Attentional Discounted Utility

Zijian Zark Wang

July 11, 2023

1 Introduction

2 The Model

2.1 The Decision Process

Suppose time is discrete. Let X_T denote the sequence of rewards $[x_0, x_1, ..., x_T]$, which yields reward x_t in time period t.[1] The time length of this sequence, denoted by T, is finite. For any $t \in \{0, 1, ..., T\}$, the reward level x_t is a random variable defined on $\mathbb{R}_{\geq 0}$. I assume that making an intertemporal choice involves three steps:

Step 1. (*Sampling*) The decision maker subjectively draws a few potential realizations of X_T, and from each drawn realization, she draws a few time periods and observes their rewards; then, she combines all observed rewards into a sample.

Step 2. (*Valuation*) The decision maker uses the mean utility of sampled rewards as an approximate value representation of X_T.

Step 3. (*Choice-making*) She chooses the sequence with the highest value from all the available reward sequences.

[1] I use uppercase letters to represent a sequence and lowercase letters to represent elements within the sequence.

In the decision process described above, Step 3 is standard. By Step 1-2, I take the notion that, to evaluate a stimuli, the decision maker needs to assess all the relevant information, while her information processing capacity is limited. Consequently, she selectively attends to only *a subset* of the available information (which is termed *a sample*), then aggregates the attributes observed in the sample to calculate the stimuli value. This sampling process is not unbiased; on the contrary, the decision maker aims to retain more of the information that they consider more relevant in the sample. Such a notion has a long history in psychological research.[2] In recent years, many theories grounded in this (or similar notions) have made significant progress in explaining choice anomalies, such as decision field theory (Busemeyer and Townsend, 1993), decision-by-sampling (Stewart et al., 2006), utility-weighted sampling (Lieder et al., 2018) and efficient coding theory (Heng et al., 2020). In the next subsection, I describe the sampling and valuation process in detail.

2.2 Attention Mechanism and Optimal Discounting

Let $S_T = [s_0, s_1, ..., s_T]$ be a potential realization of X_T, and $\mathcal{S}(X_T)$ be the support of X_T, i.e. the smallest set containing any potentially realized sequence S_T, where $\mathcal{S}(X_T) \subseteq \mathbb{R}_{\geq 0}^{T+1}$. Let $w(s_t)$ be the probability that the reward of the t-th period in S_T is in the sample, $u(s_t)$ be the utility obtained by reward s_t ($t \in \{0, 1, ..., T\}$), where $u' > 0$, $u'' < 0$. The function $w(.)$ and $u(.)$ are termed as weight function and instantaneous utility function respectively. The approximate value of X_T, which I term by $U(X_T)$, is calculated by $U(X_T) = \sum_{S_T \in \mathcal{S}(X_T)} \sum_{t=0}^{T} w(s_t)u(s_t)$.

The sampling process is sequential. At the very beginning, the decision maker has no information about which period in a potentially realized sequence S_T has a larger reward. After each sampling, she acquires new information in this regard, and adjusts the sampling weights for different time periods based on such information. Sampling and processing information require mental effort from the decision maker, thus trigger a cost C at the cognitive level. I assume the decision maker's objective in this process is to find a sampling strategy, denoted by weight function $w(.)$, that can maximize the approximate value of the given reward se-

[2] Weber and Johnson (2009) and Chun et al. (2011) provide good reviews for such studies.

quence (hereafter referred as "overall utility") minus the cost of information, i.e. maximizing $U(X_T) - C$. As the decision maker pays a higher cost in sampling as well as processing information, these sampling weights will change in a way that increases the overall utility. Therefore, we consider the cognitive cost C as a functional of weight function $w(.)$.

The problem of determining the weight function under the above setting is termed as the constrained optimal discounting problem in Noor and Takeoka (2022; 2023). I follow their terminology.

Definition 1: Let W be the smallest set containing all possible weight functions. Given a stochastic reward sequence X_T, the following optimization problem is called the *constrained optimal discounting* problem for X_T:

$$\begin{aligned} \max_{w \in W} \quad & \sum_{S_T \in \mathcal{S}(X_T)} \sum_{t=0}^{T} w(s_t) u(s_t) - C(w) \\ s.t. \quad & \sum_{S_T \in \mathcal{S}} \sum_{t=0}^{T} w(s_t) = 1 \\ & w(s_t) \geq 0, \forall t \in \{0, 1, \ldots, T\} \end{aligned}$$

where $C : [0,1]^{T+1} \to \mathbb{R}_{>0}$ is called a *information cost* function, $\partial C / \partial w(s_t) > 0$ and $\partial^2 C / \partial w(s_t)^2 > 0$. That is, the cost of information is increasing and convex in $w(s_t)$.

The assumption that $C(.)$ is a convex function ensures the constrained optimal discounting problem has an interior solution. Notably, the objective function in Definition 1 implies that the decision maker tends to pay a cost to make any period or state with a larger reward be sampled more frequently. There are two reasons for using this objective function. First, there is substantial evidence indicating that people selectively attend to desirable information and avoid unpleasant information.[3] For instance, it is found investors are more likely to check their brokerage accounts when stock market goes up (Sicherman et al., 2016). In the intertemporal choice setting, this suggests that the periods or states with more desirable outcomes may receive more attention. Second, as is pointed out in Lieder et al. (2018), the sampling algorithm in the brain may share certain principles with optimal importance

[3] This includes a wide range of behavioral biases, e.g. ostrich effect, confirmation bias. For additional evidence, see Golman et al. (2017).

sampling. In importance sampling, to minimize the variance, the events with greater absolute utilities should be assigned more importance, i.e. being sampled with a higher probability. If the same principle is applicable to the brain, it also provides a rationale to this specific objective function.

After determining the weight function, the decision maker calculates the overall utility that she can obtain from a reward sequence. Let $p(S_T)$ be the probability of S_T being sampled in the brain. I define $w_t(S_T) = \frac{w(s_t)}{p(S_T)}$. Thus, the overall utility of X_T can be calculated using the expected discounted utility (EDU) framework, i.e. $U(X_T) = E_p[\sum_{t=0}^{T} w_t(S_t)u(s_t)]$. Under EDU framework, the $w_t(S_T)$ in braces is commonly referred to as the *discounting factor* for period t in S_T, a certain realization of sequence X_T. Given that attentional mechanism plays a prominent role in the valuation of X_T, I call any $U(X_T)$ calculated in this way the *attentional discounted utility*. When each alternative sequence has been valuated, I assume the decision maker will choose the sequence with highest attentional discounted utility (ADU). This is shown in Definition 2.

Definition 2: For any sequence of rewards X_T, $X'_{T'}$, preference relation $\succsim$ has an *attentional discounted utility* (ADU) representation if and only if

$$X_T \succsim X'_{T'} \Longleftrightarrow U(X_T) \geq U(X'_{T'})$$

and

$$U(X_T) = \sum_{S_T \in \mathcal{S}(X_T)} \sum_{t=0}^{T} w(s_t)u(s_t), \quad U(X'_{T'}) = \sum_{S_{T'} \in \mathcal{S}(X'_{T'})} \sum_{t=0}^{T'} w'(s_t)u(s_t)$$

where S_τ denotes $[s_0, s_1, \ldots, s_\tau]$, τ is the time length of S_τ, $w(.)$ and $w'(.)$ are the solutions to constrained optimal discounting problems for X_T and $X'_{T'}$.

2.3 ADU with Shannon Cost Function

Hereafter I focus on a well-known specification of information cost function, which I term Shannon cost function, proposed by Matějka and McKay (2015). The Shannon cost function was originally used to justify the multinominal logit model in discrete choice analysis, and so far has been topical in rational inattention literature. To construct this style of information cost function, Matějka and McKay (2015) introduce two assumptions. The first assumption is, before acquiring any information, the decision maker estiablishes an initial allocation of weights for different attributes, which remains invariant over states. The weights are then updated in a manner consistent with Bayes rule. Suppose the initial weight assigned to period t is d_t, then $d_t = \sum_{S_T \in \mathcal{S}(X_T)} w(s_t)$. The second assumption is, the cost of information is linear to the information gains, measured by Shannon mutual information. That is,

$$C(w) = \lambda \sum_{S_T \in \mathcal{S}(X_T)} \sum_{t=0}^{T} w(s_t) \log \left(\frac{w_t(S_T)}{d_t} \right)$$

where λ is a parameter denoting unit cost of information ($\lambda > 0$). With the Shannon cost function, the constrained optimal discounting problem can be easily solved by Lagrangian method.[4] In its solution, the discounting factor is calculated by

$$w_t(S_T) = \frac{d_t e^{u(s_t)/\lambda}}{\sum_{\tau=0}^{T} d_\tau e^{u(s_\tau)/\lambda}}$$

In this case, $w_t(S_T)$ follows a logistic-like distribution. It is increasing with $u(x_t)$, indicating that the decision maker tends to pay more attention to the periods with larger rewards; and is "anchored" in the initial weight d_t, inidicating the adjustment of attention allocation is costly. Meanwhile, note that the sum of $w_t(S_T)$ for any given S_T is fixed at 1, which implies the decision maker's capacity of information processing is limited. If Shannon cost function is used in determining weight function, I call the overall utility calculated in the subsequent step as *ADU with Shannon cost function* (hereafter referred to as ADUS).

In a static choice setting, the rationale for the usage of Shannon cost function can be interpreted with independence of irrelevant alternatives (Matějka and McKay, 2015), or data compression (Caplin et al., 2022). However, this is not applicable when weights can be allocated across time periods. Hence, I propose two axioms that can characterize Shannon cost function in the intertemporal choice setting.

Axiom 1: For any reward sequence X_T, X'_T, X''_T and real number $\alpha \in (0, 1)$, $X_T \succ X'_T$

[4] For how to solve this optimization problem, see Matějka and McKay (2015) or Maćkowiak et al. (2023).

implies $\alpha X_T + (1-\alpha)X''_T \succ \alpha X'_T + (1-\alpha)X''_T$.

Axiom 1 can be interpreted by Lemma 1.[5]

Lemma 1: (*state independence*) Suppose preference relation $\succsim$ has an ADU representation and satisfies Axiom 1. If $p_1 + p_2 + ...p_n = 1$, and for any $i \in \{1, 2, \ldots, n\}$, $0 < p_i < 1$, then for any deterministic reward sequence S_T^1, S_T^2, $\ldots$, S_T^n, we have

$$U(p_1 S_T^1 + p_2 S_T^2 + ... + p_n S_T^n) = p_1 U(S_T^1) + p_2 U(S_T^2) + ... + p_n U(S_T^n)$$

Lemma 1 implies that, the determination of discounting factors for one potential realization of a reward sequence will not interfere with that for another potential realization of it. Suppose S_T and S'_T are both potential realizations of a reward sequence X_T, one can solve $w_t(S_T)$ and $w_t(S'_T)$ by constructing a constrained optimal discounting problem for each then solving it independently.

Axiom 2: For any non-negative real number c and deterministic reward sequence S_T, let cS_T denote $[c, s_0, s_1, ..., s_T]$, then $c \sim S_T$ implies $S_T \sim cS_T$.

Axiom 2 can be interpreted by Lemma 2.

Lemma 2: (**spread-consistency correlation**) Suppose preference relation $\succsim$ has an ADU representation and satisfies Axiom 2. If there exist c and S_T such that, for any c' and S'_T, $cS_T \succsim c'S'_T$, where $c + \sum_{t=0}^{T} s_t = c' + \sum_{t=0}^{T} s'_t$, then for any S'_T, we have

$$S_T \succsim S'_T \iff c \sim S_T$$

where $\sum_{t=0}^{T} s_t = \sum_{t=0}^{T} s'_t$.

Lemma 2 implies that, when allocating a consumption budget across time periods, the decision maker keeps her choice dynamically consistent if and only if she performs a strong preference for spread. Given that people are typically assumed to be impatient (preferring a declining sequence), one intuitive interpretation of Lemma 2 is that the less impatient a

[5]The preference relation $x \succ y$ is defined by the opposite of $y \succsim x$; the preference relation $x \sim y$ is defined by the joint satisfaction of $x \succsim y$ and $y \succsim x$.

decision maker is in the present, the less inclined she is to deviate from the original choice in the future.

Proposition 1: $\succsim$ *has an ADUS representation if and only if it has an ADU representation and satisfies Axiom 1-2.*

3 Implications in Time Preferences

To illustrate how ADU with Shannon cost function can account for a broad set of anomalies about time preferences, imagine that a decision maker receives a positive detereminstic reward in period j (and no reward in other periods). That is, she receives a sequence of rewards $X_T = [x_0, x_1, \ldots, x_T]$, where $x_j > 0$ and is certain, and $x_t = 0$ for all $t \neq j$ (both j and t are in $\{0, 1, ..., T\}$).

For the convenience of illustration, I assume the decision maker holds stationary time preferences before acquiring any information, that is, $d_t = \delta^t$. Meanwhile, $\delta \in (0, 1]$, where $\delta = 1$ implies the initial attention is uniformly distributed across periods. For simplicity, I define $v(x_t) = u(x_t)/\lambda$, and set $v(0) = 0$. Let $w_t(X_T)$ denote the discounting factor for period t. From the formula of ADUS we can infer that

$$w_j(X_T) = \begin{cases} \delta^j \cdot \dfrac{1}{1 + \frac{\delta}{1-\delta}(1-\delta^T)e^{-v(x_j)}}\ , & 0 < \delta < 1 \\ \dfrac{1}{1 + T \cdot e^{-v(x_j)}}\ , & \delta = 1 \end{cases}$$

Clearly, w_j is decreasing in T. This offers an account for a phenomenon called *hidden zero effect*.

3.1 Hidden Zero Effect

The most direct evidence that could support the ADUS model is likely the hidden zero effect (Magen et al., 2008). The hidden zero effect means, supposing people face a small sooner reward (SS) and a large later reward (LL), they tend to exhibit more patience when SS and

LL are framed as sequences rather than being framed as single-period rewards. For instance, suppose SS is “receive £100 today” and LL is “receive £120 in 6 months”, and we have

SS_0: “receive £100 today and £0 in 6 months”

LL_0: “receive £0 today and £120 in 6 months”

people will be more likely to prefer LL_0 over SS_0 than preferring LL over SS. Subsequent research (e.g. Read et al. (2017)) suggests that the hidden zero effect is asymmetric. That is, shifting SS to SS_0 and keeping LL unchanged leads to an increase in patience, whereas shifting LL to LL_0 and keeping SS unchanged cannot increase patience. ADUS assumes that, within a sequence, attention is limited and the weight assigned to each period is anchored in an initial positive weight. These properties naturally explain the hidden zero effect. To illustrate, in SS, the decision maker perceives the length of sequence as “today” and allocate no attention to future. Whereas, in SS_0, she perceives the length as “6 months”. This makes some attention be paid to future periods with no reward, and decreases the attention paid to the only period with positive reward (given attention is limited); thus, the overall utility of sequence decreases. By contrast, shifting from LL to LL_0 does not change the length of sequence, thus does not change overall utility.

The existence of hidden zero effect also provides a hint in selection of time length T. When evaluating a reward delivered in period j, the range of T is $[j, +\infty)$. Any increase in T will reduce the overall utility. Thus, when comparing SS and LL, the decision maker may tend to set $T = j$ (the minimum length she can set), in order to maximize the overall utility. Any period out of this length can be perceived as irrelevant to the decision; so, she does not need to sample from the periods after j, when evaluating the given reward. Though, explicitly mentioning the periods after j will direct her attention to those periods, and lead to the hidden zero effect. By setting $T = j$, we have

$$w_T(x_T) = \frac{1}{1 + G(T)e^{-v(x_T)}}$$

where

$$G(T) = \begin{cases} \dfrac{1}{1-\delta}(\delta^{-T} - 1)\ , & 0 < \delta < 1 \\ T\ , & \delta = 1 \end{cases}$$

Given period T is now the only period with a non-zero reward within the sequence, I use x_T to directly represent the whole sequence, and let $w_T(x_T)$ denote the discounting factor for period T. Interestingly, when $\delta = 1$, $w_T(x_T)$ takes a form similar with hyperbolic discounting.

3.2 Common Difference Effect

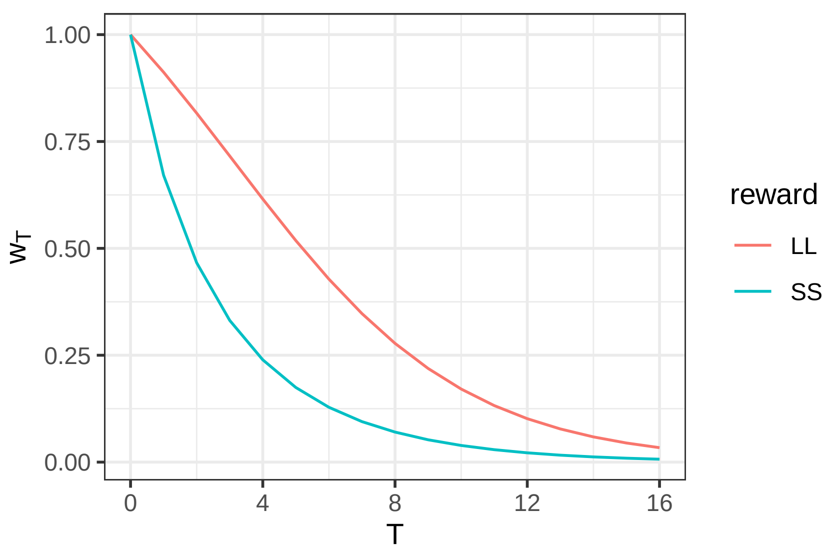
A well-known anomaly about time preferences is *common difference effect*, firstly defined by Loewenstein and Prelec (1992). Suppose there are a large later reward x_l arriving at period t_l (denoted by LL) and a small sooner reward x_s arriving at period t_s (denoted by SS), where $x_l > x_s > 0$, $t_l > t_s > 0$. Define $V(x,t) = w_t(x_t)v(x_t)$. The common difference effect means, supposing$V(x_l, t_l) = V(x_s, t_l)$, we must have $V(x_l, t_l + \Delta t) > V(x_s, t_s + \Delta t)$ for any positive integer Δt.

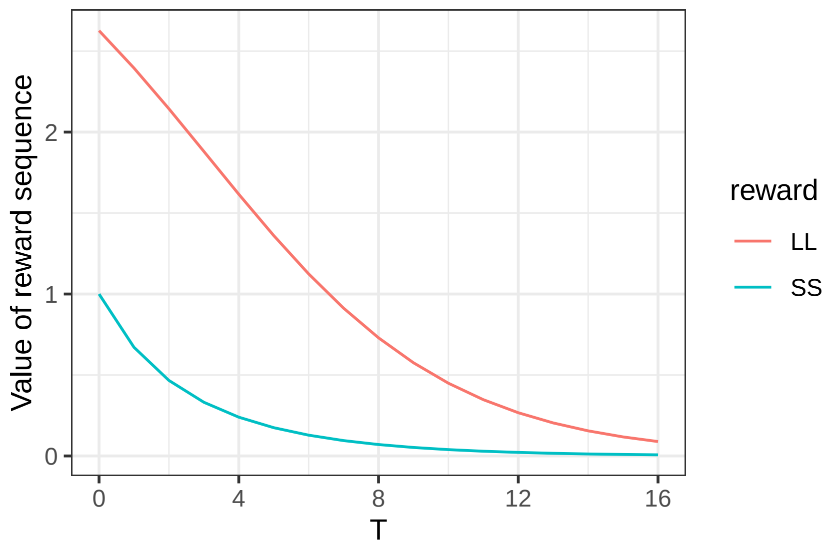
ADUS predicts that, if people are impatient, to observe the common difference effect, the difference between SS and LL in reward level must be set significantly larger than the difference in time delay. This is shown in Proposition 2.

Proposition 2: *In ADUS, if the initial weights are uniformly distributed, then the common difference effect always holds; if the initial weights exponentially declines over time, the common difference effect holds when* $v(x_l) - v(x_s) + \ln \frac{v(x_l)}{v(x_s)} > -(t_l - t_s) \ln \delta$.

Proposition 2 is interpreted as follows. When $\delta = 1$, ADUS predicts the decision maker always performs the common difference effect. This is obvious because discounting factor $w_T(x_T)$ takes a hyperbolic-like form. When $\delta < 1$, there are four factors jointly deciding whether we could observe the common difference effect or not. First, without considering attentional mechanism, when we extend time delay, each of $w_{t_l}(x_l)$ and $w_{t_s}(x_s)$, i.e. the discounting factor for (and attention paid to) the only period with positive reward, declines in an exponential fashion. Second, without considering newly added time interval, due to the decline of $w_{t_l}(x_l)$ and $w_{t_s}(x_s)$, the decision maker frees up some attention and can reallocate it across periods. Given that in LL, the decision maker has to wait longer for reward, the periods where she wait can grab more attention from the released capacity of attention, compared with those in SS. In other words, an extension of delay makes she focus more on the waiting time in LL than in SS, which decreases the preference for LL. Third, the newly added time interval also grabs some attention from other periods. Note the time delay is extended by $[t_l, t_l + \Delta t]$ in LL and by $[t_s, t_s + \Delta t]$ in SS; given $t_l > t_s$, if people are impatient, the newly added time interval will receive less attention in LL than in SS, without considering other factors. This increases the preference for LL. Fourth, ADU generally assumes that the decision maker tends to pay more attention to periods with larger rewards. Given $x_l > x_s$, the newly added interval grabs less attention from the period where x_l is positioned (in LL) than from the period where x_s is positioned (in SS). That is, the decision maker focuses comparatively more on reward level in LL than in SS, which mitigates the impact of discounting factor declining. This also increases the preference for LL. When the impact of the later two factors succeeds that of the second factor, the decision maker will perform the common difference effect.

Notably, if we explicit mention the zeros in LL and SS, extending time delay always lead to the common difference effect.





3.3 Magnitude Effect

The *magnitude effect* is another well-known anormaly about time preferences. Assuming we have t_l, t_s, x_s fixed, and want to find a x_l such that $V(x_l, t_l) \equiv V(x_s, t_s)$, the magnitude effect implies that, if we increase x_s, then the x_l/x_s that makes the equality valid will decrease.

In standard discounted utility model, the magnitude effect requires the elasticity of utility function to increase with the reward level (Loewenstein and Prelec, 1992). This requirement might be too restrictive, so that many commonly used utility functions (such as power or CARA utility function) does not satisfy it. By contrast, in ADU model, decision maker is generally assumed to attend more to periods with larger rewards. This implies that when comparing SS and LL, she exhibits more patience towards larger reward level, which is naturally compatible with the magnitude effect (Noor, 2011; Noor and Takeoka, 2022). By Proposition 3, I focus on ADU with Shannon cost function, and show how this requirement for curvature of utility function can be relaxed in this setting.

Proposition 3: *Define $v(x) \equiv u(x)/\lambda$ as the utility function. In ADUS, the magnitude effect always holds true when function $v(x)$ satisfies*

$$RRA_v(x) \leq 1 - \frac{e_v(x)}{v(x)+1}$$

where $RRA_v(x)$ is the relative risk aversion coefficient of $v(x)$, $e_v(x)$ is the elasticity of $v(x)$ to x.

Note that Proposition 3 is a very broad condition. In Corollary 1 and Corollary 2, I show that power utility function and CARA utility function both satisfy this condition in most cases.

Corollary 1: Suppose $v(x) = x^\gamma/\lambda$, where $0 < \gamma < 1$ and $\lambda > 0$. Then magnitude effect holds true for any $x \in \mathbb{R}_{>0}$.

Corollary 2: Suppose $v(x) = (1 - e^{-\gamma x})/\lambda$, where $\gamma > 0$ and $\lambda > 0$. The magnitude effect holds true for any $x \geq \frac{1+\eta}{\gamma}$, where $\eta > 0$ and $\eta e^{1+\eta} - \eta = 1$ (it can be calculated that $\eta \approx 0.35$).

3.4 Concavity of Time Discounting

Many time discounting models assumes discount function is convex in time delay, e.g. exponential and hyperbolic discounting. This style of discount function predicts decision maker is *risk seeking over time lotteries*. That is, suppose a deterministic reward of level x is delivered in period t_l with probability π and delivered in period t_s with probability $1 - \pi$ ($0 < \pi < 1$, $c > 0$); while another deterministic reward, of the same level, is delivered in a certain period t_m, where $t_m = \pi t_l + (1 - \pi)t_s$. The decision maker should prefer the former reward to the latter reward. However, some experimental studies, such as Onay and Öncüler (2007) and DeJarnette et al. (2020), suggest that people are often *risk averse over time lotteries*, i.e. preferring the reward delivered in a certain period.

One way to accommodate the evidence about risk aversion over time lotteries, as is suggested by DeJarnette et al. (2020), is to modify the convexity (concavity) of discount function. Under a general EDU framework, decision maker is risk averse over time lotteries when $\pi w_{t_l}(x) + (1 - \pi)w_{t_s}(x) < w_{t_m}(x)$. Fixing t_s and t_l, the inequality suggests $w_{t_m}(c)$ is concave in t_m. In reverse, being risk seeking over time lotteries suggests $w_{t_m}(x)$ is convex in t_m. Notably, Onay and Öncüler (2007) find that people are more likely to be risk averse over time lotteries when π is small, and to be risk seeking over time lotteries when π is large.

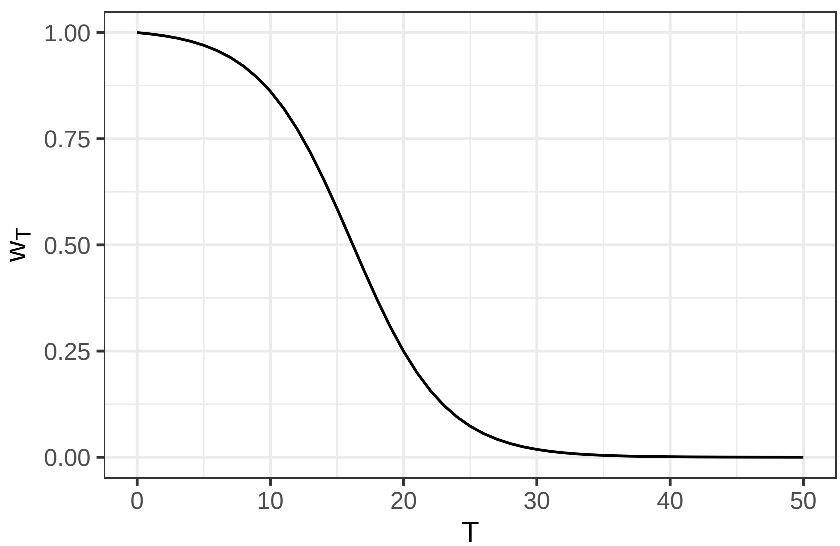
Given that when π gets larger, t_m is also larger, we can conclude that the discount function may be concave in delay for the near future but convex for the far future. Moreover, Takeuchi (2011) also find evidence that support this shape of discount function.

In Proposition 4, I show that ADUS can produce such a shape of discount function as long as the reward level x is large enough.

Proposition 4: In ADUS, *if* $\delta = 1$*, then the discount function is convex in* t*. If* $0 < \delta < 1$*, then there are a reward threshold* $\underline{x}$ *and a time threshold* $\underline{t}$ *such that*

1) *when* $x \leq \underline{x}$*, the discount function is convex in* t*;*
2) *when* $x > \underline{x}$*, the discount function is convex in* t *given* $t \geq \underline{t}$*, and it is concave in* t *given* $t < \underline{t}$*.*

It can be derived that $v(\underline{x}) = \ln(\frac{2}{1-\delta})$*, and* $\underline{t} = \frac{\ln[(1-\delta)e^{v(x)}-1]}{-\ln\delta}$.



3.5 S-Shaped Value Function

In prospect theory, Kahneman and Tversky (1979) propose an S-shaped value function that is convex for losses and concave for gains. Since that, S-shaped value functions have been widely embraced by behavioral economists. More recent theories have provided further justifications for it, including reference-dependent utility in a broad sense (Koszegi and Rabin, 2006), and efficient coding of values (Frydman and Jin, 2021). Here, I provide an account based on selective attention to time periods.

Suppose a decision maker is faced with a choice between a risky lottery and a fixed amount of money. When making this choice, she does not obtain any money from either option. Thus, she perceives the outcome of each option as something that will happen in the future. She allocate her attention between the present period and the period when she may receive the money. Assume that she perceives the outcome will be realized in period t, and in a certain state, the option she chooses yields reward x, then we can use the attentional discounted utility $V(x,t)$ to represent the value function. I derive the conditions in which ADUS can produce a S-shaped value function in Proposition 5.

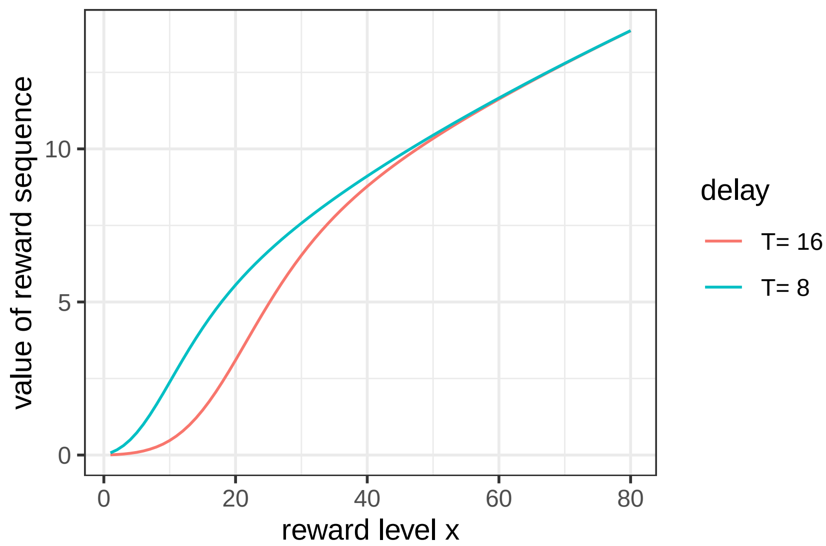
Proposition 5: *Suppose* $t \geq 1$, $\frac{d}{dx}\left(\frac{1}{v'(x)}\right)$ *is continuous in* $(0,+\infty)$, *in ADUS,*

1) *there exists a threshold* $\bar{x}$ *in* $(0,+\infty)$ *such that* $V(x,t)$ *is strictly concave in* x *when* $x \in [\bar{x},+\infty)$*;*

2) *if* $\frac{d}{dx}\left(\frac{1}{v'(x)}\right)$ *is right-continuous at* $x = 0$, *and* $\frac{d}{dx}\left(\frac{1}{v'(0)}\right) < 1$, *then there exists a threshold* x^* *in* $(0,\bar{x})$ *such that, for any* $x \in (0,x^*)$, $V(x,t)$ *is strictly convex in* x*;*

3) *there exist a hyper-parameter* λ^* *and an interval* (x_1,x_2) *such that, if* $\lambda < \lambda^*$, *for any* $x \in (x_1,x_2)$, $V(x,t)$ *is strictly convex in* x, *where* $\lambda^* > 0$ *and* $(x_1,x_2) \subset (0,\bar{x})$.

Proposition 5 implies, if the derivative of $\frac{1}{v'(x)}$ converges to a small number when $x \to 0^+$, or the unit cost of information λ is small enough, value function $V(x,t)$ will perform an S shape in some interval of x. At the intuition level, note that $V(x,t) = w_t(x)v(x)$. When the level of reward x grows, both the instantaneous utility of it, i.e. $v(x)$, and the discounting factor assigned to it, i.e. $w_t(x)$, can increase. These functions are both concave in x: when the level of reward is small, they both grow fast. So, it is possible that their product is convex in this case. By contrast, when the level of reward is large, they grow slowly, so their product keeps concave.



3.6 Inseparability of Sequences

Let x and y denote two 2-period risky reward sequences. For x, the realized sequence is [£100,£100] with probability 1/2, and is [£3,£3] with probability 1/2. For y, the realized sequence is [£3,£100] with probability 1/2, and is [£100,£3] with probability 1/2. Classical models of intertemporal choice typically assume the separability of potentially realized sequences. This implies that the decision maker is indifferent between x and y. However, Andersen et al. (2018) find evidence of *intertemporal correlation aversion*, that is, people often prefer y to x.

ADU can naturally yield intertemporal correlation aversion. For simplicity, suppose the initial attention is uniformly distributed across the two periods. For x, under each potentially realized sequence, the decision maker equally weights each period. For y, decision maker tends to assign more weight to the period with a reward of £100 (suppose that weight is w). Then the value of x is $\frac{1}{2}u(100) + \frac{1}{2}u(3)$ and the value of y is $w \cdot u(100) + (1 - w) \cdot u(3)$. Given that $x > \frac{1}{2}$, the decision makers should strictly prefer y to x.

- Other evidence related to inseparability: common sequence effect, (reverse) mere token effect, magnitude-increasing temporal sensitivity

4 The Role of Attention in Inconsistent Planning

5 Empirical Analysis

6 Discussion

7 Conclusion

Reference

Andersen, S., Harrison, G. W., Lau, M. I., and Rutström, E. E. (2018). Multiattribute Utility Theory, Intertemporal Utility, and Correlation Aversion. *International Economic Review*, 59(2):537–555.

Busemeyer, J. R. and Townsend, J. T. (1993). Decision field theory: A dynamic-cognitive approach to decision making in an uncertain environment. *Psychological Review*, 100(3):432–459.

Caplin, A., Dean, M., and Leahy, J. (2022). Rationally Inattentive Behavior: Characterizing and Generalizing Shannon Entropy. *Journal of Political Economy*, 130(6):1676–1715.

Chun, M. M., Golomb, J. D., and Turk-Browne, N. B. (2011). A Taxonomy of External and Internal Attention. *Annual Review of Psychology*, 62(1):73–101.

DeJarnette, P., Dillenberger, D., Gottlieb, D., and Ortoleva, P. (2020). Time Lotteries and Stochastic Impatience. *Econometrica*, 88(2):619–656.

Frydman, C. and Jin, L. J. (2021). Efficient Coding and Risky Choice. *The Quarterly Journal of Economics*, 137(1):161–213.

Golman, R., Hagmann, D., and Loewenstein, G. (2017). Information Avoidance. *Journal of Economic Literature*, 55(1):96–135.

Heng, J. A., Woodford, M., and Polania, R. (2020). Efficient sampling and noisy decisions. *eLife*, 9:e54962.

Kahneman, D. and Tversky, A. (1979). Prospect Theory: An Analysis of Decision under Risk. *Econometrica*, 47(2):263.

Koszegi, B. and Rabin, M. (2006). A Model of Reference-Dependent Preferences. *The Quarterly Journal of Economics*, 121(4):1133–1165.

Lieder, F., Griffiths, T. L., and Hsu, M. (2018). Overrepresentation of extreme events in decision making reflects rational use of cognitive resources. *Psychological Review*, 125(1):1–32.

Loewenstein, G. and Prelec, D. (1992). Anomalies in Intertemporal Choice: Evidence and an Interpretation. *The Quarterly Journal of Economics*, 107(2):573–597.

Magen, E., Dweck, C. S., and Gross, J. J. (2008). The Hidden-Zero Effect: Representing a Single Choice as an Extended Sequence Reduces Impulsive Choice. *Psychological Science*, 19(7):648–649.

Matějka, F. and McKay, A. (2015). Rational Inattention to Discrete Choices: A New Foundation for the Multinomial Logit Model. *American Economic Review*, 105(1):272–298.

Maćkowiak, B., Matějka, F., and Wiederholt, M. (2023). Rational Inattention: A Review. *Journal of Economic Literature*, 61(1):226–273.

Noor, J. (2011). Intertemporal choice and the magnitude effect. *Games and Economic Behavior*, 72(1):255–270.

Noor, J. and Takeoka, N. (2022). Optimal Discounting. *Econometrica*, 90(2):585–623.

Noor, J. and Takeoka, N. (2023). Constrained Optimal Discounting.

Onay, S. and Öncüler, A. (2007). Intertemporal choice under timing risk: An experimental approach. *Journal of Risk and Uncertainty*, 34(2):99–121.

Read, D., Olivola, C. Y., and Hardisty, D. J. (2017). The Value of Nothing: Asymmetric Attention to Opportunity Costs Drives Intertemporal Decision Making. *Management Science*, 63(12):4277–4297.

Sicherman, N., Loewenstein, G., Seppi, D. J., and Utkus, S. P. (2016). Financial Attention. *Review of Financial Studies*, 29(4):863–897.

Stewart, N., Chater, N., and Brown, G. D. (2006). Decision by sampling. *Cognitive Psychology*, 53(1):1–26.

Takeuchi, K. (2011). Non-parametric test of time consistency: Present bias and future bias. *Games and Economic Behavior*, 71(2):456–478.

Weber, E. U. and Johnson, E. J. (2009). Mindful Judgment and Decision Making. *Annual Review of Psychology*, 60(1):53–85.