Actor-Critic Algorithms

CS 294-112: Deep Reinforcement Learning

Sergey Levine

Class Notes

1. Homework 1 due today (11:59 pm)!
    - Don’t be late!
2. Homework 2 is out today
    - Start early!
3. Remember to start forming final project groups

Today's Lecture

1. Improving the policy gradient with a critic
2. The policy evaluation problem
3. Discount factors
4. The actor-critic algorithm

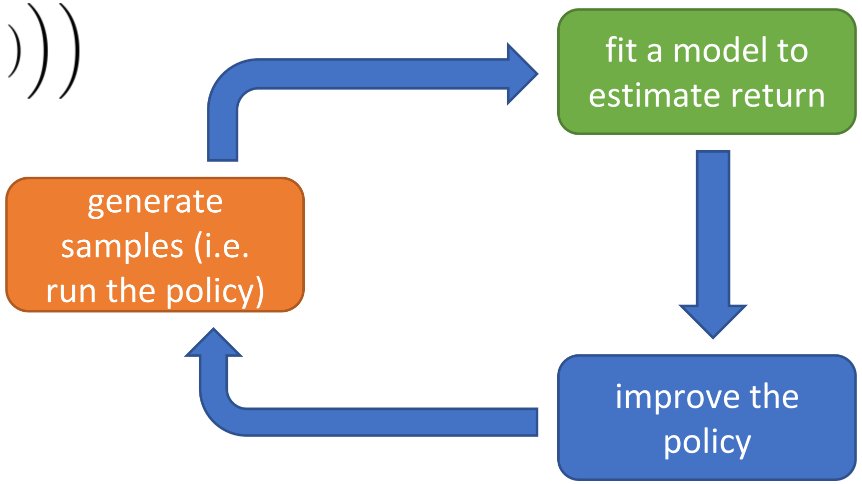
- Goals:
  - Understand how policy evaluation fits into policy gradients
  - Understand how actor-critic algorithms work

Recap: policy gradients

REINFORCE algorithm:

1. sample $\{\tau^i\}$ from $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$ (run the policy)
2. $\nabla_\theta J(\theta) \approx \sum_i \left(\sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_t^i|\mathbf{s}_t^i)\left(\sum_{t'=t}^T r(\mathbf{s}_{t'}^i, \mathbf{a}_{t'}^i)\right)\right)$
3. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

$$\hat{Q}^\pi(\mathbf{x}_t, \mathbf{u}_t) = \sum_{t'=t}^T r(\mathbf{x}_{t'}, \mathbf{u}_{t'})$$



$$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \underbrace{\hat{Q}^\pi_{i,t}}_{\text{“reward to go”}}$$

Improving the policy gradient

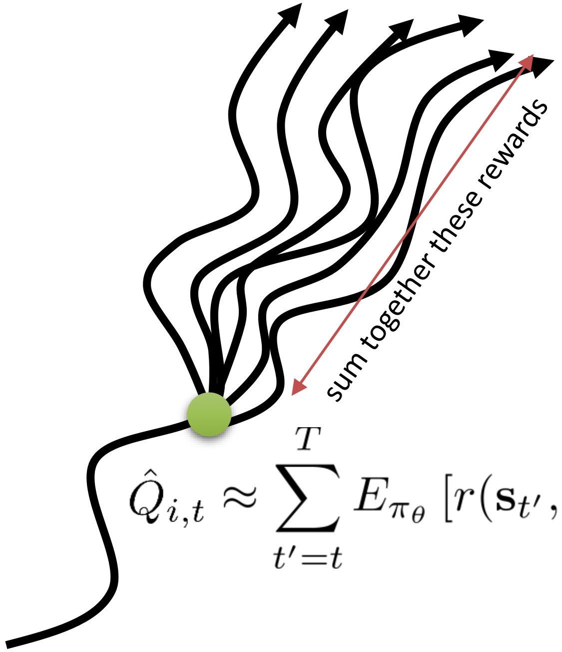
$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \underbrace{\left(\sum_{t'=1}^{T} r(\mathbf{s}_{i,t'},\mathbf{a}_{i,t'})\right)}_{\text{“reward to go”}}$$

$$\hat{Q}_{i,t}$$

$\hat{Q}_{i,t}$: estimate of expected reward if we take action $\mathbf{a}_{i,t}$ in state $\mathbf{s}_{i,t}$

can we get a better estimate?

$Q(\mathbf{s}_t,\mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'},\mathbf{a}_{t'})|\mathbf{s}_t,\mathbf{a}_t\right]$: true *expected* reward-to-go



$$\hat{Q}_{i,t} \approx \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'},\mathbf{a}_{t'})|\mathbf{s}_t,\mathbf{a}_t\right]$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(Q(\mathbf{s}_{i,t},\mathbf{a}_{i,t}) - V(\mathbf{s}_{i,t})\right)$$

$$V(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t|\mathbf{s}_t)}[Q(\mathbf{s}_t,\mathbf{a}_t)]$$

What about the baseline?

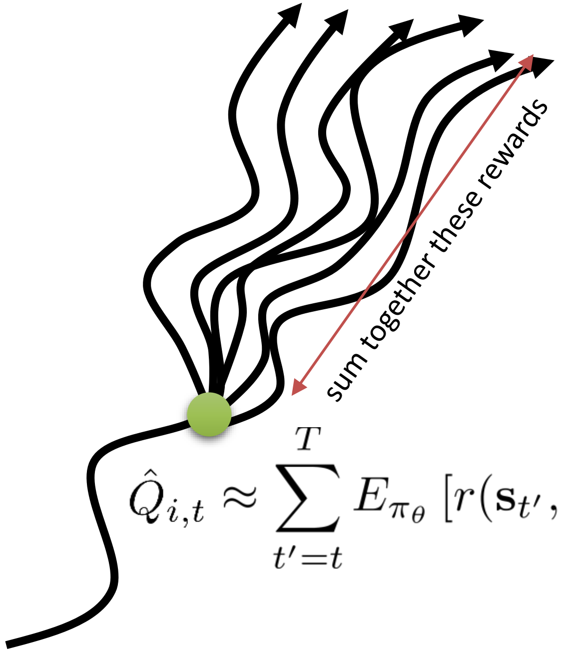
$Q(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$: true *expected* reward-to-go

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\left(Q(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) - V(\mathbf{s}_{i,t})\right)$$

$$b_t = \frac{1}{N}\sum_i Q(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

average what?

$$V(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t|\mathbf{s}_t)}[Q(\mathbf{s}_t, \mathbf{a}_t)]$$



$$\hat{Q}_{i,t} \approx \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$$

State & state-action value functions

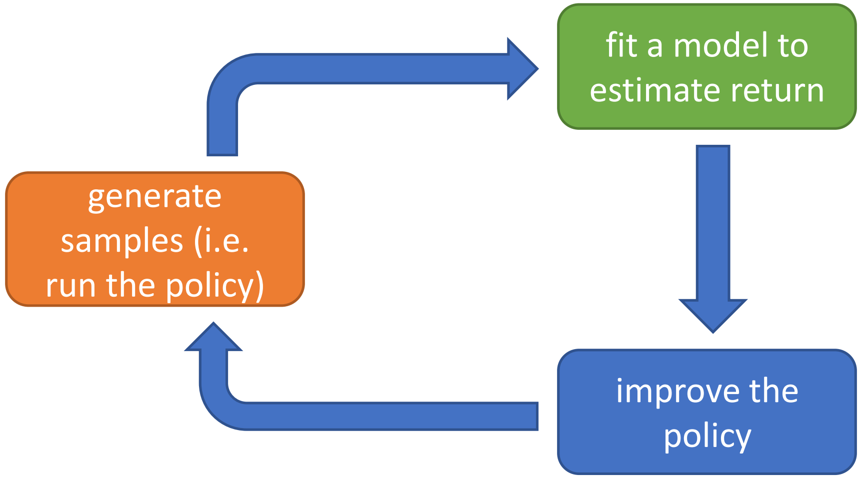
$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t\right]$: total reward from taking $\mathbf{a}_t$ in $\mathbf{s}_t$

$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t|\mathbf{s}_t)}[Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$: total reward from $\mathbf{s}_t$

$A^\pi(\mathbf{s}_t, \mathbf{a}_t) = Q^\pi(\mathbf{s}_t, \mathbf{a}_t) - V^\pi(\mathbf{s}_t)$: how much better $\mathbf{a}_t$ is

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) A^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

fit Q^π, V^π, or A^π



$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

the better this estimate, the lower the variance

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(\sum_{t'=1}^{T} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'}) - b \right)$$

unbiased, but high variance single-sample estimate

Value function fitting

what is the sum total of all the future rewards expected if we take action a at time t and continue to take actions in the future until time T in accordance with policy π?

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$$

what is the sum of rewards we can expect under policy π from state s_t until time T, if we were to sum individual rewards from each action sampled from a fixed policy π? What is the sum total of future rewards that can be expected if start in state t and continue until the end?

$$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t|\mathbf{s}_t)}[Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$$

$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) = Q^\pi(\mathbf{s}_t, \mathbf{a}_t) - V^\pi(\mathbf{s}_t)$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) A^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

what is the advantage of taking action a at time t in state s over the baseline, the baseline being the average action in state s at time t until time T. Note that the V^π(S_t) must be smaller (worse) than (Q^π(S_t, a_t) if a_t is the correct action that leads to maximum reward. Therefore, for the correct action in state s_t the advantage should be positive, for incorrect action - negative.

fit *what* to *what*?

Q^π, V^π, A^π?

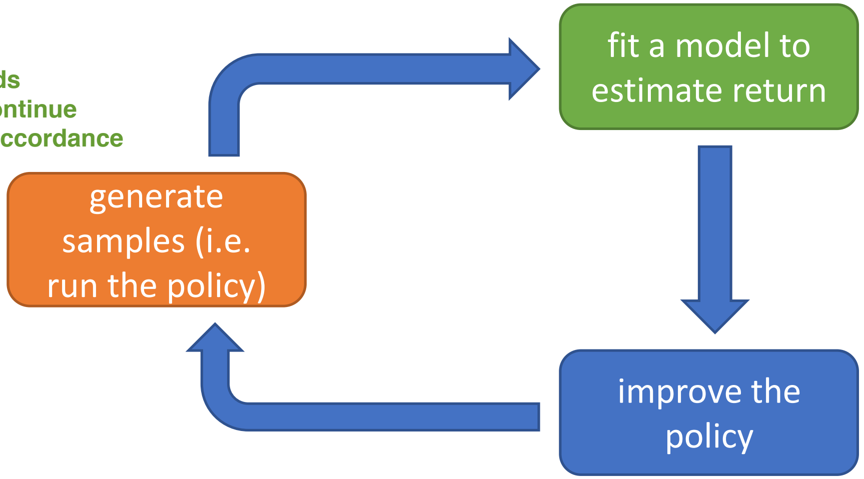
$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + E_{\mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)}[V^\pi(\mathbf{s}_{t+1})]$$

$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) \approx r(\mathbf{s}_t, \mathbf{a}_t) + V^\pi(\mathbf{s}_{t+1}) - V^\pi(\mathbf{s}_t)$$

$V^\pi(\mathbf{s}_{t+1})$

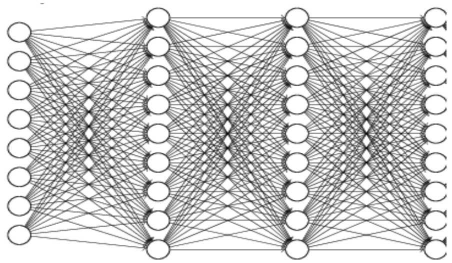
let's just fit $V^\pi(\mathbf{s})$!

fit Q^π, V^π, or A^π



$$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$$

$\mathbf{s}$



$\hat{V}^\pi(\mathbf{s})$

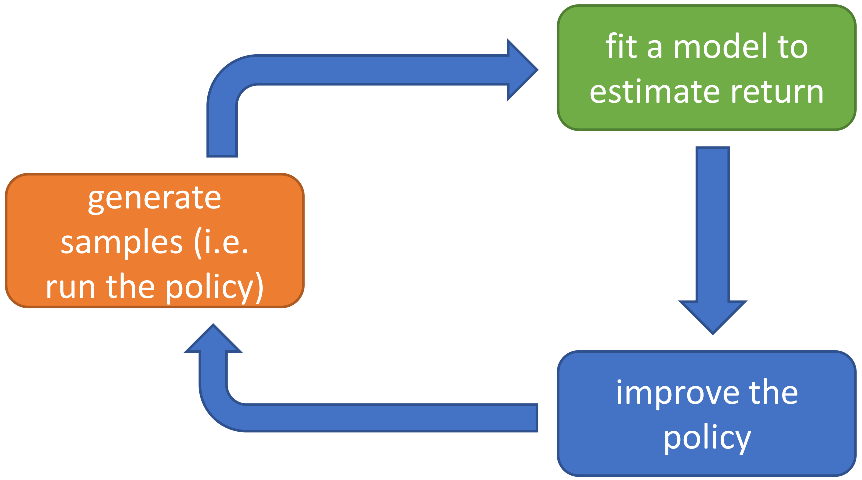
parameters ϕ

What is Q^π(s_t) in essence? It is the sum of reward at the current time step (reward being the function of state and action both at the current step, r_t+1 = r(s_t, a_t)) and the EXPECTED value at all the remaining time steps.

It is called "advantage", because it tells you what is the advantage of committing to one SPECIFIC action at the current time step compared to taking an action RANDOMLY sampled from the policy (call it average or random action). Taking random (average) actions must be worse than the best action (positive advantage), but better than committing to the worst possible action (negative advantage).

Policy evaluation

fit V^π



$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

$V^\pi(\mathbf{s}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t\right]$

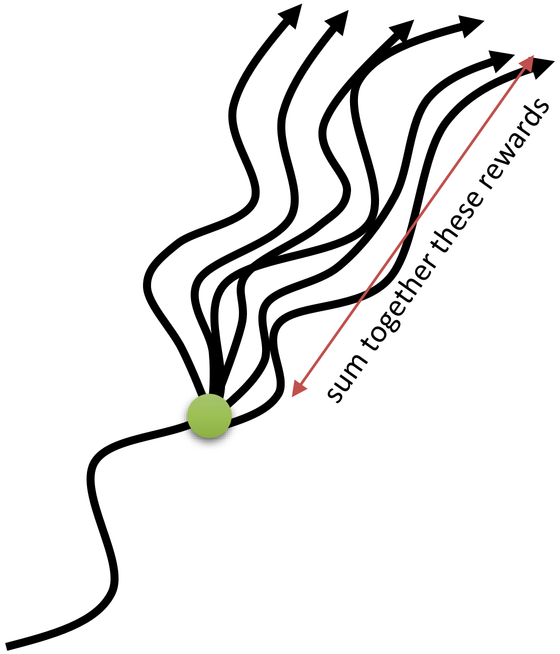
$J(\theta) = E_{\mathbf{s}_1 \sim p(\mathbf{s}_1)}[V^\pi(\mathbf{s}_1)]$

how can we perform policy evaluation?

Monte Carlo policy evaluation (this is what policy gradient does)

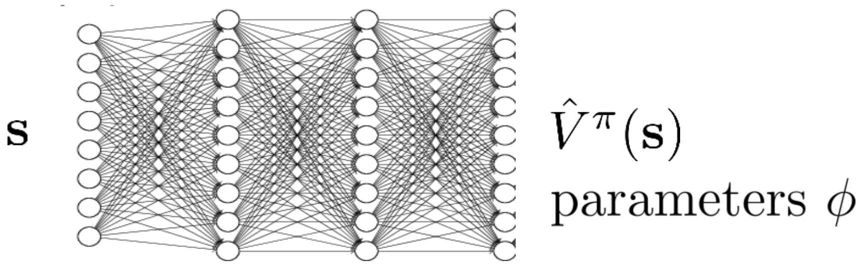
$$V^\pi(\mathbf{s}_t) \approx \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})$$

$$V^\pi(\mathbf{s}_t) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) \qquad \text{(requires us to reset the simulator)}$$



Monte Carlo evaluation with function approximation

$$V^\pi(\mathbf{s}_t) \approx \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})$$

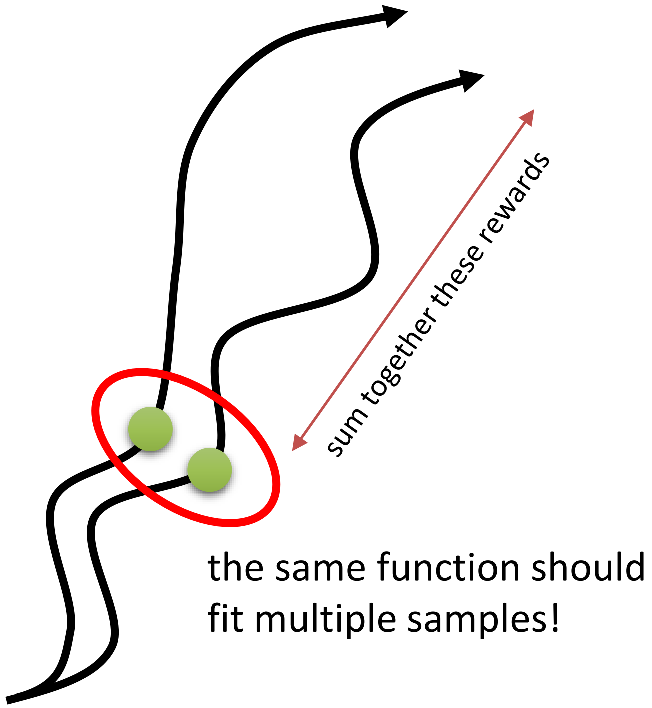


not as good as this: $V^\pi(\mathbf{s}_t) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})$

but still pretty good!

training data: $\left\{\left(\mathbf{s}_{i,t}, \underbrace{\sum_{t'=t}^{T} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'})}_{y_{i,t}}\right)\right\}$

supervised regression: $\mathcal{L}(\phi) = \frac{1}{2}\sum_i \left\|\hat{V}_\phi^\pi(\mathbf{s}_i) - y_i\right\|^2$



Can we do better?

ideal target: $y_{i,t} = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_{i,t}\right] \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + V^{\pi}(\mathbf{s}_{i,t+1}) \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \underbrace{\hat{V}_{\phi}^{\pi}(\mathbf{s}_{i,t+1})}_{\text{directly use previous fitted value function!}}$

Monte Carlo target: $y_{i,t} = \sum_{t'=t}^{T} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'})$

training data: $\left\{\left(\mathbf{s}_{i,t}, \underbrace{r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \hat{V}_{\phi}^{\pi}(\mathbf{s}_{i,t+1})}_{y_{i,t}}\right)\right\}$

supervised regression: $\mathcal{L}(\phi) = \frac{1}{2}\sum_i \left\|\hat{V}_{\phi}^{\pi}(\mathbf{s}_i) - y_i\right\|^2$

sometimes referred to as a “bootstrapped” estimate

Policy evaluation examples

TD-Gammon, Gerald Tesauro 1992

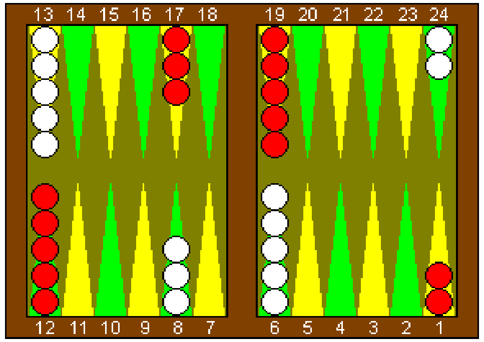


Figure 2. An illustration of the normal opening position in backgammon. TD-Gammon has sparked a near-universal conversion in the way experts play certain opening rolls. For example, with an opening roll of 4-1, most players have now switched from the traditional move of 13-9, 6-5, to TD-Gammon's preference, 13-9, 24-23. TD-Gammon's analysis is given in Table 2.

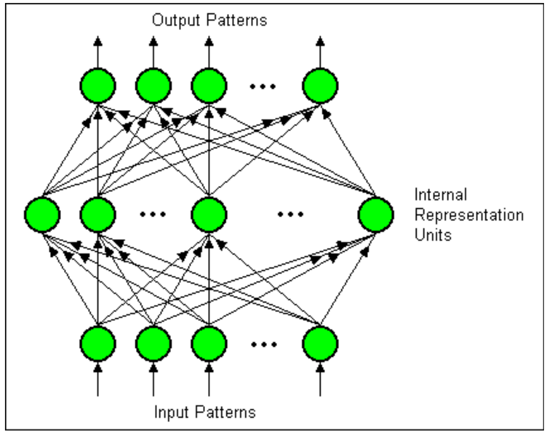
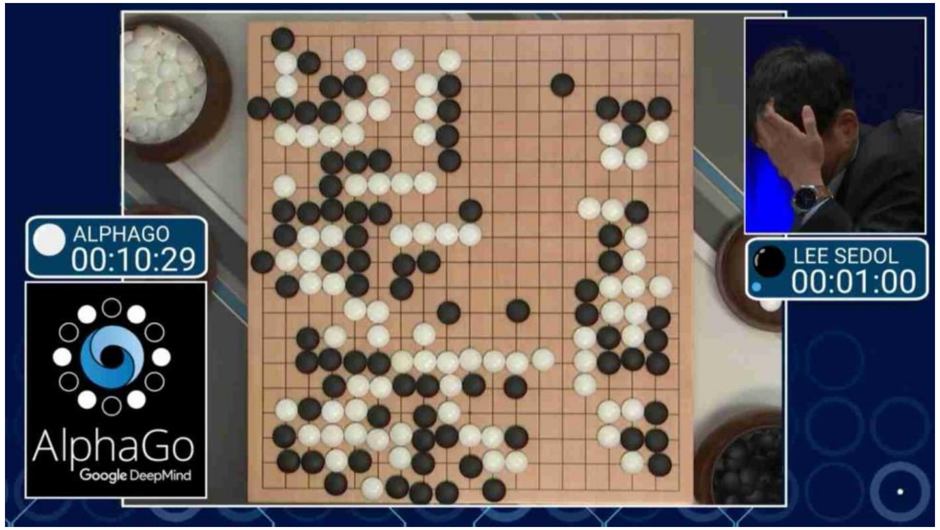


Figure 1. An illustration of the multilayer perception architecture used in TD-Gammon's neural network. This architecture is also used in the popular backpropagation learning procedure. Figure reproduced from [9].

reward: game outcome

value function $\hat{V}^{\pi}_{\phi}(\mathbf{s}_t)$:
expected outcome given board state

AlphaGo, Silver et al. 2016



reward: game outcome

value function $\hat{V}^{\pi}_{\phi}(\mathbf{s}_t)$:
expected outcome given board state

An actor-critic algorithm

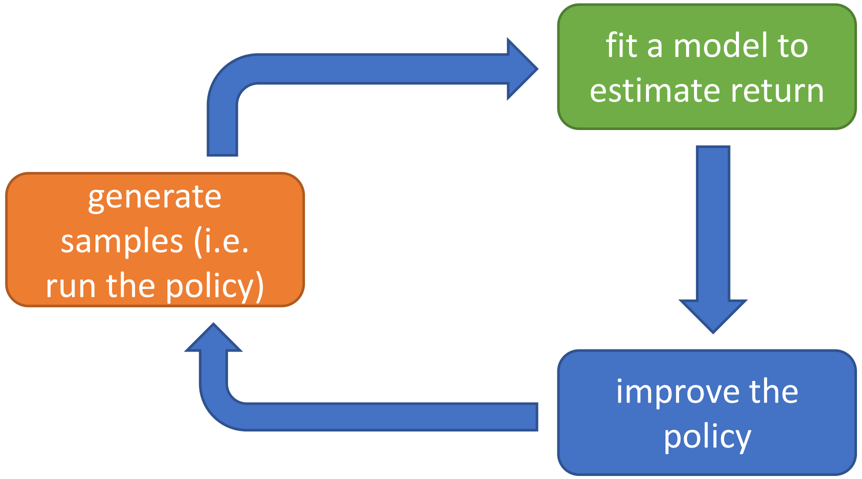
batch actor-critic algorithm:

1. sample $\{\mathbf{s}_i, \mathbf{a}_i\}$ from $\pi_\theta(\mathbf{a}|\mathbf{s})$ (run it on the robot)
2. fit $\hat{V}^\pi_\phi(\mathbf{s})$ to sampled reward sums
3. evaluate $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = r(\mathbf{s}_i, \mathbf{a}_i) + \hat{V}^\pi_\phi(\mathbf{s}'_i) - \hat{V}^\pi_\phi(\mathbf{s}_i)$
4. $\nabla_\theta J(\theta) \approx \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i) \hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

$$y_{i,t} \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \hat{V}^\pi_\phi(\mathbf{s}_{i,t+1})$$

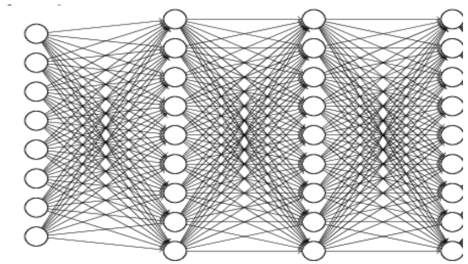
$$\mathcal{L}(\phi) = \frac{1}{2} \sum_i \left\| \hat{V}^\pi_\phi(\mathbf{s}_i) - y_i \right\|^2$$

fit $\hat{V}^\pi$



$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

$\mathbf{s}$



$\hat{V}^\pi(\mathbf{s})$

parameters ϕ

$$V^\pi(\mathbf{s}_t) = \sum_{t'=t}^T E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t\right]$$

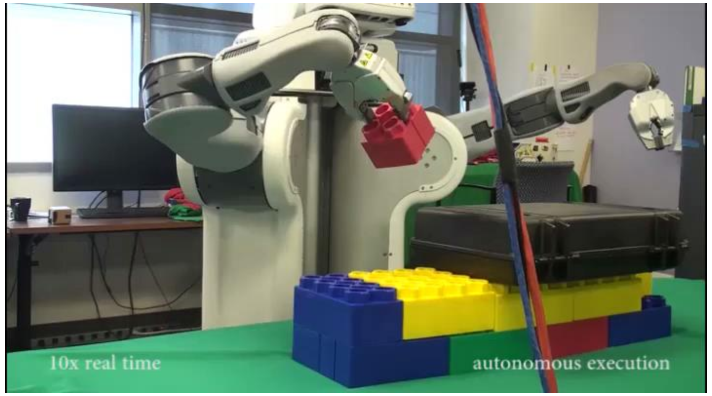
Aside: discount factors

$$y_{i,t} \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \hat{V}_\phi^\pi(\mathbf{s}_{i,t+1})$$

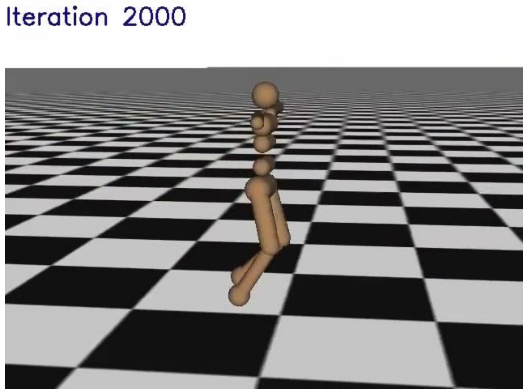
$$\mathcal{L}(\phi) = \frac{1}{2} \sum_i \left\| \hat{V}_\phi^\pi(\mathbf{s}_i) - y_i \right\|^2$$

what if T (episode length) is ∞?

$\hat{V}_\phi^\pi$ can get infinitely large in many cases



episodic tasks



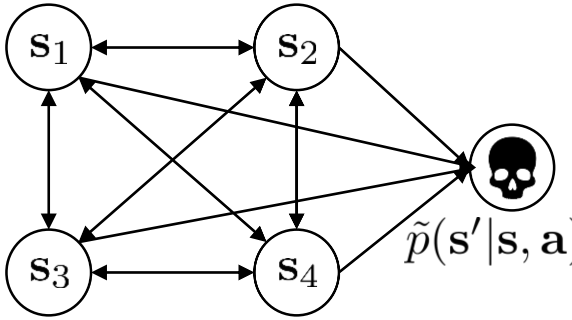
continuous/cyclical tasks

simple trick: better to get rewards sooner than later

$$y_{i,t} \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma \hat{V}_\phi^\pi(\mathbf{s}_{i,t+1})$$

discount factor $\gamma \in [0, 1]$ (0.99 works well)

γ changes the MDP:



$\tilde{p}(\mathbf{s}'|\mathbf{s}, \mathbf{a}) = (1 - \gamma)$

$\tilde{p}(\mathbf{s}'|\mathbf{s}, \mathbf{a}) = \gamma p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$

Aside: discount factors for policy gradients

$$y_{i,t} \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma \hat{V}^\pi_\phi(\mathbf{s}_{i,t+1})$$

$$\mathcal{L}(\phi) = \frac{1}{2} \sum_i \left\| \hat{V}^\pi_\phi(\mathbf{s}_i) - y_i \right\|^2$$

with critic:

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(\overbrace{r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma \hat{V}^\pi_\phi(\mathbf{s}_{i,t+1}) - \hat{V}^\pi_\phi(\mathbf{s}_{i,t})}^{\hat{A}^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})} \right)$$

what about (Monte Carlo) policy gradients?

option 1: $$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(\sum_{t'=t}^T \gamma^{t'-t} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'}) \right)$$

not the same!

option 2: $$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \left(\sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \right) \left(\sum_{t=1}^T \gamma^{t-1} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'}) \right)$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(\sum_{t'=t}^T \gamma^{t'-1} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'}) \right)$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \gamma^{t-1} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(\sum_{t'=t}^T \gamma^{t'-t} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'}) \right)$$

(later steps matter less)

Which version is the right one?

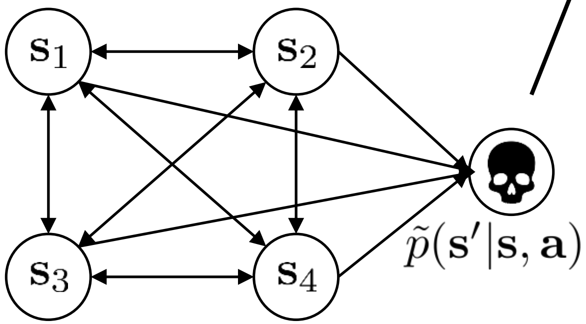
option 1: $$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\left(\sum_{t'=t}^{T}\gamma^{t'-t} r(\mathbf{s}_{i,t'},\mathbf{a}_{i,t'})\right)$$

option 2: $$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\gamma^{t-1}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\left(\sum_{t'=t}^{T}\gamma^{t'-t} r(\mathbf{s}_{i,t'},\mathbf{a}_{i,t'})\right)$$

this is what we actually use...

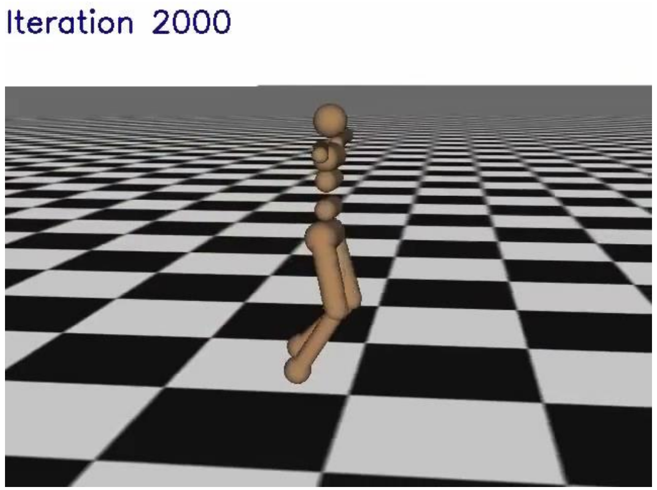
why?

later steps don't matter if you're dead!



$\tilde{p}(\mathbf{s}'|\mathbf{s},\mathbf{a}) = (1-\gamma)$

$\tilde{p}(\mathbf{s}'|\mathbf{s},\mathbf{a}) = \gamma p(\mathbf{s}'|\mathbf{s},\mathbf{a})$



Further reading: Philip Thomas, Bias in natural actor-critic algorithms. ICML 2014

Actor-critic algorithms (with discount)

batch actor-critic algorithm:

1. sample $\{\mathbf{s}_i, \mathbf{a}_i\}$ from $\pi_\theta(\mathbf{a}|\mathbf{s})$ (run it on the robot)
2. fit $\hat{V}^\pi_\phi(\mathbf{s})$ to sampled reward sums
3. evaluate $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \hat{V}^\pi_\phi(\mathbf{s}'_i) - \hat{V}^\pi_\phi(\mathbf{s}_i)$
4. $\nabla_\theta J(\theta) \approx \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i) \hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

online actor-critic algorithm:

1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$
2. update $\hat{V}^\pi_\phi$ using target $r + \gamma \hat{V}^\pi_\phi(\mathbf{s}')$
3. evaluate $\hat{A}^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma \hat{V}^\pi_\phi(\mathbf{s}') - \hat{V}^\pi_\phi(\mathbf{s})$
4. $\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(\mathbf{a}|\mathbf{s}) \hat{A}^\pi(\mathbf{s}, \mathbf{a})$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

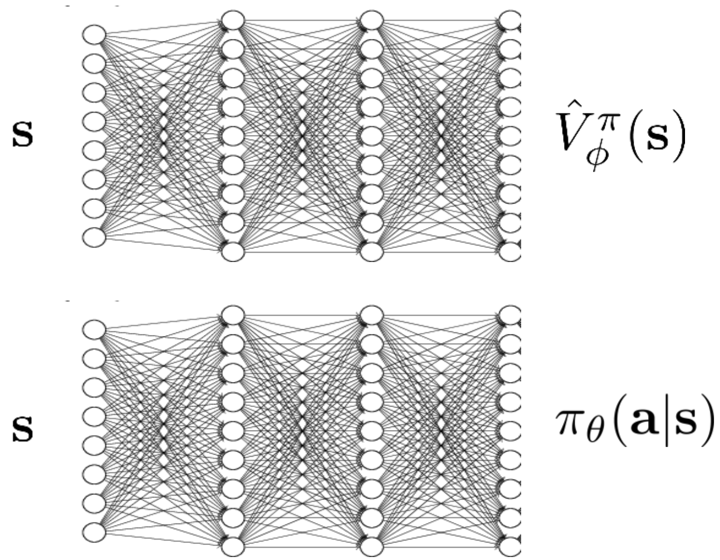
Break

Architecture design

online actor-critic algorithm:

1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$
2. update $\hat{V}^\pi_\phi$ using target $r + \gamma \hat{V}^\pi_\phi(\mathbf{s}')$
3. evaluate $\hat{A}^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma \hat{V}^\pi_\phi(\mathbf{s}') - \hat{V}^\pi_\phi(\mathbf{s})$
4. $\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(\mathbf{a}|\mathbf{s}) \hat{A}^\pi(\mathbf{s}, \mathbf{a})$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

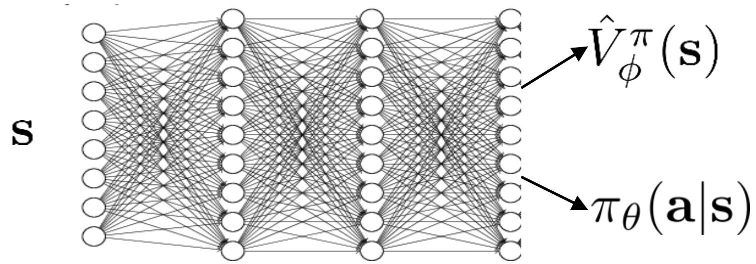
two network design



+ simple & stable

- no shared features between actor & critic

shared network design



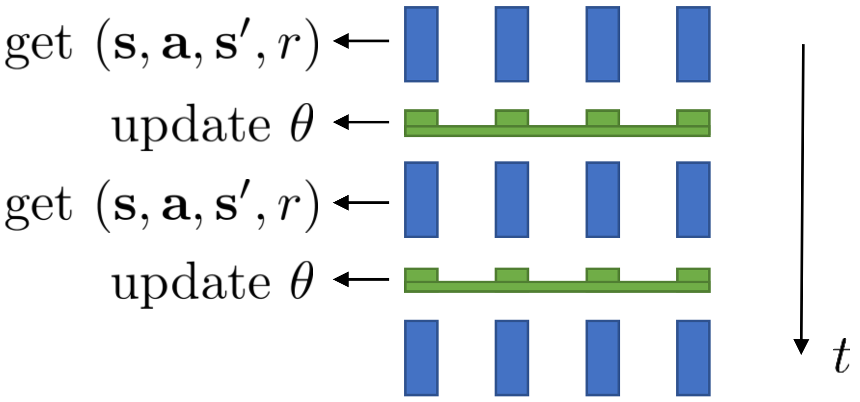
Online actor-critic in practice

online actor-critic algorithm:

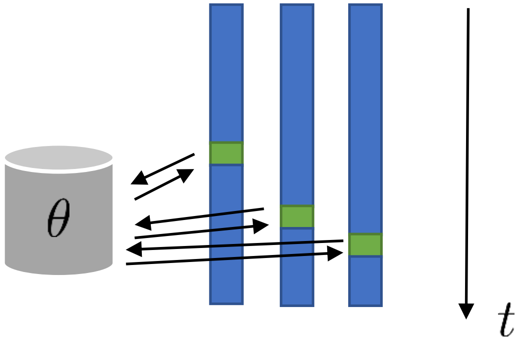
1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$
2. update $\hat{V}_\phi^\pi$ using target $r + \gamma \hat{V}_\phi^\pi(\mathbf{s}')$
3. evaluate $\hat{A}^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma \hat{V}_\phi^\pi(\mathbf{s}') - \hat{V}_\phi^\pi(\mathbf{s})$
4. $\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(\mathbf{a}|\mathbf{s}) \hat{A}^\pi(\mathbf{s}, \mathbf{a})$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

works best with a batch (e.g., parallel workers)

synchronized parallel actor-critic



asynchronous parallel actor-critic



Critics as state-dependent baselines

Actor-critic: $$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\left(r(\mathbf{s}_{i,t},\mathbf{a}_{i,t}) + \gamma\hat{V}^\pi_\phi(\mathbf{s}_{i,t+1}) - \hat{V}^\pi_\phi(\mathbf{s}_{i,t})\right)$$

+ lower variance (due to critic)

- not unbiased (if the critic is not perfect)

Policy gradient: $$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\left(\left(\sum_{t'=t}^{T}\gamma^{t'-t}r(\mathbf{s}_{i,t'},\mathbf{a}_{i,t'})\right) - b\right)$$

+ no bias

- higher variance (because single-sample estimate)

can we use $\hat{V}^\pi_\phi$ and still keep the estimator unbiased?

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\left(\left(\sum_{t'=t}^{T}\gamma^{t'-t}r(\mathbf{s}_{i,t'},\mathbf{a}_{i,t'})\right) - \hat{V}^\pi_\phi(\mathbf{s}_{i,t})\right)$$

You'll implement this for HW2!

+ no bias

+ lower variance (baseline is closer to rewards)

Control variates: action-dependent baselines

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$$

$$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t|\mathbf{s}_t)}[Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$$

$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) = Q^\pi(\mathbf{s}_t, \mathbf{a}_t) - V^\pi(\mathbf{s}_t)$$

$$\hat{A}^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{\infty} \gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) - V_\phi^\pi(\mathbf{s}_t)$$

+ no bias

- higher variance (because single-sample estimate)

$$\hat{A}^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{\infty} \gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) - Q_\phi^\pi(\mathbf{s}_t, \mathbf{a}_t)$$

+ goes to zero in expectation if critic is correct!

- not correct

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(\hat{Q}_{i,t} - Q_\phi^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})\right) + \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T} \nabla_\theta E_{\mathbf{a} \sim \pi_\theta(\mathbf{a}_t|\mathbf{s}_{i,t})}\left[Q_\phi^\pi(\mathbf{s}_{i,t}, \mathbf{a}_t)\right]$$

use a critic *without* the bias (still unbiased), provided second term can be evaluated

Gu et al. 2016 (Q-Prop) – we'll talk more about variance reduction later

Eligibility traces & n-step returns

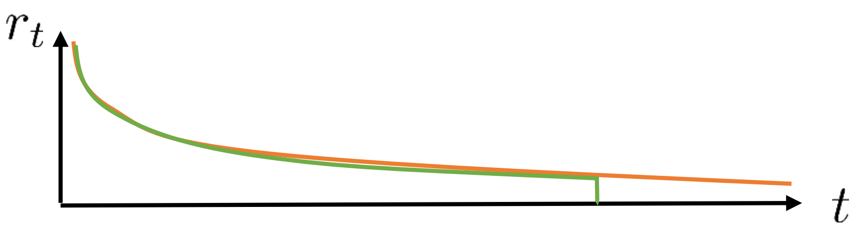
$$\hat{A}^{\pi}_{\mathrm{C}}(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + \gamma \hat{V}^{\pi}_{\phi}(\mathbf{s}_{t+1}) - \hat{V}^{\pi}_{\phi}(\mathbf{s}_t)$$

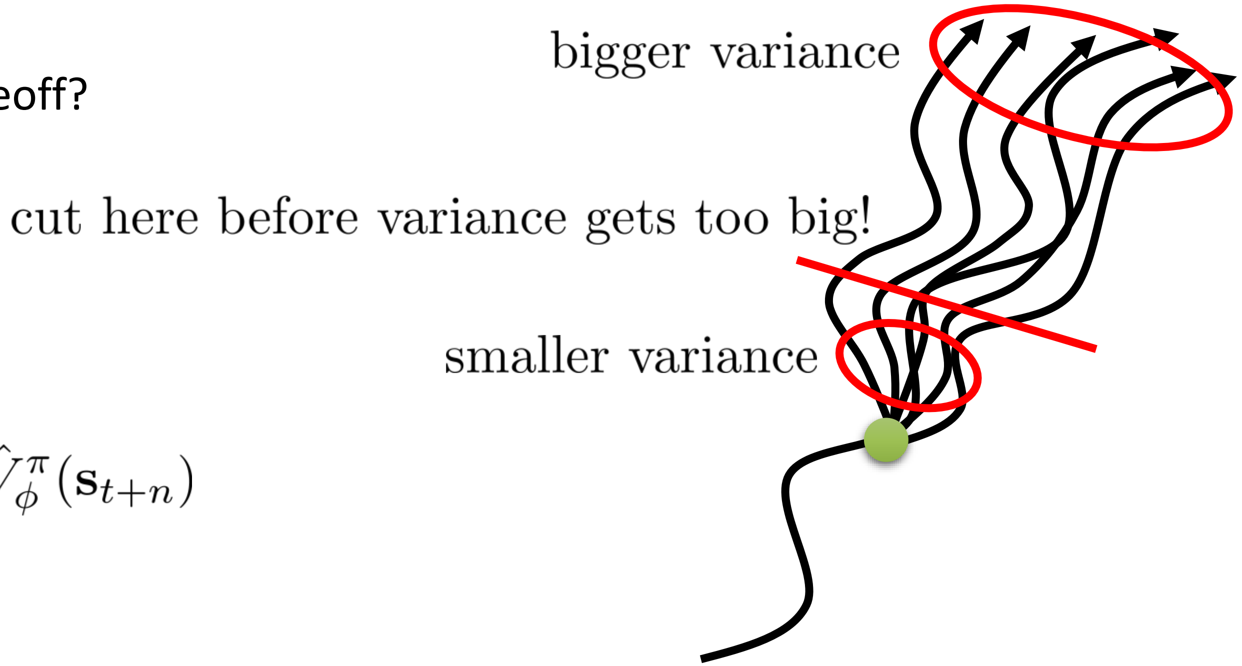
+ lower variance
- higher bias if value is wrong (it always is)

$$\hat{A}^{\pi}_{\mathrm{MC}}(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{\infty} \gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) - \hat{V}^{\pi}_{\phi}(\mathbf{s}_t)$$

+ no bias
- higher variance (because single-sample estimate)

Can we combine these two, to control bias/variance tradeoff?

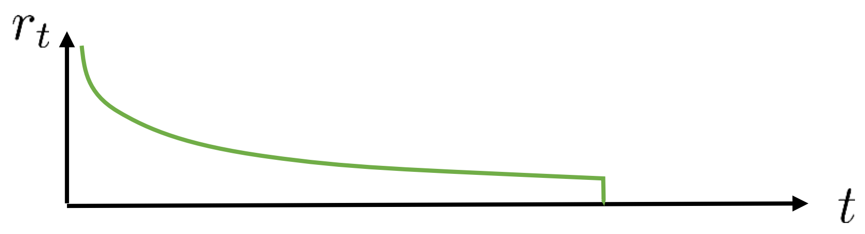




$$\hat{A}^{\pi}_{n}(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{t+n} \gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) - \hat{V}^{\pi}_{\phi}(\mathbf{s}_t) + \gamma^{n} \hat{V}^{\pi}_{\phi}(\mathbf{s}_{t+n})$$

choosing $n > 1$ often works better!

Generalized advantage estimation



Do we have to choose just one n?

Cut everywhere all at once!

$$\hat{A}_n^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{t+n} \gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) - \hat{V}_\phi^\pi(\mathbf{s}_t) + \gamma^n \hat{V}_\phi^\pi(\mathbf{s}_{t+n})$$

$$\hat{A}_{\mathrm{GAE}}^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{n=1}^{\infty} w_n \hat{A}_n^\pi(\mathbf{s}_t, \mathbf{a}_t)$$

Weighted combination of n-step returns

How to weight?

Mostly prefer cutting earlier (less variance)

exponential falloff

$$w_n \propto \lambda^{n-1}$$

$$\hat{A}_{\mathrm{GAE}}^\pi(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + \gamma((1-\lambda)\hat{V}_\phi^\pi(\mathbf{s}_{t+1}) + \lambda(r(\mathbf{s}_{t+1}, \mathbf{a}_{t+1}) + \gamma((1-\lambda)\hat{V}_\phi^\pi(\mathbf{s}_{t+2}) + \lambda r(\mathbf{s}_{t+2}, \mathbf{a}_{t+2}) + \ldots)$$

$$\hat{A}_{\mathrm{GAE}}^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{\infty} (\gamma\lambda)^{t'-t} \delta_{t'}$$

$$\delta_{t'} = r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) + \gamma \hat{V}_\phi^\pi(\mathbf{s}_{t'+1}) - \hat{V}_\phi^\pi(\mathbf{s}_{t'})$$

similar effect as discount!

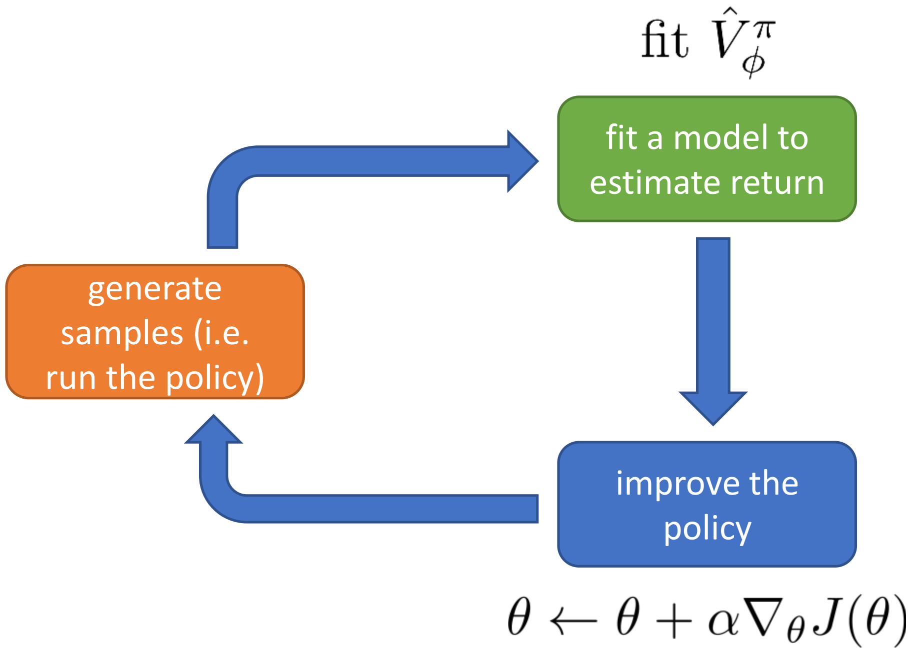
$$\text{option 1:} \quad \nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \left(\sum_{t'=t}^{T} \gamma^{t'-t} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'}) \right)$$

remember this?
discount = variance reduction!

Schulman, Moritz, Levine, Jordan, Abbeel ‘16

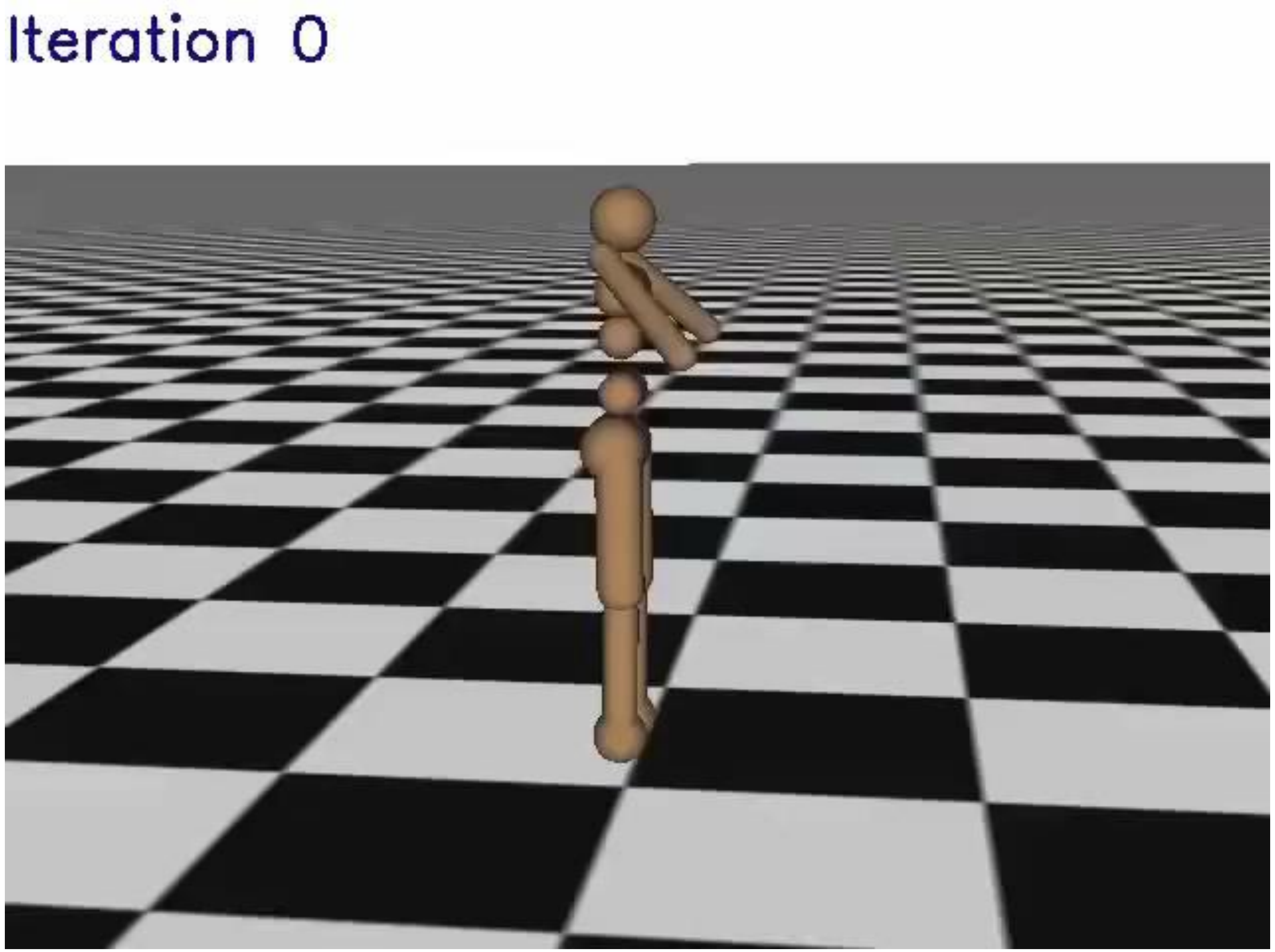
Review

- Actor-critic algorithms:
  - Actor: the policy
  - Critic: value function
  - Reduce variance of policy gradient
- Policy evaluation
  - Fitting value function to policy
- Discount factors
  - Carpe diem Mr. Robot
  - …but also a variance reduction trick
- Actor-critic algorithm design
  - One network (with two heads) or two networks
  - Batch-mode, or online (+ parallel)
- State-dependent baselines
  - Another way to use the critic
  - Can combine: n-step returns or GAE



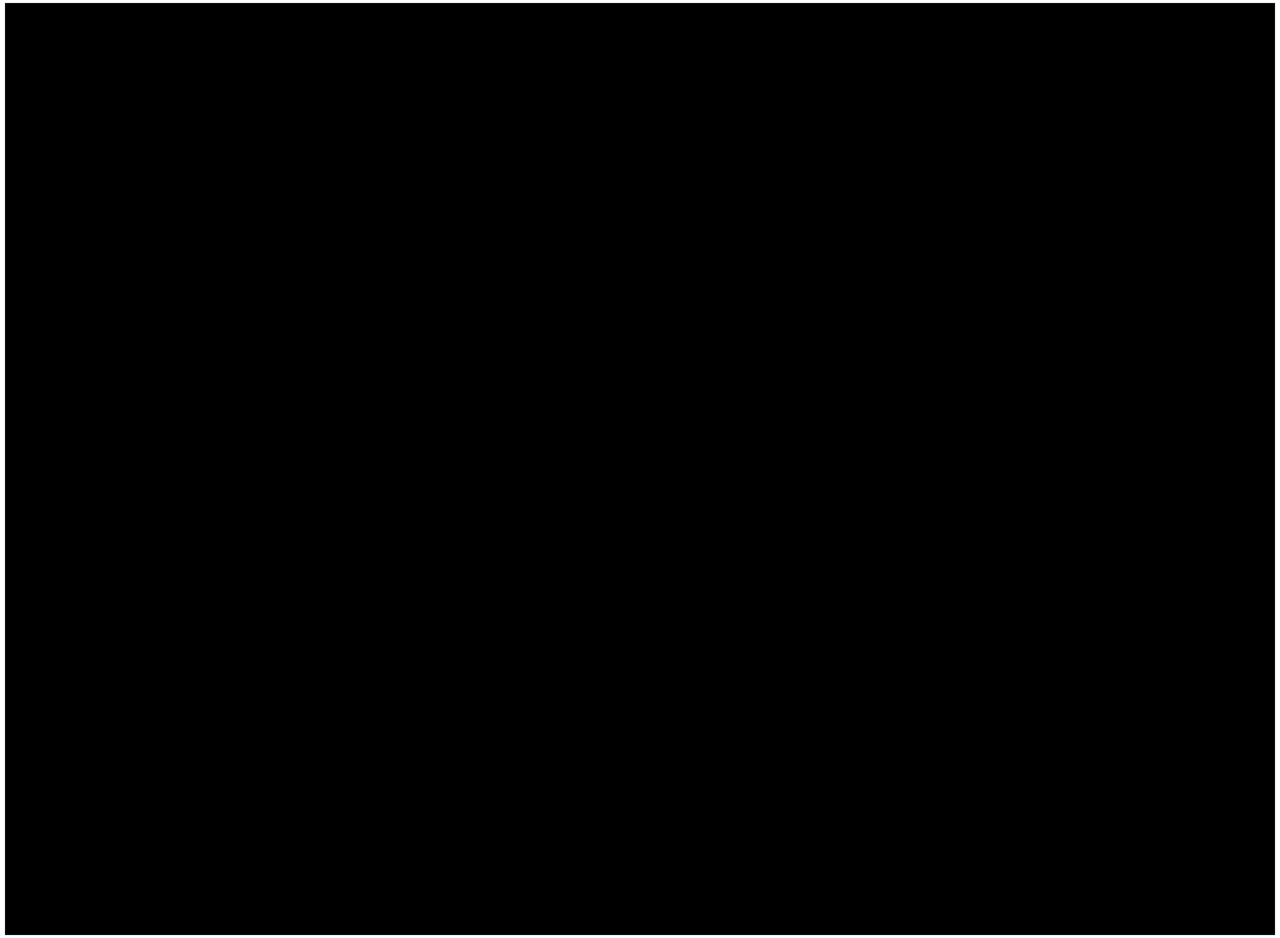
Actor-critic examples

- High dimensional continuous control with generalized advantage estimation (Schulman, Moritz, L., Jordan, Abbeel ‘16)
- Batch-mode actor-critic
- Blends Monte Carlo and function approximator estimators (GAE)



Actor-critic examples

- Asynchronous methods for deep reinforcement learning (Mnih, Badia, Mirza, Graves, Lillicrap, Harley, Silver, Kavukcuoglu ‘16)
- Online actor-critic, parallelized batch
- N-step returns with N = 4
- Single network for actor and critic



Actor-critic suggested readings

- Classic papers
  - Sutton, McAllester, Singh, Mansour (1999). Policy gradient methods for reinforcement learning with function approximation: actor-critic algorithms with value function approximation
- Deep reinforcement learning actor-critic papers
  - Mnih, Badia, Mirza, Graves, Lillicrap, Harley, Silver, Kavukcuoglu (2016). Asynchronous methods for deep reinforcement learning: A3C -- parallel online actor-critic
  - Schulman, Moritz, L., Jordan, Abbeel (2016). High-dimensional continuous control using generalized advantage estimation: batch-mode actor-critic with blended Monte Carlo and function approximator returns
  - Gu, Lillicrap, Ghahramani, Turner, L. (2017). Q-Prop: sample-efficient policy-gradient with an off-policy critic: policy gradient with Q-function control variate