Experiment Results

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1 Survey Desigm

We use a within-subjects design. The survey contains 23 choice questions, of which 20 questions are intertemporal choices and 3 questions are risky choices. Each question consists of a choice list. In an intertemporal choice question, there are 10 rows in the list. In each row, participants are required to choose between a single immediate reward (labelled as "option A") and a two-reward sequence, constituted by an immediate reward and a delayed reward (labelled as "option B"). Option A is constant within a question, while option B varies from row to row. Participants need to actively and explicitly state their preferences on each choice. The risky choice questions follow a similar pattern. In each row of a risky choice question, participants can choose either to get a large reward with a 50% chance or to get a small reward with certainty. The risky reward is constant within the question, while the safe reward varies across rows. We use these risky choice questions to characterize the participants' utility function for further analysis.

The intertemporal choice questions are presented with a random order. Two of such questions are used for attention check. In one attention check question, the immediate reward in the sequence option (option B) dominates the amount of the single-reward option (option A) in each row; in another question, the latter dominates the sum of both rewards in option B. Particpants are presumed to only choose option B for all rows on the first attention check question, and only choose option A on the second question.

The remaining intertemporal choice questions are divided into two conditions: the first condition is called "*Immediate reward varies*"; the second condition is called "*Delayed reward varies*". In each question under the "Immediate reward varies" condition, the immediate reward in the sequence option increases by £10 with each row, starting from £10 and going up to £100, whereas the delayed reward in that option remains constant for all rows. In each question under the "Delayed reward varies" condition, the amount of the delayed reward in the sequence option varies across rows, following the same pattern as the first condition, while the immediate reward in that option remains constant. Under each condition, the time length of the sequence option, e.g. when the delayed reward is delivered, is also constant across rows in a choice list.

The amount of the single-reward option is selected from {£100, £120}. Each of such amounts is paired with a combination of a constant-across-rows amount and a time length of the sequence option. The constant-across-rows amount in any sequence is selected from {£50, £70, £90}. Under the "Immediate reward varies" condition, the time length of a sequence is selected from {1 month, 9 months, 18 months}. The delayed reward is constant within a question, the lowest level of it is combined with the shortest time length (1 month), the middle level of it is combined with the shortest or middle time length (1 month or 9 months), and the highest level of it is combined with every level of time length (1 month, 9 months or 18 months). By this approach, we obtain 6 combinations and thus $6 \times 2 = 12$ questions (paired with each amount of the single-reward option) for this condition. Under the "Delayed reward varies" condition, we only examine whether the variation of the immediate reward amount in a sequence will interfere the impact of the delayed reward amount in choices. So, the time length is simply set to 3 months. This time length is combined with each level of the immediate reward amount, which is constant-across-rows under this condition. Pairing with the amounts for single-reward option, we obtain $3 \times 2 = 6$ questions for the second condition. Overall, there are 18 intertemporal choice questions for analysis.

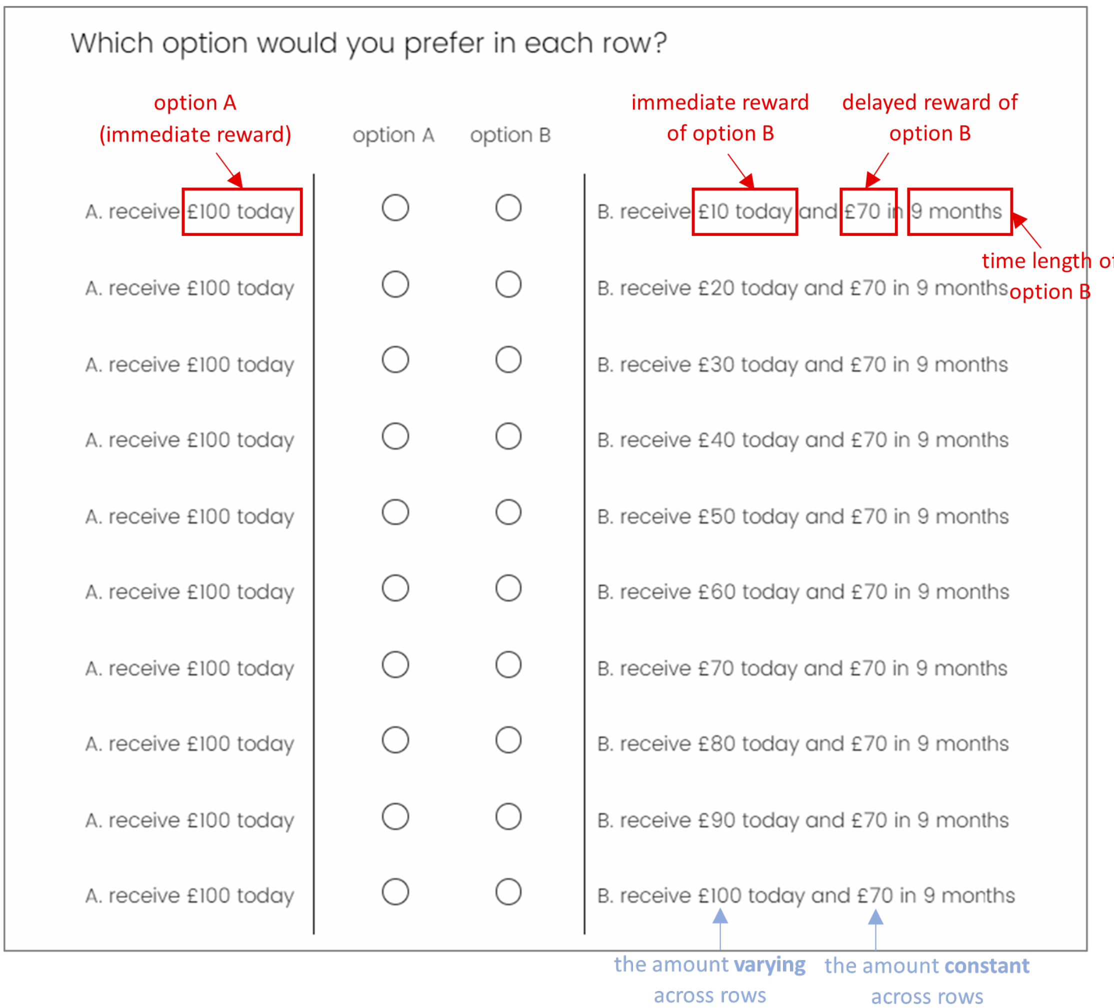

Figure 1: Screenshot of an intertemporal choice question

2 Data

We recruited 160 participants via Prolific. All the participants are British residents; 50% of them are female, and the median age is 41. On average, it took around 11.6 minutes for participants to finish the survey, with nine tenths of the participants finishing within 20 minutes. Each participants received £2 after completing the survey. Three participants failed the attention check. We remove these three participants and remain 157 participants in the sample. Each participants completed 12 questions under the “Immediate reward varies” condition and 6 questions under the “Delayed reward varies” condition. Given that each intertemporal choice question contains 10 rows, we obtain 18,840 observations (choices) for the first condition and 9,420 observations (choices) for the second condition.

3 Hypothesis

We developed two hypotheses:

(H1) *When a two-reward sequence has a larger reward amount in one specific period, people's choices are less sensitive to the changes in the other reward amount.*

(H2) *When a two-reward sequence has a longer time length, people's choices are less sensitive to the changes in the immediate reward amount.*

For illustration, suppose a set of individuals are required to both choose between “receive £100 today” (option A) and “receive £50 today and £70 in 1 month” (option B1), and choose between option A and “receive £50 today and £90 in 1 month” (option B2). Note B2 is the same as B1 except having a larger delayed reward amount. If the immediate reward (£50) in option B1 goes up to £60, each individual are more likely to prefer option B1 over option A. Then, if the immediate reward in B2 is also raised to £60, our H1 predicts that fewer individuals will act on this increase, i.e. shifting from option A to B2, compared to the responses observed for B1 (since they are “less sensitive” to the given change). Besides, consider a choice between option A and “receive £50 today and £70 in 9 months” (option B3; the same as B1 except having a longer time length). Our H2 predicts that, compared to the choices involving B1, for the same increase in the immediate reward in B3, fewer indiviuals will shift from option A to B3.

We validate our hypotheses by two approaches. First, in a descriptive analysis, we estimate the indifference point between such option A and B. A smaller variance in such indifference points among participants implies a greater sensitivity of choice to the changes in option B. In an extreme case, suppose in a choice list, every participant prefers "receive £100 today" (option A) over "receive £50 today and £70 in 1 month" (option B1), but in the next row, they all prefer "receive £60 today and £70 in 1 month". Then, every participants' indifference point is between £50 and £60; they exhibit no variance in their chocies. In other words, at the point of B1, increasing the immediate reward by £10 leads all participants to shift from option A to option B; at that point, they are extremely sensitive to the given change.

Second, by logistic regression, we estimate the probability of choosing option B in each choice. We examine the choice sensitivity by adding interactions between different elements in option B into independent variables. For instance, in the "Immediate reward varies" condition, if the regression result suggests, under a time length of 9 months, one unit increase in the immediate reward of option B, on average yields a smaller increase in choice probability for option B than that under a time length of 1 month. Then, we can conclude that people are less sensitive to the given change under the former time length.

4 Descriptive Analysis

We estimate the indifference point in varying-across-rows amounts between the single-reward option (option A) and sequence options (option B) for each intertemporal choice question. Suppose in a choice list under the "Immediate reward varies" condition, one participant prefers option A when option B is "receive £50 today and £70 in 1 month", but in the next row, when option B is "receive £60 today and £70 in 1 month", she prefers option B. Then, we set £55, i.e. the median between £50 and £60, as her indifference point for this question. Figure 2 shows the standard deviation of indifference points among participants for each question.

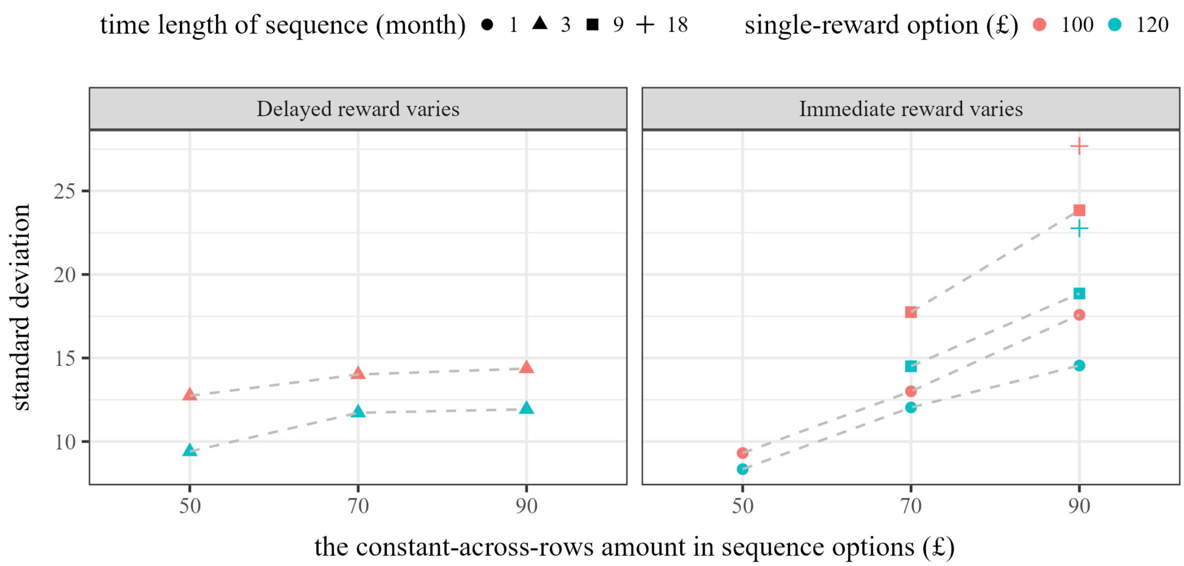

Figure 2: Standard deviation of indifference points in intertemporal choice questions

Figure 2 has three notable properties. First, under each condition, when the constant-across-rows amount in a sequence is increased (with the others being equal), the standard deviation of indifference points increases. This is consistent with H1. Second, under the "Immediate reward varies" condition, when the time length of a sequence is increased, the standard deviation of indifference points also increases. This is consistent with H2. Finally, under each condition, when the single-reward option is increased from £100 to £120, the standard deviation of indifference points increases as well.

5 Regression Analysis

5.1 Baseline Model

We set each choice as the dependent variable and run logistic regressions separately under each condition. In a choice list, let X_v denote the amount varying across rows in the sequence option (option B), X_c denote the amount constant across rows, T denote the time length of the sequence, M denote the amount of the single-reward option (option A). Note in each condition, X_c has three levels, and in the "Immediate reward varies" condition, T has three levels. Considering the potential non-linearity, both X_c and T are set as category variables, with their lowest levels being set as contrast groups. We denote the middle level and the highest level of X_c as X_{mid} and X_{high}, and the middle level and the highest level of T in the "Immediate reward varies" condition as T_{mid} and T_{high}.

Under the "Immediate reward varies" condition, M, X_v, the interaction terms between X_v and X_c as well as between X_v and T, are set as independent variables; under the "Delayed reward varies" condition, M, X_v, and the interaction terms between X_v and X_c are set as independent variables (see Table 1 for details). Considering individual-specific variation in choice probability, we also add intercept and individual-specific dummy variables to regression models.

Typically, a logistic regression model is fitted using maximum likelihood estimator (MLE). Nevertheless, in some choices of our survey, there is no variation in participants' responses. For instance, in the first row of a choice list, participants may be required to compare "receive £120 today" (option A) and "receive £10 today and £50 in 1 month" (option B). In this case, all participants will prefer option A over B. This yields the so-called "quasi-complete separation" problem in logistic regression. When the problem exists, using maximum likelihood estimator (MLE) could amplify the magnitude of coefficient estimates and standard errors. Thus, alongside with MLE, we also perform the results from Firth's penalized logistic regression method. The objective function in Firth's method is the log likelihood plus half of the log determinant of the Fisher information matrix as penalty. It penalizes the coefficients towards zero, while the penalty itself asymptotically approaches zero; overall, it is a popular way to mitigate the separation problem (Firth, 1993; Heinze and Schemper, 2002).

Table 1 shows the fixed-effect coefficients and their 95% confidence intervals for each model. Model (1) is fitted using MLE, Model (2) is the same as Model (1) in formula but is fitted using Firth's penalized logistic regression. The interaction terms shows how participants' choice sensitivity to X_v varies under different constant-across-rows amount X_c and time length T of the sequence option.

Model (1) has three notable results. First, Table 1.A shows that, under the "Immediate reward varies" condition, participants' choices are consistent with H1. The coefficient for

Table 1: Regression results for the baseline model

Panel A: Immediate reward varies

	(1) MLE		(2) Firth's estimator	
	Coef	95% CI	Coef	95% CI
M	-1.230***	[-1.302, -1.158]	-1.200***	[-1.271, -1.130]
X_v	2.559***	[2.400, 2.719]	2.496***	[2.340, 2.651]
$\mathbf{1}\{X_c = X_{mid}\}$	6.247***	[5.143, 7.351]	6.084***	[5.002, 7.166]
$\mathbf{1}\{X_c = X_{high}\}$	11.125***	[10.047, 12.204]	10.846***	[9.791, 11.902]
$\mathbf{1}\{T = T_{mid}\}$	0.020	[-0.378, 0.419]	0.019	[-0.374, 0.412]
$\mathbf{1}\{T = T_{high}\}$	-0.139	[-0.582, 0.305]	-0.136	[-0.574, 0.301]
$X_v \cdot \mathbf{1}\{X_c = X_{mid}\}$	-0.356***	[-0.523, -0.188]	-0.345***	[-0.509, -0.181]
$X_v \cdot \mathbf{1}\{X_c = X_{high}\}$	-0.817***	[-0.977, -0.657]	-0.796***	[-0.953, -0.639]
$X_v \cdot \mathbf{1}\{T = T_{mid}\}$	-0.368***	[-0.447, -0.288]	-0.359***	[-0.437, -0.280]
$X_v \cdot \mathbf{1}\{T = T_{high}\}$	-0.620***	[-0.710, -0.531]	-0.605***	[-0.693, -0.517]
observations	18840		18840	

Panel B: Delayed reward varies

	(1) MLE		(2) Firth's estimator	
	Coef	95% CI	Coef	95% CI
M	-2.908***	[-3.124, -2.692]	-2.657***	[-2.853, -2.462]
X_v	3.397***	[3.145, 3.650]	3.106***	[2.880, 3.332]
$\mathbf{1}\{X_c = X_{mid}\}$	7.828***	[6.413, 9.243]	7.161***	[5.837, 8.484]
$\mathbf{1}\{X_c = X_{high}\}$	14.283***	[12.803, 15.763]	13.065***	[11.704, 14.426]
$X_v \cdot \mathbf{1}\{X_c = X_{mid}\}$	-0.197	[-0.400, 0.006]	-0.181	[-0.372, 0.010]
$X_v \cdot \mathbf{1}\{X_c = X_{high}\}$	-0.279**	[-0.481, -0.078]	-0.257**	[-0.448, -0.067]
observations	9420		9420	

Note: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. The p-values and confidence intervals (CI) are calculated via Wald test. Each unit of M, X_v and X_c represents £10. The middle and highest levels of X_c are denoted by X_{mid}, X_{high}. The middle and highest levels of T are denoted by T_{mid}, T_{high}. The intercept and individual-specific dummy variables are omitted in the table.

$X_v \cdot \mathbf{1}\{X_c = X_{high}\}$ is -0.817, and the coefficient for $X_v \cdot \mathbf{1}\{X_c = X_{mid}\}$ is -0.356. They are both significantly smaller than zero at significance level 0.001. This indicates that, one unit increase in X_v, under the middle or highest level of X_c (i.e. $X_c = X_{high}$ or $X_c = X_{mid}$), yields a significantly smaller increase in probability of choosing the sequence option, compared to that under the lowest level of X_c. Meanwhile, the 95% confidence intervals for $X_v \cdot \mathbf{1}\{X_c = X_{high}\}$ and $X_v \cdot \mathbf{1}\{X_c = X_{mid}\}$ are [-0.977, -0.657] and [-0.523, -0.188], where the upper bound of the former is even smaller than the lower bound of the latter. This suggests, at least at significance level 0.05, the given increase in X_v under $X_c = X_{high}$ has a significantly smaller impact in the choice probability than that under $X_c = X_{mid}$. Thus, we can conclude the varying-across-rows amount X_v in sequence options has a smaller impact on the estimated choice probability as the other reward amount X_c (which is constant-across-rows) goes up.

Second, Table 1.A also shows that, the choices under the "Immediate reward varies" condition are consistent with H2. The analysis process is the same as H1. The coefficients for $X_v \cdot \{T = T_{high}\}$ and $X_v \cdot \{T = T_{mid}\}$ indicates, one unit increase in X_v under these two cases yields a significantly smaller increase in probability of choosing the sequence option, compared to that under the lowest level of T. Meanwhile, their 95% confidence intervals suggest, the effect under $T = T_{high}$ is significantly smaller than that under $T = T_{mid}$. Thus, the varying-across-rows amount X_v in sequence options has a smaller impact on the estimated choice probability as the time length T goes up.

Third, Table 1.B shows that, under the "Delayed reward varies" condition, the choices perform some patterns similar with H1 (note only H1 is tested under this condition). The coefficient for $X_v \cdot \mathbf{1}\{X_c = X_{high}\}$ is -0.279, with its 95% confidence interval being [-0.481, -0.078]; the coefficient for $X_v \cdot \mathbf{1}\{X_c = X_{mid}\}$ under this condition is -0.197, with its 95% confidence interval being [-0.400, 0.006]. The former is smaller than the latter and it is significantly smaller than zero at significance level 0.01. The latter is also smaller than zero, though the difference is not significant - one reason might be the sample size under this condition is the half as that under the "Immediate reward varies" condition. Thus, we can conclude that when X_c is at its highest level, one unit increase in the other reward amount, X_v, has a smaller impact on choice probability than when it is at the lowest level.

Model (2) penalizes the coefficients and standard errors towards zero. However, the coefficient estimates do not differ much from those we obtained in Model (1), and the main results are also the same as what we derived from Model (1). This suggests the separation problem may not have a great impact on the parameters of our interest.

5.2 Utility Models

For robustness check, we consider three issues. First, we consider the shape of utility function. Given that people are usually risk averse, when a reward amount is growing, people's sensitivity to it will be naturally diminishing. To capture this insight, we transform the reward amount into utility, using a power utility function $u(x) = x^\gamma/10$ $(0 < \gamma < 1)$. Using the risky choice questions in our survey, we estimate that γ has an average value of 0.749. We apply this average value of γ to every participant.

Second, as is shown in Figure 2, the single-reward option M also has an impact on the sensitivity of participants' choices to X_v. Thus, we set M as a category variable, and add the interaction between X_v and M into regression models. As M has two levels, the lower level of M is set as contrast group and the higher level is denoted by M_{high}.

Third, we consider removing the observations that all participants choose the same option. As the existence of these observations could amplify the magnitude of coefficient estimates, removing them from data can both mitigate the separation problem and improve the goodness of fit.

Table 2 and Table 3 present the robustness check results for the "Immediate reward varies" condition and "Delayed reward varies" condition respectively. Model (3) is the same as Model (1), except transforming the absolute amounts into utilities. Model (4) takes the same utility transformation, but also add the single-reward option M into an interaction term. Model (5) is the same as Model (4) in formula, but it removes the data that all participants choose the same option. We focus on the interactions between $u(X_v)$ and X_c, as well as between $u(X_v)$ and T. It can be shown that the results derived from Model (3)-(4) are consistent with Model (1)-(2).

Table 2: Regression with utility tranformation (Immediate reward varies)

	(3) Utility model		(4) Add interation		(5) Censored data	
	Coef	95% CI	Coef	95% CI	Coef	95% CI
$u(M)$	-5.395***	[-5.709, -5.081]				
$u(X_v)$	9.758***	[9.148, 10.368]	9.137***	[8.525, 9.750]	8.762***	[8.131, 9.394]
$\mathbf{1}\{X_c = X_{mid}\}$	8.144***	[6.682, 9.607]	7.669***	[6.226, 9.113]	7.185***	[5.705, 8.666]
$\mathbf{1}\{X_c = X_{high}\}$	14.603***	[13.177, 16.030]	13.962***	[12.552, 15.372]	13.360***	[11.914, 14.805]
$\mathbf{1}\{T = T_{mid}\}$	0.176	[-0.341, 0.694]	0.227	[-0.292, 0.746]	0.102	[-0.428, 0.632]
$\mathbf{1}\{T = T_{high}\}$	0.155	[-0.422, 0.732]	0.195	[-0.381, 0.770]	0.051	[-0.534, 0.636]
$u(X_v) \cdot \mathbf{1}\{X_c = X_{mid}\}$	-1.876***	[-2.508, -1.245]	-1.662***	[-2.288, -1.036]	-1.466***	[-2.112, -0.819]
$u(X_v) \cdot \mathbf{1}\{X_c = X_{high}\}$	-3.914***	[-4.518, -3.310]	-3.604***	[-4.205, -3.003]	-3.370***	[-3.992, -2.747]
$u(X_v) \cdot \mathbf{1}\{T = T_{mid}\}$	-1.061***	[-1.338, -0.785]	-1.109***	[-1.388, -0.831]	-1.029***	[-1.313, -0.746]
$u(X_v) \cdot \mathbf{1}\{T = T_{high}\}$	-1.808***	[-2.118, -1.498]	-1.870***	[-2.182, -1.559]	-1.772***	[-2.088, -1.455]
$\mathbf{1}\{M = M_{high}\}$			-4.421***	[-4.898, -3.944]	-4.233***	[-4.736, -3.729]
$u(X_v) \cdot \mathbf{1}\{M = M_{high}\}$			0.973***	[0.749, 1.198]	0.920***	[0.684, 1.155]
observations	18840		18840		14915	
AIC	7322.67		7250.14		7200.50	

Note: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. The p-values and confidence intervals (CI) are calculated via Wald test. The utility function is $u(x) = x^{0.746}/10$. The middle and highest levels of X_c are denoted by X_{mid}, X_{high}. The middle and highest levels of T are denoted by T_{mid}, T_{high}. The higher level of M is denoted by M_{high}. The intercept and individual-specific dummy variables are omitted in the table.

Table 3: Regression with utility tranformation (Delayed reward varies)

	(3) Utility model		(4) Add interation		(5) Censored data	
	Coef	95% CI	Coef	95% CI	Coef	95% CI
$u(M)$	-12.576***	[-13.508, -11.644]				
$u(X_v)$	13.088***	[12.112, 14.064]	11.896***	[10.904, 12.889]	11.792***	[10.792, 12.793]
$\mathbf{1}\{X_c = X_{mid}\}$	10.320***	[8.436, 12.204]	8.885***	[7.007, 10.764]	8.782***	[6.898, 10.666]
$\mathbf{1}\{X_c = X_{high}\}$	18.854***	[16.888, 20.820]	16.880***	[14.892, 18.869]	16.769***	[14.760, 18.778]
$u(X_v) \cdot \mathbf{1}\{X_c = X_{mid}\}$	-1.691***	[-2.466, -0.915]	-1.054**	[-1.839, -0.270]	-1.046**	[-1.831, -0.262]
$u(X_v) \cdot \mathbf{1}\{X_c = X_{high}\}$	-3.120***	[-3.875, -2.366]	-1.977***	[-2.789, -1.164]	-2.032***	[-2.853, -1.210]
$\mathbf{1}\{M = M_{high}\}$			-9.548***	[-10.708, -8.389]	-9.201***	[-10.388, -8.015]
$u(X_v) \cdot \mathbf{1}\{M = M_{high}\}$			1.825***	[1.330, 2.321]	1.710***	[1.205, 2.215]
observations	9420		9420		6594	
AIC	2202.54		2148.11		2137.28	

Note: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. The p-values and confidence intervals (CI) are calculated via Wald test. The utility function is $u(x) = x^{0.746}/10$. The middle and highest levels of X_c are denoted by X_{mid}, X_{high}. The higher level of M is denoted by M_{high}. The intercept and individual-specific dummy variables are omitted in the table.

In Table 2, the coefficients among Model (3)-(5) indicate that, under the "Immediate reward varies" condition, when $X_c = X_{high}$ or $X_c = X_{mid}$, one unit increase in $u(X_v)$ yields a significantly smaller increase in probability of choosing the sequence option than when X_c is at its lowest level. The upper bound of the 95% confidence interval for $u(X_v) \cdot \mathbf{1}\{X_c = X_{high}\}$ is even smaller than the lower bound of that for $u(X_v) \cdot \mathbf{1}\{X_c = X_{mid}\}$. Thus, the effect for $X_c = X_{high}$ is enhanced. This suggests, as X_c goes up, the changes in $u(X_v)$ have a smaller impact on the estimated choice probability, which is consistent with H1. The same rationale is also applicable to the time length T. So, as T goes up, the changes in $u(X_v)$ have a smaller impact on the estimated choice probability, which is consistent with H2.

In Table 3, the coefficients shows, under the "Delayed reward varies" condition, when $X_c = X_{high}$ or $X_c = X_{mid}$, one unit increase in $u(X_v)$ also yields a significantly smaller increase in choice probability for the sequence option than when X_c is at its lowest level, at significance level 0.001. This pattern is consistent with H1. Meanwhile, the coefficient for $u(X_v) \cdot \mathbf{1}\{X_c = X_{high}\}$ is also smaller than $u(X_v) \cdot \mathbf{1}\{X_c = X_{mid}\}$, though the difference is insignificant - the reason might be, while we only consider the utilities of X_v and M, the utility transformation for X_c also matters. Note that X_{high} and X_{mid} are £90 and £70 respectively, and the lowest level of X_c is £50. As the utility function is risk-averse, the difference between $u(90)$ and $u(70)$ should be much smaller than the difference between $u(70)$ and $u(50)$. As a result, increasing X_{mid} to X_{high} may not lead to a substantial change in terms of utility, so its impact on the coefficient for X_v is also not significant.

The utility transformation improves the goodness of fit for regression models. Model (1) has an AIC of 7401.59 for the "Immediate reward varies" condition and an AIC of 2203.39 for the "Delayed reward varies" condition. Compared to Model (1), each of Model (3)-(5) has a lower AIC for each condition. Among all of these models, Model (5) has the lowest AIC. Figure 3 compares the model predicted choice probabilities using Model (5) and the choice data.

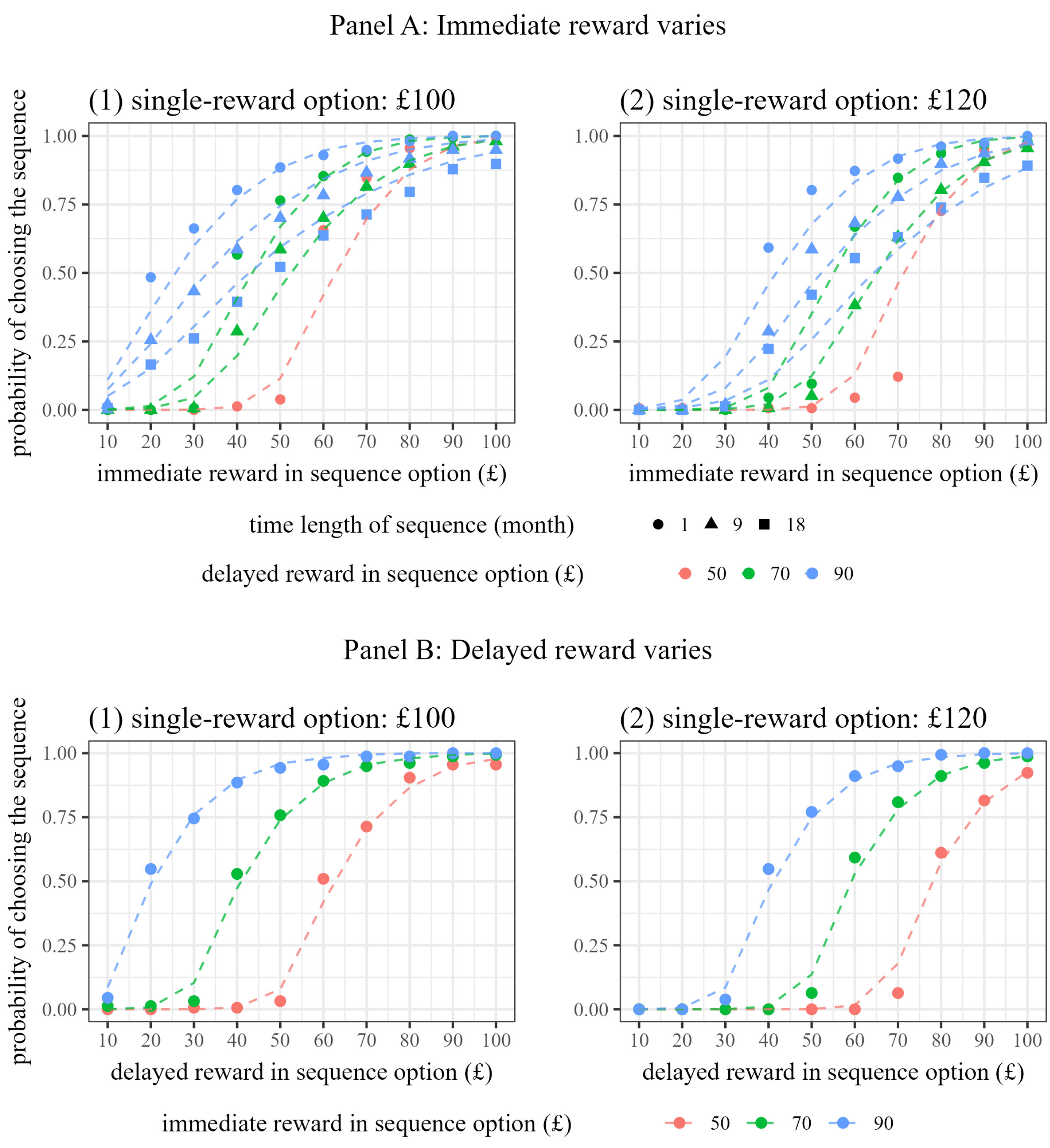

Figure 3: Data and model predicted choice probabilities.

Dots are the propotions of participants choosing the sequence option in the data, dashed curves are the mean predicted choice probabilities for sequence option. The curves are fitted by Model (5), a logit model on the censored data, with X_v being transformed to $u(X_v)$, M being added into interaction terms.

6 Alternative Mechanisms

Reference

Firth, D. (1993). Bias reduction of maximum likelihood estimates. *Biometrika*, 80(1):27–38.

Heinze, G. and Schemper, M. (2002). A solution to the problem of separation in logistic regression. *Statistics in medicine*, 21(16):2409–2419.