ASSIGNMENT 2 - ANSWER

① Question 7 Answer - discrete Uniform Distribution

$i = 1,2,3,4$

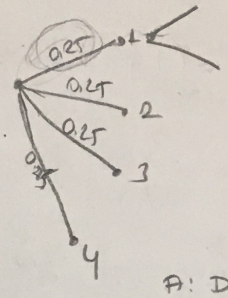
$P(X=1) = 0.25$ X: find file drawer

$P(X=2) = 0.25$

$P(X=3) = 0.25$

$P(X=4) = 0.25$

$P(X \neq 1) = 0.75$ → 1. çekmecede olmama olasılığı



$f_{x_i}(x) = \frac{1}{4}$

A: Dosyanın 1. çekmecede olması durumu

B: Dosyanın 1. çekmecede bulunmaması durumu

$P(A \cap B)$ → dosyanın 1. çekmecede olup bulunmaması $= \frac{3}{4} \times \frac{1}{4} = \frac{3}{16} = 0.1875$

② Question 8 Answer

a) Binomial Distribution

b) $q = 0.001$ fail situation

$p = 0.999$ not fail situation

$P(n \geq 100) = 1 - P(n < 100)$

$= 1 - \sum_{k=1}^{99} \binom{k}{1} (0.001)^{99-k} (0.999)^{k}$

c) q → fail prob.

p → not fail prob → 4 engine at least 2

$P(X \geq 2) = P(X=2) + P(X=3) + P(X=4) = \binom{4}{2} p^2 (1-p)^2 + \binom{4}{3} p^3 (1-p)^1 + \binom{4}{4} p^4 (1-p)^0$

$P(X \geq 2) = 6p^2(1-p)^2 + 4p^3(1-p) + p^4$ = four engine success flight

2 engine at least + 1

$P(X \geq 1) = P(X=1) + P(X=2) = \binom{2}{1} p^1 (1-p)^1 + \binom{2}{2} p^2 (1-p)^0 = 2p(1-p) + p^2$ → 2 engine plane prob.

$6p^2(1-p)^2 + 4p^3(1-p) + p^4 \geq 2p(1-p) + p^2$ → divide p both side

$6p(1-p)^2 + 4p^2(1-p) + p^3 \geq 2(1-p) + p$

$6p(1+p^2-2p) + 4p^2 - 4p^3 + p^3 \geq 2 - 2p + p$

$6p + 6p^3 - 12p^2 + 4p^2 - 3p^3 \geq 2 - p$

$3p^3 - 8p^2 + 6p + p - 2 \geq 0 \Rightarrow 3p^3 - 8p^2 + 7p - 2 \geq 0$

$(3p-2)(p-1)^2 \geq 0$ → $3p - 2 \geq 0$

↳ $p \neq 1$

$p \geq 2/3$ → 4 engine plane is safer when engine success bigger than 2/3.

d) at least 0.75 $4 \times \frac{3}{4} = 3$, $p = 0.001$

$P(X \geq 3) = P(X=3) + P(X=4) = \binom{4}{3} p^3 q^1 + \binom{4}{4} p^4 q^0 = 4p^3 + p^4$

Y → # of successful flights next 100 flights $\Rightarrow f(y) = \binom{n}{y} p^y q^{n-y}$.

$E[Y] = np \Rightarrow 100 * 0.001 = 0.1$

$V[Y] = np(1-p) = npq = 100 \times 0.001 \times 0.999 = 0.0999$

QUESTION 11

A: GS kazanma durumu

$P(A) = 0.2$

a) $(0.8)^{16}$

b) $(0.8)^{x}(0.2)$ → x yıl kazanmayacak, sonra kazanacak.

$E[x] = \mu = \frac{1}{p} \Rightarrow \frac{1}{0.2} = 5$

geometric distribution

"İlk başarının x. inci denemede gerçekleşme olasılığı"

c) 2019 − 1998 = 21 yıl

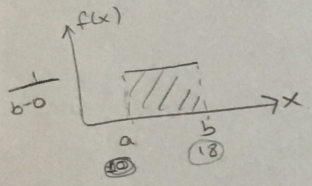
There is no difference part b. Because game's result independent. and average case calculated $1/p$ so only win's probability is important.

QUESTION 13

X : # of newspaper sold. → [10, 18]

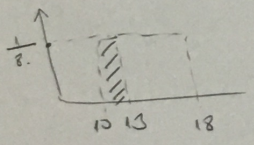
cost of newspaper : 0.8 TL
sold at " " : 1 TL
) profit for one newspaper when it sold : +0.2 TL

loss for one newspaper when it couldn't sold : −0.1 TL (0.8 − 0.7)



$$f(x) = \begin{cases} \frac{1}{18-10} & 10 \le x \le 18 \\ 0 & \text{otherwise} \end{cases}$$

$P(X=13) =$ area



$= \frac{1}{8} * (13-10) = \frac{3}{8} = 0.375$

0.375 olasılıkla satılacak

$\mu = E(x) = \frac{a+b}{2} = \frac{18+10}{2} = 14$ expected value

$0.375(0.2) - (0.625)(0.1)$

$0.075 - 0.0625 = 0.0125$

QUESTION 15

$\mu = 100$

$\sigma^2 = 40$, $\sigma = \sqrt{40} = 6.3245$

$Z = \frac{X-\mu}{\sigma}$

$P(x < 90) = P\left(\frac{X-100}{6.3245} < \frac{90-100}{6.3245}\right)$

$P(z < -1.5811) = 0.05705$

QUESTION 16

$100 → no accident during past 2 years.
$300 → accident during only one of past 2 years.
$400 → accident during both of past 2 years.

Markov Chain Model

if a customer has an accident last year,

$P(\text{customer has acc. current year} \mid \text{customer has an acc. last year}) = 0.10$

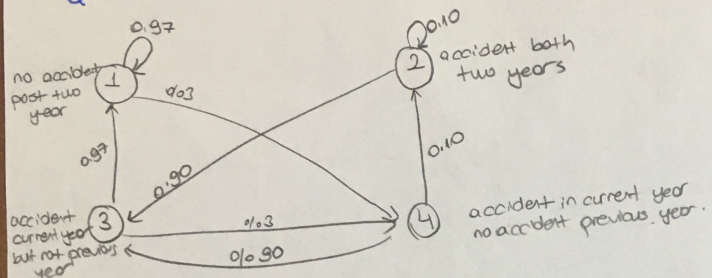
$P(\text{customer has an acc. current year} \mid \text{customer has no acc. last year}) = 0.03$

4 states

1) No accidents during the current & previous year
2) Accident during both current & previous year
3) Accident during current year, but not previous year
4) Accident during previous year, but not current year

Question 16 - devaM11 - Transition Probabilities

$1 \to 1 - 0.03 = 0.97$



$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.97 & 0 & 0 & 0.03 \\ 0 & 0.10 & 0.90 & 0 \\ 0.97 & 0 & 0 & 0.03 \\ 0 & 0.10 & 0.90 & 0 \end{bmatrix} \end{array} \begin{array}{l} \to 1 \\ \to 1 \\ \to 1 \\ \to 1 \end{array}$$

Steady state dist: $\pi = \pi P \Rightarrow [\pi_1\ \pi_2\ \pi_3\ \pi_4] = [\pi_1\ \pi_2\ \pi_3\ \pi_4] \begin{bmatrix} 0.97 & 0 & 0 & 0.03 \\ 0 & 0.1 & 0.9 & 0 \\ 0.97 & 0 & 0 & 0.03 \\ 0 & 0.1 & 0.9 & 0 \end{bmatrix}$

$\pi_1 = 0.97\pi_1 + 0.97\pi_3 \Rightarrow 0.03\pi_1 = 0.97\pi_3$

$\boxed{3\pi_1 = 97\pi_3}$

$\pi_2 = 0.1\pi_2 + 0.1\pi_4$

$\pi_3 = 0.9\pi_2 + 0.9\pi_4$

$\pi_4 = 0.03\pi_1 + 0.03\pi_3$

$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$

$0.9\pi_2 = 0.1\pi_4$

$\boxed{9\pi_2 = \pi_4}$

$\pi_1 = 97k$
$\pi_3 = 3k$
$\pi_4 = 3k$
$\pi_2 = \frac{3k}{9}$

$\pi_4 = 0.03(\pi_1 + \pi_3)$

$\pi_4 = 0.03(100k) = \boxed{3k}$

$\pi_2 = 0.1\pi_2 + 0.1\pi_4$

$0.9\pi_2 = 0.1\pi_4$

$9\pi_2 = \pi_4 \to 3k$

$\boxed{\pi_2 = \frac{3k}{9}}$

$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$

$103k + \frac{3k}{9} = 1$

$930k = 9$

$k = 9/930$

State	Prob	premium	
1	0.93870968	$100	93.870968
2	0.00322581	$400	1.2903226
3	0.02903226	$300	8709 6774
4	0.02903226	$300	~~8.709677~~4
			$112.58

The average premium is $112.58

$\Leftarrow$

$\pi_1 = 97k = 97 * \frac{9}{930} = 0.93870968$

$\pi_2 = \frac{3k}{9} = \frac{3}{9} * \frac{9}{930} = 0.00322581$

$\pi_3 = 3k = 0.02903226$

$\pi_4 = 3k = 0.02903226$