each arc, the flow trough the arc cannot exceed the arc's capacity. Lines 8 and 9 create the conservation of flow constraints. For each node I, they ensure that the flow into node I equals the Flow out of node I.

Line 10 begins the DATA section. In line 11 we input the arc capacities. Note that we have given the artificial arc a large capacity of 1000. Line 12 ends the DATA section and the **END** statement ends the program. Typing GO yields the solution, a maximum flow of 3 previously described. The values of the variable FLOW(I,J) give the flow through each arc.

Note that this program can be used to find the maximum flow in any network. Begin by listing the network's nodes in line 2. Then list the network's arcs in line 3. Finally, list the capacity of each arc in the network in line 11, and you are ready to find the maximum flow in the network!

The Ford-Fulkerson Method for Solving Maximum-Flow Problems

We assume that a feasible flow has been found (letting the flow in each arc equal zero gives a feasible flow), and we turn our attention to the following important questions:

Question 1 Given a feasible flow, how can we tell if it is an optimal flow (that is, maximizes x_0)?

Question 2 If a feasible flow is nonoptimal, how can we modify the flow to obtain a new feasible flow that has a larger flow from the source to the sink?

First, we answer question 2. we determine which of the following properties is possessed by each arc in the network:

Property 1 The flow trough arc (i,j) is below the capacity of arc (i,j). In this case, the flow trough arc (i,j) can be increased. For this reason, we let l represent the set of arcs with this property.

Property 2 The flow in the arc (i,j) is positive. In this case, the flow trough arc (i,j) can be reduced. For this reason, we let R be the set of arcs with this property.

As an illustration of the definition of l and R, consider the network in FIgure 9. The arcs in this figure may be classified as follows: (so, 1) is in l and R; (so, 2) is in l; (1, si) is in R; (2, si) is in l; and (2, 1) is in l.

We can now describe the Ford-Fulkerson labeling procedure used to modify a feasible flow in an effort to increase the flow from the source to the sink.

Step 1 Label the source

Step 2 Label nodes and arcs (except for arc a_0) according to the following rules: (1) if node x is labeled, then node y is unlabeled abd arc (x,y) is a member of l; then label node y and arc (x,y). In this case, arc (x,y) is called a **forward arc.** (2) If node y is unlabeled, node x is labeled and arc (y,x) is a member of R; label node y and arc (y,x). In this case, (y,x) is called **backward arc.**