, $EU^C = \int_0^r (v - r)\frac{2(\bar{c}-c)}{\bar{c}^2} dc$, which results iN $EU^C(r) = \frac{(v-r)(2\bar{c}r-r^2)}{\bar{c}^2} = \frac{r^3-r^2(v+2\bar{c})+2v\bar{c}r}{\bar{c}^2}$.Taking the first order condition of $EU^C(r)$, the optimal reserve price r^C is such that $0 = 3r^2 - 2r(v + 2\bar{c}) + 2v\bar{c}$

Note that this is a quadratic equation and therefore has two roots, although it can only be applied if $r^C \leq \bar{c}$. When we evaluate it at $r = \bar{c}$ we obtain that the right hand side is equal to $-\bar{c}^2 < 0$. Since this is a convex function, the solution must be the smaller root of the quadratic equation. This is, $r^C = \frac{2(v+\bar{c})+\sqrt{4(v+\bar{c})^2-24\bar{c}}}{6}$. Additionally, if we evaluate it at $r = \frac{v}{2}$, the right hand sind of the equation is equal to $-\frac{v}{4} < 0$, which again implies that r^C is smaller than $\frac{v}{2}$, proving that $r^C < r^C$. Also, note that if $v = c$, then this is equal to $r^C = \left(1 - \frac{1}{\sqrt{3}}\right)\bar{c} < \frac{\bar{c}}{2}$ ∎

A.8 Proposition 8

Given a reserve price r, the dishonest supervisor chooses to "burn money", which reduces its utility in $\psi > 0$. When firms see this, they decide to bribe $b(c|S) = \frac{(r-c)^2}{2(\bar{c}-c)}$ if their costs are smaller than the reserve price and set a price $p(c|S) = r$. If they do not see this, then they bid a price $p(c|0) = \frac{c+\bar{c}}{2} - \frac{(\bar{c}-r)^2}{2(\bar{c}-c)}$ with $b(c|0) = 0$. In this case, firms have no incentive to deviate, since they would lose the procurement for sure if they bid honestly to a corrupt supervisor and would pay a penalty if they bribe an honest supervisor.

Note that the supervisor expected revenue depends on this bribe. Since $b(c|S)$ is negatively related to the cost c, the supervisor is looking for the minimum value c which provides the largest bribe and therefore his expected revenue is equal to $\int_0^r \frac{(r-c)^2}{2(\bar{c}-c)} \frac{2(\bar{c}-c)}{\bar{c}^2} dc = \frac{r^3}{3\bar{c}}$. Therefore, given the firms belief, the dishonest supervisor effectively wants to signal his type if there is a positive expected benefit from it, this is as long as $\frac{r^3}{3\bar{c}} > \psi > 0$. In that case, neither the dishonest supervisor has incentive to deviate (because the signal provides a positive increase in his expected utility) nor the honest supervisor, because it would only burn money but would not get any of the benefit.

Finally, given the supervisor and firms strategies, the government has an expected utility equal to the revenue that it would get with each type of supervisor. Therefore, its expected utility is equal to $EU^S(r) = \alpha EU^H(r) + (1-\alpha)EU^C(r) = \frac{\alpha\bar{c}}{3} + \frac{r^3-r^2(v+2\bar{c})+2v\bar{c}r}{\bar{c}^2}$. If $\alpha = 0$ then $EU^S(r) = EU^C(r)$ and if $\alpha = 0$ then $EU^S(r) = EU^H(r)$, and since this is a cubic equation, the solution must lie between the solutions $r^S(\alpha = 1) = r^H$ and $r^S(\alpha = 0) = r^C$. Therefore, $r^S \in \left(r^C, r^H\right)$. ∎