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1 Introduction

In this document, I introduce three pieces of work.

The first is a model I term "attentional discounted utility". I leverage attentional mechanism to explain the anomalies in intertemporal choice. The model is developed based on expected discounted utility framework.

The second is an experimental test on whether (and how) attentional mechanism can influence intertemporal choice.

The third is an experiment aiming to test how people valuate risky outcomes and efforts. One prominent application of the findings from this experiment is that to help us validate a key hypothesis in rational inattention theories. I have designed the experimental program.

2 Attentional Discounted Utility

2.1 The Model

Expected discounted utility framework has been widely used in behavioral and economic research. Given a sequence of rewards $X_T = [x_0, x_1, ..., x_T]$, which yields reward x_t in time period t,[1] $t \in \{0, 1, ..., T\}$, the expected discounted utility of X_T can be calculated by

$$EDU(X_T) = \mathbb{E}\left[\sum_{t=0}^{T} d_t u(x_t)\right]$$

where d_t is the discounting factor of time period t, reward level x_t is a random variable defined on $\mathbb{R}_{\geq 0}$, $u(.)$ denotes the decision maker's instantaneous utility function, $u' > 0$, $u'' < 0$. The time length of this sequence, denoted by T, is finite.

The aim of this section is to incorporate attentional mechanism into this valuation process of reward sequences. I assume that a decision maker allocates attention across sequential outcomes when processing information about rewards, and attends more to the outcomes with larger rewards. I define the models that contain such features as *Attentional Discounted Utility* (ADU). Previous studies exploring the ADU setting include Gershman and Bhui (2020) and Noor and Takeoka (2022). Specifically, I focus on a particular form of ADU, which I term as *ADU with Shannon cost function* (ADUS). ADUS retains the architecture of expected discounted utility, but simply modifies the conventional discounting factor d_t to w_t, which I refer to as attention weight hereafter, where

$$w_t = \frac{d_t e^{u(x_t)/\lambda}}{\sum_{\tau=0}^{T} d_\tau e^{u(x_\tau)/\lambda}} \tag{1}$$

In Equation (1), w_t adopts a form resembling logistic function. It indicates three properties of attention. First, the sum of all w_t equals 1, indicating the attention that is available to be allocated across sequential outcomes is limited. Second, w_t is increasing in x_t, indicating the decision maker is inclined to allocate more attention to periods with larger potential rewards. This is in line with existing evidence, suggesting people often pay much attention to pleasant information and avoid to know unpleasant information (e.g. ostrich effect). Third, it appears that w_t is anchored in d_t. This indicates that the conventional discounting factor d_t can represent the initial attention weight assigned to each period, and the psychological process of attention adjustment (converting d_t to w_t) is costly. The parameter λ quantifies the cost

[1] I use uppercase letters to represent a sequence and lowercase letters to represent each element within the sequence.

of attention adjustment. Notably, w_t is positive for every period t within the sequence.

I provide two rationales for Equation (1). The first is based on rational inattention theories. The second is an axiomatic theory. I present the first here and discuss the second in Subsection 2.3.

The necessary notations and definitions are described as follows. Let $S_T = [s_0, s_1, ..., s_T]$ be a potential realization of X_T, and $\mathcal{S}(X_T)$ be the support of X_T, i.e. the smallest set containing any potentially realized sequence S_T, where $\mathcal{S}(X_T) \subseteq \mathbb{R}^{T+1}_{\geq 0}$. In each S_T, the attention weight assigned to period t is decided by a weight function $w(s_t)$. I follow Noor and Takeoka (2022; 2023) to define the process that generates $w(s_t)$ as *constrained optimal discounting*. The key assumption of constrained optimal discounting process is that, when evaluating a reward sequence, the decision maker wants to find an allocation policy for attention weights, in order to maximize the anticipatory utility that she can obtain from the sequence. Nonetheless, this attention re-allocation process triggers a cognitive cost. She needs to balance the benefit of focusing on periods containing larger rewards and the cognitive cost of shifting attention. The formal definition is given by Definition 1.

Definition 1: Let W be the support of all possible weight functions. Given a stochastic reward sequence X_T, the following optimization problem is called the *constrained optimal discounting* problem for X_T:

$$
\begin{aligned}
\max_{w \in W} \quad & \sum_{S_T \in \mathcal{S}(X_T)} \sum_{t=0}^{T} w(s_t) u(s_t) - C(w) \\
s.t. \quad & \sum_{S_T \in \mathcal{S}} \sum_{t=0}^{T} w(s_t) = m \\
& w(s_t) > 0, \forall t \in \{0, 1, \ldots, T\}
\end{aligned}
$$

where m is a constant, $C : [0, m]^{T+1} \to \mathbb{R}_{>0}$ is called *information cost* function, $\partial C / \partial w(s_t) > 0$ and $\partial^2 C / \partial w(s_t)^2 > 0$. That is, the information cost is increasing and convex in $w(s_t)$.

In Definition 1, the cognitive cost of shifting attention is decided by an information cost function $C(.)$. This implies that the cognitive cost is generated by focusing on certain reward information. A well-known specification of $C(.)$ is the Shannon cost function, proposed by

Matějka and McKay (2015). The Shannon cost function was originally used to justify the multinominal logit model in discrete choice analysis, and so far has been topical in rational inattention literature. To construct this style of information cost function, Matějka and McKay (2015) introduce three assumptions. The first is that the sum of all weights is 1, i.e. $m \equiv 1$. The second is, before acquiring any information, the decision maker establishes an initial weight allocation for different attributes (outcomes), which remains invariant across states. The weights are then updated in a manner consistent with Bayes rule. In ADU setting, it means $d_t = \sum_{S_T \in \mathcal{S}(X_T)} w(s_t)$. The third assumption is, the information cost is linear to the information gains, measured by Shannon mutual information. That is,

$$C(w) = \lambda \sum_{S_T \in \mathcal{S}(X_T)} \sum_{t=0}^{T} w(s_t) \log \left(\frac{w(s_t)}{d_t p(S_T)} \right)$$

where λ is a parameter denoting unit cost of information ($\lambda > 0$), $p(S_T)$ is the probability that S_T occurs. With Shannon cost function, the constrained optimal discounting problem can be easily solved by Lagrangian method, and the solution is the same as Equation (1). [2]

2.2 Implications in Intertemporal Choice

2.2.1 ADU is generally consistent with "hidden zero effect"

Suppose a decision maker faces a choice between a small sooner reward (SS) and a larger later reward (LL). The hidden zero effect (Magen et al., 2008) implies she should exhibit more patience if both SS and LL are framed as sequences rather than being framed as single-period rewards. For instance, consider SS_0: "receive £100 today", and LL_0: "receive £120 in 6 months". Suppose we also have

SS_1: "receive £100 today and £0 in 6 months"

LL_1: "receive £0 today and £120 in 6 months"

The hidden zero effect suggests people may be more likely to prefer LL_1 over SS_1 than preferring LL_0 over SS_0. Subsequent research (e.g. Read et al. (2017)) suggests the effect is

[2] In each certain state S_T, w_t in Equation (1) can be defined by $w_t \equiv w(s_t)/p(S_T)$.

asymmetric. That is, shifting SS_0 to SS_1 and keeping LL_0 unchanged do increase patience, whereas shifting LL_0 to LL_1 and keeping SS_0 unchanged cannot increase patience. This is naturally compatible with attentional discounted utility.

To illustrate, imagine that the framing of sequence length can influence how the decision maker may perceive it. For SS_0, the decision maker perceives the length of sequence as "today"; for SS_1, she perceives the length as "6 months". In the former case, she can allocate no attention to the future; while in the latter case, she needs to pay some attention to future periods, which contain no reward. Thus, the attention allocated to the current period decreases when shifting from SS_0 to SS_1 (given total amount of attention is limited). This leads to a decrease in the value of sequence. By contrast, shifting from LL_0 to LL_1 does not change the length of sequence, thus does not change the value.

One might wonder for a period with no reward, why the attention weight may still be positive ("why should I pay attention to a period when there is no reward present"). The hidden zero effect provides an empirical reason. Additionally, from the theoretical view, attention acts as a filter that determines which information could enter awareness or working memory.[3] If the decision maker is aware about the information that the reward for a certain period is zero, it implies attention may have already been directed to that period.

Hereafter, I focus on ADU with Shannon cost function (ADUS) and document five implications of the ADUS model in intertemporal choice. Each of them can be precisely proved.

2.2.2 The requirement of magnitude effect on utility function can be relaxed under ADUS

In conventional discounted utility models, to make the magnitude effect holds true, the elasticity of utility function needs to be increasing with reward level (Loewenstein and Prelec, 1992). This requirement might be too strong so that many commonly used utility functions (such as power and CARA utility functions) does not satisfy it. By contrast, note in Equation (1), w_t is increasing in x_t. This implies that when comparing SS and LL, the decision maker

[3] Such a view can be traced back to the bottleneck theories of attention, which starts from Broadbent (1958).

could exhibit more patience toward larger reward levels, which relaxes the requirement for the magnitude effect (Noor, 2011).

To illustrate how specifically ADUS relaxes this requirement, we should derive how a delayed reward is valued in ADUS. Suppose the value of "receive a certain reward x_T in period T" is calculated by $w_T \cdot u(x_T)$. Then assuming the initial attention weight is in exponential style, that is $d_t \equiv \delta^t$ $(0 < \delta \leq 1)$, we should have

$$w_T = \frac{1}{1 + G(T)e^{-u(x_T)/\lambda}} \tag{2}$$

and

$$G(T) = \begin{cases} \dfrac{1}{1-\delta}(\delta^{-T} - 1)\ , & 0 < \delta < 1 \\ T\ , & \delta = 1 \end{cases} \tag{3}$$

It can be proved that, if Equation (2) and (3) are both satisfied, for all $x_T > 0$, power utility function $u(x_T) = {x_T}^{\gamma}$ with $0 < \gamma < 1$, satisfies the curvature requirement for magnitude effect, and when x_T is greater than a positive finite number, CRRA utility function $u(x_T) = 1 - e^{-\gamma x_T}$ with $\gamma > 0$, also satisfies it.

2.2.3 ADUS specifies a novel condition for common difference effect

According to the common difference effect (Loewenstein and Prelec, 1992), suppose a decision maker is indifferent between SS and LL, increasing the time delay before receiving the reward in both options by a same amount can lead the decision maker to prefer LL over SS.

The ADU models provide an alternative account for common difference effect. Given attention is limited, when the delay of reward is extended, that is, we insert some periods with zero reward into the existing sequence, these newly inserted periods will grab attention from other periods. As people tend to attend more to periods with larger rewards, the period when the reward is delivered in LL will experience less attention being diverted compared to that in SS. This disparity in attention weighing can lead to a preference for LL over SS.

Regarding Shannon cost function, assuming initial time preference is stationary, i.e. $d_t = \delta^t$, we can derive from Equation (2) and (3) the conditions for common difference effect. Clearly, if $\delta = 1$, the attention weight w_T will follow a hyperbolic style. In this case, the common difference effect will always hold true.

If $0 < \delta < 1$, the ADUS specifies a novel condition for common difference effect. To illustrate, suppose SS is "receive a certain reward x_s in period t_s" and LL is "receive a certain reward x_l in period t_l", where $x_l > x_s > 0$, $t_l > t_s > 0$. To make the common difference effect hold true, we need $v(x_l) - v(x_s) + \ln \frac{v(x_l)}{v(x_s)} > -(t_l - t_s) \ln \delta$. That is, when the decision maker is impatient, she performs the common difference effect if and only if *the difference between SS and LL in reward level is significantly larger than that in time delay.*

2.2.4 Discount function may be concave for the near future and convex for the far future

Most models of time discounting (e.g. hyperbolic and quasi-hyperbolic discounting) assume a convex discount function. However, some studies suggest the discount function may be concave for the near future and convex for the far future (Onay and Öncüler, 2007; Takeuchi, 2011; DeJarnette et al., 2020).

In Equation (2), w_T produces this style of discount function when x_T is greater than a positive finite number.

2.2.5 ADUS offers an alternative account for S-shaped value function

The S-shaped value function has been widely adopted by behavioral economists. Some theories justifies it by reference-dependent utility (Kahneman and Tversky, 1979; Koszegi and Rabin, 2006), while some others by efficient coding of numerical values (Frydman and Jin, 2021). I offer an account based on selective attention to sequential outcomes.

Suppose a decision maker faces a choice between a risky lottery and a fixed amount of money. At the period when she makes this choice, she does not obtain any money from either option. Thus, she perceives the outcome of each option as something that will happen in the future.

She allocates attention between the current period which has no reward and the period when she may receive the money. Assume that she perceives the outcome will be realized in period T, and in a certain state, the option she chooses yields reward x_T, then we can use Equation (2) to derive the value function, i.e. $w_T \cdot u(x_T)$. I find that when the curvature of utility function satisfies a certain condition, or the unit cost of information λ is small enough, we can obverse a S-shaped value function.

2.2.6 Inattentive decision makers perform less inconsistent behaviors

Imagine that a decision maker faces a task of allocating consumption budget B across multiple periods, and in each period, she makes a new consumption plan to optimize her overall attentional discounted utility. Suppose she starts in period 0, then the task can be formulated as the following optimization problem:

$$\max_{c_0,\dots,c_T} \sum_{t=0}^{T} w_t \cdot u(c_t) \qquad s.t. \sum_{t=0}^{T} c_t = B$$

where c_t is consumption in period t, w_t is the attention weight, which is calculated with ADUS. I develop a numerical method to solve this optimization problem.

Previously, researchers often employ "present bias" (Laibson, 1997) or "naiveté" (O'Donoghue and Rabin, 1999) to account for the phenomenon of dynamic inconsistency in this context. I provide an account based on attention weight updating.

I categorize each decision maker into two types based on how they process information in this task - *attentive* and *inattentive*. A decision maker is considered *attentive* to the decision task if, in each period, she remembers the information, including anticipatory utility, acquired in the last period and use it to update attention weights. Otherwise, she is classified as *inattentive*.

Suppose in period 0, the decision maker's initial attention weight is proportional to δ^t, where $0 < \delta < 1$. Given the decision maker is impatient ($\delta < 1$), she tends to consume more in earlier periods within the planning horizon. Then in period 1, an *attentive* decision maker may update her initial attention weight into w_t formed in period 0, which is proportional to $\delta^t \exp\{\frac{u(c_t)}{\lambda}\}$, and so on. Consequently, the earlier periods, which originally has more planned consumption, will now receive more (initial) attention weights than they did in the last period. This leads to an increased preference for raising consumption levels in such periods.

By contrast, if the decision maker is *inattentive*, that is, she does not mind much about the task and always forgets the information acquired in the last period, then in period 1 and any subsequent period, her initial attention weight will keep proportional to δ^t. Thus, she could perform less inconsistent behaviors than the attentive decision makers. This contrast may shed light on why some nudges that reduce the demand for attention commitment, such as a default choice, can improve the consistency in choices.

2.3 Axiomization of ADUS

In this subsection, I provide an axiomatic rationale for using Shannon cost function in ADU. I firstly define the preference relation $\succsim$ that can be represented by ADU, then propose four axioms to characterize the Shannon cost function.

Definition 2: Preference relation $\succsim$ has an ADU representation if and only if, for any stochastic reward sequence X_T, $X'_{T'}$, we have

$$X_T \succsim X'_{T'} \Longleftrightarrow U(X_T) \geq U(X'_{T'})$$

where $U(X_T) = \sum_{S_T \in \mathcal{S}(X_T)} \sum_{t=0}^{T} w(s_t)u(s_t)$, $U(X'_{T'}) = \sum_{S_{T'} \in \mathcal{S}(X'_{T'})} \sum_{t=0}^{T'} w'(s_t)u(s_t)$. S_T and $S'_{T'}$ are the potential realizations of sequence X_T and $X'_{T'}$, $w(.)$ and $w'(.)$ are the solutions to the constrained optimal discounting problems for X_T and $X'_{T'}$.

Axiom 1: (*state independence*) For any reward sequence X_T, X'_T, X''_T and $\alpha \in (0,1)$, $X_T \succ X'_T$ implies $\alpha X_T + (1-\alpha)X''_T \succ \alpha X'_T + (1-\alpha)X''_T$.

Axiom 1 implies that the determination of attention weights in one potential realization of reward sequence will not interfere that in another. If Axiom 1 is satisfied, each state in the constrained optimal discounting problem has an independent solution.

Axiom 2: *(sequential outcome betweenness)* For any non-negative real number b and deterministic reward sequence S_T, let $S_T b$ denote $[s_0, s_1, ..., s_T, b]$, there always exists $\alpha \in (0,1)$ such that $S_T b \sim \alpha S_T + (1-\alpha)b$.

Axiom 2 implies that if we add a new element to a given sequence, the value of the new sequence will lies between the value of the original sequence and the utility of the newly added element. The evidence of "violation of dominance" (Scholten and Read, 2014) may provide support for this axiom.

Axiom 3: (*sequential bracket independence*) For any non-negative real number b, c and deterministic sequence S_T, if there exist α_1, α_2, α_3, β_1, $\beta_2 \in \mathbb{R}_{>0}$ such that

$$S_T bc \sim \alpha_1 S_T + \alpha_2 b + \alpha_3 c \quad \text{and} \quad S_T bc \sim \beta_1 S_T + \beta_2 (bc)$$

where bc denotes a sequence with immediate reward b and period-1 reward c, then we must have $\alpha_1 = \beta_1$.

Axiom 3 implies that if we segment a given sequence into different elements, and find that the value of a linear combination of these elements is equivalent to the value of the overall sequence, then in this linear combination, the weight for any specific element can hold constant no matter how we segment or bracket the other elements.

Axiom 4: (*aggregate invariance of constant sequences*) For any deterministic sequences S_T, S'_T, given non-negative real number c', c and $\alpha \in (0,1)$, if $\alpha s'_t + (1-\alpha)c' \succ \alpha s_t + (1-\alpha)c$ holds for every period t, then $\alpha S'_T + (1-\alpha)c' \succ \alpha S_T + (1-\alpha)c$.

Axiom 4 implies that in a given sequence, if the utility of every element plus an equal amount, then the overall utility of the sequence should plus the same amount. Note in conventional discounted utility models, sequences are typically assumed to be separable from each other. In that case, any sequence can be *aggregate invariant*. Thus, Axiom 4 can be viewed as a weaker version of "separability of sequences".

Theorem 1: $\succsim$ has an ADUS representation if and only if it has an ADU representation and satisfies Axiom 1-4.

2.4 Empirical Analysis

3 Experimental Tests of Attention over Sequential Outcomes

4 Valuation of Risk and Effort

Time Schedule

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