Attentional Discounted Utility

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1 Introduction

The discounted utility framework has been widely employed to model intertemporal choices. According to this framework, decision makers evaluate a sequence of rewards by assigning a weight to each time period and summing up the weighted utilities of the rewards across these periods. Typically, the weights are referred to as the discounting factors and are assumed to decline over time, indicating a preference for receiving a sooner reward over a later reward. However, recent research suggests that, due to limited information processing capacity, prior to making a decision, people tend to allocate more attention to the information most relevant to the decision. We propose that the determination of weights in discounted utility framework is also influenced by such an attentional mechanism. For instance, when an investor consider whether to invest \$100 now and get \$110 in 100 days, she may focus more on the amount of money she can obtain on the 100th day (i.e. £110). Consequently, when calculating the utility of "receiving £110 in 100 days", the investor may want to assign a higher weight to the 100th day. And doing that will reduce weight assigned to the days prior to the 100th day. Based on this perspective, we develop a novel model of intertemporal choice, which we term "*attentional discounted utility*". We demonstrate how this model can better accommodate certain empirical findings. To distinguish it from the discounting factor, we refer to the weights in our model as "attention weights."

Let $\mathbf{x} = [x_0, x_1, \ldots, x_T]$ denote a sequence of rewards and $u(.)$ denote the utility function.

The overall utility of this sequence is calculated by $\sum_{t=0}^{T} w_t u(x_t)$, where w_t is the attention weight assigned to period t. In our model, the attention weight is calculated by

$$w_t = \frac{d_t e^{u(x_t)}}{\sum_{\tau=0}^{T} d_\tau e^{u(x_t)}}$$

where d_t is the initial weight (discounting factor) allocated to period t. The weight for each period, i.e. w_t, is increasing with $u(x_t)$, indicating that the decision maker is motivated to shift attention to the periods with larger rewards. w_t is "anchored" in the initial weight d_t, because the attention adjustment process is costly. The sum of weights is fixed at 1, indicating the decision maker's capacity of focusing is limited.

We draw inspirations on three fields of research. The first is motivated beliefs. The theory of motivated beliefs states that people will adjust beliefs to subjectively maximize their utility. When the choice made by a decision maker does not deliver the maximum utility, she may tend to adjust the belief to safeguard the choice, rather than shift to another option. This is usually applied to cognitive dissonance and overconfidence. The similar mechanism may exist in the allocation of weights to each time period. **ostrich effect**

The second is rational inattention. The model of rational inattention states that, prior to making choices between options, the decision maker has to learn the information about each option. Under the assumption that she wants to maximum the expected utility minus the cost of learning, which is linear to the information gains, we can derive that the probability of each option being chosen follows a logistic-like distribution. In our model, we assume that when allocating weights across time periods, the decision maker has a similar objective function, thus the attention weights also follows a logistic-like distribution.

The third is the related evidence in intertemporal choices. (1) hidden zero effect (2) concentration bias (3) models of magnitude-increasing patience.

In the attention-adjusted discounted utility model (hereafter referred to as "ADU"), the underlying cognitive process is efficient sampling. We assume that the decision maker initially has no information about which period in a reward sequence has a larger reward. She implements a costly sampling strategy to draw some rewards from the sequence to learn the information, then choose the attention weight to each period accordingly. Therefore, when evaluating the given reward sequence, she tends to assign more weights (pay more attention) to the time periods with larger rewards, in order to subjectively maximize her overall utility. This attention adjustment process incurs a cognitive cost; and the more the weight allocation deviates from the initial allocation, the greater the cost is. The decision maker optimally re-allocates the weights across time periods.

Our contribution is three-fold. First, the model can accommodate a lot of empirical evidence with just one to two parameters.

In this paper, I show that a set of intertemporal choice anomalies can be attributed to such attention adjustment processes (that is, can be explained by ADU to some extent), including common difference effect and magnitude effect (**?**), risk aversion over time lotteries (**??**), non-additive time intervals (**??**), intertemporal correlation aversion (**?**), and dynamic inconsistency. The model can also offer insights on the preferences for sequences of outcomes (**?**) and the formation of reference-dependent preferences (**?**). In an empirical test, I find ADU outperforms a set of time discounting models in predicting human intertemporal choices. Therefore, I think there is a need to rethink the foundation of many behavioral phenomena.

Second, we link the literature of attention to time preferences.

Third, we make some novel predictions based on the model.

The remaining part of this document is organized as follows. Section 2 outlines the model of attention-adjusted discounted utility (ADU). Section 3 explains how the model can help explain some empirical findings in intertemporal choice.

2 The Model

2.1 Rational Inattention and Time Discounting

Consider a reward sequence $x = [x_0, x_1, ..., x_T]$ that yields reward x_t in time period t. The time length of this sequence, denoted by T, is finite. For any $t \in \{0, 1, ..., T\}$, the reward level x_t is a random variable defined on R_+. The support of x is X, which is a subset of R_+^T.

Suppose a decision maker evaluates reward sequence x by three steps: At first, she randomly draws some potential realizations of x from X. Then, from each drawn realization of x, she draws some time periods at random, taking the rewards of these periods into a sample. Finally, she uses the mean utility of sampled rewards as a value representation of x. Let $s = [s_0, s_1, ..., s_T]$ be a potentially realized outcome of x and $p(s)$ be the probability that s is drawn. I use $w(.)$ and $u(.)$ to denote the decision maker's weight function and utility function, where $w(s_t)$ is the probability that the reward of the t-th period in a potentially realized sequence s is sampled, $u(s_t)$ is the utility obtained by reward s_t ($t \in \{0, 1, ..., T\}$), $u' > 0$, $u'' < 0$.

The sampling process is sequential, and the decision maker wants to find a sampling strategy, denoted by function $w(.)$, that maximizes her overall utility. In a given potentially realized sequence s, the periods with larger reward levels should be sampled more frequently. However, at the very beginning, the decision maker has no information about which period in s has a larger reward – she learns such information gradually in the process of sampling. This learning process triggers a cognitive cost. Hence, her overall utility is the mean utility of sampled rewards minus the cognitive cost of learning.

Suppose when having no information, the weight on period t across each potentially realized sequence is equal ($\equiv w_t^0$). Let W and P be the minimal sets that contain all available function w and p respectively. We can use an optimization problem to represent the described evaluation procedure.

Definition 1: $\{w_t\}$ is generated by a constrained optimal discounting process if it is the solution to the following optimization problem

$$\begin{aligned} \max_{w \in W} \quad & \sum_{s \in X} \sum_{t=0}^{T} w(s_t) u(s_t) - C(w) \\ s.t. \quad & \sum_{s \in X} \sum_{t=0}^{T} w(s_t) = 1 \\ & w(s_t) > 0, \forall s \in X, t = 0, 1, \ldots, T \end{aligned}$$

where $C : (0, 1]^T \to R_+$ is a cognitive cost function, $\partial C / \partial w(s_t) > 0$ and $\partial^2 C / \partial w(s_t)^2 > 0$. That is, the cognitive cost is increasing and convex in $w(s_t)$.

To solve this optimization problem, I add two additional assumptions. The first is that the weight updating process is consistent with Bayes rule, that is, $w_t^0 = \sum_{s \in X} w(s_t)$. The second is that the cognitive cost function takes a form similar to Shannon mutual information, that is

$$C(\mathbf{w}; \theta) = \lambda \sum_{s \in X} \sum_{t=0}^{T} w(s_t) \log \left(\frac{w(s_t)}{p(s) w_t^0} \right)$$

where $p(s)w_t^0$ is the probability of s_t being sampled when no information is learned, $w(s_t)$ is the probability of that after learning the information about x. Shannon mutual information quantifies the amount of information gain when learning about which time period has a larger reward in any initially unknown s. Consistent with **?**, I set $C(\mathbf{w}; \theta)$ linear to that. Parameter λ denotes unit cost of information ($\lambda > 0$).

Definition 2: (attentional discounted utility)

Define $w(s_t|s) = \frac{w(s_t)}{p(s)}$. As is shown in **?**, the optimization problem can be easily solved by Lagrangain method. The solution is

$$w(s_t|s) = \frac{w_t^0 e^{u(s_t)/\lambda}}{\sum_{t=0}^{T} w_\tau^0 e^{u(s_t)/\lambda}} \tag{1}$$

Note $w(s_t|s)$ reveals how the decision maker weights the utility of time period t in a drawn sequence s. It can naturally represent the discounting factor. $w(s_t|s)$ is increasing in s_t, which implies the decision maker exhibit more patience for a larger reward.

While building the model, I was mainly inspired by the theories of rational inattention (**???**). In **?**'s theory of rational inattention, the decision maker makes choices between discrete alternatives; she evaluates each alternative via a costly information acquisition process, then decides the optimal choice strategy. The theory deduces the probability of each alternative being chosen should follow a logistic-like distribution. In ADU, I assume the discounting factors are generated by a similar process; hence, she subjectively weights each time period according to a logistic-like distribution – as Equation (1) does – as well.

The reason why I use Shannon mutual information as the cognitive cost function is twofold. First, note that $w(s_t|s) \propto w_t^0 e^{u(s_t)/\lambda}$. Given a certain stream s and two time periods t_1 and t_2 ($t_2 > t_1$), the relative weight between them $\frac{w(s_{t_1}|s)}{w(s_{t_2}|s)}$ is only relevant to s_{t_1} and s_{t_2}. Therefore, changing the reward of a third period has no impact on how the reward in t_2 should be discounted relative to that in t_1. Second, under such settings, the objective function can be rewritten as

$$\sum_{s\in X} p(s)[w(s_t|s)u(s_t) - \lambda D_{KL}]$$

where D_{KL} is the KL divergence between the initial weights over time periods and the weights updated given the stream s is drawn. Clearly, the determination of $w(s_t|s)$ in each s can be separated from each other. In other words, given two potentially realized streams s and s', the changes in s' has no impact on the determination of discounting factors in s. This property is consistent with many forms of optimal sequential learning (for example, **?**). **?** and **?** show that the two properties are jointly satisfied if and only if the solution of $w(s_t|s)$ follows Equation (1).

2.2 The Rationale Behind Mutual Information

Axiom 1: (state independence) the probability weighting process and time discounting process can be separated each other.

Axiom 2: (spread-consistency correlation) when allocating a consumption budget across time periods, consumers keep their choices dynamically consistent if and only if they perform a strong preference for spread.

Proposition 1: $\succsim$ has an attentional discounted utility (ADU) representation if and only if it satisfies Axiom 1-2, and

$$X_T \succsim Y_{T'} \Longleftrightarrow \sum_{t=0}^{T} w_t(X_T)u(x_t) \geq \sum_{t=0}^{T'} w_t(Y_{T'})u(y_t)$$

where $\{w_t(X_T)\}$, $\{w_t(Y_{T'})\}$ are generated by constrained optimal discounting processes.

3 Implications in Time Preferences

3.1 Valuation of A Delayed Reward

Suppose a decision maker receives a positive detereminstic reward in time period j (and no reward in other periods), that is, for any $t \in [0, 1, \ldots, T]$ and $t \neq j$, $x_t = 0$. The decision maker evaluates the reward sequence by implementing the ADU evaluation procedure. For simplicity, I set $v(0) = 0$. I also set the decision maker initially holds stationary time preferences, i.e. $w_t^0 = \delta^t$, where $\delta \in (0, 1]$. When $\delta = 1$, we say that the initial attention is uniformly distributed across time periods. Given the reward is detereminsic, one can omit s in $w(s_t|s)$ and directly represent the weight on each time period t by w_t.

$$w_j = \begin{cases} \delta^j \cdot \dfrac{1}{1 + \frac{\delta}{1-\delta}(1-\delta^T)e^{-v(x_j)}} \ , & 0 < \delta < 1 \\ \dfrac{1}{1 + T \cdot e^{-v(x_j)}} \ , & \delta = 1 \end{cases}$$

3.2 Hidden Zero Effect

One way to validate the model is that, when we frame of the length of sequence in different ways, the decision maker's overall utility may change.

One evidence is **?**, which find that people perform greater patience when both SS and LL are framed as sequences, rather than being framed single-period rewards. They term this finding as "hidden zero effect". For instance, suppose SS is "receive £100 today" and LL is "receive £120 in 6 months", and we have

SS_0: "receive £100 today and £0 in 6 months"

LL_0: "receive £0 today and £120 in 6 months"

Then people will be more likely to prefer LL_0 over SS_0 than preferring LL over SS. The subsequent studies (e.g. **?**) show that the hidden zero effect is asymmetric. That is, shifting SS to SS_0 and keeping LL unchanged leads to an increase in patience, whereas shifting LL to LL_0 and keeping SS unchanged cannot increase patience. The attention-based explanation is that, in SS, the decision maker may conceive the length of sequence as "today"; in SS_0, she may conceive the length as "6 months". In the latter case, she allocates some attention weights to some future periods with zero reward, which decreases her overall utility.

The existence of hidden zero effect also provides some hints on the selection of time length T. As is shown in Equation **??**, when evaluating a delayed reward delivered in period j, the range of T is $[j, +\infty)$. An increase in T will reduce the overall utility. Thus, when comparing SS and LL, the decision maker may tend to set $T = j$ (the minimum length she can set), in order to optimize the overall utility. That is, without mentioning the periods after j, she does not necessarily sample from the periods later than j to evaluate the given delayed reward. In this case, i.e. $T = j$ we have

$$w_T = \frac{1}{1 + G(T)e^{-v(x_T)}}$$

where

$$G(T) = \begin{cases} \frac{1}{1-\delta}(\delta^{-T} - 1) \ , & 0 < \delta < 1 \\ T \ , & \delta = 1 \end{cases}$$

Clearly, when $\delta = 1$, the attention weight w_j takes a form similar with hyperbolic discounting.

3.3 Common Difference Effect

Suppose there are a large later reward x_l arriving at period t_l (denoted by LL) and a small sooner reward x_s arriving at period t_s (denoted by SS), where $x_l > x_s > 0$, $t_l > t_s > 0$. Assuming $w_{t_l}(x_l)v(x_l) = w_{t_s}(x_s)v(x_s)$, common difference effect implies $w_{t_l+\Delta t}(x_l)v(x_l) > w_{t_s+\Delta t}(x_s)v(x_s)$ for any positive integer Δt(**?**).

Proposition 2: *If the initial weights are uniformly distributed, then the common difference effect always holds; if the initial weights exponentially declines over time, the common difference effect holds when* $v(x_l) - v(x_s) + \ln \frac{v(x_l)}{v(x_s)} > -(t_l - t_s) \ln \delta$.

When $\delta = 1$, ADU predicts that the decision maker always performs common difference effect. This is obvious because the discounting factor w_T takes a hyperbolic-like form. When $\delta < 1$, the decision maker performs common difference effect only when the difference between x_l and x_s are much larger than the difference between t_l and t_s.

The ADU's prediction on common difference effect can be understood as follows. Note that $w_t \propto \delta^t e^{u(x_t)/\lambda}$. If we omit the constraint that the sum of weights on each time period is fixed (i.e. attention is limited), then $w_{t_l+\Delta t}(x_l) = \delta^{\Delta t} \cdot w_{t_l}$ and the same can be applied to $w_{t_s+\Delta t}$. Thus, $w_{t_l+\Delta t}/w_{t_s+\Delta t}$ keeps constant for any Δt. However, given the decision maker's attention is limited, the change from w_{t_l} to $w_{t_l+\Delta t}$ is not only driven by $\delta^{\Delta t}$, but also driven by the effect that the final period, with a positive reward, can naturally grab attention from the previous periods which has no reward. Since $x_l > x_s$, this attention-grabbing effect is greater for LL than for SS. Meanwhile, when extending the time length, the average attention that can be allocated to each period should shrink. The decision makers performs common difference effect only when the former effect exceeds the latter effect.

3.4 Magnitude Effect

Assuming we have t_l, t_s, x_s fixed, and want to find a x_l such that $w_{t_l}(x_l)v(x_l) = w_{t_s}(x_s)v(x_s)$, the magnitude effect implies that, if we increase x_s, then the x_l/x_s that makes the equality valid will decrease.

Proposition 3: *The magnitude effect always holds when the utility function $v(x)$ satisfies*

$$RRA_v(x) \leq 1 - \frac{e_v(x)}{v(x)+1}$$

where $RRA_v(x)$ is the relative risk aversion coefficient of $v(x)$, $e_v(x)$ is the elasticity of $v(x)$ to x.

Corollary 1: Suppose $v(x) = x^\gamma/\lambda$, where $\gamma > 0$ and $\lambda > 0$. Then magnitude effect always holds.

3.5 Concavity of Time Discounting

Proposition 4: *If $\delta = 1$, then the discount function is convex in t. If $0 < \delta < 1$, then there are a reward threshold $\underline{x}$ and a time threshold $\underline{t}$ such that*

1) *when $x \leq \underline{x}$, the discount function is convex in t;*
2) *when $x > \underline{x}$, the discount function is convex in t given $t \geq \underline{t}$, and it is concave in t given $t < \underline{t}$.*

It can be derived that $v(\underline{x}) = \ln(\frac{2}{1-\delta})$, and $\underline{t} = \frac{\ln[(1-\delta)e^{v(x)}-1]}{-\ln\delta}$.

Both exponential and hyperbolic discounting models predict the decision maker is risk seeking over time lotteries. That is, suppose a deterministic reward of level c ($c > 0$) is delivered in period t_s with probability π and is delivered in period t_l with probability $1-\pi$ ($0 < \pi < 1$); another deterministic reward, of the same level, is delivered in a certain period $\pi t_s + (1-\pi)t_l$. The decision maker should prefer the former case to the latter case. However, **?** find in experiments that people are only risk seeking over time lotteries when π is small and are risk averse over time lotteries when π is large. This finding can be explained by the convexity of w_T.

Let $t_m = \pi t_s + (1-\pi)t_l$. By definition, the decision makers are risk seeking over time lotteries when $\pi w_{t_s}(c) + (1-\pi)w_{t_l}(c) > w_{t_m}(c)$. First, note the LHS equals to the RHS when $\pi = 0$ or $\pi = 1$. Fixing t_s and t_l, the inequality implies $w_{t_m}(c)$ is convex in t_m. Second, it can be proved that $w_T(c)$ is convex in T if and only if T is above a certain threshold. This is also consistent with **?** that suggests the discount function should be inverse S-shaped with respect to time. By contrast, in many models such as exponential and hyperbolic discounting, discounting factors are typically decided by a convex function of T. Third, note t_m is linearly decreasing with π, thus the decision maker is more likely to be risk seeking over time lotteries when π is small. The same can be applied to the risk aversion case.

Now consider T is small enough to make w_T concave in T. In this case, adding an extension to T will increase the rate at which w_T declines with T – this property is termed "super-additive time intervals" by **?**. Moreover, ADU predicts intervals are sub-additive when the total time length T is large, and are super-additive when T is small, which is consistent with ?.

3.6 Violation of Diminishing Sensitivity

We discuss two behavioral implications of this property. The first is reference-dependent preferences. The second is sub-additivity and super-additivity of time intervals.

Proposition 5: *Suppose* $t \geq 1$, $\frac{d}{dx}\left(\frac{1}{v'(x)}\right)$ *is continuous in* $(0, +\infty)$*, then*

1) *there exists a threshold* $\bar{x}$ *in* $(0, +\infty)$ *such that* $V(x, t)$ *is strictly concave in* x *when* $x \in [\bar{x}, +\infty)$*;*

2) *if* $\frac{d}{dx}\left(\frac{1}{v'(x)}\right)$ *is right-continuous at* $x = 0$*, and* $\frac{d}{dx}\left(\frac{1}{v'(0)}\right) < 1$*, then there exists a threshold* x^* *in* $(0, \bar{x})$ *such that, for any* $x \in (0, x^*)$*,* $V(x, t)$ *is strictly convex in* x*;*

3) *there exist a hyper-parameter* λ^* *and an interval* (x_1, x_2) *such that, if* $\lambda < \lambda^*$*, for any* $x \in (x_1, x_2)$*,* $V(x, t)$ *is strictly convex in* x*, where* $\lambda^* > 0$ *and* $(x_1, x_2) \subset (0, \bar{x})$*.*

Supporting evidence: sub- and super-additive intervals

ADU predicts that the larger the unit cost of information λ or the smaller the magnitude of x_l and x_s is, the more likely it is that the decision maker performs magnitude effect.

First, note that the magnitude effect requires the decision maker's overall utility $w_T(x_T)u(x_T)$ to be a convex function of x_T. Given that $u(.)$ is concave, whether the magnitude effect holds should depend on w_T. Then, set $z = u(x_T) - \lambda \log G(T)$. We can rewrite Equation (2) as a logistic function of z, i.e. $w_T = 1/(1+e^{-z/\lambda})$. By the shape of logistic function, w_T is convex in $u(x_T)$ if and only if $u(x_T) < \lambda \log G(T)$ (that is, when x_T is small relative to T or when λ is large). Finally, it is notable that the given condition is necessary but not sufficient to yield magnitude effect.

In summary, holding the others equal, the decision makers' overall utility can be convex in a future reward when the level of it is under a certain threshold, and be concave when it is above the threshold. This is also consistent to the theories about reference-dependent preferences (?).

3.7 Inseparability of Sequences

Let x and y denote two 2-period risky reward sequences. For x, the realized sequence is [£100,£100] with probability 1/2, and is [£3,£3] with probability 1/2. For y, the realized sequence is [£3,£100] with probability 1/2, and is [£100,£3] with probability 1/2. Classical models of intertemporal choice, such as **?**, typically assume the separability of potentially realized sequences. This implies that the decision maker is indifferent between x and y. However, **?** find evidence of intertemporal correlation aversion, that is, people often prefer y to x. Such a property is also termed "weak separability" in **?**.

ADU can naturally yield intertemporal correlation aversion. For simplicity, suppose the initial attention is uniformly distributed across the two periods. For x, under each potentially realized sequence, the decision maker equally weights each period. For y, decision maker tends to assign more weight to the period with a reward of £100 (suppose that weight is w). Then the value of x is $\frac{1}{2}u(100) + \frac{1}{2}u(3)$ and the value of y is $w \cdot u(100) + (1-w) \cdot u(3)$. Given that $x > \frac{1}{2}$, the decision makers should strictly prefer y to x.

4 Implications in Dynamically Inconsistent Planning

4.1 Weight Updating and Attention-Grabbing Effect

Suppose a decision maker has budget m ($m > 0$) and is considering how to spend it over different time periods. We can use a reward sequence x to represent this decision problem, where the decision maker's spending in period t is x_t. In period 0, she wants to find a x such that

$$\max_{x} \sum_{t=0}^{T} w_t u(x_t) \quad s.t. \sum_{t=0}^{T} x_t = m \tag{3}$$

where w_t is the attention-adjusted discounting factor in period t. I assume $w_t = \delta^t e^{u(x_t)/\lambda} / \sum_{t=\tau}^{T} \delta^\tau e^{u(x_\tau)/\lambda}$ and there is no risk under this setting.

In models like exponential and hyperbolic discounting, the discounting factor of a future period is consistently smaller than that of the current period. Thus, the decision maker should spend more at the present than in the future. By contrast, in ADU, when increasing the spending in a certain period, the discounting factor corresponding to that period should also increase. So it is possible that the decision maker spends more in the future and that a future period has a greater discounting factor than the current period. This is consistent with **?** that find people sometimes prefer improving sequences to declining sequences.

ADU suggests there are two mechanisms that can help explain why people may perform dynamically inconsistent behavior. The first is *attention-grabbing effect*, that is, keeping the others equal, when we increase x_t (which lead to an increase in w_t), the discounting factor in any other period should decrease due to limited attention. After omitting a previous period from the decision problem in Equation (3), the decision maker can assign more weights to remaining periods; thus, the attention-grabbing effect is enhanced. The increased attention-grabbing effect will offset some benefit of increasing spending toward a certain period. Therefore, when the decision maker prefers improving sequences, the attention-grabbing effect will make her perform a present bias-like behavior (always feeling that she should spend more at the present than the original plan); when the decision maker prefers declining sequences, this effect will maker her perform a future bias-like behavior (always feeling she should spend more in the future).

The second mechanism is *initial attention updating*. As is assumed above, in period 0, prior to evaluating each reward sequence, the decision maker's initial weight on period t is proportional to δ^t; after evaluation, the weight becomes being proportional to $\delta^t e^{u(x_t)/\lambda}$. In period 1, if she implements the evaluation based on the information attained in period 0, the initial weight should be updated to being proportional $\delta^t e^{u(x_t)/\lambda}$; thus, the weight after evaluation should become being proportional to $\delta e^{2u(x_t)/\lambda}$. As a result, the benefit of increasing spending toward a certain period gets strengthened. The updated initial attention can make those who prefer improving sequences perform present bias and those who prefer declining sequences perform future bias.

Both the attention-grabbing effect and initial attention updating are affected by the curvature of utility function. They jointly decide which behavior pattern that people should perform in dynamics.

4.2 Excess Smoothness and Sensitivity of Consumption

5 Empirical Analysis

6 Discussion

6.1 Comparison With Other Related Models

The third is the attention mechanisms which have been widely applied in deep learning. In such models, there is often an input sequence and a query vector. Attention weights are assigned to each period of the sequence to determine their relevance to the current context. The most common approach to obtain attention weights involves computing similarity scores between the query vector and each period in the input sequence, normalizing these scores using a softmax (i.e. multinominal logistic) function.

In addition to ADU, there are other models that attempt to incorporate attention mechanism into the formation of time preferences. For example, **?** consider a decision maker adjusting the belief $p(s)$ over time but holding the discounting factor $w(s_t|s)$ constant. In each time period, given that her ability to learn new information is limited, the updated belief cannot deviate from that in a previous period by too much, which causes behavioral inertia. Instead, ADU assumes the decision maker re-allocates $w(s_t|s)$ each time period. Thus, the process of attention adjustment not only affects dynamic decision-making but also affects the choices in "Money Earlier or Later" (MEL) tasks. Besides, **?** assume the perception of future rewards is noisy and the decision maker infers the value of them by sampling from normal distributions; **?** allow the decision maker optimally chooses sample variance to minimize the mean sample squared error. Such theories, together with a certain specification on rate-distortion function, can lead to magnitude-increasing patience and hyperbolic-like discounting. Discounting factors in this style can be viewed as a special case of those in ADU. **?**, **?** construct an optimization problem similar to ADU. However, they use a different cognitive cost function. I compare the performance of ADU with models of **?**, **?** and some other papers in predicting human choices in MEL tasks.

6.2 The Sampling Process Underlying ADU

6.3 Potential Research Directions

- direct measure of discounting
- intransitive time preference
- range-dependent weighting (focusing) / consider salience and similarity

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