ELEC5460 Assignment1

February 2023

Problem 1

Suppose that a mobile station is moving along a straight line between base stations BS_1 and BS_2, as shown in Figure . The distance between the base stations is $D = 1600$ m. The received power (in dBm) at base station i, from the mobile station, is modeled as (reverse link)

$$P_{r,i}(d) = P_0 - 10n \log_{10}(d_i/d_0) + \chi_i, \quad (dBm), \quad i = 1, 2.$$

where d_i is the distance between the mobile and base station i, in meters, P_0 is the received power at distance d_0 from the mobile antenna, and n is the path loss exponent. The term $P_0 - 10\gamma \log_{10}(d_i/d_0)$ is usually called *local area mean power*. The terms χ_i are zero-mean Gaussian random variables with standard deviation σ, in dB, that model the variation of the received signals due to shadowing. Assume that the random components χ_i of the signals received at different base stations are independent of each other. n is the path loss exponent.

The minimum usable signal for acceptable voice quality at the base station receiver is $P_{r,min}$, and the threshold level for handoff initiation is $P_{r,HO}$, both in dBm.

Assume that the mobile is currently connected to BS_1. A handoff occurs when the received signal at the base station BS_1, from the mobile, drops below threshold $P_{r,HO}$, and the signal received at candidate base station BS_2 is greater than the minimum acceptable level $P_{r,min}$. Using the parameters in Tabel 1, determine:

Parameter	Value
n	4
σ	6 dB
P_0	0dBm
d_0	1 m
$P_{t\ \min}$	-118dBm
P_{rHO}	-112dBm

Table 1: Mobile moving along straight line between BS_1 and BS_2

(a) The probability that a handoff occurs (Pr[handoff]), as a function of the distance between the mobile and its serving base station. Show your result in a plot Pr [handoff] vs. distance d_1.

(b) The distance d_{ho} between base station BS_1 and the mobile, such that the probability that a handoff occurs is equal to 80%.

Problem 2

Consider a SISO communication link under flat fading channel. The received signal Y is given by:

$$Y = HX + Z$$

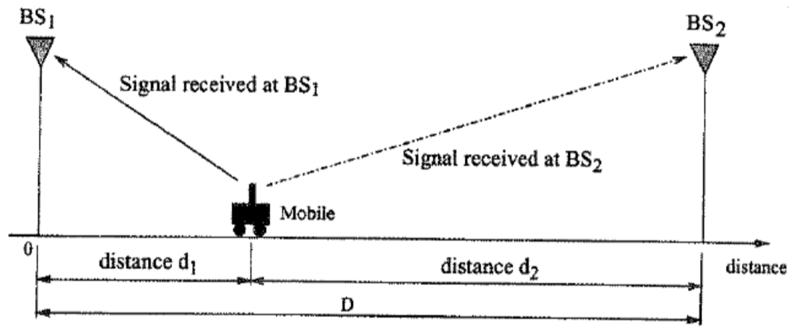


Figure 1: Problem 1(a)

where X is the transmitted symbol, H is the complex Gaussian channel fading and Z is the channel noise. Assume that an encoding frame spans across ergodic realization of channel fading. The channel fading is memoryless across symbols and is available to the receiver only. The transmitted symbol satisfies the average transmit power constraint:

$$\mathbb{E}[|X|^2] \leq P_{tx}$$

and the channel noise satisfies the following constraints:

$$\mathbb{E}[|Z|^2] = 1, \ \mathbb{E}[ZX^*] = 0, \ \mathbb{E}[ZH^*] = 0$$

(a) Define C_{worst} as the worst case ergodic channel capacity, which is the minimum of the SISO link capacity (over all possible noise distribution $p(Z)$ satisfying the three aforementioned constraints.) That is:

$$C_{worst} = \min_{p(Z)} \{ \max_{p(X)} I(X; Y|H) \}$$

Prove the following inequality

$$C_{worst} \leq \mathbb{E}[\log_2(1 + P_{tx}|H|^2)]$$

(b) Prove the following equation:

$$\text{cov}(X \mid HY) = \mathbb{E}\left[|X - \mathbb{E}(X \mid HY)|^2)\right] \leq \sigma_X^2 - \sigma_{XY}^2 \sigma_Y^{-2} \sigma_{YX}^2$$

(c) Using results in (b), prove the following two inequalities

$$H(X \mid HY) \leq \mathbb{E}\left[\log_2\left(\frac{\pi e \sigma_X^2}{(1 + |H|^2 \sigma_X^2)}\right)\right]$$
$$C_{\text{worst}} \geq \mathbb{E}\left[\log_2\left(1 + P_{tx}|H|^2\right)\right]$$

Hence, show that the worst-case channel noise is complex Gaussian.

Problem 3

(a)Consider the coherent detection of BPSK in the Rayleigh fading channel, where the received signal is given by (for convenience, we drop the time index)

$$y = hx + n$$

where $h \sim CN(0,1)$ and $n \sim CN(0, N_0)$.

(i)Show that the error probability is

$$p_e = E\left\{Q\left(\sqrt{2|h|^2 SNR}\right)\right\} = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{1+SNR}}\right)$$

where $SNR = Ex^2/N_0$.

(ii) Find $\lim_{SNR\to\infty} p_e \cdot SNR$.

(b)We investigate the use of the repetition coding to obtain the diversity. The system model is given by

$$y = hx + n$$

where $\boldsymbol{y} = [y_1, y_2, \ldots, y_L]$, $\boldsymbol{h} = [h_1, h_2, \ldots, h_L]$, $\boldsymbol{n} = [n_1, n_2, \ldots, n_L]$, and $x = \pm a$ is the BPSK symbol. It is assumed that $h_i \sim \mathrm{CN}(0,1)$ the component n_i is i.i.d. AWGN noise with $n_i \sim CN(0, N_0)$.

(i)Show that given a particular realization of $\boldsymbol{h}$, the error probability is given by

$$p_{e|\boldsymbol{h}} = Q\left(\sqrt{2|\boldsymbol{h}|^2 SNR}\right)$$

where $SNR = Ex^2/N_0$.

(ii)Suppose MRC is used, Show that

$$p_e = \left(\frac{1-\mu}{2}\right)^L \sum_{l=0}^{L-1} \binom{L-1+l}{l} \left(\frac{1+\mu}{2}\right)^l$$

or equivalently show

$$p_e = \frac{1}{2} - \frac{\mu}{2} \sum_{k=0}^{L-1} \binom{2k}{k} \left(\frac{1-\mu^2}{4}\right)^2$$

where $\mu = \sqrt{\frac{SNR}{1+SNR}}$.

Hint: $\|h\|^2$ is Chi-square distributed with 2 L degrees of freedom, and the density is given by

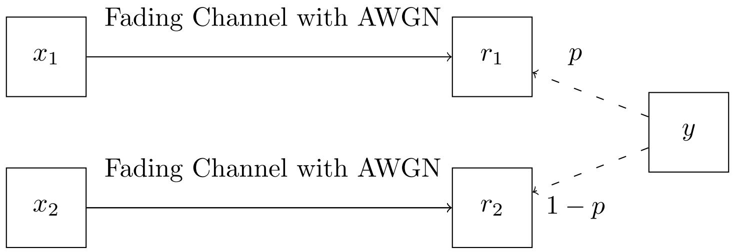
$$f(x) = \frac{1}{(L-1)!} x^{L-1} e^{-x}, \quad x \geq 0.$$

(iii) Show that for high SNR, $p_e \approx \binom{2L-1}{L} \frac{1}{(SNR)^L}$ (Hint: you may use a fact in high school mathematics $\binom{a}{b} + \binom{a}{b+1} = \binom{a+1}{b+1}$ in relating the term in (ii) to some desired terms in (iii).)

Problem 4

A two-input (x_1, x_2) and one-output (y) channel is given by the figure where the switch toggles between r_1 and r_2 randomly with probability p (to be in p_1) and $1-p$ (to be in r_2). The fading channel between x_1, r_1 and x_2, r_2 are given by $r_1 = H_1 x_1 + N_1$ and $r_2 = H_2 x_2 + N_2$, where H_1 and H_2 are the random fading coefficients $H_1 \sim \mathcal{CN}(0, \sigma(|H_1|^2))$, $H_2 \sim \mathcal{CN}(0, \sigma(|H_2|^2))$ and the two AWGN noise terms $N_1 \sim \mathcal{CN}(0, \sigma^2_{N_1})$, $N_2 \sim \mathcal{CN}(0, \sigma^2_{N_2})$.

In answering the following questions we may assume the above channel is operating at high SNR regime, both the transmitter and the receiver know the statistics on the channel.



(a) Express analytically the above situation into a canonical channel model with inputs $(x1, x2)$ and output y and a certain random state variable. Identify what is the state.

(b) Suppose the overall channel is ergodic now, the receiver knows about the instantaneous position of the switch and the corresponding instantaneous channel fading coefficient of the channel in contact. Find the ergodic capacity of the above channel with an average transmit power constraint $P0$. If there is no closed form expression, derive an algorithm so that it can converge to the ergodic capacity.

(c) Repeat (b) if besides the receiver knows about the instantaneous position of the switch and the corresponding instantaneous channel fading coefficient of the channel in contact, the transmitter also knows about the switch position (but not the instantaneous channel fading coefficient)?

(d) Repeat (b) if besides the receiver knows about the instantaneous position of the switch and the corresponding instantaneous channel fading coefficient of the channel in contact, the transmitter knows about the instantaneous channel fading coefficient (but not the switch position)?

(e) Repeat (c) if further transmitter knows both the switch position and the corresponding instantaneous channel fading coefficient of the channel in contact.

(f) Assuming we have high SNR, compare the result in (c) and (d), which piece of information (the switch position or the instantaneous fading coefficient) is a more important side information at the transmitter. Explain your choice. How much additional benefit we can obtain if the transmitter knows about both the switch position and the fading coefficient?

(g) Suppose the overall channel is not ergodic now and the transmitter always transmit with a fixed rate R regardless of the CSIT. Usng the framework in (f), show analytically which piece of information knowledge at the transmitter (switch position or the fading coefficient) is more important.