A Modern Conceptual Framework for the Analysis of Factors in Retirement Decisions

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Abstract

This article focuses on analytical methodologies useful for analyzing and forecasting turnover in large organizations. We discuss a variety of biases that can occur in retirement databases along with modelling strategies and factors that may play a roll in retirement decision making with employees. The models presented are applied to prediction of both retirement and quitting behavior in a large single site industrial organization and include analysis of prior early retirement incentives as well as demographic, behavioral and external economic factors. Model predictions are based on the Cox proportional hazard model and closely related methods. Simulation is used to demonstrate the potential impact of sampling biases on predictions using. We find that a key factor in retirement among defined benefit employees is achieving full vesting but that those that do not retire immediately maintain a reduced hazard after qualifying for retirement. We also find that external economic factors related to S&P 500 real earnings are beneficial in predicting retirement while dividends are most associated with quitting behavior. Both censored data bootstrap and large sample estimates are applied in order to estimate the variance of the predictions.

Key words: Cox model, proportional hazards, defined benefit pension, early retirement incentive, left truncation, censoring

1 Introduction

Employee turnover is a topic that has drawn the attention of management researchers and practitioners for decades because it is both costly and disruptive to the functioning of most organizations (Staw, 1980; Mueller and Price, 1989; Kacmar et al., 2006). Both private firms and governments spend billions of dollars every year managing the issue according to Leonard (2001). In addition to identifying factors that lead to employee satisfaction and productivity, the ability to identify both causes and timing of turnover are key goals of human resource analytics systems (IBM, 2013). For many mature firms with large workforces, an important piece of this puzzle is the development predictive models for both retirement and quitting. While commercial tools may exist in this space, very little discussion of applied predictive models has appeared in the academic literature. From an operational perspective, the ability to accurately predict turnover across a range of organizations and job types is a highly beneficial to both the front line management of these organizations as well as financial, human resources, and actuarial concerns of the company and its supporting partners. The ability to forecast turnover becomes even more valuable in specialized industries and government agencies with long hiring lead times. From a research perspective, a predictive retirement model is a platform that can allow investigators to evaluate both external economic and demographic factors as well as internal policies that can influence retirement decisions.

The current study focuses on behavior of individuals between the years 2000-2012 employed by a large industrial organization located at a single site that provided employees with a defined benefit retirement plan. The study has four objectives: 1) Develop a probabilistic model of the employee lifetime as a function of basic demographic, employment, and external factors. 2) Evaluate the aggregate predictive accuracy of the model for 1 and 2 year time frames as a tool to facilitate planning. 3) Determine the impact of internal and external economic variables on retirement. 4) Quantify the impact of an early retirement incentive on retirement behavior. Because of the sampling approach used to collect the data, an integral part of the study was to ensure that the modelling strategy was robust to biases introduced by left truncation and right censoring.

In order to analyze the data, we used the Cox proportional hazards models (PH). The strength of the Cox model is the semiparametric form that incorporates a non-parameteric baseline estimate and a parametric term that determines the relative impact of the other factors. As we discuss in Section 4.2, this model is not sensitive to truncation bias, can be viewed as a modelling approach for non-homogeneous poisson processes and can include the effects of covariates that change over time.

[EXPLAINS AVOIDS THE IMPACT OF TRUNCATION ... ALSO DISCUSS COUNTING PROCESS FORMULATION, TIME VARYING COVARIATES].

1.1 Motivation of the Study

In many industries with older worker populations, retirement is a major source of HR disruption causing delays and other problems in processing the flow of work. Replacing retired workers can also be a major expense both in HR staff time and recruiting costs. Effective predictive models can also help managers identify divisions, departments, or potentially individuals in an organization that are likely to leave giving the organization more lead time for planning and recruitment of replacements. For example, in large organizations that utilize skilled workers in both white collar and blue collar jobs, accurate forecasts of predictive models of openings in the next 6 to 12 months can be extremely beneficial in maintaining continuity of operations. Similarly, other firms with younger employee populations may be more impacted by employee quitting and would benefit from predictions.

While aggregate forecast models of turnover and, more specifically, retirement exist (Zhu et al., 2015), such models have limited ability to provide estimates at division or job category levels. Such models also do not take into account demographic structure of the population such as age, years of service, pension type, and potentially numerous other factors that can influence the probability of retirement. By modeling the distribution of time until retirement at the individual level we can include much more relevant information such as age, retirement plan type, job classification, organizational division, years of service, pension benefit details, individual survey responses if they exist, as well as external social and economic trends that are geographically relevant. This model could potentially provide both accurate predictions as well as giving managers and researchers feedback on how different factors and incentives may influence retirement and other HR decisions such as early retirement incentives.

2 Literature Review

Wang and Shultz (2010) summarizes key theoretical and empirical research between 1986 and 2010 as well as identifying inconsistent findings. This work draws from the literature in a variety of social science fields. Most of the work cited in that review focuses on the process of retirement and factors driving the retirement decision from the individual perspective. Most relevant to the current work is research in retirement decision making and human resource management.

The current work bridges the gap between individual retirement decision making and the factors involved and the importance of workforce management from the perspective of human resources. Here, the focus is on the prediction of retirement for strategic planning in large organizations such as government agencies, large corporations, large academic institutions in order to determine changes in workforce size and plan for eventual loss of critical skills. Particularly applicable to corporations that have a majority of working with defined benefit retirement plans. Can also be used by human resources, benefits managers, and actuaries to determine how much funding is left?

2.1 Methods for Retirement Forecasting

Employee turnover is a general term and captures the loss of employees to a wide range of causes such as retirement, death, quitting, termination, and potentially promotion or reassignment. Each of these modes of turnover has different foundational causes and may be more or less prevalent during different points in ones career. Forecasting or prediction of turnover may be accomplished at both the aggregate level or may be broken down by organizational factors or by the mode of loss. For example, using a similar data source to that considered here, Zhu et al. (2015) use a time series approach to predict future aggregate turnover based on losses in previous years. The weakness of such an approach is that it does not use known characteristics of the employee population such as age, skill set, performance evaluation, salary, years of service, or numerous other factors to attempt to predict turnover. Obviously, one would expect that the use of these factors as well as further factors identified as critical in the retirement decision making process. Wang and Shultz (2010) would be beneficial in predicting future patterns, most obviously in areas such as retirement.

Given access to internal human resource data, regression models for lifetime data offer the potential to make use of this valuable information when making predictions. Such models have been widely applied in academic settings such as engineering (reliability) (Lawless, 2011) , social sciences (event studies) (Allison, 2010; Long and Freese, 2006), and medicine and epidemiology (Kalbfleisch and Prentice, 2011; Moeschberger and Klein, 2003). In the organizational and business settings both academic and professional researchers have began to use these methods to solve practical problems in industry. While actuarial scientists have been using these methods since their inception to create models in risk and insurance

(Brockett et al., 2008), more recently, researchers in finance have explored the use of these models to model lifetimes of banks (Lane et al., 1986), as well as time until default of financial instruments such as fixed income securities (LeClere, 2005). However, until now, little has been done in the area of human resources. While Berger and Chen (1993) considers statistical modelling of retirement of tenured faculty within a university setting, the emphasis is on applying the Bayesian statistical approach to modelling retirement outcome. More recently, major analytics consulting firms such as IBM and PWC have begun to offer human resource analytics software and services (IBM, 2013; PWC, 2015). Within this area, some consultants have proposed basic survival models for employee churn (Briggs, 2014) but few details are available and complexities of real world situations tend to be avoided.

2.2 Survival Analysis Applications

As discussed above survival analysis is widely used to analyze lifetime data in many areas, particularly in the health care and engineering areas. These method have been applied to thousands of epidemiological studies retrospective biomedical studies, and clinical trials over the past 40 years. [PROVIDE A DETAILED REVIEW OF THE METHODOLOGY.] For example, Claus et al. (1991) investigated the familial risk of breast cancer in a large population-based, case-control study using recurrent life time analysis and found that the risks of breast cancer are a function of women's age. Moeschberger and Klein (2003) provide a thorough book length overview of the methods and include specific case studies focused on medical applications. Survival analysis is also widely used in reliability area. For example, Carrión et al. (2010) estimates the time to failure of the pipes in water supply network dataset under left-truncation and right-censoring by using the extend Nelson estimator (Pan and Chappell, 1998). Book length treatments of these methods in engineering applications include Lawless (2011); Meeker and Escobar (2014).

Although less frequent than the other applications, the method is also heavily utilized in the business area also, Lu (2002) applied survival analysis techniques to predict customer churn by using data from a telecommunications company. Their study provided a tool for telecommunication companies to make retention plan to reduce the customer churn. Also, Braun and Schweidel (2011) used a hierarchical competing risks analysis to model when and why customers terminate their service by using the data from a provider of land-based telecommunication services.

3 Data Preparation

The dataset analyzed was provided by a large multipurpose research organization in the U.S. and consists 4316 active and 3782 former full-time employees. This population of employees is followed across a 12 year window from November 2000 to December 2012. Records of employees that retired or left before November 2000 or that began employment after December 2012 truncated from the dataset. In addition, for 4316 current employees there is no termination date. The sampling approach taken, capturing only employees active in a fixed window creates two forms of bias in the sample that must be accounted for, right censoring and left truncation. A subject is right censored if their endpoint, retirement in this case, is unknown at the time of the study since they are still actively employed. Right censored observations do provide information and should not be dropped but must be analyzed differently than complete observations. Left truncation results from a failure to include cases that fail before the beginning of the study window. This results in a biased sample since only those cases that survive long enough will be represented in a sample. Both of these potential biases will be considered during the discussion of models below.

A number of static employee attributes are provided in the list below:

- Payroll (PR): hourly, weekly, or monthly payroll,
- Gender (GENDER): male, female
- Division (DIV): used to distinguish the departments, including ten departments. In this study, the definition of each division may not be static over the entire period of observation. Over the course of time, divisions can be renamed, reduced, or dismissed in reorganizations. Furthermore, no transfer of employees between divisions is recorded. Therefore, for purposes of prediction, the division variable indicates the organization level that an employee is associated with at the time of final observation.
- Occupational Code (OC): a standardized code used to describe the job category in the organization for reporting purposes including, Crafts(C), Engineers (E), General Administrative (G), Laborers (L), General Managers (M), Administrative (P), Operators (O), Scientists (S), Technicians (T). In this study, occupational code are highly correlated with payroll category: managers, engineers, administrative, and scientists are monthly payroll, general administrative employees and technicians have weekly payroll, and other categories are paid on an hourly basis.
- Age at hire (AGEH): age at most recent time that an employee is hired or their age in November, 2000.
- Age at credit (AGEC): age at most recent time that an employee is credited for their pension.
- Years of Current Service (YCS): the years of service which accounts for pension credit
- Years of service at hire (YCSH): the years of service which accounts for pension credit at the most recent hiring of the employee.
- Termination date (TD): the date when an employee left the organization.
- Termination type: the reason for employee leaving the organization, such as retirement (RE), voluntary quitting (VQ), or other reasons.
- Points: An employee's points are the sum of their age and YCS. YCS can be larger than 0 at hire if an employee has credit from earlier employment in the same organization.

In this study, all of the employees are eligible for a defined benefit retirement plan. Beginning in January of 2012 the organization began to offer new employees defined contributions plans but such plans were not available during the period of the current study. An employee is eligible for full pension benefits after exceeding either 65 years of age or 85 points of retirement credit.

Beyond employee level information, we also investigate the potential impact of external exogenous economic factors on retirement decision making and employee turnover. A number of financial indices were selected to capture the various economic factors that may play a role in decisions to retire or leave a position. Because factors such as job market, housing market, and financial markets potentially influence high impact financial decisions. List of financial variables.

- Unemployement Rate (Unemployement), seasonally adjusted unemployment rate published by the U.S. Bureau of Labor Statistics (U.S Bureau of Labor Statistics, 2015)
- Monthly Housing Price(MHP), unadjusted monthly U.S housing price index (Federal Housing Finance Agency, 2015),
- Seasonal Adjusted Monthly Housing Price (SAMHP), seasonal adjusted monthly housing price.
- Southeastern Monthly Housing Price (SEMHP), unadjusted southeastern monthly purchase-only index
- Seasonal Adjusted Southeastern Monthly Housing Price (SESAMHP), seasonal adjusted southeastern monthly purchase-only index,
- S&P 500 stock index and Dow Jones Indices: S&P 500, dividend, earnings, consumer index, long interest rate, real price, real dividend, real earnings, P/E 10 (S&P Dow Jones Indices, 2015)
- Wilshire 5000: total market full cap index published by Wilshire Associates (Wilshire5000) (Wilshire Associates, 2015).

All twelve of these indices were operationalized for testing using their using a twelve month lag of their one year averages. This approach ensures that the variable can be useful in forecasting since the one year lag will be known at the time of forecasting. The economic indices are originally reported at the daily or monthly level. The yearly average is computed as the average value of the index over the previous twelve months.

4 Model Development and Evaluation

This study aims to develop accurate predictive models of retirement and quitting behavior. The model will be used to address several key questions:

1. How accurately can retirement(quitting) be predicted?
2. What factors indicate an individual is more likely to retire(quit)?
3. Which external economic factors are most predictive of retirement(quitting)?

4. What is the magnitude of the impact of an Early Retirement Incentive Programs (ERIP)?

5. How do the tenure and age impact retirement?

6. How many employees will retire(quit) next year by occupational category and division?

Survival or lifetime data analysis is methodology that is used to study the distribution of time that it takes for a subject from a population to experience an event such as mechanical failure, death or recovery. Survival regression models relate aspects of the lifetime distribution such as the hazard function to a linear function of explanatory variables. Statistical survival models are often separated into two categories: parametric survival models and semi-parametric proportional hazards (PH) models or Cox models. In this study, the Cox PH model is employed to build predictive models of retirement and quitting, to estimate a employees' baseline hazard of retirement or quitting, and to identify significant factors that may impact turnover. The parametric models are not appropriate for this study for several reasons. First, it is unlikely that hazards for events like retirement would match common parametric distributions such as weibull or log-normal since the risk should stay close to zero until the usual range of retirement at which time it spikes and then drops quickly again. Furthermore, as mentioned in the introduction, the sampling scheme used in this data involved several biases, which can most easily be adjusted for using the Cox PH model. A third advantage of the cox model is the ability to incorporate time dependent covariates in the model. In the case of the current model these are required to incorporate the impact of a 2008 early retirement intervention promotion (ERIP) in the organization, as well as examining the effects of two retirement key variables and outside economic indicators. A special version of the model known as the competing risks analysis is applied for modeling a population which can experience two types of events, in this case employee retirement and voluntary quitting. In addition to the model fitting, a simulation study is performed to examine the impact of data bias on the forecasting capability of Cox proportional hazard model.

4.1 Missing Data Biases: Right Censoring and Left Truncation

Right censoring and left truncation are both commonly observed forms of missing data in survival analysis data sets. In the current, the study window is from November 2000 to December 2012 as shown in the Figure 1. Anyone that was an active employee during this period has their complete record included in the dataset whether or not their tenure began or ended outside of the study window. Conversely, those employees whose tenure ended before the study window or for whom the start date occurred after the study window ended are not included in the study.

Let T be the time at which an individual experiences the event of interest and let C denote the final time at which the individual is observed. An observation is called *right censored* if $T > C$, indicating that actual event time for the individual is not recorded but is only known to be greater than C. Thus, employees that are currently active at the end of the observation window are right censored. These right censored observations do contain

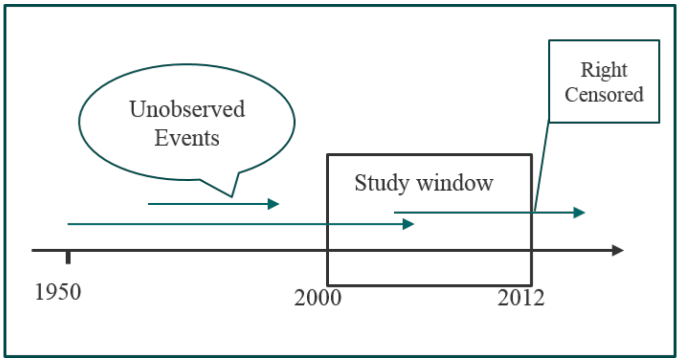


Figure 1: Right Censoring and Left Truncation

information, although incomplete, about a subjects lifetime and require special treatment in order to draw proper inference.

$$\delta_i = \begin{cases} 1 & \text{if } t_i \leq c_i \text{ (uncensored)}, \\ 0 & \text{if } t_i > c_i \text{ (censored)}, \end{cases}$$

where, i denotes the i^{th} observation, and the failure time of event for i^{th} observation is minimum time between t_i and c_i, i.e., $min(t_i, c_i)$, that is when $c_i < t_i$, c_i is taken as end time of the i^{th} observation in order to do next analysis.

Left truncation is another interesting artifact of the window sampling scheme. Let T again denote the time that the event of interest occurs and let X denotes the time an individual enters the study. Only the individuals with $T \geq X$ are observed in the study window. Those individuals with $T \leq X$ are referred to as left truncated because they cannot be included in this study based on the sampling window as shown in the Figure 1. Left truncation exaggerates the number of longer life individuals leading to a biased sample. To see this consider Figure 1. The longest arrow represents a life span for an employee hired in 1950 and retiring in 2006. While this employee and any in his cohort that are still active are in the sample, others that began in 1950 but retired in 1998, for example, are not. Hence, because of the sampling window approach the longer someone continues working, the more likely they are to appear in our dataset. We therefore see an overabundance of longer lived individuals in our data. The presence of left truncation and the associated bias in the data must be taken into account to achieve accurate estimation of survival analysis (Carrión et al., 2010).

4.2 Cox Proportional Hazards Regression Model

The Cox proportional hazards (PH) regression model is the most widely used method for modelling lifetime data. The model was introduced in a seminal paper by Cox (1972), one of the most cited papers in history. The Cox PH model is the canonical example of the semi-parametric family of models, specifying a parametric form for the effect of the covariates on an unspecified baseline hazard rate which is estimated non-parametrically. The form of hazard model formula as shown in the equation 1:

$$h(t, x) = h_0(t) e^{(\sum_{i=1}^{k} \beta_i x_i)} \tag{1}$$

where $x_i = (x_{i1}, x_{i2}, \ldots, x_{ik})$ are characteristics of individual i, $h_0(t)$ is the baseline hazard, and β is a vector of regression coefficients. The model provides an estimator of the hazard at time t for an individual with a given set of explanatory variables denoted by x_i. In the standard Cox model, the linear combination $\sum_{i=1}^{k} \beta_i x_i$, is not a function of time t, and is called time-independent. If $x_i(t)$ is a function of time the model is called the extended Cox model and is discussed in Section 4.3. A key assumption for the model is the proportional hazards assumption. Proportional hazards assumes that explanatory factors have a strictly multiplicative impact on the hazard function so that different groups maintain a constant hazard ratio at all times. However, Cox regression can be extended to handle non proportional hazards using time-dependent variables or stratification; see Klein and Moeschberger (2003).

The Cox PH regression is "robust" and popular, because the baseline hazard function $h_0(t)$ is an unspecified function and its estimation can closely approximate correct parametric model (Kleinbaum, 1998). Taking the logarithm of both sides of the equation, the Cox PH model is rewritten in the equation 2:

$$\log h(t, x) = \alpha(t) + \sum_{i=1}^{k} \beta_i x_i \tag{2}$$

where $\alpha(t) = \log h_0(t)$. If $\alpha(t) = \alpha$ i.e. constant, then the model reduces to the exponential distribution. As noted earlier, the general Cox PH model puts no restrictions on $\alpha(t)$. The partial likelihood method is used to estimate the model parameters (Allison, 1995). [NEED TO CHECK]

4.3 Time Dependent Variable and Counting Process

Some explanatory variable values change over the course of the study. The extended Cox PH regression is a modification of the model that incorporates both unchanging time-independent variables as well as variables that change with time or time-dependent variables,

$$h(t, x) = h_0(t) e^{(\sum_{i=1}^{k_1} \beta_i x_i + \sum_{j=1}^{k_2} \gamma_j x_j(t))} \tag{3}$$

where $x = (x_1, x_2, \ldots, x_{k_1}, x_1(t), x_2(t), \ldots, x_{k_2}(t))$, $h_0(t)$ is the baseline hazard occurring when $x = 0$, β and γ are the coefficients of x. In order to fit this model, a modification of the partial likelihood is required and we often present the data set in a format called the *counting process* format in order to facilitate this calculation. The current study considers three internal time dependent variables that are functions of the individuals characteristics.

- Early Retirement Incentive Program (ERIP): a specific time during the study window, part of the 2008 calendar year, when the organization offered an early retirement incentive program; see (Clark, 2002). It is time varying in the sense that it occurs at a different age for each individual in the study population and is set at 0 during the period when no program exists and 1 during the period when the program does exist.

- Points 85 (P85): an indicator that an employee has amassed 85 service points, the sum of their years of service and age, and qualifies for full retirement benefits,

- Age at 65 (A65): an indicator that an employee qualifies for retirement by exceeding the age 65 threshold

P85 and A65 are two additional time varying variables that capture important changes in individuals hazard level throughout the study.

The counting process format allows software packages to handle time dependent variables, by creating multiple intervals for each employee. Each interval is defined so that the time varying variables are constant within the interval. For example, for an individual that remained active until the end of the study and that achieved 85 points in December 2003, exceeded age 65 in December 2007, and received a retirement incentive in the 2008 calendar year, our study would include 5 records for the individual, November 2000 - November 2003, December 2003 - November 2007, December 2007, January 2008-December 2008, and January 2009 - December 2012, which is the end of the study.

Economic indicators represent another form of time dependent variables. These are referred to as external variables because they depend upon factors external to the employee. In models considering economic variables, monthly observations of economic indicators are aggregated at the yearly level leading to a larger number of time periods in the counting process format. The economic variables are included at a one year lag since it is assumed that retirement and quitting decisions are made a significant time in advance of the actual event and therefore depend on older values of these indicators. Importantly, using lagged quantities allows the model to be used for forecasting for a 12 month period since these numbers will be known at the time of forecast.

The start and end points of each interval are given in the definitions below.

$$\begin{aligned}(\text{start point}, \text{end point}) = (&max(\text{hired date}, \text{January 1 of a certain year}),\\ &min(\text{terminated date}, \text{December 31 of a certain year}))\end{aligned} \tag{4}$$

4.4 Stratification and Multiple Baselines

An alternative for handling nonproportional hazards is stratification. A stratified model allows each subgroup of data as defined by a grouping variable to have its own baseline hazard while sharing parameters for other variables across. If the proportional hazards assumption holds within these subgroups then this model allows us to get valid common estimates of variable effects using all of the observations. Equation 5 below represents the hazard function for strata z;

$$h(t, x, z) = h_0^z(t)e^{(\sum_{i=1}^{k} \beta_i x_i)} \tag{5}$$

where z represents the grouping variable, and $h^z\sigma_0(t)$ is a baseline hazard based for stratam z and β_i are common effects of variables. Note that the strata variables cannot be the variables in the Cox PH model.

4.5 Testing the PH Assumption

Three common approaches areavailable for testing the validity of the proportional hazard assumption: The first approach is to investigate the Schoenfeld residuals. A second alternative is to test the interaction between time-dependent and time-independent variables in the Cox PH model. The PH assumption is valid if the interaction is not statistically significant ($P > 0.05$). Finally, including separate baseline hazards for each strata defined by the analyst can also capture variation in changes of the hazard rate. See Allison (2010); Collett (2015) for more details on these tests.

4.6 Competing Risks

One of the many nuances observed within this data set is the fact that currently employees can leave employment in several mutually exclusive ways. These include leaving due to quitting voluntarily, being laid off, dismissed for cause, transferred, retired, or being unable to continue due to disability or death. A competing risk is an event whose occurrence either precludes the occurrence of the event of interest or fundamentally alters the probability of occurrence of this event of interest (Tableman and Kim, 2003). A competing risks model is a common approach when studying a single mode of leaving such as retirement if subjects at risk may also exit through an alternative mode such as quitting. In the current study, when considering retirement as the event of interest and voluntary quitting as a competing risk, all observations are initially included in the study and outcomes that end in a quitting event are treated as censored which allows their observed work period to be used informatively.

4.7 Variable Selection and Model Choice

In the current study two equivalent time measurements were considered as response variables for modelling, AGE in years and YCS. Because of the inclusion of time varying explanatory variables, and the need to estimate the baseline hazard for purposes of forecasting, the data must be formulated as a counting process , see Section 4.3. After some consideration, age was considered the better option for analysis because the more condensed distribution of values allows more accurate estimates of the baseline.

The model selection initiated by considering DIV, AGC, GENDER and other time independent variables as well as ERIP, P85, and A65. We removed non-significant variables using the criteria that p-values should be less than .05 and starting with the largest p-value first, i.e. backwards selection. This continued until only statistically significant variables remained. We also tested stratifying the baseline using occupational code, which did not improve the model. Finally we tested external time varying covariates, which capture economic factors, one at a time and noted the impact on model performance.

4.8 Model Evaluation and Comparison

In order to evaluate the models considered in this study, the data were first split into two sets, a training set containing all of the observations from years 2000-2010 and a testing set containing events on the same individuals that occurred in calendar years 2011 and 2012. The testing (holdout) sample was included to get an accurate measure of how well the model would forecast beyond the observed data. Because the testing data set is not included in the model fitting process, this out of sample evaluation provides a better estimate of predictive accuracy, see Kuhn and Johnson (2013) for further discussion of this approach.

All of the fitted models considered in this study are first evaluated by four statistical criteria: Akaikes Information Criterion (AIC), Schwartzs Bayesian Criterion (SBC), mean absolute percentage error (MAPE) and likelihood based goodness of fit G^2. Ideally, the optimal model should minimize the values of AIC, SBC, MAPE, and G^2 when fit to the training data. In this study, the model performance on holdout dataset is considered more important than that on the training dataset. AIC and SBC both assess model fit by balancing a larger likelihood value with a penalty that increases with the number of variables included. The inclusion of the penalty term diminishes the potential for over-fitting (Allison, 2010; Hosmer et al., 2013). These measures are generated automatically by the model fitting process.

In order to asses predictive measures such as MAPE and G^2 we first predict the probability of the event of interest, e.g. retirement, for each active individual during each calendar year of the training or testing data set. For each employee, we compute the conditional probability of the event occurring between time t_j and t_{j-1}, given that the employee is active at time t_{j-1}. It is calculated using the baseline hazard and coefficients from Cox PH models as shown in Equation 6 below

$$\begin{aligned} P\{t_{j-1} < T < t_j | T \geq t_{j-1}\} &= 1 - P\{T > t_j | T \geq t_{j-1}\} \\ &= 1 - \frac{S_k(t_j)}{S_k(t_{j-1})} \\ &= 1 - \frac{S_0(t_j)^{(\sum_{i=1}^{p} \beta_i x_i)}}{S_0(t_{j-1})^{(\sum_{i=1}^{p} \beta_i x_i)}} \end{aligned} \tag{6}$$

where, T_k is survival time of the k^{th} individual , t_j is a specific time value, $S_k(t) = S_0(t_j)^{(\sum_{i=1}^{p} \beta_i x_i)}$ is the survival function for the k^{th} individual, $S_0(t)$ is the baseline function generated by Cox PH model, x_i are the individual explanatory variables, and β_i are the respective regression coefficients for the variables.

MAPE is common measure for computing the accuracy of predictions from a forecast model and is often used to compare models since it measures relative performance (Chu, 1998). MAPE is calculated as the average percent deviation of a forecast from the actual observation,

$$MAPE = \sum_{t=1}^{n} \left| \frac{y_t - \hat{y}_t}{y_t} \right| \frac{1}{n} \% \tag{7}$$

Our implementation of MAPE for retirement predictions used the yearly actual and predicted numbers of retirements number as y_t and $\hat{y}_t$. The predicted yearly retirement number, $\hat{y}_t$, is the expected retirement count for a particular year and is computed as the sum of conditional probabilities given by Equation 6 over currently active employees. This follows from the fact that the probability of each employee retiring can be viewed as an independent Bernoulli random variable.

Although less common in forecasting, G^2 is another useful criteria for evaluating model prediction for dichotomous events (Simonoff, 2013). The calculation takes the form,

$$G^2 = 2 \sum_t [y_t \log(\frac{\bar{p}_t}{\hat{p}_t}) + (n_t - y_t) \log(\frac{1 - \bar{p}_t}{1 - \hat{p}_t})] \tag{8}$$

where, y_t is the number of employees retired in year t, n_t is the workforce number in year t, $\bar{p}_t = y_t/n_t$ is the observed proportion of events, and $\hat{p}_t = \hat{y}_t/n_t$ is the model predicted number of events. Small values of G^2 indicate close agreement of observed and predicted numbers of events.

5 Simulation Studies of Proportional Hazards Models

The goal of the proposed data analysis is to create a predictive model for retirement and other types of turnover based on a database of employees. Section 4.1, pointed out two sources of bias present in the data, left truncation and right censoring. Because the Cox PH model is estimated using a partial likelihood that depends only upon the cases at risk at the specific failure times, estimates of regression coefficients should remain unbiased and efficient in the presence of left truncation and right censoring (Harrell, 2013). What is less clear is the impact of the bias on model predictions. This stems from the fact that model predictions depend upon both the baseline, estimated using non-parametric methods, and the parametric estimates of the regression coefficients. In addition, a third ever-present challenge, model selection, may also impact predictions to a significant degree.

In order to better understand the effect of various levels of truncation and censoring on the predictions we perform 3 simulation studies based on Weibull simulated data with varying amounts of bias.

The basic setup in all three simulations is the same. Data sets of sizes $n = 100, 200, 500, 1000, 2000$, and 4000 were generated from a Weibull regression model which was a function of 1 explanatory variable, which we refer to as *age*. In the simulation, *age* is uniformly distributed from 22 to 70 years, a range that is chosen to mimic the actual distribution of worker ages observed in our sample.

The baseline hazard for Weibull distribution with shape α and scale λ is $h_0(t) = \alpha(\lambda)^\alpha t^{\alpha-1}$. Extending this to a hazard from a proportional hazards regression model for *age*, we simply multiply by the exponentiated linear predictor shifting the baseline up or down,

$$\begin{aligned} h(t|age) &= h_0(t)exp(\beta \times age) \\ &= \alpha(\lambda(exp(\beta \times age)^{\frac{1}{\alpha}})^\alpha t^{\alpha-1} \\ &= \alpha(\tilde{\lambda})^\alpha t^{\alpha-1} \end{aligned} \tag{9}$$

where, $\tilde{\lambda} = \lambda(exp(\beta \times age)^{\frac{1}{\alpha}}$.

The survival times T_i are randomly generated from the Weibull distribution with shape parameter $\alpha = 1.5$ and $\tilde{\lambda} = exp(1.5 + 0.025 \times age)^{\frac{1}{\alpha}}$. It follows that $\lambda = e^1$.

The simulations are performed using the `coxreg` and `phreg` functions from the `R`-package `eha` (Brostrm, 2015) for model fitting. Function `coxreg` performs a standard Cox PH regression using the partial likelihood to fit the model. The `phreg` function performs a parametric proportional hazards regression using both Weibull, Extreme value (EV) baselines.

5.1 Simulation 1: Right Censoring

The first study focuses on understanding the impact of right censoring, a significant effect in the turnover data analyzed later due to the many employees that remain active for the entire observation window. For each of the sample sizes above, survival times T_i are simulated from the Weibull distribution as described earlier. Four censoring times C_j are defined as the first, second, third, and fourth(maximum) quartiles of the simulated sample of lifetimes and refer to 75%, 50%, 25% and 0% censoring proportions respectively. If the survival time T_i of the i^{th} observation is below the censoring time (C_i), then the lifetime T_i is observed and the censoring indicator $\delta_i = 1$. When the survival time T_i for i^{th} observation is greater than the censoring time (C_i), then the censoring time C_i is observed and censoring indicator $\delta_i = 0$.

The results of 100 simulations at each combination of sample size and censoring proportion are shown on the left side of Table 1. Column 1 gives the censoring proportion, column 2 the observed number of events before censoring, and columns 3 and 4 are the average β_{age} estimates over 100 simulations for both `coxreg` and `phreg`. The values in columns 5 & 6 are the average estimates of λ and α from the parametric fit of `phreg`. The simulation results show censoring proportion and the number of events are two influential factors for the coefficient estimation. The model overestimates the coefficients of age, λ, and α, when the dataset has a high proportion of censoring. For example, when 75% of the data are censored with only 25 events, the estimates for three parameters are 0.028, 4.043, and 1.564, respectively, which are the highest among all the estimates. As the event number increases, the estimates approach the actual value. For example, the estimation of age, λ, and α are close to 0.025, 2.7, and 1.5, respectively, as the number of events exceeds 500.

5.2 Simulation 2: Right Censoring with Staggared Entry Times

In order to more accurately capture the nuances and complexities of the data sampling scheme and understand the impact of right censoring we modify the above simulation by staggering the entry times. Starting with the Weibull simulated failure times, we add offset factor S that follows a uniform distribution from 0 to 10 and represents variation in the starting times of the employees within the study window. The event time is equal to the summation of start point and survival time: $S + T$. The censoring time is a single fixed value that ensures a fixed proportion (25%, 50%, and 75%) of censored observations. The observation and censoring indicator are then determined as in the first simulation with the survival time for an individual being $min(C, T_i + S_i) - S_i$. Because some observations start after the cutoff point (censor time) the sample sizes would vary for different censoring proportions. To ensure constant sample size, 6000 observations are initially simulated and 400 whose start point occur before the censoring point are randomly selected.

The results of the simulation are shown in Table 2. As discussed above, both a correctly parametric proportional hazards regression model with a Weibull baseline `phreg` and a semi-parametric Cox regression `coxreg` are fit in order to evaluate differences in efficiency. The estimation of age and α are all close to the true values (0.025 and 1.5) when averaged over 100 simulations. Based on 400 events, the estimates of λ increase from 2.694 with no censoring to over 2.8 when censoring is above 50%. In general, the results indicate that right censoring has little impact on coefficient estimation and that the semi-parametric estimates show similar efficiency to fully parametric estimates. However, right censoring does impact estimation of the baseline function for the Cox PH model as shown in the 6th and 7th columns of the Table and Figure 2. Panel (a) shows no censoring and the parametric baseline matches the non-parametric although as time increases and the number at risk decreases we see that

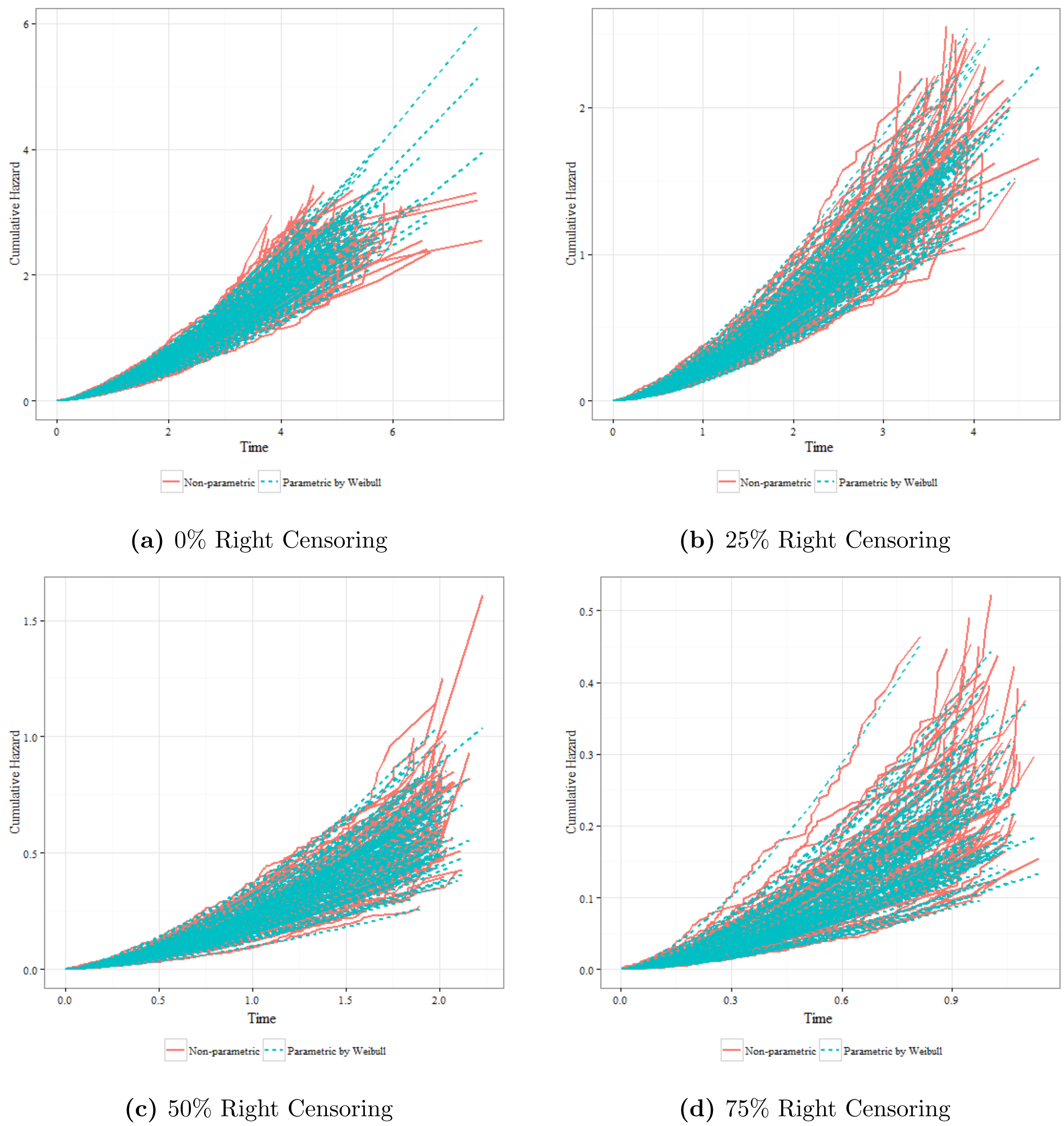
Table 1: Right Censoring and Left Truncation Simulation Statistics

Right Censor						Left Truncation					
Right Censor Proportion	Events	β_{coxreg}	β_{phreg}	λ_{ph}	α_{ph}	Left Truncation Proportion	Events	β_{coxreg}	β_{phreg}	λ_{ph}	α_{ph}
0%	100	0.026	0.027	2.931	1.509	0%	100	0.027	0.027	2.865	1.534
25%	100	0.027	0.027	2.962	1.527	25%	75	0.027	0.027	2.917	1.546
50%	100	0.028	0.028	3.237	1.530	50%	50	0.027	0.027	2.899	1.577
75%	100	0.028	0.028	4.043	1.564	75%	25	0.029	0.029	3.280	1.757
0%	200	0.026	0.026	2.841	1.508	0%	200	0.025	0.025	2.777	1.506
25%	200	0.026	0.026	2.856	1.513	25%	150	0.025	0.025	2.756	1.515
50%	200	0.026	0.026	2.925	1.527	50%	100	0.025	0.025	2.825	1.532
75%	200	0.026	0.026	3.167	1.540	75%	50	0.026	0.026	2.927	1.572
0%	500	0.025	0.025	2.731	1.500	0%	500	0.025	0.025	2.732	1.509
25%	500	0.025	0.025	2.718	1.508	25%	375	0.025	0.025	2.737	1.514
50%	500	0.025	0.025	2.744	1.514	50%	250	0.026	0.026	2.778	1.514
75%	500	0.025	0.025	2.787	1.525	75%	125	0.026	0.026	2.835	1.547
0%	1000	0.025	0.025	2.748	1.509	0%	1000	0.025	0.025	2.710	1.504
25%	1000	0.025	0.025	2.747	1.512	25%	750	0.025	0.025	2.709	1.504
50%	1000	0.025	0.025	2.748	1.514	50%	500	0.025	0.025	2.715	1.506
75%	1000	0.026	0.026	2.844	1.509	75%	250	0.025	0.025	2.694	1.524
0%	2000	0.025	0.025	2.714	1.502	0%	2000	0.025	0.025	2.740	1.503
25%	2000	0.025	0.025	2.713	1.503	25%	1500	0.025	0.025	2.731	1.502
50%	2000	0.025	0.025	2.742	1.500	50%	1000	0.025	0.025	2.724	1.503
75%	2000	0.025	0.025	2.733	1.502	75%	500	0.025	0.025	2.718	1.508
0%	4000	0.025	0.025	2.719	1.504	0%	3999	0.025	0.025	2.720	1.500
25%	4000	0.025	0.025	2.718	1.505	25%	3000	0.025	0.025	2.722	1.501
50%	4000	0.025	0.025	2.724	1.503	50%	2000	0.025	0.025	2.710	1.502
75%	4000	0.025	0.025	2.729	1.513	75%	1000	0.025	0.025	2.703	1.503

Table 2: Right Censor Simulation Results by Various Start Time

Censoring Pct.	Events	Variable Estimates			Predicted Events	
		age	λ	α	"coxreg"	"phreg"
0%	400	0.025	2.694	1.508	398.52	400.44
25%	400	0.026	2.802	1.518	394.24	401.72
50%	400	0.026	2.828	1.514	340.73	398.51
75%	400	0.025	2.821	1.518	215.92	400.80

non-parametric estimate deviates more strongly from the parametric fit. Panel (b) shows 25% censoring for 100 replications. We see that, as the range of the data decreases, the duration of the non-parametric baseline estimate is more restricted, while the parametric fit can be extrapolated with the usual caveats. In panel (d), the 75% censoring level, we see increased variability even with the diminished range due to the more restrictive censoring time. Note that these results are indicative of monotonically increasing hazards and may differ if we consider other baseline distributions



(a) 0% Right Censoring

(b) 25% Right Censoring

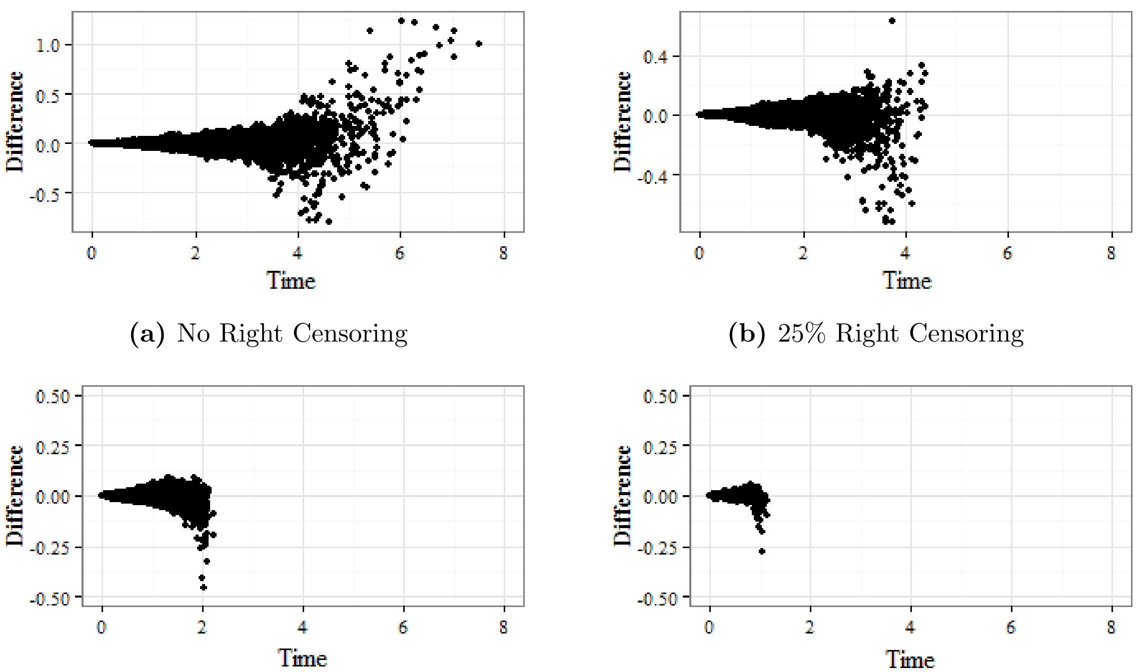
(c) 50% Right Censoring

(d) 75% Right Censoring

Figure 2: Baseline Comparison by Various Censoring

[PARAMETRIC BOOTSTRAP DISTRIBUTION-LOOK AT RESEARCH, ALSO SIMONOFF PAPER]

Figure 3, panels (a-d) illustrate the difference between parametric and non-parametric estimates of the baseline cumulative hazard function, $H_0(t) = -log(\hat{S}(t))$ at different time points over 100 simulated data sets. Panel (a) indicates that without censoring, overall survival estimates including both baseline and covariates are very similar across parametric and semi-parametric approaches. As lifetime, t increases, we see increasing deviations between the two estimates. This due both to the cumulative nature of the estimates and to the increasing variability of hazard estimation of the non-parametric baseline, the Breslow estimator, (Davison and Hinkley, 1997; Burr, 1994) as the number of observations at risk diminishes over time.



(a) No Right Censoring

(b) 25% Right Censoring

(c) 50% Right Censored

(d) 75% Right Censoring

Figure 3: Differences in Estimates of Baseline Cumulative Hazard Functions, Parametric - Non-parametric, for 4 Levels of Censoring

When censoring is introduced in panel (b) we see that the range of the x axis is restricted. Although this varies slightly across simulations, it is clear that the non-parametric estimator cannot estimate survival probabilities beyond the maximum observed lifetime due to the lack of a parametric model for the baseline. In the current simulation, this was dictated by the censoring time, which was set to ensure a fixed proportion of censored cases. In real studies, such as the one introduced in Section 6, the maximum observed lifetime will be dictated by the population and the sampling window. Panels (c) and (d) reiterate both of these factors. As the censoring level increases to 50% and finally 75%, the range of non parametric baseline estimate is further restricted but the accuracy improves due to the increasing concentration of observations before the censoring time. Because all simulations had 400 observations, 75% censoring ensures 100 events before the censoring point, which is close to 1 providing a very accurate estimate of the baseline and less dispersion than the uncensored data. Parametric models do not suffer from limitations on the baseline estimate, a seeing advantage, but instead require careful model selection steps which introduce other challenges.

5.3 Left Truncation Simulation

Finally, we perform a simulation to evaluate the impact of left truncation bias on parameter estimation and prediction. Again, we begin by generating a simulated sample of employment times $T_1, \ldots, T_n$. For each observation, i, a uniform random variable $S_i \sim U(0, max(T))$ is then generated and represents the simulated employee starting time. The event time is then $R_i = S_i + T_i$. Figure 4 provides a histogram of the simulated population of R_i. Four levels of truncation are introduced by shifting the beginning of the sampling window, L from 0 across the quartiles of R . When the starting for ith observation, S_i, is less than truncation time l_i, the observation starting point is reset to l_i which is the first point that the employee is observed in the sampling window. If $S_i > l_i$, then the employee is first observed at S_i, which remains the starting point. If $R_i > C$, where C represents the end of the observation window, then the employee is still active at the end of the sampling window and their turnover time is censored. In order to isolate the impact of truncation bias, no censored values were generated in this study.

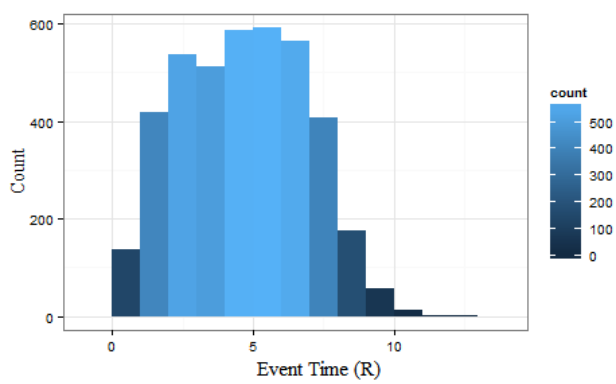
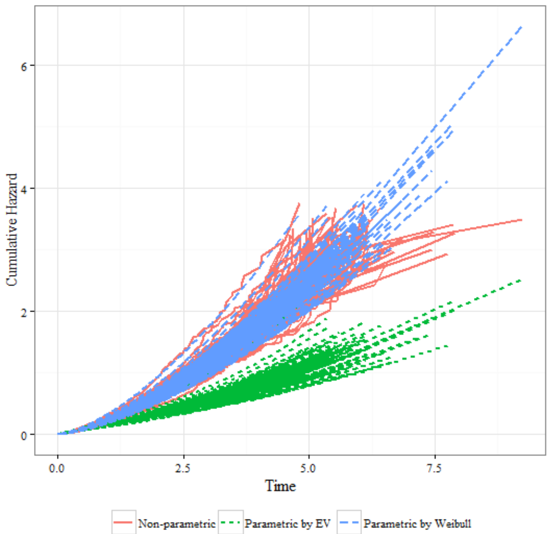


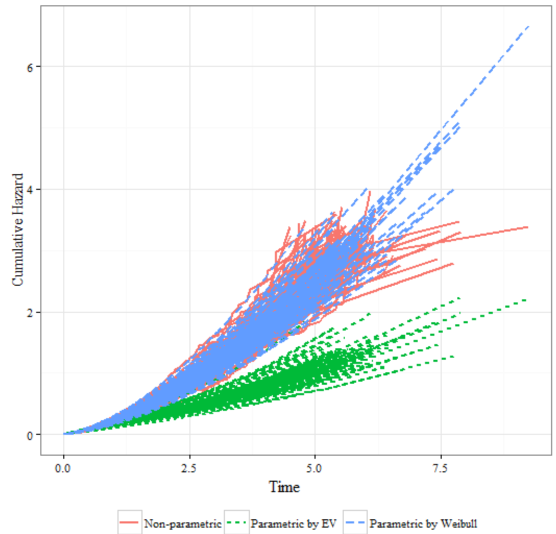
Figure 4: Histogram of Simulated Lifetimes

Sample size and censoring proportions and results follow the same protocol given in Section 5.1. The results of the simulation are shown on the right side of Table 1. As before, values shown represent average parameter estimates over 100 replications for the Cox model and a Weibull PH. The general pattern suggests that as truncation proportion increases for a fixed number of events, the parameter estimates become slightly biased. In the Cox PH case when the sample size is 100, the coefficient for age increases from .027 to .029 as the truncation increases from 0% to 75%. The scale parameter for the parametric model, `phreg`, also increases with truncation percentage. For samples of size 100, with $\lambda = e^1 \approx 2.718$ the average estimate increases from 2.865 with no truncation to 3.280 with 75% truncation. Estimates for $\alpha = 1.5$ increase from 1.534 to 1.757. Such effects disappear as the number of events increase.

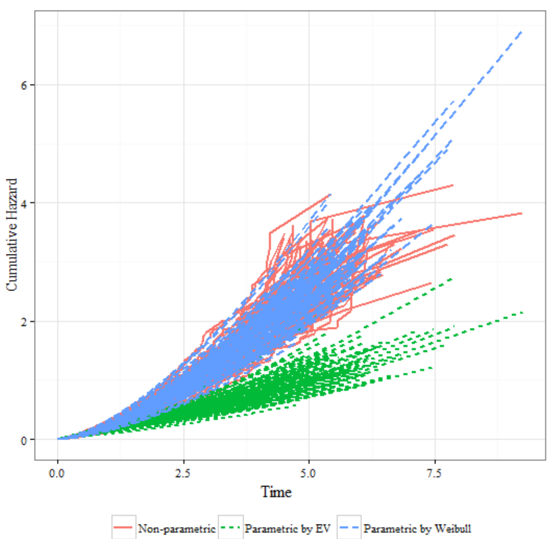
Figure 5 compares the non-parametric estimates of the baseline from the Cox PH model with two parametric proportional hazards fits estimated using `eha` (Brostrm, 2015). The first



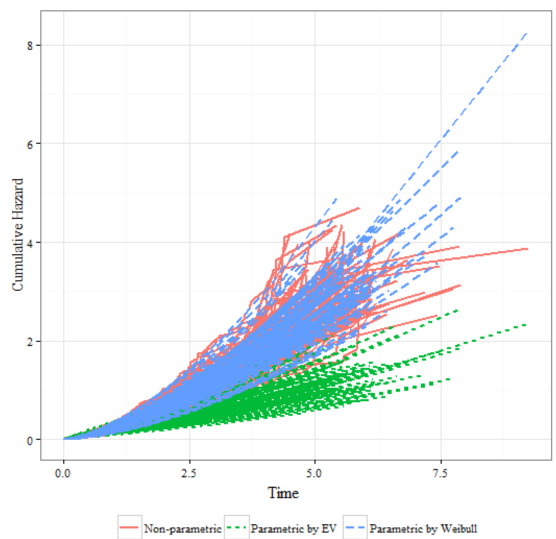
(a) No Left Truncation



(b) 25% Left Truncation



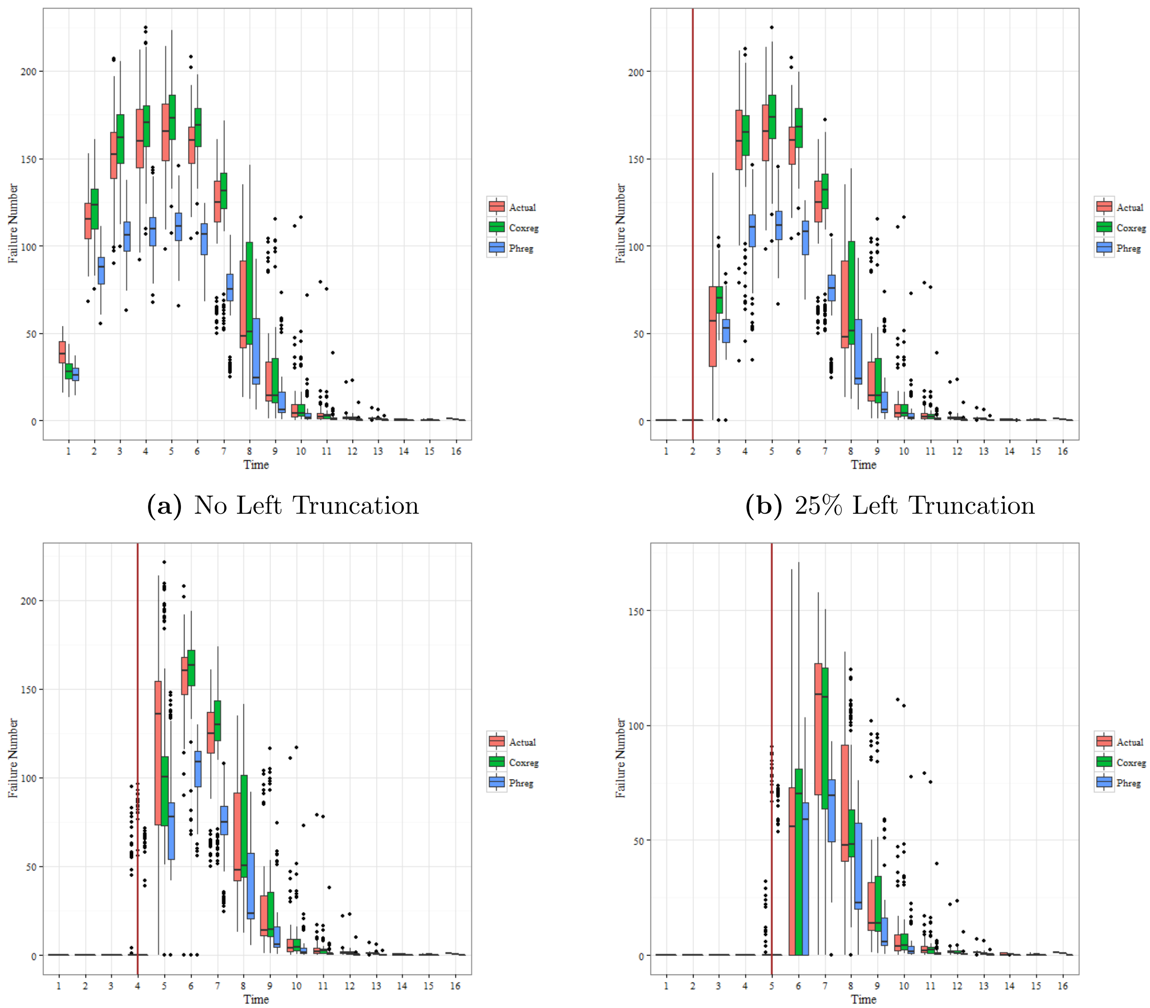
(c) 50% Left Truncation



(d) 75% Left Truncation

Figure 5: Left Truncation Simulation Predictions: Comparison of Actual vs. Cox PH and Parametric PH with EV baseline.

parametric fit is properly specified and assumes a Weibull distribution as in the prior simulations. The second fit assumes that data follow a type-I extreme value distribution[CHECK MANUAL ON THIS]. Unlike the simulation of Section 5.2 the duration of non-parametric baseline estimates are not limited by the four left truncation proportions. Both Weibull and Cox PH fits overlap with the variability of the Cox baselines showing increased variability with amount of truncation. EV based fits all significantly underestimate the cumulative hazard indicating a potential risk of parametric modelling. However, it is interesting to note that the EV fits converge slightly with increasing amounts of truncation. This is a conse-



(a) No Left Truncation

(b) 25% Left Truncation

(c) 50% Left Truncation

(d) 75% Left Truncation

Figure 6: Left Truncation Simulation Results: Comparison of Actual vs. Cox PH and Parametric PH with EV baseline Predicted Failure Number

quence of extreme-value theory and the theory of exceedances; more information about this phenomenon can be found in Coles et al. (2001).

Figure 6 focuses on the number of events predicted by the models. Here 2000 events are generated using the protocol of this section. Both parametric and nonparametric models are fit to the data and this procedure is repeated 100 times. The red vertical line is the average left truncation time across the simulations for each truncation percentage. Using each model, the expected number of failures is then predicted for each 1 unit time interval and compared to the observed number of failures. Boxplots indicate the range of observed values across the 100 replications. The non-parametric Cox PH model tends to slightly overestimate the number of events in each interval indicating that baseline estimation is not affected by the left truncation. The overestimation can be explained by the convention used to compute the survival probability over each interval[ADD BY JULIA: The overestimation is because of jagged baseline function which is lack of information of certain points. The survival probability of those points is the previous closest failure point]. Matching parametric estimates are generated using a PH model with extreme value (EV) baseline instead of the true Weibull model. The underestimation of the baselines by the EV fit shown in Figure 5 lead to drastic under-predictions of the number of events.

Simulations were performed to better understand the effects of truncation and censoring bias on predictions.

In the situations considered, we found that, if sample size are large enough, in particular if they exceeded 250 events, all parameter estimates showed very little bias under both parametric and semi-parametric models. If samples were small, regression coefficients were slightly overestimated in both types of models. For data sets with small numbers of events due to heavy right censoring, parametric models tend to overestimate both shape and scale both leading to an underestimate of the baseline. Although the impact of right censoring on the non-parametric baseline is not directly quantified, Figure 3 suggests that baseline estimates are often smaller than the parametric versions and become more variable near the maximum lifetime. Simulations show similar impacts of left truncation on covariate and baseline effects. A major difference is that the length of the non-parametric baseline is potentially less limited due to the lack of censoring time. This may not be relevant in real world data, which has both effects.

Consequences of simulation study on Retirement. Although this study has more than 50% right censoring and an unknown proportion of left truncated cases, it still has more than 1000 retirement events, many with long duration (around 50 years length). Based on the simulation, we expect the Cox model to provide very accurate estimates of covariate effects in this case and highly accurate baseline estimates for most employees, particularly those with age less than 70.

6 Results & Analysis

The construction of predictive models for turnover involves consideration of several factors. Among the variables available for analysis, we must choose the set that offers the maximum predictive power, i.e. a model that includes variables that provide the best possible predictions on out of sample testing data sets and not simply on data used to train the model. We must also evaluate potential strengths and weaknesses of the baseline hazard estimate and its impact on prediction accuracy. The bootstrap will be a useful tool in assessing uncertainty in model predictions. From the perspective of administration it will also be useful to contemplate the ERIP implications of the predictors found to have an impact on prediction.

6.1 Descriptive Analysis

As described in Section 3, the current data provides five demographic and career history variables: PR (hourly, weekly, or monthly payroll), GENDER (M, F), DIV (ten levels of division), OC (crafts, engineers, general administrative, laborers, managers, administrative, operators, scientists, technicians), AGEH, and YCSH. Table 3 provides marginal counts of the number of workers in the sample within each category. We see that among occupation codes there are four large categories (C, E, M, &P) with over one thousand employees observed throughout the data sample, four medium sized groups with 500-700 employees (G, L, R, &T), and one small group, S, with 208 employees. Payroll data shows that the largest group is paid monthly, followed by hourly, and weekly. In terms of gender, approximately 72% of employees are male. Finally, divisions, while not fixed over the life of the employee, are distributed in a similar fashion to occupational codes with four larger groups and a number of smaller groups.

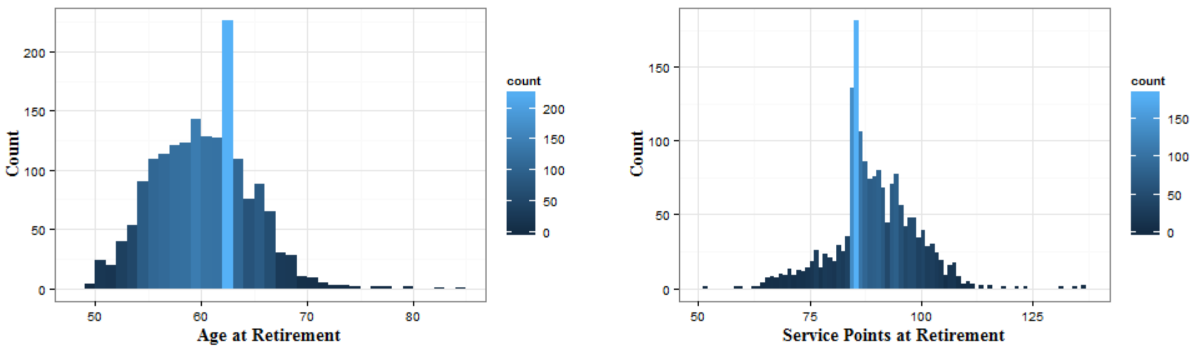
Table 3: Descriptive Statistics

Variable	Count	N %	Variable	Count	N %
OC			**GENDER**		
C	1295	16.0%	F	2296	28.4%
E	1361	16.8%	M	5802	71.6%
G	574	7.1%	**DIV**		
L	613	7.6%	Div1	1542	19.0%
M	1178	14.5%	Div2	751	9.3%
P	1621	20.0%	Div3	1042	12.9%
R	595	7.3%	Div4	369	4.6%
S	208	2.6%	Div5	398	4.9%
T	652	8.1%	Div6	1199	14.8%
Missing	1	0.0%	Div7	302	3.8%
PR			Div8	823	10.2%
Hourly	2503	30.9%	Div9	404	5.0%
Monthly	4369	54.0%	Div10	1268	15.7%
Weekly	1226	15.1%			

Table 4: Discriptive Statistics 2

	Count	Mean	Median	Mode	Minimum	Maximum	Std. Deviation
Age at Retire	1757	59.72	60.00	62.00	49.00	84.00	4.56
Years of Service at Retire	1757	29.72	30.90	30.06	0.05	55.68	7.74
Points at Retire	1757	89.44	88.67	85.47	51.05	136.66	9.15

Beyond these demographic and career factors, our models also include behavioral variables derived from policy requirements for retirement and early retirement incentives that occur throughout the observation period. Histograms of retirement age and accumulated pension points are shown in Figure 7. The histogram of retirement age is right skewed and shows an anomalous spike at age equals 62 which is the mode. We find that 79% of the employees that retired at 62 also reached or exceeded 85 points. [COMMENTS:WHY IS THIS SUCH A POPULAR AGE? MEDICARE, SOCIAL SECURITY?] The average retirement age is 59.72 demonstrating that many individuals retire before 62 and most before age 70. In terms of points accrued at time of retirement, recall that points are the sum of years of service plus current age, we see an irregular distribution with the vast majority retiring with point totals in the range of 85 to 100. Relatively few elect to take a reduced pension and retire with diminished benefits with points below 85. Again we see that 85 points is



(a) **(b)**

Figure 7: Histogram of Age and Point at Retire

the mode indicating that retiring immediately after become fully vested in the pension is a popular choice.

6.2 Models for Retirement without External Economic Variables

Section 4.7 discussed two potential response variables that are suitable for time until turnover models, age and years of service from hire. Because of the existence of time varying covariates and the need for a baseline estimate to facilitate predictions, we chose to model age at retirement arranging the data in a counting process format (SAS will only estimate the baseline if the counting process formulation is used).

Table 5: Models Statistics for Retirement

No.	Model	LR	AIC	SBC	Pred. MAPE	Holdout MAPE	Pred. G^2	Holdout G^2
1	DIV GENDER PR OC YCSH AGEH	1194.3	19271.3	19377.3	39.44	56.78	381.77	85.19
2	DIV OC YCSH AGEH	1193.8	19269.8	19370.5	39.45	56.84	381.92	85.29
3	DIV ERIP YCSH AGEH	1451.6	18998	19061.6	25.91	15.51	128.75	2.64
4	DIV ERIP YCSH AGEH OC	1469.92	18995.7	19101.7	25.91	15.24	129.47	2.56
5	DIV ERIP YCSH AGEH P85	1826.95	18624.6	18693.6	25.59	19.04	109.17	3.40
6	DIV ERIP YCSH AGEH P85 P85*A65	1873.69	18579.9	18654.1	25.38	7.97	111.55	0.79
7	DIV ERIP YCSH AGEH P85 P85*A65 P85*ERIP	1881.02	18574.6	18654.0	25.42	4.20	112.27	0.81
8	Time series	N/A	N/A	N/A	11.17	34.38	32.10	8.54

Extensive model selection using a variety of metrics including log-likelihood, AIC, BIC, and out of sample predictive scoring (MAPE and G^2) was then applied to identify key predictive factors in the model as shown in Table 5. The first four models listed are all standard Cox PH models with various sets of explanatory variables. Among these four, the third model offers the lowest BIC value, 19061, while the fourth has the lowest AIC, 18,996 indicating these two models are the best of this subset. Overall, both models perform almost identically on MAPE and holdout MAPE which is error when the model is used on data from years 2011 and 2012 which was not used to train the model. G^2 and holdout G^2 were also nearly identical. From this perspective, the variable OC seems to provide very little predictive impact. Model 5 uses the same variables as 1 - 4 but uses OC as a stratification variable meaning that we create a separate baseline for each job classification variable. Creating multiple baselines for subgroups makes the model significantly more complex leading to a huge reduction in log likelihood and traditional model fit but providing very poor predictive and holdout fits due to the significant overfitting of the data.[THIS DOESN'T MAKE SENSE DUE TO THE PREVIOUS FINDING-OC IS NOT A GOOD TARGET-NEED TO DECIDE HOW TO REWORK THIS].

Models 5-7 build on model 3 and add various interaction terms. Here again we find models 6 and 7 being roughly equivalent with the more complex model having a smaller AIC and holdout MAPE, and very slightly higher predicted G^2 and predicted MAPE. In the end, we chose to explore the more complex model 7 due to interest in interpreting the interacting model coefficients.[MAYBE NOT A GOOD REASON, CHECK IF SBC IS RIGHT]

Finally, for comparison purposes we include a time series prediction model discussed in Zhu et al. (2015). This forecast is based on a four component time series decomposition model, which includes terms for trend, cycle, seasonality, and irregularity. This model is fit to monthly aggregate retirement counts for the period of November 2000 to December 2010 based on the same data used here. The model produces an aggregate forecast by month leading to a very accurate MAPE on the training data but much poorer performance out of sample. It does not provide accurate forecasts by occupational code (OC) due to the small individual samples for each subgroup. Furthermore, it is not possible to include the effects of other explanatory facors in this model.

Excluding aggregate economic factors, the optimal modelling variables for the prediction of age at retirement include DIV, YCSH, and AGEH. In addition, based on our understanding of the covenants and parameters of the retirement program we found several additional variables that increase the predictive power of the model. These include ERIP, P85, A65*P85, an interaction term that moderates the impact of the P85 effect after the individual has exceeded 65 years of age, and ERIP*P85, an interaction term that moderates the impact of the P85 effect while the ERIP is in place. These variables were introduced in Section 3.

Table 6 describes the fit parameters and hazard ratios. As noted above, we did not find that gender, occupational code and payroll category were statistically significant predictors indicating that employees' gender, job types, and payroll status are not associated with choice of retirement age conditional on the other variables in the model. P85 is an indicator that a person is eligible for maximum retirement benefits and naturally this has a strong impact on the probability that a person will retire. From a quantitative point of view, with all other factors held constant, for those that achieve 85 points before the age of 65, the hazard ratio is $e^{1.44} = 4.22$ meaning that the hazard of retirement becomes 4.22 times more likely. While not surprising, this quantification is important in predicting individual and aggregate retirement and reflects the modal spike observed in the histogram in Figure 7b.

An alternative eligibility criteria for retirement occurs when individuals exceed an age

Table 6: Parameter Estimates for Retirement Models

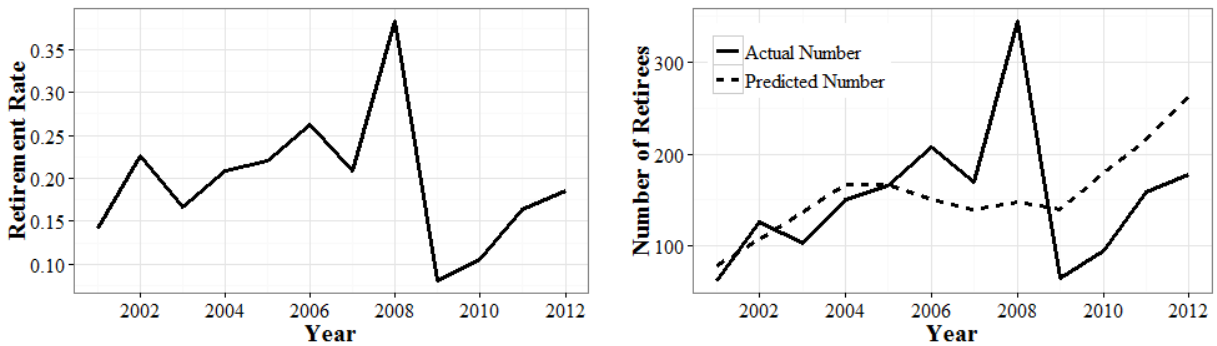
		Model W/O External Variable		**Model with Real Earnings**	
Parameter	Label	Parameter (Standard Error)	Hazard Ratio	Parameter (Standard Error)	Hazard Ratio
DIV	Div2	-0.965 (0.179)***	0.381	-0.969 (0.177)***	0.38
DIV	Div3	-0.241 (0.112)*	0.786	-0.242 (0.111)*	0.785
DIV	Div4	0.078 (0.195)	1.081	0.013 (0.195)	1.013
DIV	Div5	-0.131 (0.19)	0.877	-0.216 (0.19)	0.806
DIV	Div6	2.136 (0.095)***	8.463	2.261 (0.093)***	9.594
DIV	Div7	2.435 (0.129)***	11.418	2.515 (0.128)	12.363
DIV	Div8	0.864 (0.106)***	2.373	0.855 (0.106)***	2.352
DIV	Div9	-3.023 (0.581)***	0.049	-2.731 (0.504)***	0.065
DIV	Div10	0.793 (0.093)***	2.211	0.726 (0.093)***	2.068
YCSH	1	0.019 (0.004)***	1.019	0.023 (0.004)***	1.023
ERIP		0.859 (0.169)***	.	0.489 (0.133)***	.
AGEH		-0.172 (0.013)***	0.842	-0.193 (0.014)***	0.825
P85	1	1.435 (0.091)***	.	0.756 (0.073)***	.
P85*A65	1	-1.61 (0.206)***	.	-1.019 (0.197)***	.
ERIP*P85	1	0.469 (0.179)**	.	0.506 (0.141)***	.
Real Earnings				0.013 (0.001)***	1.013

[1] * denotes $P < 0.05$, ** denotes $P < 0.01$, and *** denotes $P < 0.001$.

of 65 years and so we would anticipate the hazard increasing at this point in an employees career. Because the response variable in our model is age, the impact of this 65 year effect is included in the baseline hazard, which should increase after this point. Figure 9 shows the baseline survival and cumulative hazard function for a standard case. Independent of the P85 effect, we see a further steep increase in the cumulative hazard/decrease in survival between age 62 and 65. By including an interaction between the indicators of age greater than 65 and points greater than 85, A65*P85, we can estimate how the impact of reaching 85 points changes when a person exceeds regular retirement age. In this case, the interaction term is estimated at -1.61 indicating a diminishing effect on the P85 criteria to $e^{1.44-1.61} = 0.84$. In addition, as can be seen from the trend of the cumulative hazard, the baseline hazard seems to return lower levels. Hence, those that exceed both criteria actually have a reduced hazard of retiring over those have only met the age 65 criteria. It seems that the fact that the individual remains on the job after hitting both criteria indicates a diminished intention to retire.

According to our model, retirement can also be influenced by an employee's age at time of hire and their years of service at the time of hiring. The coefficient estimate for age is -0.17. As the reference age is 45.49, this means that the hazard ratio for retirement of an employee that started working at age 46.49 is $e^{-.017} = .84$ indicating a 16% drop in hazard for each additional year later that an employee started. The employee's survival probability at any time, t, can be computed as $S(t)^{1.19} = (S(t)^{e^{0.17}})$ when age at hire is one year below 45.49, where $S(t)$ is the baseline survival probability for a reference employee of average age at the time of hiring. Moving in the other direction, an employee's survival probability is $S(t)^{0.84} = (S(t)^{e^{-0.17}})$ for a one year increase beyond 45.49 in the employee's starting age. Together, this implies that at any given retirement age, the employee who starts earlier is more likely to retire because they have more years of service and are closer to vesting full benefits (85 points) than an equivalent employee who starts working at an older age. Similarly, the employee's years of service at hire show a positive estimate (0.019) with a hazard ratio (1.019) indicating that each year of service at hire beyond the population average of 2.75 is associated with an approximately 2% increase in the hazard of retirement. This leads to a survival probability $S(t)^{0.98}$ for a one year decrease in the reference years of service at time of hiring. On the other hand, the survival probability is $S(t)^{1.019}$ for an employee with one year of service above average at the time of hire. Together, both age at hire and YCSH effects reflect the intuitive fact that, all else being equal, an employee who has more years of service and therefore is closer to full vesting is more likely to retire. What is non-intuitive about this finding is that while one might suspect that the effects should be of similar magnitude, we actually see that the effect of one year difference in age seems to have about 8 times the impact that one year of previous service does on the hazard of retirement.

In the fiscal year 2008, the employer in this study created a temporary early retirement incentive program (ERIP). The response window for this option was 3 months although the specific details beyond this are unknown. In order to deal with the increased level of retirement during this period we include a time dependent indicator variable for each employee that indicates their age when this program was in effect. The coefficient for this indicator was 0.86 leading to a hazard ratio of $e^{.86} = 2.21$ which indicates that, on average, an individuals hazard of retirement increased by almost 2.2 times during this period. If more



(a) Retirement Rate

(b) Prediction w/o Variable ERIP

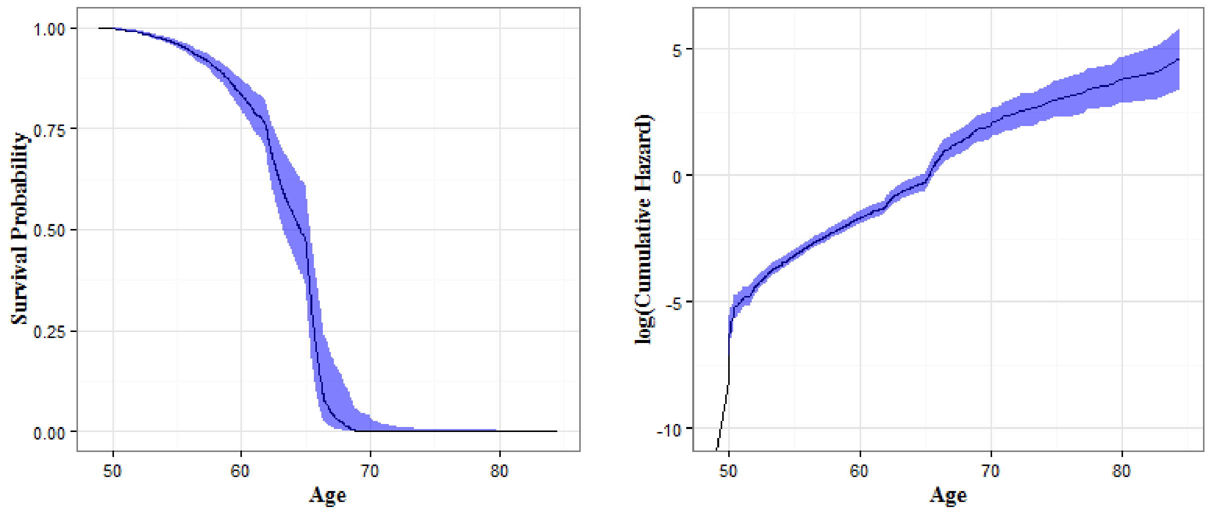
Figure 8: Retirement Rate and Prediction Plot

information were known about the requirements or targets of this ERIP, a more case specific estimate may be possible. The effect of the ERIP is considerable as indicated by the huge uptick in events in 2008; see Figure 10. If this one-time effect were not included in the model it may bias the other estimates parameters considerably.

As a further step, we test the ERIP effect on the employees who are eligible for getting a pension, i.e. points greater than 85. After adding an interaction term between ERIP and the indicator that a person exceeds 85 points, the hazard ratio for the ERIP increases substantially to 15.85 from 2.21, more than seven times the basic ERIP effect. In contrast, employees that were eligible only for partial retirement, an interaction between ERIP and indicator variables that employees achieve only 65 or 75 points, were not statistically significant.

The DIV variable was also a significant predictor. For analysis, the baseline level was chosen arbitrarily as division 1 so that its hazard rate is determined by the baseline. Relative to this baseline, divisions 6 and 7 have very high hazard ratios, 8.363 and 11.405 respectively, which indicates, other factors being equal, that the employees in division 6 and 7 are much more likely to retire at any age than those from division 1. Conversely, division 9 has a hazard ratio of $e^{-3.023} = .049$ indicating that individuals within this group have 1/20 the hazard of group 1. This may indicate that the division is new and contains younger employees. In general, differences in retirement rates could be caused by differences in age demographics, departmental and job functions, or departmental leadership.

The baseline survival function and log hazard function are shown in Figure 9. The survival probability is 1 before age 49 as shown in Figure 9a, which indicates that no employees retire before this age. The survival probability starts to slowly decease from age 50 to age 62. By age 62 the survival probability has decreased by nearly 25%, which indicates that 75% of employees retire at an age greater than 62 years assuming that they are at average or baseline levels for other factors included in the model. The slope of the survival function decreases sharply at this point indicating the increased retirement rates for workers between age 62 and 65. After age 65 the probability drops off even further as most of the remaining population retires by age 68 or 69. Accompanying the survival function is the log of the cumulative hazard ratio. Again, the steep rise in the cumulative hazard between age 62 and 65 indicates the increased retirement activity during this period. Afterward the cumulative hazard levels off, indicating a drop in the instantaneous hazard rate at these future points.



(a) Survival Function

(b) Cumulative Hazard

Figure 9: Baselines with 95% Confident Intervals

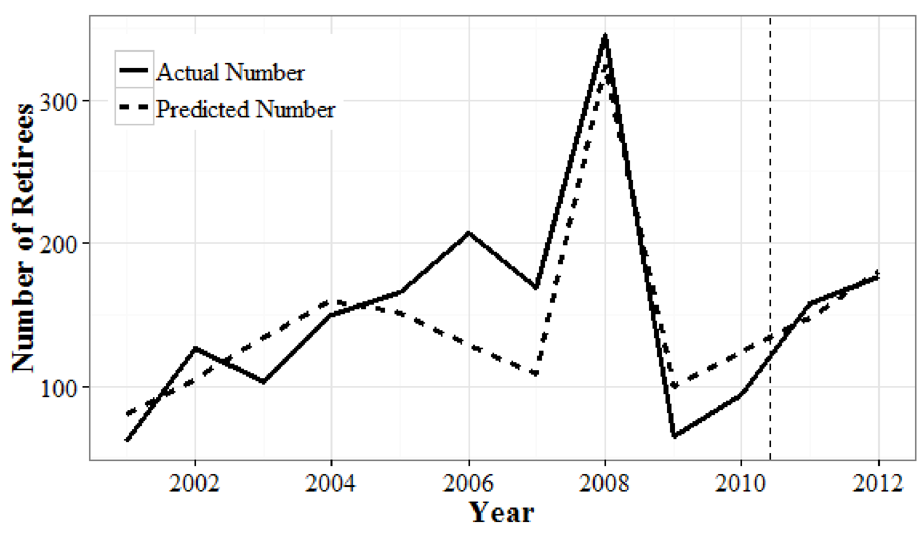


Figure 10: Retirement Forecasting

Aggregate predictions for employee retirement are shown in Figure 10 and Tables 7. The model predictions capture the fluctuations in actual retirement, and also capture the peak year of 2008 when the ERI was introduced. The out-of-sample predictions for holdout years (2011 and 2012) are very close to the actual number indicating that the model performs well on both the training and holdout samples. Besides predicting the overall retirement,

Table 7: Retirement Predictions by Occupational Code (OC) without External Variables

Year	Crafts	Engineers	General Admin.	Laborers	Managers	Prof. Admin.	Operators	Scientists	Technicians	Total
2001	23[1](16)[2]	12 (10)	4 (1)	6 (5)	11 (14)	11 (9)	8 (2)	2 (1)	4 (4)	81 (62)
2002	31 (29)	15 (16)	5 (15)	7 (11)	15 (26)	13 (18)	11 (6)	2 (1)	6 (4)	105 (126)
2003	37 (26)	18 (13)	6 (5)	9 (6)	17 (21)	18 (13)	16 (15)	4 (2)	8 (2)	133 (103)
2004	43 (32)	23 (23)	8 (9)	11 (7)	21 (30)	23 (25)	15 (15)	4 (3)	10 (6)	158 (150)
2005	40 (39)	24 (17)	8 (13)	9 (7)	20 (27)	24 (31)	12 (15)	3 (4)	11 (12)	151 (165)
2006	31 (58)	20 (29)	6 (10)	8 (9)	19 (32)	25 (37)	9 (13)	2 (4)	9 (15)	129 (207)
2007	19 (44)	14 (25)	7 (9)	6 (9)	18 (26)	27 (40)	8 (6)	3 (4)	7 (6)	109 (169)
2008	55 (71)	30 (33)	19 (20)	22 (12)	64 (63)	79 (84)	27 (32)	6 (7)	21 (23)	323 (345)
2009	16 (14)	9 (6)	7 (3)	7 (7)	20 (8)	25 (10)	8 (11)	1 (1)	5 (5)	98 (65)
2010	18 (19)	11 (17)	9 (1)	7 (8)	28 (23)	34 (16)	8 (4)	1 (3)	7 (3)	123 (94)
2011	22 (36)	13 (25)	11 (8)	9 (9)	34 (27)	40 (34)	9 (5)	2 (1)	8 (13)	148 (158)
2012	24 (29)	16 (23)	14 (11)	12 (10)	41 (46)	49 (36)	12 (4)	3 (2)	9 (16)	180 (177)

[1] the number before the parentheses is predicted retirement number.
[2] the number inside the parentheses is actual retirement number.

the model can also provide predictions by category. Table 7 shows the yearly predictions by occupation code, which are computed by summing the retirement probabilities of individuals by job classification. The predicted values match the actual values in this category well.

6.3 Models for Retirement with External Economic Variables

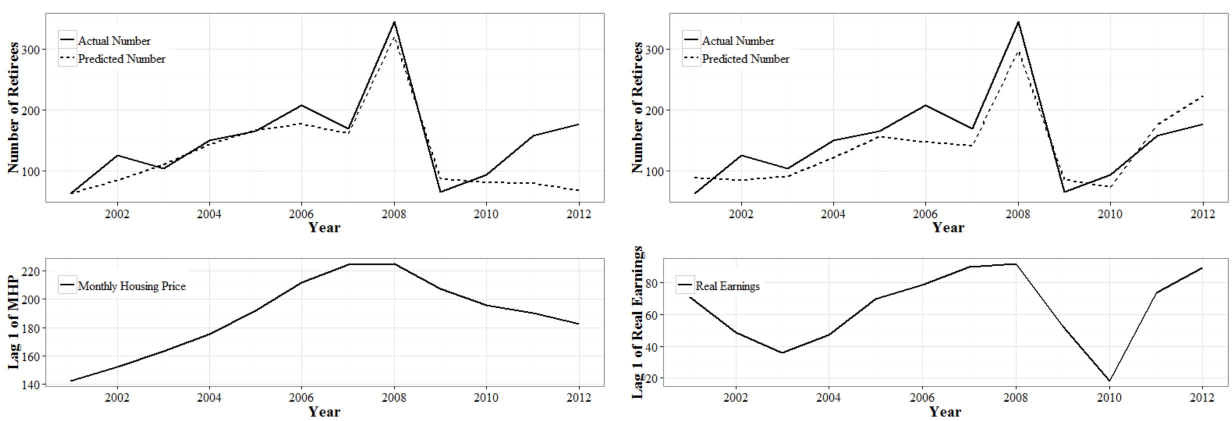
The impact of the external economy on retirement decision making is a topic of considerable interest. To explore these effects we include lagged versions of a number economic factors using the counting process data formulation with calendar year based intervals to set up and test the impact on retirement. Parameter estimates for common effects across models with and without economic variables remain constant as can be observed in Table 6. As shown

Table 8: Economic Index Test Statistics

Economic Inidcator	χ^2	**P-value**	**Hazard ratio**	**MAPE**	G^2
Without Econmic indicator[1]				21.31	147.43
MHP	129.614	$< .001$	1.020	17.84	220.65
SAMHP	129.516	$< .001$	1.020	17.86	221.66
SEMHP	68.055	$< .001$	1.030	23.37	178.38
SESAMHP	67.871	$< .001$	1.030	23.40	179.87
S&P500	13.319	$< .001$	1.001	20.79	129.83
Dividend	1.045	0.307	1.015	22.63	150.03
Earnings	84.895	$< .001$	1.016	21.40	105.06
Consumer Price Index	5.404	0.020	1.013	21.36	133.57
Real Price	5.522	0.019	1.000	21.22	138.72
Real Dividend	1.925	0.165	1.022	23.39	154.84
Real Earnings	80.358	$< .001$	1.013	20.33	95.02
Long Interest Rate	1.539	0.215	1.082	22.03	149.01
Unemployment Rate	32.212	$< .001$	0.849	25.08	179.49
P/E 10	0.041	0.839	0.998	21.31	147.97
Wilshire5000	22.392	$< .001$	1.028	20.83	121.58

[1] it is the selected model without economic indicator.

in Table 8, among economic indices tested, S&P500, Real Earnings, and Wilshire5000 were statistically significant and also improved the model forecast leading to lower MAPE and G^2. Although Real Prices is also statistically significant, its coefficient estimate of 0.0004 leads to a hazard ratio of 1 indicating little if any practical impact. As shown in Table 8, Real Earnings is the most important factor among all the indicators leading to the lowest G^2 value and showing the strongest impact on retirement behavior. Figure 11b plots the fluctuation of retirement against a 1 year lag of Real Earnings. Two highly correlated equity market indices, S&P500 and Wilshire5000, were also statistically significantly with hazard ratios > 1 indicating that employees of this organization are more likely to retire when the stock market is strong.



(a) Monthly Housing Price

(b) Real Earnings

Figure 11: Economic Indicators and Retirement Predicting Plot

Table 9: Retirement Predictions by Occupational Code (OC) with External Variable.

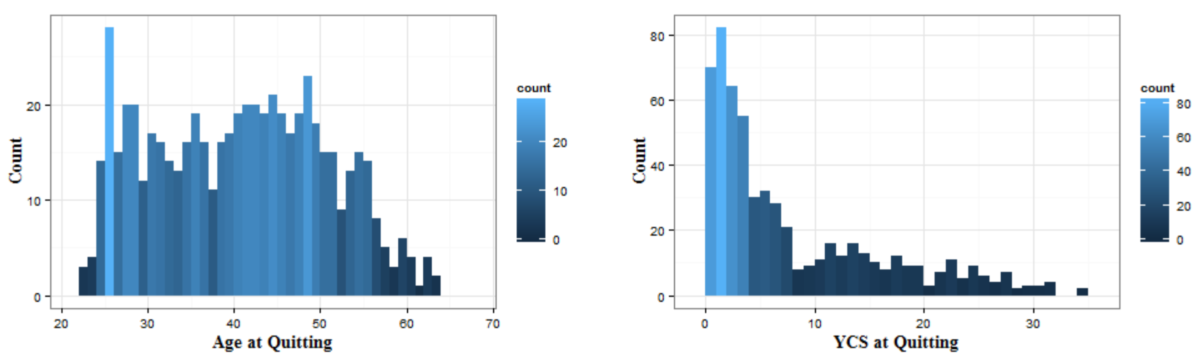
Year	Crafts	Engi-neers	General Admin.	Laborers	Man-agers	Prof. Admin.	Opera-tors	Scien-tists	Techni-cians	Total
2001	23 (16)	13 (10)	4 (1)	7 (5)	15 (14)	11 (9)	8 (2)	2 (1)	5 (4)	88 (62)
2002	23 (29)	12 (16)	4 (15)	6 (11)	15 (26)	10 (18)	8 (6)	2 (1)	5 (4)	85 (126)
2003	23 (26)	13 (13)	5 (5)	6 (6)	13 (21)	12 (13)	10 (15)	3 (2)	6 (2)	91 (103)
2004	32 (32)	18 (23)	7 (9)	9 (7)	16 (30)	18 (25)	11 (15)	3 (3)	8 (6)	122 (150)
2005	40 (39)	24 (17)	8 (13)	10 (7)	21 (27)	26 (31)	12 (15)	4 (4)	11 (12)	156 (165)
2006	35 (58)	25 (29)	6 (10)	9 (9)	21 (32)	29 (37)	10 (13)	3 (4)	9 (15)	147 (207)
2007	24 (44)	19 (25)	9 (9)	8 (9)	24 (26)	35 (40)	10 (6)	3 (4)	9 (6)	140 (169)
2008	50 (71)	29 (33)	19 (20)	20 (12)	57 (63)	72 (84)	27 (32)	5 (7)	19 (23)	298 (345)
2009	13 (14)	8 (6)	6 (3)	6 (7)	17 (8)	22 (10)	7 (11)	1 (1)	5 (5)	85 (65)
2010	10 (19)	8 (17)	5 (1)	4 (8)	16 (23)	19 (16)	5 (4)	1 (3)	4 (3)	72 (94)
2011	25 (36)	16 (25)	14 (8)	10 (9)	40 (27)	47 (34)	11 (5)	2 (1)	10 (13)	175 (158)
2012	29 (29)	21 (23)	16 (11)	12 (10)	51 (46)	64 (36)	14 (4)	3 (2)	13 (16)	223 (177)

[1] the number before the parentheses is predicted retirement number.
[2] the number inside the parentheses is actual retirement number.

Unadjusted Monthly Housing Price (MHP) is another influential index and inclusion in the model resulted in the lowest MAPE value. Fluctuations in the lag also correlate strongly with the retirement number as shown in Figure 11a. However, the retirement number does not decrease coinciding with the decreasing of MHP after the 2008 financial crisis.

6.4 Models for Voluntary Quitting without External Economic Variables

A second area of interest in analyzing turnover is voluntary quitting. Using the same methodology and set of variables applied to retirement modelling, we constructed a predictive model for quitting. The challenge of modelling quitting in the current context is that the current data has approximately 600 out of 8000 employees quitting during the 10-year study window resulting in a very high proportion of censored data. With this data density, the model cannot generate a smooth baseline to achieve a good forecasting model using age as the dependent variable because employees quit across such wide range of ages (20 to 64); see Figure 12a and Table 10. Modelling years of service (YCS) as the dependent variable compresses the reference frame leading to better estimates with employees usually quitting within the first 10 years of service as shown in Figure 12b. Since an employee will not quit if they are eligible for pension, we remove those employees from the risk set when they meet either one of the requirements for retirement. With those cases now censored, the indicators P85 and A65 are no longer useful for modelling. Among all the models shown in Table 10, model



(a) Age at Quitting

(b) YCS at Quitting

Figure 12: Histogram of Age and YCS at Quitting

[FILL IN NUMBER] with DIV, OC, AGEC, and ERIP as explanatory variables fits best. A discrete version of the Cox model based on logistic regression also performs well with both models showing similar MAPE and G_2 values; see Allison (2010) for more detail on discrete models.

Our results suggest that voluntary quitting is influenced by an employee's age at the start of their service. The coefficient estimate for age is -0.025. As the reference age is 35.44, this means that the hazard of quitting for an employee that started working at age 36.44 is $exp(-.025) = .975$ that of an employee starting at age 35.44. For each additional year of age at beginning of service we estimate 2.5% drop in hazard of quitting. The employee's survival probability at any time, t, can be computed as $S(t)^{1.025} = (S(t)^{e^{0.025}})$ if age is one year below average, where $S(t)$ is the baseline survival probability for a reference employee of average age at initial employment. Moving in the other direction, the employee's survival probability increases to $S(t)^{0.975} = (S(t)^{e^{-0.025}})$ for a one year increase in the employee's starting age. Together, this implies that at any given voluntary quitting age, the employee who starts

Table 10: Voluntary Quitting Models Statistics

No.	Dependent Variables	Independent Variables	LR	AIC	SBC	Pred. MAPE	Holdout MAPE	Pred. G^2	Holdout G^2
1	AGE	DIV GENDER OC YCSH ERIP	1226.8	5960.1	6041.4	83.7	91.35	1213.96	197.21
2	YCS	DIV OC AGEC ERIP	1012.4	6848.5	6929.7	23.90	21.05	65.63	2.48
3	YCS reduced riskset	DIV OC AGEC ERIP	838.4 1	6911.2	6992.5	15.16	18.77	26.25	2.08
4	Logistic regression	DIV OC AGEC ERIP	1870.7	4712.6	4938.6	15.98	17.55	23.03	3.21
5	Time series		NA	NA	NA	26.41	61.32	30.88	18.33

earlier is more likely to quit than an equivalent employee who starts working later.

The early retirement incentive option ERIP also shows a significant impact on an employee's quitting behavior. The coefficient for this indicator was 1.111 leading to a hazard ratio of $e^{\cdot 1.111} = 3.04$. This indicates that, on average, an individual's hazard of quitting increased by almost 3 times during this period. It is unclear why an optional early retirement program would influence quitting but it is possible that the program led to leadership or organizational disruptions [NEED CITATION HERE] or simply that it took place during 2008, a time of significant economic upheaval due to rapidly changing external economic conditions.

The DIV variable was also a significant predictor. For analysis, the baseline level was chosen arbitrarily as division 6 so that it's hazard rate is determined by the baseline. Relative to this baseline, division 7 has a similar hazard of quitting according to the model estimates. Conversely, the other divisions all have negative coefficients with hazard ratios less than 1 indicating that individuals within these groups have lower hazard of quitting than group 6.

The OC variable was another significant predictor. For analysis, the baseline level was engineer for this variable so that it's hazard rate is determined by the baseline. Relative to this group, all the other divisions have negative coefficients with hazard ratios less than 1. In particular, the crafts group with coefficient of -1.165 and a hazard ratio of 0.312 shows a much lower hazard of quitting than the engineering group. This may reflect that differences in sociological or demographic factors among workers in these job categories, differences in compensation relative to other opportunities, or other differences in local or national economic mobility.

In general, differences in quitting rates could be caused by differences in age demographics, leadership, departmental and job function, or departmental leadership.

The baseline survival function and log hazard function for quitting are shown in Figure 13. The survival probability decreases steeply between 0 and 10 years of service. By 10 years of service at 10 the survival probability has decreased to close to 0.25, which indicates that 75% of employees quit within the first 10 years of service. The slope of survival function flattens to 0 from 10 to 30 years of service. Accompanying the survival function is the log of the cumulative hazard ratio. Again, the steep rise in the cumulative hazard between years of service at 0 to 10 indicates the increased quitting activity during this period. After this the cumulative hazard levels off indicating a drop in the hazard rate at these future points.

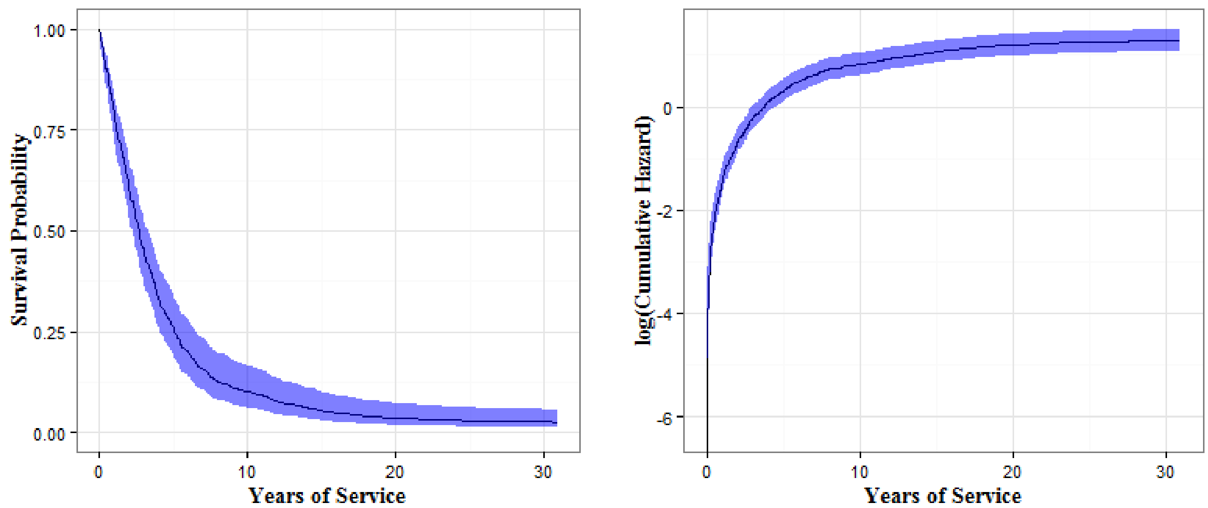
Table 11: Parameter Estimates for Voluntary Quitting Models

		Model w/o external variable		**Model with Real Dividend**	
Parameter	Label	Parameter (Standard Error)	Hazard Ratio	Parameter (Standard Error)	Hazard Ratio
DIV	Div1	-3.066 (0.263)***	0.047	-3.305 (0.269)***	0.037
DIV	Div2	-2.652 (0.198)***	0.071	-2.875 (0.204)***	0.056
DIV	Div3	-3.015 (0.253)***	0.049	-3.258 (0.258)***	0.038
DIV	Div4	-2.533 (0.288)***	0.079	-2.769 (0.292)***	0.063
DIV	Div5	-2.739 (0.314)***	0.065	-2.934 (0.317)***	0.053
DIV	Div7	0.028 (0.145)	1.029	0.101 (0.146)	1.107
DIV	Div8	-0.806 (0.157)***	0.447	-0.968 (0.163)***	0.38
DIV	Div9	-3.985 (0.586)***	0.019	-4.207 (0.588)***	0.015
DIV	Div10	-1.325 (0.136)***	0.266	-1.5 (0.142)***	0.223
OC	C	-1.163 (0.274)***	0.313	-1.11 (0.275)***	0.329
OC	G	-0.794 (0.198)***	0.452	-0.796 (0.198)***	0.451
OC	L	-0.711 (0.228)**	0.491	-0.619 (0.229)**	0.539
OC	M	-0.541 (0.15)***	0.582	-0.497 (0.15)***	0.609
OC	P	-0.53 (0.135)***	0.588	-0.506 (0.136)***	0.603
OC	R	-0.95 (0.308)**	0.387	-0.886 (0.309)**	0.412
OC	S	-0.691 (0.275)*	0.501	-0.682 (0.276)*	0.506
OC	T	-0.608 (0.203)**	0.544	-0.564 (0.204)**	0.569
AGEC		-0.025 (0.005)***	0.975	-0.026 (0.005)***	0.974
ERIP	1	0.851 (0.135)***	2.343	0.479 (0.149)**	1.614
Real Dividends				0.085 (0.017)***	1.089

[1] * denotes $P < 0.05$, ** denotes $P < 0.01$, and *** denotes $P < 0.001$.

6.5 A Model for Voluntary Quitting including external economic variables

i. tested which variable does significantly impact on employee voluntary quit. Because voluntary quitting is more sensitive affected by macro economics, we further examined the external variables using the counting process model with yearly interval based on calendar year to test their effects on voluntary quitting. This model has the similar parameter estimation as the selected model as shown in the right part of the Table 11. We found one indicators are statistically significant and also improve the model forecasting due to lower MAPE and G^2 than the values of selected model without economic indicator, which is Real Dividend as shown in Table 13. According to the estimates of Table 8, Real dividend is the most important factor among all the indicators as it has lowest G^2 and MAPE. The test results show that it has strong impact on the quitting behaviors. As shown in Figure 14, the



(a) Survival Function **(b)** Log of Cumulative Hazard

Figure 13: Voluntary Quitting Model Baselines with 95% Confident Intervals

Table 12: Voluntary Quitting Predictions by Occupational Code (OC) without external variables

Year	Crafts	Engi-neers	General Admin.	Laborers	Man-agers	Prof. Admin.	Opera-tors	Scien-tists	Techni-cians	Total
2001	3 (2)	12 (14)	4 (2)	4 (3)	8 (12)	11 (16)	2 (0)	2 (2)	4 (1)	49 (52)
2002	2 (1)	13 (10)	3 (3)	3 (2)	7 (8)	11 (11)	1 (0)	1 (4)	3 (5)	45 (44)
2003	2 (0)	17 (13)	3 (1)	4 (2)	7 (14)	12 (10)	1 (1)	1 (5)	3 (2)	51 (48)
2004	2 (2)	23 (20)	4 (4)	3 (2)	7 (5)	12 (12)	1 (0)	1 (1)	4 (3)	57 (49)
2005	1 (5)	23 (23)	3 (3)	2 (2)	7 (8)	11 (18)	1 (0)	1 (0)	3 (4)	53 (63)
2006	1 (1)	22 (32)	2 (2)	1 (2)	6 (6)	10 (16)	1 (3)	1 (1)	2 (2)	46 (65)
2007	1 (2)	17 (29)	2 (3)	1 (2)	6 (6)	9 (14)	1 (2)	1 (0)	2 (2)	39 (60)
2008	1 (2)	23 (34)	5 (5)	2 (0)	11 (4)	19 (18)	2 (5)	3 (1)	4 (5)	69 (74)
2009	1 (1)	9 (16)	2 (4)	1 (5)	5 (4)	9 (6)	1 (0)	1 (1)	2 (3)	29 (40)
2010	1 (1)	9 (13)	3 (7)	1 (4)	4 (4)	9 (2)	1 (2)	1 (1)	2 (4)	30 (38)
2011	1 (0)	8 (9)	3 (4)	1 (2)	3 (3)	9 (3)	1 (0)	1 (2)	1 (1)	29 (24)
2012	1 (1)	8 (6)	3 (7)	1 (0)	3 (4)	10 (14)	1 (0)	1 (2)	2 (2)	30 (36)

[1] the number before the parentheses is predicted retirement number.
[2] the number inside the parentheses is actual retirement number.

fluctuation of voluntary quitting plot is corresponding with 1 year lag of the trend of Real dividend.

7 Conclusions and Managerial Implications

Generally, we found that using the Cox proportional hazards model along with appropriately chosen internal and external variables led to accurate predictions of retirement. In the training sample we found that prediction error as measured by MAPE was approximately 25% while the predictions in the smaller 2 year holdout window were approximately 5%. Although this included only two validation points, these results indicate that the method

Table 13: Economic Index Test Statistics for Voluntary Quitting

Economic Inidcator[1]	χ^2	**P-value**	**Hazard ratio**	**MAPE**	G^2
Without Econmic indicator				15.71	28.33
MHP	37.93	<.001	1.012	13.62	18.44
SAMHP	37.75	<.001	1.012	13.64	18.50
SEMHP	39.80	<.001	1.020	13.74	18.67
SESAMHP	39.63	<.001	1.020	13.75	18.68
S&P500	0.02	0.879	1.000	15.46	27.79
Dividend	31.21	<.001	1.077	13.01	18.01
Earnings	8.83	0.003	1.009	15.81	23.88
Consumer Price Index	35.84	<.001	1.024	24.26	42.95
Real Price	5.34	0.021	1.000	17.94	34.52
Real Dividend	26.66	<.001	1.089	12.40	16.37
Real Earnings	3.71	0.054	1.005	14.16	23.34
Long Interest Rate	12.04	0.001	0.789	19.10	38.73
Unemployment Rate	2.99	0.084	1.068	17.04	34.96
P/E 10	16.22	<.001	0.968	17.95	35.19
Wilshire5000	10.75	0.001	1.031	15.84	21.31

[1] it is the selected model without economic indicator.

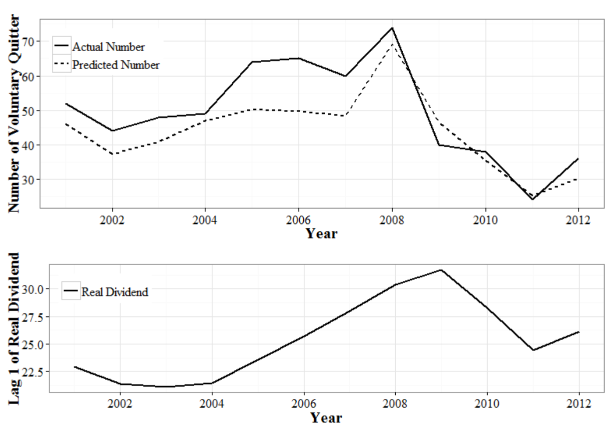


Figure 14: Voluntary Quitting with Real Dividend

Table 14: Voluntary Quitting Predictions by Occupational Code (OC) with External Variables

Year	Crafts	Engineers	General Admin.	Laborers	Managers	Prof. Admin.	Operators	Scientists	Technicians	Total
2001	3 (2)	12 (14)	3 (2)	4 (3)	7 (12)	10 (16)	2 (0)	1 (2)	4 (1)	46 (52)
2002	2 (1)	11 (10)	2 (3)	3 (2)	6 (8)	8 (11)	1 (0)	1 (4)	3 (5)	37 (44)
2003	2 (0)	14 (13)	2 (1)	3 (2)	6 (14)	9 (10)	1 (1)	1 (5)	3 (2)	41 (48)
2004	1 (2)	20 (20)	3 (4)	3 (2)	6 (5)	10 (12)	1 (0)	1 (1)	3 (3)	47 (49)
2005	1 (5)	22 (23)	3 (3)	2 (2)	6 (8)	10 (18)	1 (0)	1 (0)	3 (4)	50 (63)
2006	1 (1)	24 (32)	2 (2)	1 (2)	7 (6)	10 (16)	1 (3)	1 (1)	2 (2)	50 (65)
2007	1 (2)	21 (29)	2 (3)	1 (2)	7 (6)	11 (14)	1 (2)	2 (0)	2 (2)	48 (60)
2008	1 (2)	22 (34)	5 (5)	2 (0)	11 (4)	20 (18)	2 (5)	3 (1)	4 (5)	69 (74)
2009	1 (1)	13 (16)	4 (4)	1 (5)	7 (4)	15 (6)	1 (0)	2 (1)	2 (3)	47 (40)
2010	1 (1)	10 (13)	3 (7)	1 (4)	5 (4)	11 (2)	1 (2)	1 (1)	2 (4)	35 (38)
2011	1 (0)	7 (9)	2 (4)	1 (2)	3 (3)	8 (3)	1 (0)	1 (2)	1 (1)	25 (24)
2012	1 (1)	8 (6)	3 (7)	1 (0)	3 (4)	11 (14)	1 (0)	1 (2)	2 (2)	30 (36)

[1] the number before the parentheses is predicted retirement number.
[2] the number inside the parentheses is actual retirement number.

has good potential [LETS USE A GAINS CHART OR PRECISION RECALL OR SOMETHING]. The key internal variables that improved model predictions include division, years of service at hire (YCSH), and age at hire (AGEH). In addition, S&P 500 real earnings also showed a significant association with risk of retirement although the magnitude at approximately 1.3% was (greater or less) than other effects for small changes. We also find that retirement hazard increases significantly when an individual hits 85 points of retirement credit and is eligible for full benefits but that this reverts back to a lower hazard after age 65. Furthermore, the the early retirement incentive plan implemented by the organization in 2008 had a major impact with a large increase in the hazard of retirement, particularly for those that were eligible for full benefits with points exceeding 85. In terms of prediction, in out-of-sample tests, taking into account individual and external information provides a significant improvement over more traditional forecasting methods. It also provides useful predictions on subgroups that are not possible for standard forecasting models.

Quitting behavior differed significantly by division, occupation, and age at start of service, which reflects both differences in worker satisfaction across the organization, and by job type. Age connects logically since, for a given number of years of service, the employee that starts earlier, will be younger and, therefore, may see a more significant long term opportunity in a new position elsewhere. The 2008 early retirement incentive program was also correlated with a significant amount of quitting. It is unclear if the higher hazard of quitting was directly due to disruptions, management or otherwise, caused by the accelerated retirements or was related somehow to the drastic economic changes during that time period. In terms of predictive power, the quitting model performs well with MAPE in the training sample estimated at 15.16% and 18.77% in the holdout sample[GAINS CHART HERE ALSO]. Both estimates are far superior to traditional forecasting techniques. As with retirement, external information in the form of S&P real corporate dividends were positively correlated with quitting indicating that as profits at large private sector companies increase, the hazard of quitting rises significantly.

Drawing on the this list of findings, this work provides a number of valuable managerial implications. Foremost, this work shows that fairly accurate survival model based retirement prediction is feasible in large organizations when defined benefit plans are in place. These models are accurate enough to provide useful predictive visibility for human resources management professionals. Such models are of particular value when a large portion of the workforce has specialized skills that require an extensive search process to replace or if the organization requires long lead times in the hiring process due to security or other concerns. The model also provides guidance as to expected retirement and quitting behavior across subgroups of the organization. Significant deviations from these expectations, particularly in terms of abnormal quitting behavior, may indicate the need for management intervention. Finally, such models can alert managers to key external factors that may indicate increases in retirement or quitting behavior. The use of lagged variables may allow management to craft incentive policies in response to changes in the external economy.

Although this study brings to light much, it is important to consider a number of limitations due to the data and models used. The data were sampled based on a specific time window leading to a truncated sample which limits the modeling approaches. In addition, it would be beneficial to have have a longer prediction window to validate the model. Variables such as salary, and more carefully documented division data, which would be available within the sponsoring organization, could strengthen the results presented here. In the same vein, more complete internal organizational details on early retirement incentive program as well as details of the defined benefit plan could improve the quality of inferences in this study. Finally, the accuracy of results for the retirement portion of this study are based on the specific nature of the defined benefit pension dominant during the period of study. The accuracy of predictions observed in this study may not generalize to organizations offering defined contribution retirement plans although a similar methodology may still prove useful in generating predictions.

In terms of the model, although tested, the proportional hazards assumption may be violated for some effects. In addition, variability in the baseline estimator can cause overestimation of hazard and more sophisticated smoothed baseline models may be beneficial in this case.

This work contributes significantly to both practical and academic literature on factors affecting retirement and management of staffing through predictive models. We expect that future work will continue to develop and improve the modeling techniques and result in better explanatory models for academic research as well as functioning systems for industry. Furthermore, we believe that the current methods can be extended to include qualitative and survey based longitudinal feedback from employees through the time varying covariate methodology. The addition of such attitudinal data may provide significant additional value to both academic and industry practitioners.

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