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Multi-commodity flow network model of the flight gate assignment problem

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ABSTRACT

The flight gate assignment problem is encountered by gate managers at an airport on a periodic basis. This assignment should be made so as to balance carrier efficiency and passenger comfort, while providing buffers for unexpected events that cause assignment disruptions. In this paper, a binary integer multi-commodity gate flow network model is presented with the objective of minimizing the fuel burn cost of aircraft taxi by type and expected passenger discomfort for "tight" connections as a function of inter-gate distance and connection time. This approach is shown to be computationally efficient within a decomposition approach for large problem instances. A numerical application of this approach is given for the gating of Continental Airlines at George W. Bush Intercontinental Airport in Houston (IAH).

1. Introduction

Gates are one of the important resources at the disposal of an airline in a hub-spoke airport, and with an air-traffic increase in recent years (nearly doubled since early 1980s), the need to efficiently use these to reduce operating costs, increase passenger satisfaction, and alleviate congestion has become more prevalent in modern times. It is clear that there are straightforward integer optimization models that could be used to make these assignments while minimizing several objectives of interest, yet these become impractical in even moderately scaled problems, which are not reflective of large-carrier operations. A model that balances the viewpoints of the consumer and carrier in making assignments that can be solved in reasonable time could have a broad impact on airline industry.

A survey paper by Dorndoff, Drexl, Nikulin, and Pesh (2007) provides a variety of mathematical models and up-to-date solution techniques that have been developed for the flight gate scheduling problem, yet it is interesting to examine the evolution of work in this area as it follows that of the airline industry. In particular, initial research into the flight gate assignment problem was focused solely on the passenger, particularly minimizing walking distances within an airport for departing passengers to departure gates or the distance to baggage claim for arrival passengers, etc. In a seminal paper, Braaksma (1977) demonstrated the possibility of reducing the walking distance through an assignment policy without changing the layout of the airport terminal, which resulted in an average distance reduction of over 100 ft per passenger. Babic, Teodorovic, and Tosic (1984) formulated a linear integer mathematical model to minimizing walking distances for both arriving and departing airplane passengers simultaneously, which they subsequently solved using branch and bound methodology, upon which (Mangoubi & Mathaisel, 1985) provided a similar formulation that was solved using a linear relaxation. Yan and Chang (1998) further extended this vein of work through a multi-commodity flow network formulation of the walking distances problem with a solution based on a Langrangian relaxation, which was shown to be much more efficient when compared to that of Mangoubi and Mathaisel (1985). Several researchers have used quadratic formulations to model and solve the gate assignment problem with an objective that includes the distances of terminating passengers to baggage claim in addition to those connecting, including Haghani and Chen (1998), Xu and Bailey (2001), Ding, Lim, Rodrigues, and Zhu (2005) using different types of heuristics. Recently, Jaehn (2010) developed a dynamic programming approach around a passenger preference metric that solves in linear time with respect to number of flights.

With increasing number of air traffic over time, however, research emphasis in gate modeling shifted from concentration on the passenger alone to that which addressed flight delay, with a notable early paper in this area by Wirasinghe and Bandara (1990). Following along this line, Bard, Yu, and Argello (2001) developed a transformed time-based network formulation that captures flight delays and cancelation costs for each flight in making gate assignments, and Yan and Tang (2007) developed a heuristic approach for airport gate assignments that considers stochastic flight delays. Recently, Dorndoff, Jaehn, and Pesh (2010) presented

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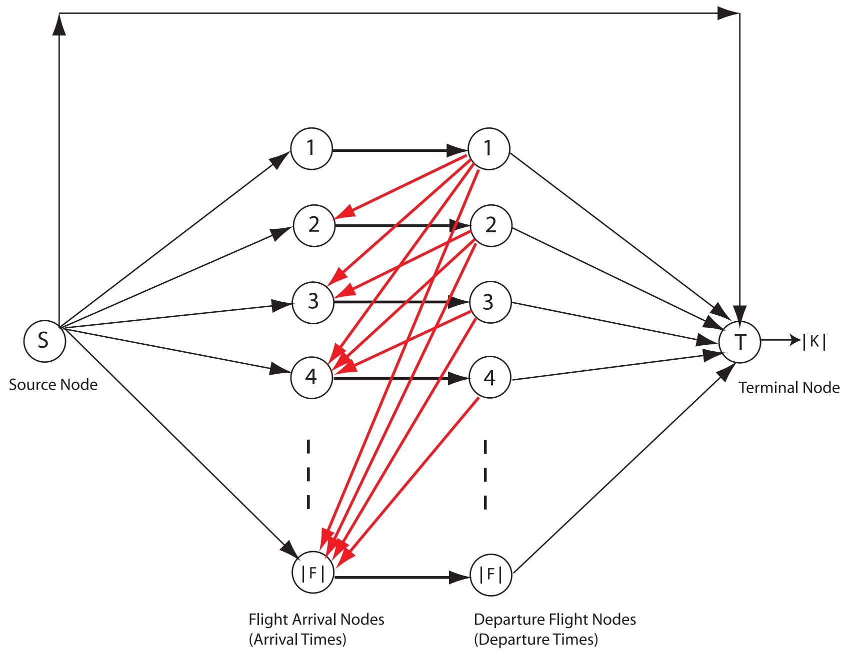

Fig. 1. Multi-commodity network model for flight gate assignment problem.

a model and heuristic solution approach based on the clique partitioning problem that robustly optimizes several detailed facets of aircraft turn. There have been several authors that have considered multi-objective or weighted formulations that balance metrics related to both passenger comfort and carrier efficiency, which are often competing. Along these lines, Drexl and Nikulin (2007) used simulated annealing methods in a formulation that minimized both the number of flights not assigned to a gate and the total passenger walking distance while maximizing a user preference metric. More recently, Hu and Paolo (2007) demonstrated a genetic algorithm approach to a multi-objective problem that minimizes passenger walking distance, baggage transport distance, and the time spent by aircraft waiting for a gate, and Zheng, Hu, and Deng (2011) used tabu search methods in a formulation that maximizes gate occupied efficiency in terms of utilization, space, and time and minimizes passenger walking distance.

In this paper, we develop a binary multi-commodity network flow formulation of a weighted gate assignment problem that minimizes both passenger discomfort for "rushed" connections and fuel burn costs of aircraft taxi. These competing objectives logically follows as the discomfort passengers experience by walking (or running) a given distance between gates is a function of how "tight" the connections is going to be, and from the airline perspective, the cost of unnecessary fuel burn is a real dollar loss. This formulation and solution approach is shown to be computationally efficient, and we show how this may be applied in a practical manner for large-scale problems through decomposition based on airport zoning strategies. In particular, gates are treated as a commodity that originate at a source node and flow through various arcs and intermediate nodes that represent flight demand on its way to the terminal node. The routing of these flows satisfy flight demands while simultaneously minimizing taxi in and out fuel burn costs and a non-linear two dimensional cost function with arguments that represent the distance connecting customers traverse and the time between these connections. As such gate assignment models are used for making static assignments, and not real time gate reassignments in response to aircraft delays, the expectation of passenger connections is used based on historical data. Notwithstanding, buffers to compensate for variability in planes arriving and departing from gates are built into the constraints of the model to alleviates excessive day-of reassignments and idling of aircraft for blocked gates.

2. Model

2.1. Mathematical notation

The mathematical notation that will be used throughout this paper is as follows.

2.1.1. Sets

$\mathbf{F}$	Set of integers mapped to the aircraft tail number of sequential flight landings
$\mathbf{K}$	Set of integers mapped to available gates
$\mathbf{A} = \mathbf{F}$	Set of arriving aircraft
$\mathbf{D} = \mathbf{F}$	Set of departing aircraft

2.1.2. Nodes

S	Source node
T	Terminal node

2.1.3. Parameters

A_i	Flight arrival time corresponding to aircraft $i \in \mathbf{A}$
D_j	Flight departure time corresponding to aircraft $j \in \mathbf{D}$
C_k	Fixed cost associated with gate k, $k \in \mathbf{K}$
d_{k1}	Expected runway distance from arrival runway to gate $k1 \in \mathbf{K}$
d_{k2}	Expected runway distance from departure runway to gate $k2 \in \mathbf{K}$

$d_{kk'}$	Intergate/interzone distance (ft) where $k \in \mathbf{K}$ to gate $k' \in \mathbf{K}$ s.t. $k \neq k'$
f_i	Fuel burn rate (kg/s) for flight $i \in \mathbf{F}$
f_s	Average runway speed for an aircraft (ft/s)
f_c	Fuel cost ($/kg)
p	Buffer time between sequential flight assignments to a gate (h)
$C_{kk'}^{ii'}$	Per passenger cost for transferring passenger from gate $k \in \mathbf{K}$ to gate $k' \in \mathbf{K}$ s.t. $k \neq k'$
N'_{ii}	Number of passenger transfer from flight $i \in \mathbf{F}$ to flight $i' \in \mathbf{F}$ s.t. $i \neq i'$
$\lvert\mathbf{K}\rvert$	Cardinality of set $\mathbf{K}$
$\lvert\mathbf{F}\rvert$	Cardinality of set $\mathbf{F}$

2.1.4. Decision variables

X_{Si}^k	Binary variable representing initial assignment of gate $k \in \mathbf{K}$ to aircraft $i \in \mathbf{F}$
X_{ji}^k	Binary variable representing assignment of gate $k \in \mathbf{K}$ to aircraft $j \in \mathbf{D}$ followed by $i \in \mathbf{A}$

2.1.5. Other variables

X_{jT}^k	Binary variable representing last assignment of gate $k \in \mathbf{K}$ to aircraft $j \in \mathbf{D}$
X_{ST}^k	Binary variable representing no assignment of gate $k \in \mathbf{K}$ to any aircraft

2.2. Mathematical formulation

An initial mathematical formulation of the network flow model, as shown in Fig. 1, is given as follows.

$$\text{Min } Z = \sum_{i \in A} \sum_{k \in K} (C_k + 2f_i f_c / f_s \{d_{k1} + d_{k2}\}) \left(X_{s,i}^k + \sum_{j \in D} X_{j,i}^k \right) + \sum_{\substack{(i \in F, i=j, \\ i'=j' i \neq i' \\ j \neq j')}} \sum_{\substack{(i' \in F, i=j, \\ ,i'=j' i \neq i', \\ j \neq j')}} \sum_{k \in K} \sum_{k' \in K} \tag{1}$$

Subject to

$$\sum_i X_{s,i}^k + X_{S,T}^k = 1 \quad \forall k \in K, \quad i \in F \tag{3}$$

$$X_{s,i}^k + \sum_j In_{ji} X_{j,i}^k = X_{m,n}^k \quad \forall i,\ m \in A, j,\ n \in D, \quad i \neq j,\ m = n \tag{4}$$

$$\sum_j In_{ji} X_{j,i}^k + X_{j,T}^k = X_{m,n}^k \quad \forall i,\ m \in A, j,\ n \in D, \quad i \neq j,\ m = n \tag{5}$$

$$\sum_j X_{jT}^k + \sum_k X_{ST}^k = 1 \quad i \in F, \quad k \in K \tag{6}$$

$$\sum_k X_{ij}^k = 1 \quad \forall i \in A, \quad j \in D,\ i = j \tag{7}$$

$$X_{s,i}^k, X_{j,i}^k, X_{jT}^k, X_{m,n}^k, X_{S,T}^k = \{0, 1\} \tag{8}$$

Note that the indicator function In_{ji} is equal to 1 if and only if the arrival time corresponding to aircraft i is greater than departure time corresponding to aircraft j plus some buffer p, i.e.

$$In_{ji} = \begin{cases} 1 & \text{if } (A_i - D_j) \geqslant p \quad \forall i \neq j \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

In other words, if $In_{ji} = 1$, then an arc exits which allows aircraft j and i to share the same gate since the departure time of aircraft j is at least p hours before the arrival time of aircraft i. Note also that infeasibility of the model may occur if either the number of flights on the ground at a particular time exceeds the number of available gates or the buffer time p for which a gate is idle between sequential flight assignments is too large.

2.2.1. Objective function

The objective function represents a linear combination of costs associated with fuel burn related to the taxi in and out of planes and the "discomfort" of passengers connecting to flights in other gates. In particular, the first term of Eq. (1) represents the *expected* taxi in and out fuel burn cost of assigning a plane to a particular gate based on the expected runway distance corresponding to arrival and departure cities for the flight. Note that most airports have multiple runways and those used vary depending upon the direction of approach and departure of the aircraft, and the cost of fuel burn varies depending on aircraft type. Due to constraint Eq. (4) and the unit capacity of arcs connecting each node i, this fuel burn objectives summed over all arrival flight nodes and available gates gives the total taxi in fuel burn cost for a particular assignment.

The second part of Eq. (1) is related to the connecting passenger objective. Once again, X_{ij}^k and $X_{i'j'}^{k'}$ must be equal to 1 for $\forall k$, $i = j$, $i' = j'$ and $i \neq i'$ since a unit flow is forced through each node, and there are potentially connecting passengers if and only if arrival time of flight i is less than the departure time of flight j ($A_i < D_j$) such that $i \neq j$, as given by Eq. (9). The cost associated with these connections, however, is multidimensional as it is determined by both how far the connecting gates are apart and the time until the departure of the connecting flight. For example, if the time between connections is tight, the cost of passenger discomfort for assigning a gate far away is much greater than one that is close, yet as this interval of time grows, the cost differential between distances is much less. Indeed if this interval of time is long enough, the differential of discomfort for connection distances is the same as the passenger has plenty of time to walk between gates. A functional representation of this connecting passenger cost $C_{ii'}^{kk'}$ is given in Eq. (16) of Section 2.2.4. Note that the magnitude of these costs may be scaled to line up relatively with those of fuel burn in the first part of the objective.

2.2.2. Constraints

The first constraint indicated by Eq. (3) is referred to as a flow-in constraint because it deals with the gate flow from the source node to the arrival flight node. It forces a unit flow of a certain gate to the arrival flight node or any unused will then be by-passed to the terminal node via arc (S, T). The second constraint Eq. (4) is referred to as conservation of flow at the arrival node, i.e. flow-in from arc (S, i) and arc (j, i), $i \in \mathbf{F}$, $j \in \mathbf{D}$ is forced to flow-out from arc (i, j), $i \in \mathbf{F}$, $j \in \mathbf{D}$ such that $i = j$. The constraint defined by Eq. (5) is conservation of flow at the departure node, i.e. the flow in from arc (i, j), $i \in \mathbf{F}$, $j \in \mathbf{D}$ such that $i = j$ is forced to get out from either flow-out arc (j, T), $j \in \mathbf{D}$ or arc (j, i), $j \in \mathbf{D}$, $i \in \mathbf{F}$. The fourth constraint Eq. (6) is the flow-out constraint that forces all the flow to leave the departure node to the terminal node, and note that the sum of all flows to the departure node must be equal to cardinality of the set $\mathbf{K}$. The fifth constraint Eq. (7) is referred to as a unit flow serving arc constraint as it allows only one unit gate $k \in \mathbf{K}$ to flow through serving arc. The sixth constraint is the binary constraints of Eq. (8).

2.2.3. Linearization of the quadratic objective function

The quadratic term in Eq. (2) of the objective function, i.e. connecting passenger cost, is cumbersome and difficult to solve, as noted by Obata (1979) who showed that quadratic flight gate assignment problem is an NP hard. Therefore, a linearization of the quadratic objective function is considered in this paper, similar to that of Xu and Bailey (2001) who have shown a similar method for linearizing the quadratic objective function. In particular, the quadratic cost function

$$\text{Min } Z_2 = \sum_{\substack{(i\in F, i=j,\\ i'=j', i\neq i',\\ j\neq j')}} \sum_{\substack{(i'\in F, i=j,\\ i'=j', i\neq i',\\ j\neq j')}} \sum_{k\in K}\sum_{k'\in K} N_{ii'} C^{ii'}_{kk'} X^{k}_{i,j} X^{k'}_{i',j'} \tag{10}$$

may be linearized by replacing the quadratic term $(X^{k}_{ij}X^{k'}_{i'j'})$ with a new variable $Y^{kk'}_{ii'}$ defined as

$$Y^{kk'}_{ii'} = \begin{cases} 1 & \text{if } (X^{k}_{ij} = 1 \text{ and } X^{k'}_{i'j'} = 1 \quad \forall i = j,\ i' = j', i \neq i', j \neq j' \text{ and } k \neq k') \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

The above relationship in Eq. (11) can be expressed with the following constraints expressed as in Eq. (12) through Eq. (15) to convert the quadratic objective into an equivalent linear objective model.

$$Y^{kk'}_{ii'} - X^{k}_{ij} \leqslant 0 \quad \forall i = j,\ i' = j', i \neq j' \text{and} k \neq k' \tag{12}$$

$$Y^{kk'}_{ii'} - X^{k'}_{i'j'} \leqslant 0 \quad \forall i = j,\ i' = j',\ i \neq j' \text{ and } k \neq k' \tag{13}$$

$$X^{k}_{ij} + X^{k'}_{i'j'} - Y^{kk'}_{ii'} \leqslant 1 \quad \forall i = j,\ i' = j', i \neq j' \text{ and } k \neq k' \tag{14}$$

$$X^{k}_{ij} + X^{k'}_{i'j'} - Y^{kk'}_{ii'} \geqslant 0 \quad \forall i = j,\ i' = j',\ i \neq j' \text{ and } k \neq k' \tag{15}$$

The inequalities of Eqs. (12) and (13) represent the constraints that variable $Y^{kk'}_{ii'}$ is equal to 1 if and only if binary variable X^{k}_{ij} and $X^{k'}_{i'j'}$ is equal to 1. Eq. (14) specifies that $Y^{kk'}_{ii'}$ cannot be greater than 1, and Eq. (15) further specifies that $Y^{kk'}_{ii'}$ cannot be less than zero. Due to the binary nature of X^{k}_{ij} and $X^{k}_{i'j'}$ with the above constraints, $Y^{kk'}_{ii'}$ is forced to be a binary variable.

2.2.4. Connecting passenger cost ($C^{kk'}_{ii'}$)

The passenger connecting cost is a function of both the expectation of time available and the inter-gate distance for a passenger making a connecting flight. In particular, it is a three dimensional penalty cost function for passenger discomfort that discourages the assignment of far away gates for tight connections. This function is represented as

$$C^{kk'}_{ii'} = \begin{cases} s\sqrt{d_{kk'}}(2 - \Delta t_{ii'})^2 & \forall 0 < \Delta t_{ii'} \leqslant t_{max} \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

where $d_{kk'}$ (ft) is the integrate (or interzone) distance between gates k and k', $\Delta t_{ii'}$ (h) is the time available for making a transfer from flight i to flight i', t_{max} is the upper limit on the time window beyond which no penalty for distance is incurred, and s is a scaling factor. A surface plot of the cost function defined in Eq. (16) letting t_{max} = 2 h and s = 1 is given in Fig. 2. Note the non-linear nature of this surface plot, which penalizes the distance between gate assignment severely as the connecting time for passengers is reduced. In some regards, this function may be thought of as expected loss based on the probability of a missed connection, which would be maximized at \$40 with a scaling factor of s = 1. The cost function of Eq. (16) is novel to this work and deemed plausible in structure by representatives of Continental Airlines.

2.3. Mathematical formulation using zoning strategies

The formulation of Section 2.2 is a mixed integer program (MIP), which for larger size problems of the magnitude typically encountered in practice is difficult to solve due to (1) insufficient memory and (2) non-polynomial computational times. In particular, the branch and cut tree frequently becomes so large that insufficient memory is available to solve the LP sub-problems, and in the case that memory is available, the computation times are prohibitively long. Due to these pragmatic issues, an alternative approach is to breakdown the problem into a series of sub-problems, and then solve these sub-problems independently using a solution terminating criteria for "good" solution. In the case of the flight gate assignment problem, the airport gates were divided into different zones and then sub-zones, which are then solved hierarchically. The basic model described in Section 2.2 remains the same, yet the definitions of parameters $\left(C_k, d_{k1}, d_{k2}, C^{kk'}_{ii'}\right)$, objective function, constraints (Eqs. (3) and (7)) are modified to reflect the grouping of gates into zones. Note that this hierarchical approach is a heuristic and may not solve the entire problem of Section 2.2 to optimality, yet provides an approach that is scalable to large problems and often coincides with the physical layout of an airport.

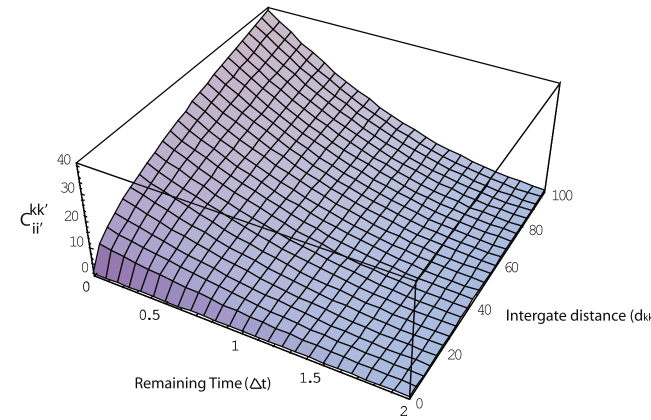

Fig. 2. Surface plot for the penalty cost function for transfer passenger.

2.3.1. Sets

K_z	Set of zones/ set of sub-zones
F_z	Set of arrival flights at zones/set of sub-zones

2.3.2. Parameters

d_{k1}	Expected arrival distance from runway to zone/sub-zone k, $k \in K_z$
d_{k2}	Expected departure distance from runway to zone/sub-zone k, $k \in K_z$
$C^{ii'}_{kk'}$	Per passenger connecting cost from zone/sub-zone k, $k \in K_z$

Note that the expected arrival (d_{k1}) and departure (d_{k2}) distances from the runway to each zone or subzone are assessed from their respective median gate.

2.3.3. Objective Function

A minor modification in an objective function is needed to accommodate the possible connecting passengers from gates that belong to a different zones/sub-zones. Hence, the following equation sums on $i \in \mathbf{F_z}$, $i' \in \mathbf{F}$, $k \in \mathbf{K_z}$, and $k' \in \mathbf{K}$ as

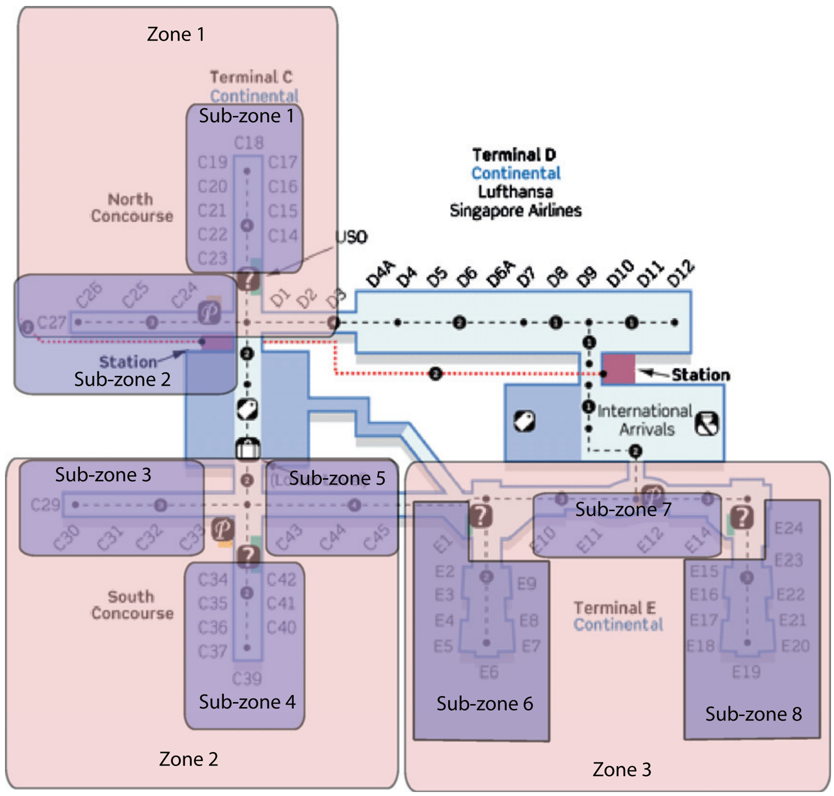

Fig. 3. Zone and subzones division for Terminal C (North and South Concourse) and Terminal E.

$$\text{Min } Z = \sum_{i\in F}\sum_{k\in K_z}(2f_i f_c/f_s\{d_{k1}+d_{k2}\})\left(X^k_{s,i}+\sum_{j\in D}X^k_{j,i}\right) + \sum_{\substack{(i\in F_Z, i=j,\\ i'=j', i\neq i',\\ j\neq j')}}\ \sum_{\substack{(i'\in F, i=j,\\ i'=j' i\neq i',\\ j\neq j')}}\ \sum_{k\in K_z}\sum_{k'\in K} N_{ii'}C^{ii'}_{kk'}X^k_{ij}X^{k'}_{i'j'} \tag{17}$$

There is no fixed cost associated with gates (C_k) on the first part of an objective function because this modification is done only for zone/subzone assignment only.

2.3.4. Constraints

The constraints of Eqs. (3) and (7) are modified and replaced with

$$\sum_i X^k_{si} + X^k_{ST} = |K_z| \quad \forall k\in K_z,\ i\in F \tag{18}$$

$$\sum_j X^k_{jT} + \sum_k X^k_{ST} = |K_z| \quad \forall i\in F,\ k\in K_z. \tag{19}$$

3. Numerical illustration

An illustration of the application of this multi-commodity flow formulation is presented using the operations of Continental Airlines at the Houston George Bush Intercontinental Airport (IAH). A snapshot of February 1, 2010 was used in specifying several of the parameters, which corresponded to operations on a typical Monday for the airline. Model parameter specifications of flight arrival and departure times (A_i, D_j), average taxi/idling fuel burn cost per unit distance (f_i, f_s, F_c), fixed cost for operating a gate (C_k), inter-gate distances ($d_{kk'}$), distance of gates to/from runways (d_{k1}, d_{k2}), and the number of connecting passengers ($N_{ii'}$) that were used in this paper are given in Appendix A.

Continental Airlines schedules and operates its narrow body and wide body airplanes in Terminal C and Terminal E at IAH, which will be the focus of this paper.[1] Hence, the gates at Terminals C and E at IAH are grouped into zones and sub-zones based on their relative proximity to each other. Note that these groupings define mathematical formulations, which will be solved in hierarchical order building upon assignments made at the previous level.

1. *Level 1:* Zone assignment.
   The zone assignment problem will assign flights to either the North Terminal C (Zone 1), South Terminal C (Zone 2), or Terminal E (Zone 3) zone.
2. *Level 2: Sub-zone assignment.*
   The sub-zone assignment problem will assign flights assigned to a particular zone in the Level 1 problem to either 2 or 3 respective sub-zones that are defined by gates in the same corridor.
3. *Level 3:* Gate assignment.
   The Gate assignment problem will assign flights assigned to a particular sub-zone in the Level 2 problem to a specific gate within the sub-zone.

A pictorial representation of Terminals C and E at IAH with overlaid zoning assignments is given in Fig. 3, with a corresponding skeletal representation given in Fig. 4.

In addition, a mix of narrow-body (737,757) and wide-body (767,777) with variations in the physical dimensions of the aircraft restrict the assignment of some flights to a subset of available gates. Likewise, restrictions on international travel restrict assignment of some flights to a subset of gates. These restrictions are simply modeled into the problem by partitioning the sets **F** and

[1] Flights at Terminal A is operated by Continental Connection, Air Canada, United, and US Airways, flights at Terminal B is operated by Continental Express and flights at Terminal D is operated by Lufthansa and Singapore Airlines. Adapted from http://www.continental.com/web/en-US/content/travel/airport/maps/iah.aspx retrieved August 08, 2010

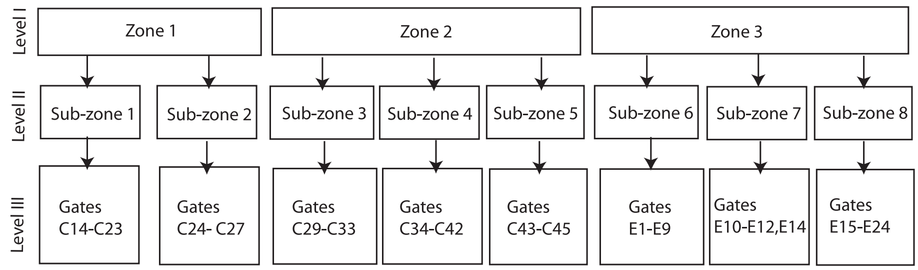

Fig. 4. Computational Skeleton of the flight-gate assignment problem.

Table 1
Computational summary on zone/subzone/gate assignment.

Computational heiraracy level	$\lvert F\rvert$	$\lvert K\rvert$ or $\lvert K_z\rvert$	Number of variables	Number of constraints	Optimal objective	Solve elapsed time (s)
Zone assignment[a] (Level I)	226	3	520,059	827,148	47,208.56	145,518
Subzone assignment (Level II)						
Subzones on Zone 1[a]	93	2	69,566	68,918	23,465.16	57.5
Subzones on Zone 2[a]	67	3	67,740	106,604	17,252.18	6036.33
Subzones on Zone 3[a]	66	3	65,739	103,312	9970.10	80.42
Total	226	8				
Flight gate assignment (Level III)						
Gates on Subzone 1[b]	70	10	8,484,010	1,740,290	193,760.73	151,889
Gates on Subzone 2[a]	23	4	1,106,400	24,503	62,487.91	39.29
Gates on Subzone 3[a]	20	5	1,202,000	30,630	54,588.06	1498.11
Gates on Subzone 4[a]	35	8	3,374,010	267,172	97,590.67	155,618
Gates on Subzone 5[a]	12	3	432,147	3258	32,442.87	1.25
Gates on Subzone 6[c]	10	9	1,080,010	26,137	26,931.22	0.31
Gates on Subzone 7[a]	4	4	191,812	624	10,665.42	0.02
Gates on Subzone 8[d]	52	10	6,283,690	955,859	139,822.23	93,600.5
Total	226	53				

[a] Gap % = 0%.
[b] Gap % = 2.37%.
[c] Gap % = 2.64%.
[d] Gap %=0.38%.

Table 2
Comparison of un-optimized/optimized gate assignment for February 01, 2010 at Terminal C-North Concourse.

Gate	Optimized assignment		Unoptimized assignment	
	Coded flight number	Count	Coded flight number	Count
C14	161, 188	2	–	–
C15	16, 32, 56, 66, 81	5	–	–
C16	33, 37, 52, 84, 219	5	–	–
C17	4,21,70, 88, 91, 136, 172, 180, 215	9	–	–
C18	27, 41, 130, 148, 170, 171, 177	7	56, 212	2
C19	72, 134, 160, 193, 205	5	51, 211	2
C20	12,20,115,141, 155, 176, 187, 191	8	77, 150	2
C21	7,18,31,61,122,145,186,225	8	66, 204	2
C22	3, 28, 34, 45, 126, 128, 131, 144, 147, 163, 169, 175, 179	13	92	1
C23	36, 43, 150, 166, 168, 181, 195, 206	8	8, 153	2
C24	30, 77, 95, 135, 142, 203	6	214	1
C25	6, 14, 86, 156, 185, 213	6	–	–
C26	19, 48, 90, 140, 152, 223	6	–	–
C27	24, 46, 112, 204, 222	5	–	–
Total		93		12

Table 3
Comparison of un-optimized/optimized gate assignment for February 01, 2010 at Terminal C-South Concourse.

Gate	Optimized assignment		Unoptimized assignment	
	Coded flight number	Count	Coded flight number	Count
C29	8, 15, 76, 200	4	3, 6, 10, 89, 101, 156, 175, 179	8
C30	49, 50, 60, 74, 143	5	23, 97, 144, 154, 170, 177	6
C31	9, 54, 101, 133	4	49, 73, 85, 95, 172	5
C32	59, 92, 116, 151	4	9, 28, 39, 44, 222	5
C33	10, 162, 196	3	35, 41, 64, 69, 80, 102, 191	7
C34	40, 51, 211	3	15, 27, 46, 94, 208	5
C35	62, 117, 167, 202	4	68, 116, 125, 157, 193, 203	6
C36	13, 109, 139, 159, 190, 197	6	21, 161, 164, 174, 186, 187	6
C37	75, 154, 208	3	31, 37, 160, 176	4
C39	68, 93, 123, 124, 192	5	24, 29, 61, 195, 205, 213, 221	7
C40	17, 25, 38, 44, 67, 89, 158	7	7, 50, 55, 134, 207, 217	6
C41	29, 113, 127, 153	4	19, 47, 72, 122, 142, 151, 152	7
C42	73, 87, 173	3	40, 58, 75, 93, 223	5
C43	11, 55, 64, 199, 207	5	14, 139, 181, 199	4
C44	23, 39, 174, 184	4	4, 34, 83, 123, 168, 180	6
C45	22, 69, 80	3	43, 54, 62, 84, 98, 128, 163, 225	8
Total		67		95

K into subsets that reflect the restricted aircraft and gates and adding to the mathematical program as assignment constraints. Specific restrictions for CO aircraft at IAH are given in Appendix A.

3.1. Computational solution

For computational purpose, we used Intel (R) Xenon (R) CPU E5450 processors installed at High Performance Computing Center (HPCC) at the Experimental Sciences Building at Texas Tech University. Each node contains two Intel 5450 Quad Core 64bit processors with a core frequency of 3.0 GHz with 16 GB of memory. We used AMPL/CPLEX 11.2 to code our program without parallel computing due to licensing restrictions, which was solved

Table 4
Comparison of un-optimized/optimized gate assignment for February 01, 2010 at Terminal E.

Gate	Optimized assignment		Unoptimized assignment	
	Coded flight number	Count	Coded flight number	Count
E1	96	1	5, 36, 104, 149, 183, 189	6
E2	106	1	32, 91, 112, 117, 218, 226	6
E3	121	1	71, 126, 137	3
E4	220	1	17	1
E5	157	1	70, 113, 146, 148, 190	5
E6	100,110,149	3	42, 57, 59, 121, 124, 141, 158, 216	8
E7	224	1	1, 220	2
E8	165	1	22, 30, 88, 127, 145, 188, 206, 219	8
E9	–	0	18, 78, 96, 135, 143, 162, 171, 215	8
E10	182	1	11, 79, 119, 133, 138, 147, 197	7
E11	209	1	48, 99, 155, 166, 185, 196, 198	7
E12	218	1	60, 81, 87, 178, 202	5
E14	132	1	16, 53, 108, 165, 209, 210	6
E15	102, 42, 71, 103	4	100, 105, 111, 136, 173, 192, 201	7
E16	94, 217, 78, 114, 178	5	25, 26, 67, 82, 103, 159, 200	7
E17	47, 79, 138, 63, 99, 194	6	13, 74, 109, 130, 169, 182	6
E18	83, 111, 214, 221, 2, 82, 210	7	2	1
E19	5, 97, 119, 129, 226, 104, 108, 118, 120, 137	10	115, 131	2
E20	58, 125, 1, 26, 183, 201	6	52, 110, 224	3
E21	35, 216, 57, 65, 189, 198	6	33, 38, 63, 65, 132, 184, 194	7
E22	107, 105, 146	3	90, 107, 114	3
E23	53, 85, 212	3	20, 76, 106, 120, 129, 167	6
E24	98, 164	8	12, 45, 86, 118, 140	5
Total		66		119

in the hierarchial zoning pattern as shown previously (Fig. 4). As summary of the number of flights assigned to zones/subzones/gates shown, total number of variables, total number of constraints, and solution elapsed time for each problem are given in Table 1. The solution search was terminated at optimality or when the "Gap %" reached an acceptable level (ILOG AMPL CPLEX System, 2008), which was less than 3% in this case. The final assignments of flights to gates in the North and South Concourses of Terminal C and in Terminal E are given in Tables 2–4 respectively. As a matter of reference, the flight numbers shown in Tables 2–4 are coded flight numbers to ensure confidentiality, though most of the data used in this paper is publicly accessible.

3.1.1. Gate assignment comparisons

The comparison of assignments for un-optimized flight gate assignment and optimized flight gate assignment with wide-body/international flight gate constraints for Terminal C-North Concourse, Terminal C-South Concourse, and Terminal E are given in Tables 2–4 respectively.

3.1.2. Comparison of fuel-burn costs and connecting passenger costs

The comparisons of objective function by fuel-burn costs and connecting passenger costs for un-optimized and optimized flight gate assignment summarized by subzones are given in Tables 5 and 6 respectively.

Table 5
Un-optimized (as existed) flight gate assignment for February 01, 2010.

Subzones	Fuel burn cost	Gate cost	Connecting passenger cost	Total cost
Gates on Subzone 1	$2505.10	$26,950.00	$18,035.01	$47,490.11
Gates on Subzone 2	$233.85	$2450.00	$318.20	$3002.05
Gates on Subzone 3	$7464.21	$75,950.00	$50,770.55	$134,184.76
Gates on Subzone 4	$12,199.68	$112,700.00	$77,948.80	$202,848.48
Gates on Subzone 5	$4730.69	$44,100.00	$31,038.72	$79,869.42
Gates on Subzone 6	$11,571.85	$115,150.00	$62,815.39	$189,537.24
Gates on Subzone 7	$5390.28	$61,250.00	$37,028.00	$103,668.29
Gates on Subzone 8	$11,418.05	$115,150.00	$123,057.33	$249,625.38
Total	$55,513.71	$553,700.00	$401,012.02	$1,010,225.73

Table 6
Optimized flight gate assignment for February 01, 2010 with wide-body flight constraints/international flight constraints.

Optimized	Fuel cost	Gate cost	Passenger	Total
Gates on Subzone 1	$17,836.61	$171,500.00	$4424.13	$193,760.73
Gates on Subzone 2	$6094.01	$56,350.00	$43.90	$62,487.91
Gates on Subzone 3	$5169.67	$49,000.00	$418.39	$54,588.07
Gates on Subzone 4	$8905.80	$85,750.00	$2934.87	$97,590.67
Gates on Subzone 5	$2935.59	$29,400.00	$107.28	$32,442.87
Gates on Subzone 6	$2637.06	$26,950.00	$206.09	$29,793.15
Gates on Subzone 7	$865.42	$9800.00	$0.00	$10,665.42
Gates on Subzone 8	$11,661.71	$124,950.00	$187.85	$136,799.56
Total	$56,105.86	$553,700.00	$8322.51	$618,128.37

When comparing un-optimized fight gate assignment (refer Table 5) with optimized flight gate assignment with wide-body/international flight constraints (refer Table 6), fuel burn cost increased by 3.6% compared to an un-optimized flight gate assignment scenario. However, the connecting passenger cost is reduced by 97.9%, with a reduction of 38.81% in total cost.

4. Discussion and conclusion

The specification of the indicator variable defined in Eq. (11) has noticeable impact on the model. Increasing the value of p will force flights to be spread across zones more uniformly, yet at a cost of increasing the objective function and possibly leading to infeasibility. Decreasing such reduces cost, yet may lead to greater congestion in a particular zone (terminal) and hence the need for more day-of gate reassignment due flight delays. In practice, this value will often be chosen based on operational needs, yet may be modified to balance loads within the airport. Note that in this paper, it was specified that $A_i - D_j \geqslant 0.75$ (45 min), from whence 66 flights out of the 226 total flights are assigned to Terminal E, 67 flights are assigned to the South Concourse of Terminal C, and 93 flights are assigned to the North Concourse of Terminal C. If this were reduced to 30 min, however, testing has shown that most all flights would be concentrated in Terminal E, suggesting the capacity of IAH for an increased number of Continental flights.

In conclusion, we have formulated a binary multi-commodity flow network representation of the flight gate assignment problem that minimizes both fuel burn of aircraft and the comfort of connecting passengers. A three dimensional penalty function for

Table 7
Airplane model data with jet engine model taxi/idling fuel burn cost.

Model	Engine type	Engine manufacturer	Seating capacity	Fleet in CO	Taxi/idling fuel per engine (kg/s)
737-300	CFM56-3B1	CFM International*	124	23	0.1140
737-500	CFM56-3B1	CFM International	114	42	0.1140
737-700	CFM56-7B24	CFM International	124	36	0.1090
737-800	CFM56-7B26	CFM International	157	108	0.1130
737-900	CFM56-7B26	CFM International	173	12	0.1130
737-900ER	CFM56-7B26	CFM International	173	27	0.1130
757-200	RB211-535	Rolls Royce	175	41	0.2000
757-300	RB211-535E4B	Rolls Royce	216	17	0.1900
767-200ER	GECF6-80C2B4F	General Electric	174	10	0.1926
767-400ER	GECF6-80C2B8F	General Electric	235	12	0.2050
777-200ER	GE90-90B	General Electric	285	20	0.3130

Table 8
Runway dimensions.

Runways	Dimensions	Type
15L/33R	12,000 ft by 150 ft	R/W 33 ILS Category I
9/27	10,000 ft by 150 ft	R/W 27 – ILS Category I/III
8R/26L	9400 ft by 150 ft	R/W 26 – ILS Category I/III
15R/33L	10,000 ft by 150 ft	R/W 15 – ILS Category I
8L/26R	9000 ft by 150 ft	R/W 26 ILS Category III

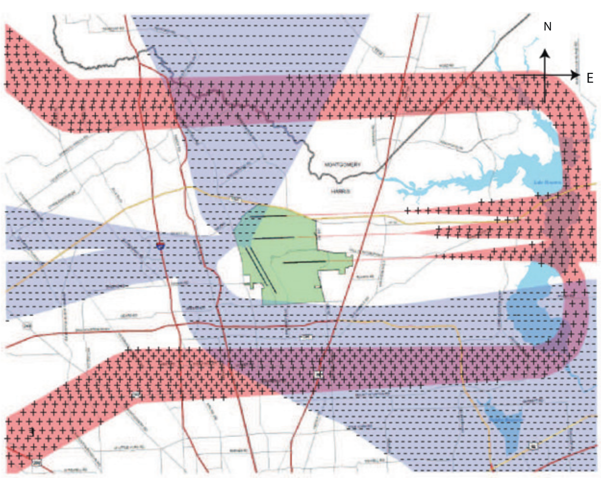

Fig. 5. West-flow arrival and departure corridor for IAH (Departures are shown in (–) pattern and arrivals are shown in (+) pattern.)

connecting passengers with arguments of connection time and distance was collaboratively specified and is given in this paper. A decomposition approach for large problems based on zoning strategies that often coincide with the physical layout of an airport provides for scalability of this methodology. An numerical illustration is given for operations of Continental Airlines at Houston Intercontinental Airport using actual data from a snapshot of operations on Feb 1, 2010, whereupon a comparison of the heuristically optimal assignments from this methodology versus the un-optimized assignments used by the airline on that day are displayed.

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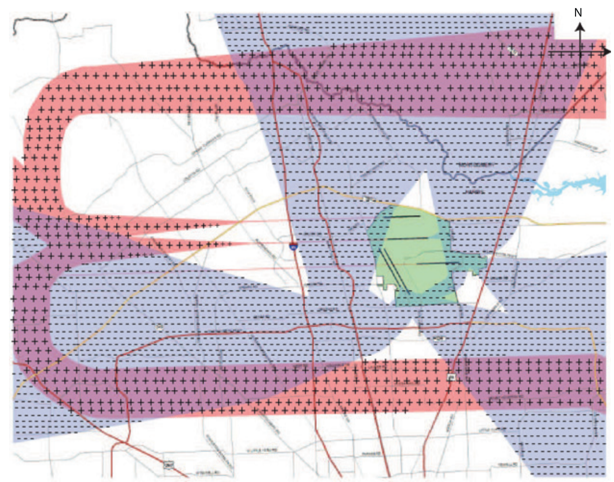

Fig. 6. East-flow arrival and departure corridor for IAH (Departures are shown in (–) pattern and arrivals are shown in (+) pattern.)

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Appendix A

A.1. Arrival and departure times (A_i, D_i)

The arrival and departure times for Continental (CO) aircraft at IAH on Feb 1, 2010 were obtained from the publicly accessible database (Bureau of Transportation Statistics, 2010) and used in this paper.

A.2. Fuel burn (f_i)

Fuel burn during take-off, climb out, approach and idle for the CO aircraft is given in Table 7.[2] It is also assumed that power remain constant at 7% thrust during taxi and idling operations, as noted by Kim, Waitz, Stouffer, and Locke (2005).

[2] Adapted from http://www.continental.com/web/en-US/content/travel/inflight/aircraft/default.aspx & ICAO Engine Exhaust Emissions Databank.

Table 9
FEIS guidelines for runway usage.

Operation hours	Flow direction	Arrival runways	Departure runways
6:00 AM–9:59 PM	West flow	26L, 26R, 27, 15L and 15R	15L, 15R, 26L, 26R and 8L
	East flow	8R, 8L, 9, 15L and 15R	15L, 15R,9, 8R and 8L
10:00 PM–5:59 AM	West flow	26L, 26R, 27, 15L and 15R	15L, 15R, 26L, 26R and 27
	East flow	8R, 8L, 15L, 15R, and 9	15L, 15R,9, 8R and 8L

Table 10
Runway usage in percentage of arrival flights, departure flights and total flights.

S.N.	Runway	Runway usage (%) arrival flight	Runway usage (%) departure flight	Total Runway usage (%)
1	15L	0.23	52.14	26.18
2	15R	0.25	27.19	13.72
3	26L	35.44	2.34	18.89
4	26R	7.83	0.53	4.18
5	27	34.85	1.19	18.33
6	9	0.60	8.05	3.93
7	8R	13.87	0.06	7.06
8	8L	6.89	0.05	3.48
9	33L	0.00	3.63	1.81
10	33R	0.03	4.80	2.41
11	Unknown	0.00	0.04	0.02

Table 11
Other parameter values.

Parameter	Value
C_k	2450 $\forall k$
f_s	25 knots
f_c	$1.614/kg

Table 12
Gate restrictions based on aircraft type.

Aircraft type	Available gates
Boeing 737 and 757	All Terminal C gates and E1–E23
Boeing 737-200 and 737-300	E24
Boeing 767 & 777	E4, E7, E18, E20, C14, C16, C19, C20

A.3. Gate operating costs (C_k)

The gate operations cost for each gate at Terminals C and E in IAH are assumed to be equal and fixed.

A.4. Runway distances to gates (d_{k1}, d_{k_2})

There are five runways at IAH whose respective dimensions and usage are given in Table 8. Wind flow direction is a critical factor in determining on which runway an airplane will land and takeoff, and at IAH, there are two airport configurations due to prevailing wind conditions given by West-flow and East-flow as depicted in Figs. 5 and 6 respectively. (Houston Airport System: Noise Management, 2010) Guidelines for runway usage based on wind conditions at IAH is given in Table 9, and actual runway usage from October 1, 2009 to January 1, 2010 is summarized in Table 10.

Google Earth™ (Google Earth (Version 5.1.3533.1731) [Software], 2009) was used to measure the physical distance from the end/beginning of runways to each gate and using the data from Table 10, the expected values of d_{k1} and d_{k2} were calculated and used in this paper.

In calculating the expected runway length to zones/subzones and interzonal/subzonal distances in the Levels 1 and 2 zoning subproblems, the median gate[3] of the grouping is chosen. In particular, the median gates for zones 1, 2, and 3 are C21, C36, and E12 respectively, and the median gates for subzones 1–8 are C19, C26, C31, C39, C44, E5, E12, and E20 respectively.

A.5. Connecting passengers ($N_{ii'}$)

The number of passengers connecting from one gate to another were used from a snapshot of operations on February 1, 2010. This data is not reported in this paper for confidentiality reasons, but was used in generating the results as reported in this paper.

A.6. Other parameter specifications

Other parameter specifications used in this paper are given in Table 11.

A.7. Restrictions on gate assignment due to wide-body aircraft and international flights

All international flights both arriving or departing from IAH for CO airlines must pass through Terminal E, and the corresponding aircraft are restricted to such gates. Restrictions based on the type and corresponding dimensions of the aircraft on available gates are as given in Table 12.

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[3] Median gate is a middle gate separating gates into two halves.

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