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Zijian Zark Wang

1 Current Progress

I develop a novel model of intertemporal choice, which I term "*attention-adjusted discounted utility*" (ADU). I postulate that the overall utility a decision maker can obtain from a reward sequence is decided by the weighted sum of utilities that can be obtained in each time period. The initial weight allocation is exponential, i.e. the decision maker is initially time-stationary. However, when evaluating the given reward sequence, she tends to assign more weights (pay more attention) to the time periods with larger rewards, in order to subjectively maximize her overall utility. This attention adjustment process incurs a cognitive cost; and the more the weight allocation deviates from the initial allocation, the greater the cost is. The decision maker optimally re-allocates the weights across time periods. I term the discounting factors that can be represented by such weights as *attention-adjusted* discounting factors.

In this document, I show that a set of intertemporal choice anomalies can be attributed to such attention adjustment processes (that is, can be explained by ADU to some extent), including common difference effect and magnitude effect (Loewenstein and Prelec, 1992), risk aversion over time lotteries (Onay and Öncüler, 2007; DeJarnette et al., 2020), non-additive time intervals (Read, 2001; Scholten and Read, 2006), intertemporal correlation aversion (Andersen et al., 2018), and dynamic inconsistency. The model can also offer insights on the preferences for sequences of outcomes (Loewenstein and Prelec, 1993) and the formation of reference-dependent preferences (Koszegi and Rabin, 2006). In an empirical test, I find ADU outperforms a set of time discounting models in predicting human intertemporal choices. Therefore, I think there is a need to rethink the foundation of many behavioral phenomena.

To understand ADU better, let's consider a simple example. Suppose a decision maker wants to estimate the value of receiving £10 on Day 3, with no reward on Day 1 and Day 2. Using an exponential discounting model with a discounting parameter of 0.8, the value of this reward sequence can be calculated as follows: $0 \times 1 + 0 \times 0.8 + 10 \times 0.64$.

The underlying evaluation procedure may involve a sampling and estimation process. Imagine a black box with three types of balls: "Day 1's reward," "Day 2's reward," and "Day 3's reward." Initially, the box contains 1000 balls labeled as Day 1's reward, 800 balls labeled as Day 2's reward, and 640 balls labeled as Day 3's reward. The decision maker randomly samples from the box to evaluate the sequence. An exponential discounting model would assume the number of balls for each type keeps constant throughout the process.

By contrast, ADU assumes that, before she draws, an "attention mechanism" determines how many balls of each type should be included in the black box. Considering that "Day 3's reward" is the greatest, the decision maker wants to include more of these balls in the box to exchange for more rewards later. However, the total number of balls in the box is fixed (indicating "limited attention"), and it requires significant cognitive effort to change all the balls in the box to this type ("costly attention adjustment"). Therefore, the decision maker can only change a portion of the balls.

Suppose the final distribution of balls for Day 1, Day 2, and Day 3 is 970, 770, and 700, respectively. This means the decision maker successfully changed 60 balls to "Day 3's reward." Consequently, the discounting factors for each time period shift to 1, 0.79 (770/970), and 0.72 (700/970), reflecting a "hyperbolic"-style discounting. If the magnitude of Day 3's reward is increased to £20, the benefit of putting more "Day 3's reward" balls into the box can offset a greater cost of attention adjustment. Thus, the decision maker would exhibit more patience for a larger delayed reward.

The remaining part of this document is organized as follows. Section 2 outlines the model of attention-adjusted discounted utility (ADU). Section 3 explains how the model can help explain some empirical findings in intertemporal choice. Section 4 performs an empirical test of ADU and compare it with other models. Section 5 introduces my plan for the next step.

2 Attention-Adjusted Discounted Utility

Consider a reward sequence $x = [x_0, x_1, ..., x_T]$ that yields reward x_t in time period t. The time length of this sequence, denoted by T, is finite. For any $t \in \{0, 1, ..., T\}$, the reward level x_t is a random variable defined on R_+. The support of x is X, which is a subset of R_+^T.

Suppose a decision maker evaluates reward sequence x by three steps: At first, she randomly draws some potential realizations of x from X. Then, from each drawn realization of x, she draws some time periods at random, taking the rewards of these periods into a sample. Finally, she uses the mean utility of sampled rewards as a value representation of x. Let $s = [s_0, s_1, ..., s_T]$ be a potentially realized outcome of x and $p(s)$ be the probability that s is drawn. I use $w(.)$ and $u(.)$ to denote the decision maker's weight function and utility function, where $w(s_t)$ is the probability that the reward of the t-th period in a potentially realized sequence s is sampled, $u(s_t)$ is the utility obtained by reward s_t ($t \in \{0, 1, ..., T\}$), $u' > 0$, $u'' < 0$.

The sampling process is sequential, and the decision maker wants to find a sampling strategy, denoted by function $w(.)$, that maximizes her overall utility. In a given potentially realized sequence s, the periods with larger reward levels should be sampled more frequently. However, at the very beginning, the decision maker has no information about which period in s has a larger reward – she learns such information gradually in the process of sampling. This learning process triggers a cognitive cost. Hence, her overall utility is the mean utility of sampled rewards minus the cognitive cost of learning.

Suppose when having no information, the weight on period t across each potentially realized sequence is equal ($\equiv w_t^0$). Let W and P be the minimal sets that contain all available function w and p respectively. We can use an optimization problem to represent the described evaluation procedure:

$$\begin{aligned} \max_{w \in W} \quad & \sum_{s \in X} \sum_{t=0}^{T} w(s_t) u(s_t) - C(w; \theta) \\ s.t. \quad & \sum_{s \in X} \sum_{t=0}^{T} w(s_t) = 1 \\ & w(s_t) > 0, \forall s \in X, t = 0, 1, \dots, T \end{aligned}$$

where $C(.)$ is a cognitive cost function with θ as its parameters. To solve this optimization problem, I add two additional assumptions. The first is that the weight updating process is consistent with Bayes rule, that is, $w_t^0 = \sum_{s \in X} w(s_t)$. The second is that the cognitive cost function takes a form similar to Shannon mutual information, that is

$$C(\mathbf{w};\theta) = \lambda \sum_{s \in X} \sum_{t=0}^{T} w(s_t) \log \left(\frac{w(s_t)}{p(s) w_t^0} \right)$$

where $p(s)w_t^0$ is the probability of s_t being sampled when no information is learned, $w(s_t)$ is the probability of that after learning the information about x. Shannon mutual information quantifies the amount of information gain when learning about which time period has a larger reward in any initially unknown s. Consistent with Matějka and McKay (2015), I set $C(\mathbf{w};\theta)$ linear to that. Parameter λ denotes unit cost of information ($\lambda > 0$).

Define $w(s_t|s) = \frac{w(s_t)}{p(s)}$. As is shown in Matějka and McKay (2015), the optimization problem can be easily solved by Lagrangain method. The solution is

$$w(s_t|s) = \frac{w_t^0 e^{u(s_t)/\lambda}}{\sum_{t=0}^{T} w_\tau^0 e^{u(s_t)/\lambda}} \tag{1}$$

Note $w(s_t|s)$ reveals how the decision maker weights the utility of time period t in a drawn sequence s. It can naturally represent the discounting factor. $w(s_t|s)$ is increasing in s_t, which implies the decision maker exhibit more patience for a larger reward.

While building the model, I was mainly inspired by the theories of rational inattention (Matějka and McKay, 2015; Jung et al., 2019; Maćkowiak et al., 2023). In Matějka and McKay (2015)'s theory of rational inattention, the decision maker makes choices between discrete alternatives; she evaluates each alternative via a costly information acquisition process, then decides the optimal choice strategy. The theory deduces the probability of each alternative being chosen should follow a logistic-like distribution. In ADU, I assume the discounting factors are generated by a similar process; hence, she subjectively weights each time period according to a logistic-like distribution – as Equation (1) does – as well.

The reason why I use Shannon mutual information as the cognitive cost function is twofold. First, note that $w(s_t|s) \propto w_t^0 e^{u(s_t)/\lambda}$. Given a certain stream s and two time periods t_1 and t_2

$(t_2 > t_1)$, the relative weight between them $\frac{w(s_{t_1}|s)}{w(s_{t_2}|s)}$ is only relevant to s_{t_1} and s_{t_2}. Therefore, changing the reward of a third period has no impact on how the reward in t_2 should be discounted relative to that in t_1. Second, under such settings, the objective function can be rewritten as

$$\sum_{s \in X} p(s)[w(s_t|s)u(s_t) - \lambda D_{KL}]$$

where D_{KL} is the KL divergence between the initial weights over time periods and the weights updated given the stream s is drawn. Clearly, the determination of $w(s_t|s)$ in each s can be separated from each other. In other words, given two potentially realized streams s and s', the changes in s' has no impact on the determination of discounting factors in s. This property is consistent with many forms of optimal sequential learning (for example, Zhong (2022)). Matějka and McKay (2015) and Caplin et al. (2022) show that the two properties are jointly satisfied if and only if the solution of $w(s_t|s)$ follows Equation (1).

In addition to ADU, there are other models that attempt to incorporate attention mechanism into the formation of time preferences. For example, Steiner et al. (2017) consider a decision maker adjusting the belief $p(s)$ over time but holding the discounting factor $w(s_t|s)$ constant. In each time period, given that her ability to learn new information is limited, the updated belief cannot deviate from that in a previous period by too much, which causes behavioral inertia. Instead, ADU assumes the decision maker re-allocates $w(s_t|s)$ each time period. Thus, the process of attention adjustment not only affects dynamic decision-making but also affects the choices in “Money Earlier or Later” (MEL) tasks. Besides, Gabaix and Laibson (2017) assume the perception of future rewards is noisy and the decision maker infers the value of them by sampling from normal distributions; Gershman and Bhui (2020) allow the decision maker optimally chooses sample variance to minimize the mean sample squared error. Such theories, together with a certain specification on rate-distortion function, can lead to magnitude-increasing patience and hyperbolic-like discounting. Discounting factors in this style can be viewed as a special case of those in ADU. Noor and Takeoka (2022), Noor and Takeoka (2023) construct an optimization problem similar to ADU. However, they use a different cognitive cost function. In Section 4, I compare the performance of ADU with models of Gershman and Bhui (2020), Noor and Takeoka (2022) and some other papers in predicting human choices in MEL tasks.

3 Explaining Behavioral Biases

3.1 Evaluating Delayed Rewards

Suppose a decision maker receives a positive detereminstic reward in time period T (and no reward in other periods). I assume she evaluates the reward by implementing the ADU evaluation procedure on a reward sequence $x = [x_0, x_1, \ldots, x_T]$, where $x_0 = \ldots = x_{T-1} = 0$ and $x_T > 0$. For simplicity, I set $u(0) = 0$, and the decision maker initially holds stationary time preferences, i.e. $w_t^0 = \delta^t$. $\delta \in (0, 1]$, where $\delta = 1$ implies the initial attention is uniformly distributed across time periods. Given the reward is detereminsic, one can omit s in $w(s_t|s)$ and directly represent the weight on each time period t by w_t. Therefore, the value of this delayed reward is $w_T u(x_T)$. By Equation (1),

$$w_T = \frac{\delta^T e^{u(x_T)/\lambda}}{\sum_{t=0}^{T} \delta^t e^{u(x_t)/\lambda}} = \frac{1}{1 + G(T) \cdot e^{-u(x_T)/\lambda}} \tag{2}$$

where

$$G(T) = \begin{cases} T \ , & \delta = 1 \\ \dfrac{1}{1-\delta}(\delta^{-T} - 1) \ , & 0 < \delta < 1 \end{cases}$$

Equation (2) implies w_T is increasing in x_T and decreasing in T. Hereafter, I use $w_T(x_T)$ to represent w_T. I will show that Equation (2) can explain a series of experimental findings. Due to the limitation on word number, I omit mathematical proof.

3.1.1 Common Difference Effect

Suppose there are a large later reward x_l arriving at period t_l (denoted by LL) and a small sooner reward x_s arriving at period t_s (denoted by SS), where $x_l > x_s > 0$, $t_l > t_s > 0$. Assuming $w_{t_l}(x_l)u(x_l) = w_{t_s}(x_s)u(x_s)$, common difference effect implies $w_{t_l+\Delta t}(x_l)u(x_l) >$

$w_{t_s+\Delta t}(x_s)u(x_s)$ for any positive integer Δt, magnitude effect implies $w_{t_l}(x_l)u(x_l + \Delta x) > w_{t_s}(x_s)u(x_s + \Delta x)$ for any positive real number Δx (Loewenstein and Prelec, 1992).

When $\delta = 1$, ADU predicts that the decision maker always performs common difference effect. This is obvious because the discounting factor w_T takes a hyperbolic-like form. When $\delta < 1$, the decision maker performs common difference effect only when the difference between x_l and x_s are much larger than the difference between t_l and t_s.

The ADU's prediction on common difference effect can be understood as follows. Note that $w_t \propto \delta^t e^{u(x_t)/\lambda}$. If we omit the constraint that the sum of weights on each time period is fixed (i.e. attention is limited), then $w_{t_l+\Delta t}(x_l) = \delta^{\Delta t} \cdot w_{t_l}$ and the same can be applied to $w_{t_s+\Delta t}$. Thus, $w_{t_l+\Delta t}/w_{t_s+\Delta t}$ keeps constant for any Δt. However, given the decision maker's attention is limited, the change from w_{t_l} to $w_{t_l+\Delta t}$ is not only driven by $\delta^{\Delta t}$, but also driven by the effect that the final period, with a positive reward, can naturally grab attention from the previous periods which has no reward. Since $x_l > x_s$, this attention-grabbing effect is greater for LL than for SS. Meanwhile, when extending the time length, the average attention that can be allocated to each period should shrink. The decision makers performs common difference effect only when the former effect exceeds the latter effect.

3.1.2 Magnitude Effect and Reference-Dependent Preferences

ADU predicts that the larger the unit cost of information λ or the smaller the magnitude of x_l and x_s is, the more likely it is that the decision maker performs magnitude effect.

First, note that the magnitude effect requires the decision maker's overall utility $w_T(x_T)u(x_T)$ to be a convex function of x_T. Given that $u(.)$ is concave, whether the magnitude effect holds should depend on w_T. Then, set $z = u(x_T) - \lambda \log G(T)$. We can rewrite Equation (2) as a logistic function of z, i.e. $w_T = 1/(1 + e^{-z/\lambda})$. By the shape of logistic function, w_T is convex in $u(x_T)$ if and only if $u(x_T) < \lambda \log G(T)$ (that is, when x_T is small relative to T or when λ is large). Finally, it is notable that the given condition is necessary but not sufficient to yield magnitude effect.

In summary, holding the others equal, the decision makers' overall utility can be convex in a future reward when the level of it is under a certain threshold, and be concave when it is above the threshold. This is also consistent to the theories about reference-dependent preferences (Koszegi and Rabin, 2006).

3.1.3 Risk Attitude over Time Lotteries and Non-additive Time Intervals

Both exponential and hyperbolic discounting models predict the decision maker is risk seeking over time lotteries. That is, suppose a deterministic reward of level c ($c > 0$) is delivered in period t_s with probability π and is delivered in period t_l with probability $1-\pi$ ($0 < \pi < 1$); another deterministic reward, of the same level, is delivered in a certain period $\pi t_s + (1-\pi)t_l$. The decision maker should prefer the former case to the latter case. However, Onay and Öncüler (2007) find in experiments that people are only risk seeking over time lotteries when π is small and are risk averse over time lotteries when π is large. This finding can be explained by the convexity of w_T.

Let $t_m = \pi t_s + (1-\pi)t_l$. By definition, the decision makers are risk seeking over time lotteries when $\pi w_{t_s}(c) + (1-\pi)w_{t_l}(c) > w_{t_m}(c)$. First, note the LHS equals to the RHS when $\pi = 0$ or $\pi = 1$. Fixing t_s and t_l, the inequality implies $w_{t_m}(c)$ is convex in t_m. Second, it can be proved that $w_T(c)$ is convex in T if and only if T is above a certain threshold. This is also consistent with Takeuchi (2011) that suggests the discount function should be inverse S-shaped with respect to time. By contrast, in many models such as exponential and hyperbolic discounting, discounting factors are typically decided by a convex function of T. Third, note t_m is linearly decreasing with π, thus the decision maker is more likely to be risk seeking over time lotteries when π is small. The same can be applied to the risk aversion case.

Now consider T is small enough to make w_T concave in T. In this case, adding an extension to T will increase the rate at which w_T declines with T – this property is termed "super-additive time intervals" by Read (2001). Moreover, ADU predicts intervals are sub-additive when the total time length T is large, and are super-additive when T is small, which is consistent with Scholten and Read (2006).

3.2 Intertemporal Correlation Aversion

Let x and y denote two 2-period risky reward sequences. For x, the realized sequence is [£100,£100] with probability 1/2, and is [£3,£3] with probability 1/2. For y, the realized sequence is [£3,£100] with probability 1/2, and is [£100,£3] with probability 1/2. Classical models of intertemporal choice, such as Fishburn and Rubinstein (1982), typically assume the separability of potentially realized sequences. This implies that the decision maker is indifferent between x and y. However, Andersen et al. (2018) find evidence of intertemporal correlation aversion, that is, people often prefer y to x. Such a property is also termed "weak separability" in Noor and Takeoka (2023).

ADU can naturally yield intertemporal correlation aversion. For simplicity, suppose the initial attention is uniformly distributed across the two periods. For x, under each potentially realized sequence, the decision maker equally weights each period. For y, decision maker tends to assign more weight to the period with a reward of £100 (suppose that weight is w). Then the value of x is $\frac{1}{2}u(100) + \frac{1}{2}u(3)$ and the value of y is $w \cdot u(100) + (1 - w) \cdot u(3)$. Given that $x > \frac{1}{2}$, the decision makers should strictly prefer y to x.

3.3 Dynamic Inconsistency

Suppose a decision maker has budget m ($m > 0$) and is considering how to spend it over different time periods. We can use a reward sequence x to represent this decision problem, where the decision maker's spending in period t is x_t. In period 0, she wants to find a x such that

$$\max_{x} \sum_{t=0}^{T} w_t u(x_t) \quad s.t. \sum_{t=0}^{T} x_t = m \tag{3}$$

where w_t is the attention-adjusted discounting factor in period t. I assume $w_t = \delta^t e^{u(x_t)/\lambda} / \sum_{t=\tau}^{T} \delta^\tau e^{u(x_\tau)/\lambda}$ and there is no risk under this setting.

In models like exponential and hyperbolic discounting, the discounting factor of a future period is consistently smaller than that of the current period. Thus, the decision maker should spend more at the present than in the future. By contrast, in ADU, when increasing the spending in a certain period, the discounting factor corresponding to that period should also increase. So it is possible that the decision maker spends more in the future and that a future period has a greater discounting factor than the current period. This is consistent with Loewenstein and Prelec (1993) that find people sometimes prefer improving sequences to declining sequences.

ADU suggests there are two mechanisms that can help explain why people may perform dynamically inconsistent behavior. The first is *attention-grabbing effect*, that is, keeping the others equal, when we increase x_t (which lead to an increase in w_t), the discounting factor in any other period should decrease due to limited attention. After omitting a previous period from the decision problem in Equation (3), the decision maker can assign more weights to remaining periods; thus, the attention-grabbing effect is enhanced. The increased attention-grabbing effect will offset some benefit of increasing spending toward a certain period. Therefore, when the decision maker prefers improving sequences, the attention-grabbing effect will make her perform a present bias-like behavior (always feeling that she should spend more at the present than the original plan); when the decision maker prefers declining sequences, this effect will maker her perform a future bias-like behavior (always feeling she should spend more in the future).

The second mechanism is *initial attention updating*. As is assumed above, in period 0, prior to evaluating each reward sequence, the decision maker's initial weight on period t is proportional to δ^t; after evaluation, the weight becomes being proportional to $\delta^t e^{u(x_t)/\lambda}$. In period 1, if she implements the evaluation based on the information attained in period 0, the initial weight should be updated to being proportional $\delta^t e^{u(x_t)/\lambda}$; thus, the weight after evaluation should become being proportional to $\delta e^{2u(x_t)/\lambda}$. As a result, the benefit of increasing spending toward a certain period gets strengthened. The updated initial attention can make those who prefer improving sequences perform present bias and those who prefer declining sequences perform future bias.

Both the attention-grabbing effect and initial attention updating are affected by the curvature of utility function. They jointly decide which behavior pattern that people should perform in dynamics.

4 Comparing Models of Intertemporal Choice

4.1 Data

I test the ability of ADU in predicting human intertemporal choices on two open datasets. The sources of data are Ericson et al. (2015) and Chávez et al. (2017). In each study, participants are recruited to do the classical MEL tasks (choosing between a SS reward and a LL reward). The data from Ericson et al. (2015) contains 23,131 observations from 939 participants, the data from Chávez et al. (2017) contains 34,515 choices from 1,284 participants.

4.2 Empirical Strategy

I mainly focus on out-of-sample model performance. For each dataset, I randomly draw 20% of the participants as the test sample, and set the rest as the train sample. To mitigate the overfitting issue, I implement a 10-fold cross-validation procedure on the train sample.

I fit the train sample with ADU and 8 other time discounting models, the intertemporal trade-off model (Scholten et al., 2014), and a decision-tree heuristic model, namely XGboost (Chen and Guestrin, 2016). Table 1 describes the models used for comparison. For each time discounting model and the intertemporal trade-off model, I use Logit model to predict the probability of participants choosing LL. A temperature parameter is added to control the entropy of the probability distribution. Besides, I test each model with two types of utility functions: CARA utility, $u(x) = 1 - e^{-\gamma x}$; power utility, $u(x) = x^{\gamma}$ (γ is a parameter). For the heuristic model, I use the same features as Read et al. (2013) and Ericson et al. (2015).

Table 1: Models for Comparison

model	**description**	**#parameter**	**source**
attention	ADU	2	-
attention_uni	ADU with uniformly allocated initial attention	1	-

Table 1: Models for Comparison

model	description	#parameter	source
expo	exponential discounting	1	-
hb	hyperbolic discounting	1	-
expo2	double-exponential discounting	3	van den Bos and McClure (2013)
hb2	dual-parameter hyperbolic discounting	2	Loewenstein and Prelec (1992)
hbmd	magnitude-dependent hyperbolic discounting	1	Gershman and Bhui (2020)
quasihb	quasi-hyperbolic discounting	2	Laibson (1997)
quasihb_fc	quasi-hyperbolic discounting plus a fixed delay cost	3	Benhabib et al. (2010)
hce	homogeneous costly empathy model of discounting	2	Noor and Takeoka (2022)
trade	intertemporal trade-off	4	Scholten et al. (2014)
heuristic	a decision-tree learning algorithm	by tunning	Chen and Guestrin (2016)

I use the maximum likelihood method to estimate the parameters, and apply L-BFGS-B method for optimization. As the solution of L-BFGS-B is sensitive to initial points, I use the basin-hopping algorithm for global optimization. For details about empirical analysis, see https://github.com/zarkwang/attention_discount_project

4.3 Results

In summary, I find the heuristic model outperforms the other models in predictive accuracy. ADU and the intertemporal trade-off model rank either the second or the third (dependent on the dataset and empirical strategy), and their performance is very close. Each of the two models can accurately predict about 95% of the simulated responses generated by XGBoost. Note that ADU has only one additional parameter compared with the exponential or hyperbolic discounting, whereas the intertemporal trade-off model has three additional parameters, researchers who want to save the number of parameters can consider ADU as a candidate in model fitting. Table 2 present the out-of-sample test results on the data from Chávez et al. (2017).

Table 2: Out-of-Sample Test Results

model	**utility**	**mse**	**mae**	**logLoss**	**accuracy**	**predLL**
heurstic	–	0.163	0.322	0.500	0.767	0.295
trade	power	0.165	0.316	0.504	0.766	0.258
attention	power	0.165	0.326	0.505	0.763	0.332
attention_uni	power	0.165	0.325	0.506	0.763	0.332
hb	power	0.167	0.327	0.508	0.757	0.332
quasihb_fc	power	0.206	0.448	0.604	0.757	0.332
quasihb	power	0.180	0.397	0.543	0.757	0.332
hce	power	0.167	0.335	0.510	0.757	0.332
hbmd	power	0.165	0.327	0.505	0.757	0.332
hbmd	cara	0.167	0.329	0.509	0.757	0.332
hb2	power	0.167	0.341	0.509	0.757	0.332
expo	power	0.167	0.334	0.510	0.757	0.332
trade	cara	0.248	0.498	0.689	0.700	0.222
attention	cara	0.179	0.356	0.535	0.685	0.037
attention_uni	cara	0.191	0.414	0.567	0.685	0.037
hb2	cara	0.184	0.344	0.545	0.667	0.000
hb	cara	0.207	0.322	0.622	0.667	0.000
expo2	power	0.333	0.333	3.764	0.667	0.000
expo2	cara	0.326	0.333	1.605	0.667	0.000
hce	cara	0.202	0.316	0.634	0.667	0.000
quasihb	cara	0.222	0.333	0.650	0.667	0.000

Table 2: Out-of-Sample Test Results

model	**utility**	**mse**	**mae**	**logLoss**	**accuracy**	**predLL**
expo	cara	0.203	0.314	0.653	0.667	0.000
quasihb_fc	cara	0.198	0.336	0.582	0.667	0.000

Note: **mae** denotes mean absolute error, **mse** denotes mean squared error, **predLL** denotes the ratio of LL in predicted choices. The data source is Chávez et al. (2017).

5 Next Step

The ADU model draws inspiration from theories of rational inattention. In line with most literature in rational inattention (e.g. Matějka and McKay (2015), Jung et al. (2019), Maćkowiak et al. (2023)), I set the cognitive cost function linear to Shannon mutual information. The rationale, according to Matějka and McKay (2015) and Caplin et al. (2022), is twofold. First, Shannon mutual information can be written as $\sum_{s\in X} p(s)D(s)$, where $D(s)$ is a function of $w(s_t|s)$ and w_t^0. This indicates that how the decision maker allocates attention across periods under a given outcome, denoted by $w(s_t|s)$, is independent of the belief regarding how likely it is that a potential outcome s is realized, denoted by $p(s)$. Second, when the cognitive cost function is in this style, the relative discounting factor between two periods depends merely on their own rewards, and is irrelevant from the reward in any other period, regardless of the rewards in any other period.

I am designing an experiment to test the first property. The experimental design does not draw on intertemporal choice setting. Instead, it asks participants to explicitly implement the sampling and attention adjustment processes.

The design is as follows. Suppose there are 10 red balls and 10 blue balls in a black box. The decision maker randomly draws one ball. If a red ball is drawn, she can get v_r; if a blue ball is drawn, she can get v_b. Prior to the draw, the decision maker can change the color of balls as her will. However, every time she changes the color of a ball, she has to complete a real-effort task (such as entering a CAPTCHA code). Suppose $v_b > v_r$, and she changes a

balls from red to blue, then the corresponding decision problem can be written as:

$$\max_{a} \frac{10+a}{20} v_b + \frac{10-a}{20} v_r - C(a)$$

where $C(a)$ is the cognitive cost of completing a tasks. $\frac{10+a}{20}$ is the probability of a blue ball being drawn, $\frac{10-a}{20}$ is the probability of a red ball being drawn. Clearly, this decision problem has the same representation as rational inattention or ADU. Suppose when $v_b = b_1$, $v_r = r_1$, we have $a = a_1$; when $v_b = b_2$, $v_r = r_2$, we have $a = a_2$. Then, set that the first case occurs with probability 0.5, and the second case occurs with probability 0.5, if the desired property holds, we should have $C(a) = 0.5C(a_1) + 0.5C(a_2)$. That is the target to test.

I am also trying to develop an axiomatic characterization of ADU. Moreover, more empirical analyses are needed for validating the model. Table 3 shows my plan for the next step.

Table 3: Plan for the Next Step

Timeline
June 2023
- Refine the theory of ADU
- Find proper ways to further validate the ADU model
- Finalize the experimental design for testing rational inattention
July-Aug 2023
- Obtain necessary approvals and run a pilot test
- Conduct the experiment and collect data
- Start analyzing the data
Sep-Dec 2023
- Complete the paper about ADU
- Begin drafting a paper on the experiment tests of rational inattention
- Plan for the final chapter of thesis

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