Attention-adjusted discounting

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I propose a new model of intermporal choice, drawing on the recent literature on rational inattention. I assume the decision makers seek to maximize the weighted sum of instantaneous utilities, where the weights depend on the attention assigned to each time point. The initial attention allocation is in line with time-consistent choice. But after knowing the temporal reward structure, they tend to assign more weights (pay more attention) to the time points with larger rewards, though the attention-adjusting process incurs a cognitive cost. I show that this process can explain a lot of behavioral biases, including common difference effect, magnitude effect, risk aversion over time lotteries, interval additivity anomalies, intertemporal correlation aversion, and inconsistent planning. Also, several discounting models can be viewed as a special case of this model, such as magnitude-dependent hyperbolic discounting and quasi-hyperbolic discounting.

1 Introduction

Suppose a decision maker is informed that she will receive a sequence of rewards $\mathbf{x} = [x_0, x_1, ..., x_T]$. Her instantaneous utility in time point t is $u(x_t)$, which is twice differentiable and strictly increasing in x_t, and $u(0) = 0$. I assume that she values the reward in each time point separately, then use the weighted sum of each instantaneous utility to represent the overall utility of her own U. Thus,

$$U(\mathbf{w}; \mathbf{x}) = \sum_{t=0}^{T} w_t u(x_t)$$

where $\mathbf{w} = [w_0, w_1, ..., w_T]'$ denotes decision weights, $w_i \geq 0$ for all $i \in \{1, .., T\}$ and $\sum_{t=0}^{T} w_t = 1$.

The idea that decision makers evaluate an option using the sum of weighted utilities is widely adopted in behavioral economic literature. [1] In intertemporal choice setting, those weights (aka $\mathbf{w}$) is decided by some discounting function. According to the classical work of Strotz (1955), when the decision maker is time-consistent, $\mathbf{w}$ should take an exponential form. We assume $\mathbf{w} = \{1/\iota, \delta/\iota, \delta^2/\iota, ..., \delta^T/\iota\}$ under this circumstance, where δ is a constant parameter and $\iota = \sum_{t=0}^{T} \delta^t$.

I use a simple example to illustrate how the model works. Suppose the decision maker is asked to choose one option between "receive x^s now" and "receive x^l in T", where $0 < x^s < x^l$, $T > 0$. By choosing to receive x^s now, the decision maker can obtain utility $u(x^s)$ at time point 0. By choosing to receive x^l later, she can gain $u(x^l)$ at time point T and 0 at other time points; thus, her overall utility is $w_T u(x^l)$.

I assume the decision maker is initially time-consistent, and for simplicity, the discounting parameter $\delta = 1$. So, when considering the "receiving later" option, her initial decision weight allocation is $\mathbf{w}^0 = [1/T, 1/T, ..., 1/T]$. She can reallocate the weights to $\mathbf{w}$, in order to maximize $w_T u(x^l)$; but the reallocation process triggers some cognitive cost, which also needs to be considered in utility maximization. When the cost is zero, she will assign full weight to time point T and zero weight to other time points; when the cost is fairly large, $\mathbf{w}$ cannot deviate from $\mathbf{w}^0$ by too much. I define the cognitive cost function with KL divergence, a measure of how much a distribution differs from another. Therefore, the decision maker's objective is

$$\max_{\mathbf{w}} \; U(\mathbf{w}; \mathbf{x}) - \lambda \cdot D_{KL}(\mathbf{w}||\mathbf{w}^0)$$

where λ is a parameter, and the definition of $D_{KL}(\mathbf{w}||\mathbf{w}^0)$ is

$$D_{KL}(\mathbf{w}||\mathbf{w}^0) = \sum_{t=0}^{T} w_i \log(\frac{w_i}{w_i^0})$$

The FOC condition of this optimization problem tells us $w_t \propto \delta^t \exp\{u(x_t)/\lambda\}$. Therefore, when considering "receive x^l in T", the decision maker will adjust her weight for time point T to

$$w_T = \frac{1}{1 + k \cdot T}$$

where $k = e^{-u(x^l)/\lambda}$. Clearly, w_T takes a similar form with hyperbolic discounting function. The literature on rational inattention (Jung et al., 2019; Maćkowiak et al., 2023; Matějka and

[1] For example, in risky choice theories, such as prospect theory, the weights are interpreted as the outputs of probability weighting function.

McKay, 2015) suggests that the determination of w_T can be interpreted as some costly top-down attention shifting process. Therefore, I term the weights calculated by such decision process as "*attention-adjusted discounting factors*".

Let us set $T = 9$, $x^s = 12$, $x^l = 16$, $u(x) = \lambda x^{0.5}$. I show that how the attention-adjusted discounting factor w_T satisfies both the magnitude effect and the common difference effect. First, if the decision maker chooses to receive x^s now, she gains $u(x^s) = 3.46\lambda$; if she chooses to receive x^l in T, she gains $w_T u(x_l) = 4\lambda/(1 + 9e^{-4}) = 3.43\lambda$. Hence, in this setting she tends to receive x_s now. Second, for magnitude effect, we can increase x^s and x^l by a same amount, say 9. Then the utility of receiving reward earlier will be 4.58λ, and that of receiving later will be 4.71λ, i.e. increasing the reward magnitude by a same amount makes her more likely to wait. Finally, for common difference effect, we can delay the reward of each option by a same interval, say 3. Then the utility of receiving reward earlier shrinks to $u(x^s)/(1 + 3e^{-u(x^s)/\lambda}) = 3.17\lambda$ and that of receiving later shrinks to 3.28λ, i.e. adding a common delay to each option makes her more likely to wait.

2 The Model

2.1 Attention Reallocation Process

Consider a decision maker evaluating a sequence of rewards, which depends on the state of world s. The time length of this sequence is T. Let t denote each time point, $t \in [0, T]$. The decision maker samples from those time points and aggregate the utilities she can obtain in each time point in her sample, to construct a value representation of s. Her objective is to find a sampling strategy $f(t, s)$, which denotes the choice probability for each time point under each certain state. I set the probability of s occurring and the unconditional probability of choosing t to be $p(s)$, $p(t)$. By the insight of motivated beliefs, I assume she wants to maximize her total utility through f; thus, time points with greater rewards should be sampled more frequently. However, processing reward information is costly. Following the rational inattention literature, the optimization problem for the decision maker is

$$\max_f \int u(t,s)f(t,s)dtds - \lambda I(t;s)$$
$$s.t. \int u(t,s)dt = p(s),\ \forall s$$

where λ is a fixed parameter and $I(t;s)$ denotes Shannon mutual information.

$$I(t;s) = E_s[D_{KL}(f(t,s)||p(t))] = \int f(t,s)\log\left(\frac{f(t,s)}{p(t)p(s)}\right)dtds$$

The solution is

$$f(t|s) = \frac{p(t)e^{u(t,s)/\lambda}}{\int_z p(z)e^{u(z,s)/\lambda}}$$

The authors of rational inattention map this to a discrete choice setting. I also consider discrete time here: $t \in \{0, 1, ..., T\}$. Therefore, the process can be viewed as the decision maker use a weighted sampling strategy to evaluate a time sequence that is evenly split into multiple periods and rewards arrive at the beginning of each period. The initial decision weights for each time period $w_t^0 \equiv p(t)$, and after processing the information, she adjusts the weights to $w_t \equiv f(t|s)$.

When facing a intertemporal choice, the decision maker evaluate each option by: (1) bracketing the time horizon; (2) processing the information about rewards within the horizon; (3) reallocating the decision weights to each period within the horizon, with an optimal sampling strategy. Then, she compare the options and make the choice. A lot of literature has documented that human make decisions by sampling approach.

For why I use Shannon mutual information here, one can refer to Caplin et al. (2022).

Posterior Separable

Changes in prior do not affect the selection of learning policy - in descriptive intertemporal choice problem, the decision maker clearly know she will receive what reward in what time, so the probability of learning such specific payoff information is 1.

Only the payoff of each action (but not the labeling of action) will affect action policy - that is, for the early time periods with payoff 0, we can either treat them as T inidividual periods or an integrated period with length T.

There are two experimental paradigms: SS vs LL, time budget. In the former, the participants are informed when and how many rewards will arrive, which is certain. In the latter, the participants decide the timing and volume of rewards by themselves, which can be variable; thus, I set that for $t > 0$, $u_t = v(x_t) + \epsilon_t$, $\epsilon_t \sim N(0, \lambda\sigma_t)$.

Throughout the paper, I assume $w_t^0 = \delta^t$, where δ is the exponential discounting parameter, $\delta \in (0, 1]$

2.2 SS or LL

Given that the reward is pre-determined, the state s is certain (or $\sigma_t = 0$).

Reward arrives only at period T. For all $t < T$, $x_t = 0$.

The optimal weight for T is

$$w_T = \frac{1}{1 + G(T) \cdot e^{-v(x_T)/\lambda}}$$

where

$$G(T) = \begin{cases} T & , \delta = 1 \\ \frac{1}{1-\delta}(\delta^{-T} - 1) & , 0 < \delta < 1 \end{cases}$$

3 Related Literature

One of the closest papers to this model may be Steiner et al. (2017). Steiner et al. (2017) mainly consider a decision maker allocating attention across discrete options and updating the attention allocation over time. In each time point, given that the decision maker's ability to process new information is limited, her attention allocation cannot deviate from the original one in the last time point by too much. This causes her action strategy to lag behind the state of world and to perform a inertial style. However, the attention-adjusted discounting model assumes the decision maker directly allocates attention across discrete time points and evaluates each action separately. This makes it convenient to analyze not only how decision makers will update action strategy in a dynamic process, but also how their will choose between a early small reward or a late large reward when these options are presented in a descriptive form.

Another relevant model is proposed by Noor and Takeoka (2022) and Noor and Takeoka (2020), which they term as "*costly empathy*". They admits a similar intertemporal choice process, in which discounting factors are determined by maximizing the sum of the utilities for each time point, where the utility for each time point is the discounted instantaneous utility of reward minus the cognitive cost of assigning a non-zero discounting factor to a future time point. They prove that when the margnial cognitive cost is increasing with both the discounting factor and time, the final discounting factors will be increasing in reward magnitude. My work differ from them by imposing a normalization condition on the weights and presume a initial weight allocation.

Gershman and Bhui (2020) propose another approach of applying rational inattention into time discounting models. They assume that the decision maker is uncertain about a future reward and runs a Bayesian estimation on it. The discounting factor is decided by minimizing the distance between the estimate and the true value of reward. They show under a certain specification of rate-distortion function[2], the discounting factor for T in SS vs LL task is $d_T = 1/(1+kT)$ given $k = (\beta v_T)^{-1}$, where β is a parameter and v_T denote the utility of reward obtained in T. Hereafter I call their model "*magnitude-dependent hyperbolic discounting*". The attention-adjusted discounting model with a logarithmic utility function and $\delta = 1$ is identical to the magnitude-dependent discounting with a power utility function.

My work may be helpful in understanding why the *Transformer* is so powerful in building artificial intelligence.

4 Behavioral Implications

4.1 Common Difference Effect

The decision-maker prefers a small reward arriving at t_1, than a large reward arriving at t_2 ($t_2 > t_1$)

However, when the same large reward arrives at $t_2 + \Delta t$ and the same small reward arrives at $t_1 + \Delta t$, the preference is reversed ($\Delta t > 0$)

Suppose the two options (A and B) delivers the equal utility to the decision-maker: *A. receive* x_1 in period t_1; B. receive x_2 in period t_2.

For simplicity, we define $v \equiv v(x_1)/\lambda$ and $\alpha \equiv v(x_2)/v(x_1) = w_{t_1}/w_{t_2}$

Note $t_2 > t_1$, thus we have $x_1 < x_2$, that is $\alpha > 1$

If common difference effect holds, then there exists Δt such that $w_{t_1+\Delta t}/w_{t_2+\Delta t} < \alpha$

Proposition 1: decision makers with attention-adjusted discounting perform common difference effect

[2] $R = \frac{1}{2}\ln(\beta u_T/T + 1)$
where u_T is the utility obtained by receiving reward in time point T.

4.2 Magnitude Effect

People are getting more patient when increasing both the small and large rewards by the same magnitude.

Noor & Takeoka (2022) provides another account, also based on finding the optimal discounting factors. Attention-adjusted discounting model admits a *General Costly Empathy (CE)* representation in their paper, but its cognitive cost function is different. In their paper, for discounting factor d_t, there exists $0 < \underline{d}_t < \overline{d}_t < 1$ such that ***CE*** cognitive cost is 0 when $d_t \in (0, \underline{d}_t]$, is strictly increasing when $d_t \in (\underline{d}_t, \overline{d}_t]$, and is ∞ when $d_t \in (\overline{d}_t, 1]$

Define $V(t, x_t) = w_t(x_t) \cdot v(x_t)$. Again, assume

$$V(t_1, x_1) = V(t_2, x_2) \equiv 1 + b \tag{2}$$

Given t_1, t_2, we can set x_1 that satisfies **eq. (2)** as a function of x_2.

By definition, magnitude effect is

$$\frac{\partial}{\partial x_2}\left(\frac{x_1}{x_2}\right) > 0 \Longrightarrow \frac{\partial x_1}{\partial V}\frac{\partial V}{\partial x_2}x_2 - x_1 > 0 \Longrightarrow \frac{\partial V}{\partial x_2}x_2 > \frac{\partial V}{\partial x_1}x_1$$

Proposition 2: decision makers with attention-adjusted discounting perform magnitude effect if

$$RRA_v - b\frac{\partial E_{vx}}{\partial v} < 1$$

where RRA is relative risk aversion coefficient of function $v(x)$, and $E_{vx} = v'(x)\frac{x}{v(x)}$ is the elasticity of v to x.

Corollary 1: If $v(x)$ admits a CRRA representation, then the decision maker perform magnitude effect when CRRA<1.

4.3 Risk Aversion over Time Lotteries

Generally, people prefer a reward arriving at a sure time t_M, rather than the same reward arriving at t_S with probability p and at t_L with probability $1-p$, where $t_M = t_S \cdot p + t_L \cdot (1-p)$.

When short delay p is smaller, the time lottery option is more attractive compared with the sure time option.

A decision-maker is risk averse over time lotteries if and only if

$$V(t_M, x) \geq V(t_S, x) \cdot p + V(t_L, x) \cdot (1 - p)$$

which implies $\frac{\partial^2 V}{\partial t^2} \leq 0$

It can be derived that:

$$t \leq \ln\left(\frac{1-z}{z}\right) / \ln(\frac{1}{\delta}) \tag{3}$$

where $\delta \in (0, 1)$ and $z \equiv 1/(1-\delta) \cdot e^{-v(x)/\lambda} \in (0, \frac{1}{2})$

People are timing risk averse when delay is short, and are timing risk seeking when delay is long.

When z gets smaller (or x gets larger), the bound for t gets relaxed.

4.4 Interval Additivity Anomalies

Super-additivity:

$V(t, x - \Delta x) \leq V(t + \tau, x)$

however, $V(t, x - 2\Delta x) \geq V(t + 2\tau, x)$

Sub-additivity:

$V(t - \tau, x) \geq V(t, x + \Delta x)$

however, $V(t - 2\tau, x) \leq V(t, x + 2\Delta x)$

Let h denote the difference of rewards that satisfies $V(t, x-h) = V(t+\tau, x)$ and is a function of τ

Supper-additivity implies that when increasing τ by a certain ratio, h needs increasing by a larger ratio to make the equation hold.

$$\frac{\partial^2 h}{\partial \tau^2} > 0 \Longrightarrow \frac{\partial^2 V}{\partial \tau^2} / \frac{\partial V}{\partial h} > 0 \Longrightarrow \frac{\partial^2 V}{\partial \tau^2} < 0$$

From **ineq. (3)** we know that ***people perform super-additivity when*** t is small. When x is larger, people are more likely to perform super-additivity.

Similarly, people perform sub-additivity when t is large. When x is smaller, people are more likely to perform super-additivity.

4.5 Intertemporal Correlation Aversion

Andersen et al. (2018)

4.6 Inconsistent Planning

Suppose the decision maker has a total reward m and needs to allocate it over T periods.

$$x_0 + x_1 + ... + x_T = m$$

The decision maker optimally set x_1 at time period 0. Planning fallacy implies that at time period 1 she tends to increase x_1. To make her performs planning fallacy, we need to introduce uncertainty. Without uncertainty, the decision maker with attention-adjusted discounting factors is always time consistent.

At time period i, the optimal decision weights are

$$\mathbf{w} = \arg\max_{\mathbf{w}} \left\{ \sum_{t=i}^{T} E_s[w_t u_t] - \lambda I(t; s) \right\}$$

For example, the classical (β, δ)-preference admits such a representation:

$$\rho = \begin{cases} \beta, \ t = 0 \\ 1, \ t > 0 \end{cases}$$

where $\beta \in (0, 1)$.

Case 1: $\sigma_t = 0$ when $t = 0$, and $\sigma_t = \sigma$ $(\sigma > 0)$ when $t > 0$.

This case is identical to (β, δ)-preference.

Case 2: $\sigma_t = t \cdot \sigma$

The decision maker performs future bias.

(proof by contradiction)

5 Empirical Test

I compare the following models in the dataset of Marzilli Ericson et al. (2015).

Using a similar method with Wulff and Bos (2018).

discounting	utility	mse	mae	accuracy	log_like
attention	power	0.208908	0.417425	0.682846	-3515.8
expo	power	0.20566	0.415521	0.691343	-3485.9
expo2	power	0.205394	0.415116	0.690721	-3490.88
hb	power	0.207483	0.416106	0.68518	-3505.01
hb2	power	0.209631	0.406708	0.680632	-3566.92
hbmd	power	0.202267	0.405576	0.700174	-3434.34
hce	power	0.208268	0.415015	0.68377	-3515.48
quasihb	power	0.20783	0.382915	0.681961	-3864.07
quasihb_fc	power	0.208875	0.395679	0.6776	-3815.24
attention_uni	power	0.206063	0.414933	0.691464	-3478.06
attention_uni	log	0.202205	0.407536	0.700601	-3431.68

6 Appendix

6.1 Proof of common difference effect

From the definition of α,

$$\alpha \cdot (1 + G(t_1) \cdot e^{-u}) = 1 + G(t_2) \cdot e^{-\alpha u} \tag{1}$$

Set up a function

$$f(\Delta t) = \alpha \cdot (1 + G(t_1 + \Delta t) \cdot e^{-u}) - (1 + G(t_2 + \Delta t) \cdot e^{-\alpha u})$$

We know that $f(0) = 0$

common difference effect implies $f(\Delta t) > 0$ when $\Delta t > 0$

if $f'(\Delta t) > 0$ then

$$\frac{G'(t_2 + \Delta t)}{G'(t_1 + \Delta t)} < \alpha e^{(\alpha - 1)u}$$

when $\delta = 1$, the right hand is 1, the common difference effect always holds

when $0 < \delta < 1$, rewrite $f(\Delta t) > 0$:

$$\delta^{-\Delta t}(\delta^{-t_1}\alpha e^{-u} - \delta^{-t_2}e^{-\alpha u}) > \alpha e^{-u} - e^{-\alpha u} - (1 - \delta)(\alpha - 1)$$

from eq. (1) we know the the left hand equals $\delta^{-t_1}\alpha e^{-u} - \delta^{-t_2}e^{-\alpha u}$. Therefore, the inequality always holds when $\Delta t > 0$

6.2 Proof of magnitude effect

Given that $G(t_1)e^{-v(x_1)/\lambda} = v(x_1)/(1+b) - 1$,

$$\frac{\partial V}{\partial x_1}x_1 = (1+b)(v(x_1)+b)\frac{v'(x_1)}{v(x_1)}x_1$$

Observing that $x_2 \cdot \partial V/\partial x_2$ admits a similar representation, we can define a function $\psi(x)$:

$$\psi(x) = (v(x)+b)\frac{v'(x)}{v(x)}x = xv'(x) + bE_{vx}$$

(Lama 1) for any $x_1 < x_2 < \infty$, there always exists $t_1 < t_2 < \infty$ that makes **eq. (2)** holds.

Thus, if $\psi'(x) > 0$, then $\psi(x_2) > \psi(x_1)$ for any , $x_1 < x_2 < \infty$. The inequality can be derived from $\psi'(x) > 0$.

6.3 Proof of inconsistent planning

Given that $w_t(s) \propto \delta^t e^{u_t(s)/\lambda}$, we have $E_s[w_t u_t] \propto \delta^t \xi(x_t, \sigma_t)$, where

$$\xi(x_t, \sigma_t) \equiv (\frac{v(x_t)}{\lambda} + \sigma_t^2)\exp\left\{\frac{v(x_t)}{\lambda} + \frac{\sigma_t^2}{2}\right\}$$

Note that if $z \sim N(\mu, \sigma)$, then $E[ze^z] = (\mu + \sigma^2)\exp\{\mu + \sigma^2/2\}$

$\xi(.)$ should be a concave function to x_t; therefore,

$$v'(x_t)\left[1 + \sigma_t^2 + \frac{v(x_t)}{\lambda}\right]\exp\left\{\frac{v(x_t)}{\lambda} + \frac{\sigma_t^2}{2}\right\}$$

is decreasing with x_t.

At period 0, the decision maker allocate her total rewards to solve the optimization problem

$$\max_{\mathbf{x}} \sum_{t=0}^{T} \delta^t \cdot \xi(x_t, \sigma_t) \quad s.t. \sum_{t=0}^{T} x_t = m$$

by FOC, we have

$$\frac{\partial \xi}{\partial x_t} = \delta \frac{\partial \xi}{\partial x_{t+1}}$$

i.e.

$$\frac{v'(x_t)}{v'(x_{t+1})} = \rho\delta$$

where

$$\rho = \frac{1 + \sigma_{t+1} + v(x_{t+1})/\lambda}{1 + \sigma_t + v(x_t)/\lambda} \exp\left\{\frac{v(x_{t+1}) - v(x_t)}{\lambda} + \frac{\sigma_{t+1} - \sigma_t}{2}\right\}$$

When ρ is constant with t, the decision maker performs no planning fallacy. When β is (weakly) increasing with t and there exists an interval $[0, \bar{t}]$ such that β is strictly increasing with t, the decision maker performs planning fallacy and is present-biased.

6.4 Proof of intertemporal correlation aversion

6.5 Model Comparison

8 discounted utility models, in comparison

1. exponential (EXPO)

$$d_t = \delta^t$$

2. double exponential (EXPO2)

$$d_t = \omega\delta_1^t + (1-\omega)\delta_2^t$$

3. hyperbolic (HB)

$$d_t = \frac{1}{1+kt}$$

4. dual-parameter hyperbolic (HB2)

$$d_t = \frac{1}{(1+kt)^\alpha}$$

5. magnitude-dependent hyperbolic (HB-MD)

$$d_t = \frac{1}{1 + (\beta u(x_t))^{-1} \cdot t}$$

6. quasi-hyperbolic (QuasiHB)

$$d_t = \begin{cases} 1, & t = 0 \\ \beta \delta^t, & t > 0 \end{cases}$$

7. quasi-hyperbolic plus fixed delay cost (QuasiHB-FC)

$$d_t = \begin{cases} 1, & t = 0 \\ \beta \delta^t - \dfrac{\alpha}{u_t}, & t > 0 \end{cases}$$

8. homogeneous costly empathy (HCE)

$$d_t = a_t u(x_t)^{\frac{1}{m-1}}$$

where $a_t = \delta^t$.

The choice probability function is given by

$$P(LL) = \frac{1}{1 + e^{-(V_l - V_s)\rho}}$$

where $V_t = d_t u(x_t)$, $t \in \{s, l\}$

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