Assignment-2 due 6.11.2020 11:59pm

(You are expected to hand-in (upload to LMS) the solutions for <u>Questions: 7, 8, 11, 13, 15 and 16</u>)

Question 1: Suppose we have a family with four children. Assuming that having a girl or a boy is equally likely, answer the following:

a) List all the possible outcomes for the four children.
b) What is the probability of having two girls and two boys?
c) What is the probability of having at least one boy?
d) What is the probability that three of the children are boys, given that at least two of them are boys?

Question 2: Suppose three people drop their hotel room keys on a coffee table. The keys are mixed up and everybody selects a key randomly. What is the probability that nobody gets their own keys?

Question 3: Assume we have two urns. The first urn contains 3 white and 6 black balls; whereas the second has 4 white and 7 black balls. To select an urn, we toss a coin. If the result of the coin toss is heads, then we select the first urn; otherwise second urn is selected. What is the conditional probability that the outcome was heads given that a white ball is selected?

Question 4: Let 0.5 be the probability that you know the answer of a given question in a test and (1-0.5=0.5) is the probability you guess the answer. If you guess the answer, you will be correct with probability 0.2 (assuming there 5 multiple choice alternatives for each question). What is the probability that you knew the answer given that you answered it correctly?

Question 5: A blood test is 90% effective in detecting a disease when the disease is present. On the other hand, with 0.01 probability, the test concludes that the person has the disease, when, in fact, she does not (in other words, the test finds a healthy person ill with 0.01 probability). If 10% of the whole population actually has the disease, what is the probability that a person has the disease given that the test concludes positive?

Question 6: Let X be a random variable and b, c are given constant parameters such that $b, c \in R$. Define $Y = cX + b$ as a new random variable. Find $E[Y]$ and $Var[Y]$ in terms of $E[X]$, b and c.

Question 7: Suppose a certain file is equally likely to be in any of the four drawers in your desk. Let 0.25 be the probability that you will find your file upon an examination of drawer i if the file is in fact in drawer i such that i = 1, 2, 3, 4. Suppose you looked in drawer 1 and cannot find the file. What is the probability that the file is in drawer 1?

Question 8: Suppose an airplane engine will fail, when in flight, with probability p independently from engine to engine and independently from flight to flight.

a- Suppose X is the number of engines remaining operative. Which distribution does X follow?
b- If the airplane has one engine and p is 0.001; what is the probability that plane will survive at least 100 flights?
c- Further suppose that the airplane will make a successful flight if at least 50% of its engines remain operative. For what values of p is a four engine plane preferable to a two-engine plane?

d- Assume p=0.001 for a four engine plane and assume the airplane will make a successful flight if at least 75% of its engines remain operative. Let *Y* be the random variable that represents number of successful flights that plane had in the next 100 flights. Write down the characteristics of variable Y (i.e. its f(Y), F(Y), E[Y] and V[Y])

Question 9: Suppose there are three sections of Calculus 101 course. The first section has 55 students, the second section has 65 and the third has 30 students. If we select a student randomly, on the average, how many students will be on his/her class?

Question 10: Suppose *X* is a continuous random variable and follows a uniform distribution within the range [p, q]. Show that:

a- The variance of *X*, V[*X*] = $\frac{(q-p)^2}{12}$

b- The cumulative distribution function, CDF, $F(X=u) = \begin{cases} 0, & x < p \\ \frac{u-p}{q-p}, & p \leq x < q \\ 1, & x \geq q \end{cases}$

c- Assume *p* =10 and *q* =20. Then what is the probability that X$\leq$ 17?

Question 11: Suppose in a soccer game between GS and FB at Saraçoğlu stadium, GS would win with *p*=0.2. Further assuming that both teams play only once in a year at Saraçoğlu and each game's result is independent of each other, answer the following questions:

a- As you might know, GS had their last in 1998 at Saraçoğlu (no wins in the past 16 games at Saraçoğlu), what is the probability of observing such a streak of 16 years?
b- Assume that year is 1998 and we're interested the very first win of GS at Saraçoğlu. What is the average number of years we have to wait for the first win?
c- Now assume the year is 2019 and GS had no wins since 1998. What is the average number of years we have to wait for the first win? Compare your result with the result in part b and discuss why there is a difference (or no difference) between the answers.

Question 12: Suppose in a hamburger restaurant customers order a burger in every two minutes. Assume that the number of burgers ordered follows a Poisson distribution.

a- Find the probability that exactly 60 burgers are ordered between 8pm-10pm.
b- Find the mean and standard deviation of number of burgers ordered between 9pm-1am.
c- Find the probability that the time between two consecutive orders is between 1 and 3 minutes.

Question 13: Suppose the number of sales of a certain newspaper in "bakkal" follows a continuous uniform distribution, such that $X \in [10,18]$, where *X* represents the number of newspapers sold. Assume a single newspaper costs 0.80 TL and sold at 1TL. If a newspaper cannot be sold, then it is returned to the distributer for 0.70TL. What is the expected profit for tomorrow, if the guy working at the "bakkal" decides to order 13 newspapers for tomorrow?

Question 14: Show that the CDF of exponential distribution is, $F(x) = 1 - e^{\theta x}$ where θ is the rate of the distribution is.

Question 15: Assume that the daily demand for a certain newspaper in a newspaper stand follows a normal distribution with mean 100 and variance 40.

a- Find the probability that the number of papers that will be sold tomorrow is less than 90?
b- Find the probability that the number of papers that will be sold tomorrow is between 90 and 110? Could you compute the probability in part b just using your result in part a? Why, why not?

Question 16: An insurance company charges a customer according to his or her accident history. A customer who has had no accident during the last two years is charged a \$100 annual premium. Any customer who has had an accident during each of the last two years is charged a \$400 annual premium. A customer who has had an accident during only one of the last two years is charged an annual premium of \$300. A customer who has had an accident during the last year has a 10% chance of having an accident during the current year. If a customer has not had an accident during the last year, there is only a 3% chance that he or she will have an accident during the current year. During a given year, what is the average premium paid by a Payoff customer?