Supplementary Material for Annual Review 22/23

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August 14, 2023

1 Introduction

In this document, I present three pieces of work.

The first is a novel model of intertemporal choice, which I term as "attentional discounted utility" (ADU). The model incorporates attentional mechanism into the expected discounted utility framework. It assumes that people have limited attention to allocate across sequential outcomes, and tend to pay more attention to the outcomes with larger rewards. The model is consistent with a known empirical phenomenon in intertemporal choice termed the "hidden zero effect". I focus on a certain variant of ADU, that is, ADU with the Shannon cost function (ADUS), and show how this can explain a broad set of decision anomalies.

The model was presented in the most recent annual review, and since then, has undergone further clarification and refinement. Specifically, I outline the following behavioral implications of ADUS: (1) the requirement for the magnitude effect on curvature of utility function can be relaxed; (2) it specifies a novel condition for the common difference effect; (3) the discount function can perform an inverse-S shape, which is consistent with some evidence; (4) it offers an alternative account for S-shaped value function; (5) whether decision makers are attentive or inattentive to decision task influences the consistency of their choices. Moreover, I present an axiomatic characterization of ADUS.

The second is an experimental test on whether (and how) attentional mechanism can influence intertemporal choice. It is a new section, added after the most recent annual review. I conducted two pilot studies and am working on pre-registration at the moment of submitting this document. The purpose of this experiment is to examine the key assumptions in my model. In this experiment, participants are required to choose between different sequences of rewards. I make two predictions for the experimental results. First, extending the sequence length will dilute the attention resources available for each specific period (given people have limited attention to allocate across the periods), which will make people less sensitive to changes in the amount of reward offered in each period. Second, increasing reward amount in one period will shift attention from other periods to this particular period; thus, people will be less sensitive to changes in reward amount in the other periods. Similar behavioral patterns were observed in a pilot study.

The third is an experiment aiming to test how people jointly evaluate risky outcomes and efforts. This experimental design originates from an idea I proposed two years ago. Findings from this experiment may help clarify some issues in rational inattention theories. I am working on the experimental program.

These three pieces of work may correspond to three chapters of my thesis. Each piece of work is presented in Section 2, Section 3, and Section 4 respectively. During the upcoming two months, I hope to complete the main analyses in the first piece of work. The experiment in the second piece of work will be implemented in one month; then, I plan to run a follow-up test in the subsequent two months. The experiment in the third piece of work may be implemented by the end of this year. Hopefully, I will be working on additional analyses early next year, then complete the thesis in July, 2024. A full schedule of work is presented at the end of the document.

2 Attentional Discounted Utility

2.1 The Model

Expected discounted utility framework has been widely used in behavioral and economic research. Given a sequence of rewards $X_T = [x_0, x_1, ..., x_T]$, which yields reward x_t in time period t,[1] $t \in \{0, 1, ..., T\}$, the expected discounted utility of X_T can be calculated by

$$EDU(X_T) = \mathbb{E}\left[\sum_{t=0}^{T} d_t u(x_t)\right]$$

where d_t is the discounting factor of time period t, reward level x_t is a random variable defined on $\mathbb{R}_{\geq 0}$, $u(.)$ denotes the decision maker's instantaneous utility function, $u' > 0$, $u'' < 0$. The time length of this sequence, denoted by T, is finite.

The aim of this section is to incorporate attentional mechanism into this valuation process of reward sequences. I assume that a decision maker allocates attention across sequential outcomes when processing information about rewards, and attends more to the outcomes with larger rewards. I define the models that contain such features as *Attentional Discounted Utility* (ADU) models. Specifically, I focus on a particular form of ADU, which I term as *ADU with the Shannon cost function* (ADUS). ADUS retains the architecture of expected discounted utility, but simply modifies the conventional discounting factor d_t to w_t, which I refer to as an attention weight hereafter, where w_t is calculated by

$$w_t = \frac{d_t e^{\frac{u(x_t)}{\lambda}}}{\sum_{\tau=0}^{T} d_\tau e^{\frac{u(x_\tau)}{\lambda}}} \tag{1}$$

In Equation (1), w_t adopts a form resembling a logistic function. It indicates three properties of attention. First, the sum of all w_t equals 1. Under this constraint, an increase in attention weight assigned to one period will result in a reduction in that for some other periods, which indicates a shift of attention. Second, w_t is increasing in x_t, indicating the decision maker is inclined to allocate more attention to periods with larger potential rewards. This is in line with existing evidence, suggesting people often pay more attention to pleasant information and less to unpleasant information.[2] Third, w_t is anchored in d_t. This indicates that the conventional discounting factor d_t can represent the initial attention weight assigned to each

[1] I use uppercase letters to represent a sequence and lowercase letters to represent each element within the sequence.

[2] Many studies have suggested information has hedonic value. Golman et al. (2017) provides a good review for the relevant literature. The tendency to avoid paying attention to unpleasant information is sometimes called ostrich effect. Galai and Sade (2006), Sicherman et al. (2016) and Quispe-Torreblanca et al. (2022) find the evidence of ostrich effect in financial markets. Moreover, Olafsson and Pagel (2017) find people are more likely to log in their bank accounts when they have more cash holdings and less debt in the accounts.

period, and the psychological process of attention adjustment (converting d_t to w_t) is costly. The parameter λ quantifies the cost of attention adjustment. Notably, w_t is positive for every period t within the sequence.

I provide two rationales for Equation (1). The first is based on rational inattention theories. The second is an axiomatic theory. I present the first here and discuss the second in Section 2.4.

The rational inattention theories investigate how the cost or capacity of acquiring information may impact relevant decision-making processes. In ADU, it is assumed that when evaluating a reward sequence, the decision makers tends to pay more attention to information with a higher hedonic value, and the information that she could receive a substantially large reward at some time is perceived as more (hedoically) valuable than the information that she could receive only a small reward at the same time. Consequently, as more information about the sequence is acquired, her allocation of attention weights may deviate more from its initial allocation. Notably, this attention adjustment process triggers a cognitive cost. The decision maker needs to balance the benefit of focusing on the information with higher value and the cognitive cost of shifting attention. Her objective thus can be viewed as to maximize an attention-weighted value of information at some cost.

The necessary notations and definitions are described as follows. Let $S_T = [s_0, s_1, ..., s_T]$ be a potential realization of X_T, and $\mathcal{S}(X_T)$ be the support of X_T, i.e. the smallest set containing any potentially realized sequence S_T, where $\mathcal{S}(X_T) \subseteq \mathbb{R}^{T+1}_{\geq 0}$. Let $w(s_t)$ denote the attention weight assigned to outcome s_t, and $\mathcal{W}$ denote the matrix consisting of all $w(s_t)$, i.e. the attention weights for all periods in all potential realizations of X_T. For simplicity, I assume the value of each piece of information is proportional to the instantaneous utility of a reward in each period. Under this assumption, we can directly use the *constrained optimal discounting* problem, termed by Noor and Takeoka (2022; 2023), to define the process that generates $\mathcal{W}$. The formal definition is given by Definition 1.

Definition 1: Let W be the set containing all possible realizations of $\mathcal{W}$. Given a stochastic reward sequence X_T, the following optimization problem is called the *constrained optimal* *discounting* problem for X_T:

$$\begin{aligned} \max_{\mathcal{W}\in W} \quad & \sum_{S_T\in\mathcal{S}(X_T)} \sum_{t=0}^{T} w(s_t)u(s_t) - C(\mathcal{W}) \\ s.t. \quad & \sum_{S_T\in\mathcal{S}} \sum_{t=0}^{T} w(s_t) = m \\ & w(s_t) > 0 \text{ for all } S_T \text{ and } t \in \{0,1,\dots,T\} \end{aligned}$$

where m is a constant, $C : [0,m]^{T+1} \to \mathbb{R}_{>0}$ is called *information cost* function, $\partial C/\partial w(s_t) > 0$ and $\partial^2 C/\partial w(s_t)^2 > 0$. That is, the information cost is increasing and convex in $w(s_t)$.

The process in Definition 1 determines the attention weight assigned to each potential outcome in each time period. After that, the decision maker' objective becomes to choose the option with the highest ADU, where ADU is calculated by $\sum_{S_T\in\mathcal{S}}\sum_{t=0}^{T} w(s_t)u(s_t)$.

A well-known specification of $C(.)$ is the Shannon cost function, proposed by Matějka and McKay (2015). The Shannon cost function was originally used to justify the multinominal logit model in discrete choice analysis, and so far has been widely discussed in rational inattention literature. To construct this style of information cost function, Matějka and McKay (2015) introduce three assumptions. The first is that the sum of all weights is 1, i.e. $m \equiv 1$. The second is, before acquiring any information, the decision maker establishes an initial weight allocation for different attributes (time periods), which remains invariant across states. The weights are then updated in a manner consistent with Bayes' rule. In ADU setting, it means $d_t = \sum_{S_T\in\mathcal{S}(X_T)} w(s_t)$. The third assumption is, the information cost is linear to the information gains, measured by Shannon mutual information. That is,

$$C(w) = \lambda \sum_{S_T\in\mathcal{S}(X_T)} \sum_{t=0}^{T} w(s_t) \log\left(\frac{w(s_t)}{d_t p(S_T)}\right)$$

where λ is a parameter denoting unit cost of information ($\lambda > 0$), $p(S_T)$ is the probability that S_T occurs. With the Shannon cost function, the constrained optimal discounting problem can be easily solved by Lagrangian method, and the solution is the same as Equation (1). [3]

[3] In each certain state S_T, w_t in Equation (1) can be defined by $w_t \equiv w(s_t)/p(S_T)$.

2.2 Related Literature

The present model is related to two kinds of models - endogenous time preference and rational inattention. In a seminal paper about endogenous time preference, Becker and Mulligan (1997) assume the decision maker can increase the discounting factor for a future period by spending resources in imagining or focusing on it. Such resources enters the budget constraint, hence reduces the total amount of rewards that the decision maker can obtain. On the contrary, the present model does not make any specific assumption about budget constraint. The cost that the decision maker pays to adjust attention is totally cognitive, and the attention weights are subject to only a cognitive constraint.

In rational inattention models, one typical setting involves a decision maker paying cognitive cost to learn the values of various options, then determining the optimal probability for selecting each option (Caplin et al., 2022; Maćkowiak et al., 2023). Steiner et al. (2017) propose a dynamic choice model to account for status quo bias and behavioral inertia based on this typical setting. The present model extends this setting by adding assumptions to how the decision maker will determine the optimal weight assigned to each specific outcome realized at each time. These assumptions contribute to the research in how the value of an option is calculated based on the information acquired in mind.

Moreover, Gabaix and Laibson (2017) assume that, in intertemporal choice, the true value of a reward in each period is unknown. The decision maker observes some noisy signals of it and estimates it using Bayes' rule. The assumption is similar to the present model; yet, they focus on how the decision maker infers the true value from the observed signals, while the present model investigates how she integrates those signals. Gershman and Bhui (2020) extend the thought of Gabaix and Laibson (2017) and investigate how the decision maker optimizes the precision of such signals under a certain information constraint. The discounting factor derived from their model is also increasing in reward magnitude, and it can be viewed as a special case of the present model.

2.3 Behavioral Implications

2.3.1 ADU is generally consistent with "hidden zero effect"

Suppose a decision maker faces a choice between a small sooner reward (SS) and a larger later reward (LL). The hidden zero effect (Magen et al., 2008) implies she should exhibit more patience if both SS and LL are framed as sequences rather than being framed as single-period rewards. For instance, consider SS_0: "receive £100 today", and LL_0: "receive £120 in 6 months". Suppose we also have

SS_1: "receive £100 today and £0 in 6 months"

LL_1: "receive £0 today and £120 in 6 months"

The hidden zero effect suggests people are more likely to prefer LL_1 over SS_1 than preferring LL_0 over SS_0. Subsequent research (e.g. Read et al. (2017)) suggests the effect is asymmetric. That is, shifting SS_0 to SS_1 and keeping LL_0 unchanged do increase patience, whereas shifting LL_0 to LL_1 and keeping SS_0 unchanged cannot increase patience. This is naturally compatible with attentional discounted utility.

To illustrate, imagine that the framing of sequence length can influence how the decision maker may perceive it. For SS_0, the decision maker perceives the length of sequence as "today"; for SS_1, she perceives the length as "6 months". According to ADU, in the former case, she can allocate no attention to the future; while in the latter case, she needs to pay some attention to future periods, which contain no reward. Thus, the attention allocated to the current period decreases when shifting from SS_0 to SS_1 (given total capacity of attention is limited). This leads to a decrease in the value of sequence. By contrast, shifting from LL_0 to LL_1 does not change the length of sequence, thus does not change the value.

One might wonder for a period with no reward, why the attention weight may still be positive ("why should I pay attention to a period when there is no reward present"). The hidden zero effect provides an empirical reason. Additionally, from the theoretical view, attention acts as a filter that determines which information could enter awareness or working memory.[4] If the

[4] Such a view can be traced back to the bottleneck theories of attention, which starts from Broadbent (1958).

decision maker is aware about the information that the reward is zero in a certain period, it implies attention may have already been directed to that period.

Hereafter, I focus on ADU with the Shannon cost function (ADUS) and document five implications of the ADUS model in intertemporal choice. Each of them can be mathematically proved.

2.3.2 The requirement of magnitude effect on utility function can be relaxed under ADUS

In conventional discounted utility models, to make the magnitude effect holds true, the elasticity of utility function needs to be increasing with reward level (Loewenstein and Prelec, 1992). This requirement might be too strong so that many commonly used utility functions (such as power and CARA utility functions) does not satisfy it. By contrast, note in Equation (1), w_t is increasing in x_t. This implies that when comparing SS and LL, the decision maker could exhibit more patience toward larger reward levels, which relaxes the requirement for the magnitude effect.

To illustrate how specifically ADUS relaxes this requirement, we should derive how a delayed reward is valued in ADUS. Suppose the value of "receive a certain reward x_T in period T" is calculated by $w_T \cdot u(x_T)$. Then assuming the initial attention weight is in exponential style, that is $d_t \equiv \delta^t$ $(0 < \delta \leq 1)$, we should have

$$w_T = \frac{1}{1 + G(T)e^{-u(x_T)/\lambda}} \tag{2}$$

and

$$G(T) = \begin{cases} \dfrac{1}{1-\delta}(\delta^{-T} - 1)\ , & 0 < \delta < 1 \\ T\ , & \delta = 1 \end{cases} \tag{3}$$

It can be proved that, if Equation (2) and (3) are both satisfied, for all $x_T > 0$, power utility function $u(x_T) = {x_T}^{\gamma}$ with $0 < \gamma < 1$, satisfies the curvature requirement for magnitude effect, and when x_T is greater than a positive finite number, CRRA utility function $u(x_T) = 1 - e^{-\gamma x_T}$ with $\gamma > 0$, also satisfies it.

2.3.3 ADUS specifies a novel condition for common difference effect

According to the common difference effect (Loewenstein and Prelec, 1992), suppose a decision maker is indifferent between SS and LL, increasing the time delay before receiving the reward in both options by a same amount can lead the decision maker to prefer LL over SS.

The ADU models provide an alternative account for common difference effect. Given attention is limited, when the delay of reward is extended, that is, we insert some periods with zero reward into the existing sequence, the newly inserted periods will grab attention from other periods. As people tend to attend more to periods with larger rewards, in LL, the period when a positive reward is delivered will experience less attention being diverted compared to that in SS. This disparity in attention weighing can lead to a preference for LL over SS.

Regarding the Shannon cost function, assuming initial time preference is stationary, i.e. $d_t = \delta^t$, we can derive from Equation (2) and (3) the conditions for common difference effect. Clearly, if $\delta = 1$, the attention weight w_T will follow a hyperbolic style. In this case, the common difference effect will always hold true.

If $0 < \delta < 1$, the ADUS specifies a novel condition for common difference effect. To illustrate, suppose SS is "receive a certain reward x_s in period t_s" and LL is "receive a certain reward x_l in period t_l", where $x_l > x_s > 0$, $t_l > t_s > 0$. To make the common difference effect hold true, we need $v(x_l) - v(x_s) + \ln \frac{v(x_l)}{v(x_s)} > -(t_l - t_s) \ln \delta$. That is, when the decision maker is impatient, she performs the common difference effect if and only if *the difference between SS and LL in reward level is significantly larger than that in time delay.*

2.3.4 Discount function may be concave for the near future and convex for the far future

Most models of time discounting (e.g. hyperbolic and quasi-hyperbolic discounting) assume a convex discount function. However, some studies suggest the discount function may be concave for the near future and convex for the far future (Onay and Öncüler, 2007; Takeuchi, 2011; DeJarnette et al., 2020). This property can emerge in ADUS due to the attention weight following a logistic-like function.

In Equation (2), w_T produces this style of discount function when x_T is greater than a positive finite number. This is illustrated in Figure 1(a).

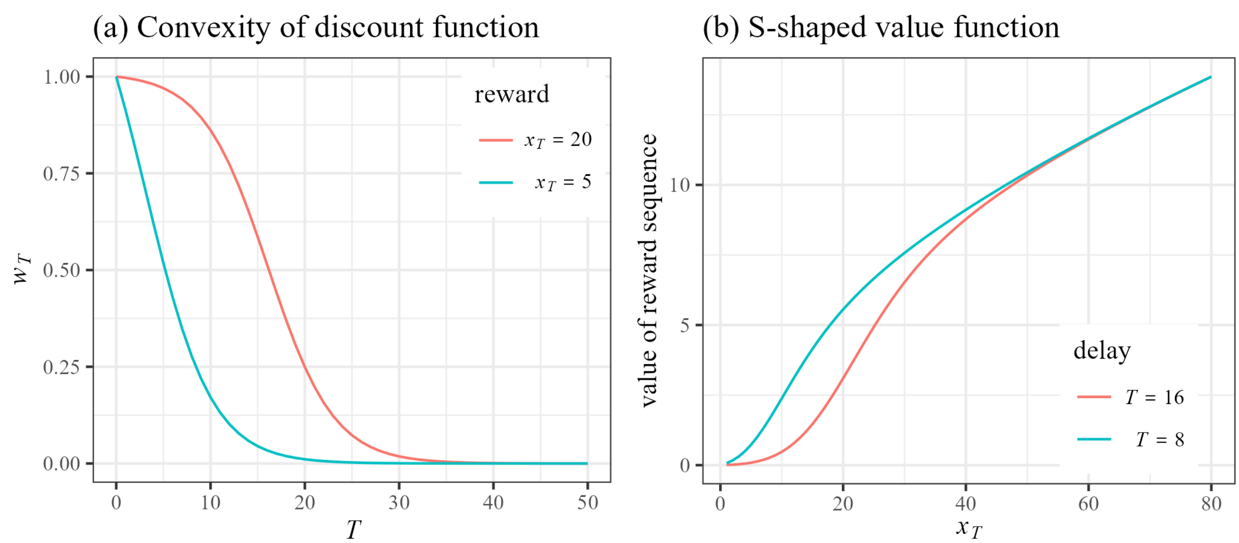


Figure 1: Simulation results for choices between SS and LL. Value of reward sequence is calculated by $w_T \cdot u(x_T)$, where w_T is calculated using Equation (2), $\delta = 0.75$, $\lambda = 1$, utility funtion $u(x) = x^{0.6}$.

2.3.5 ADUS offers an alternative account for S-shaped value function

The S-shaped value function has been widely adopted by behavioral economists. Some theories justifies it by reference-dependent utility (Kahneman and Tversky, 1979; Koszegi and Rabin, 2006), while some others by efficient coding of numerical values (Frydman and Jin, 2021). I offer an account based on selective attention to sequential outcomes.

Imagine that a decision maker faces a choice between two lotteries. At the moment when the choice is made, she does not receive any money from either option. Thus, she may perceive the outcome of each option as something that will happen in the future. She allocates attention between the current period, which offers no reward, and the future period when she could receive some money. When the amount of money that she could receive is increased, not just does its utility increases, but her attention is more directed towards that particular future period.

Assuming the decision maker perceives the outcome will be realized in period T, and in a certain state, the option she chooses yields reward x_T. We can use Equation (2) to derive the value function, i.e. $w_T \cdot u(x_T)$. Both utility function $u(.)$ and attention weight w_T exhibit diminishing sensitivity to the reward outcome x_T. However, when the reward is small enough, their product may be convex . Notably, when the curvature of utility function satisfies a certain condition, or the unit cost of information λ is small enough, we can obverse a S-shaped value function. Figure 1(b) illustrates this.

2.3.6 Inattentive decision makers perform less inconsistent behaviors

Imagine that a decision maker faces a task of allocating consumption budget B across multiple periods, and in each period, she makes a new consumption plan to optimize her attentional discounted utility. Suppose she starts in period 0, then the task can be formulated as the following optimization problem:

$$\max_{c_0,\ldots,c_T} \sum_{t=0}^{T} w_t \cdot u(c_t) \qquad s.t. \sum_{t=0}^{T} c_t = B$$

where c_t is consumption in period t, w_t is the attention weight, which is calculated with ADUS. I develop a numerical method to solve this optimization problem.[5]

Previously, researchers often employ “present bias” (Laibson, 1997) or “naiveté” (O’Donoghue and Rabin, 1999) to account for the phenomenon of dynamic inconsis-

[5]The method combines the gradient projection method and F-W algorithm as a local optimizer, and use the basin-hopping method to achieve global optimization.

tency in this context. I provide an account based on attention weight updating.

I categorize each decision maker into two types based on how they process information in this task - *attentive* and *inattentive*. A decision maker is considered *attentive* to the decision task if, in each period, she remembers the information about her consumption plan made in the last period and use it to update attention weights. Otherwise, she is classified as *inattentive*.

Suppose in period 0, the decision maker's initial attention weight is proportional to δ^t, where $0 < \delta < 1$. Given the decision maker is impatient ($\delta < 1$), she tends to consume more in earlier periods within the planning horizon. Then in period 1, an *attentive* decision maker may update her initial attention weight into w_t formed in period 0, which is proportional to $\delta^t \exp\{\frac{u(c_t)}{\lambda}\}$, and so on. Consequently, the earlier periods, which originally has more planned consumption, will now receive more (initial) attention weights than they did in the last period. This leads to an increased preference for raising consumption levels in such periods.

By contrast, if the decision maker is *inattentive*, that is, she does not mind much about the task and thus always forgets the information about her prior consumption plans, then in period 1 and any subsequent period, her initial attention weight will keep proportional to δ^t. In this case, she could perform less inconsistent behaviors than the attentive decision makers. 2 offers an illustration for the contrast. The contrast may shed light on why some nudges that reduce the demand for attention commitment, such as a default choice, can improve the consistency in choices.

2.4 Axiomization of ADUS

In this subsection, I provide an axiomatic rationale for using the Shannon cost function in ADU. I firstly define the preference relation $\succsim$ that can be represented by ADU, then propose four axioms to characterize the Shannon cost function.

Definition 2: Preference relation $\succsim$ has an ADU representation if and only if, for any

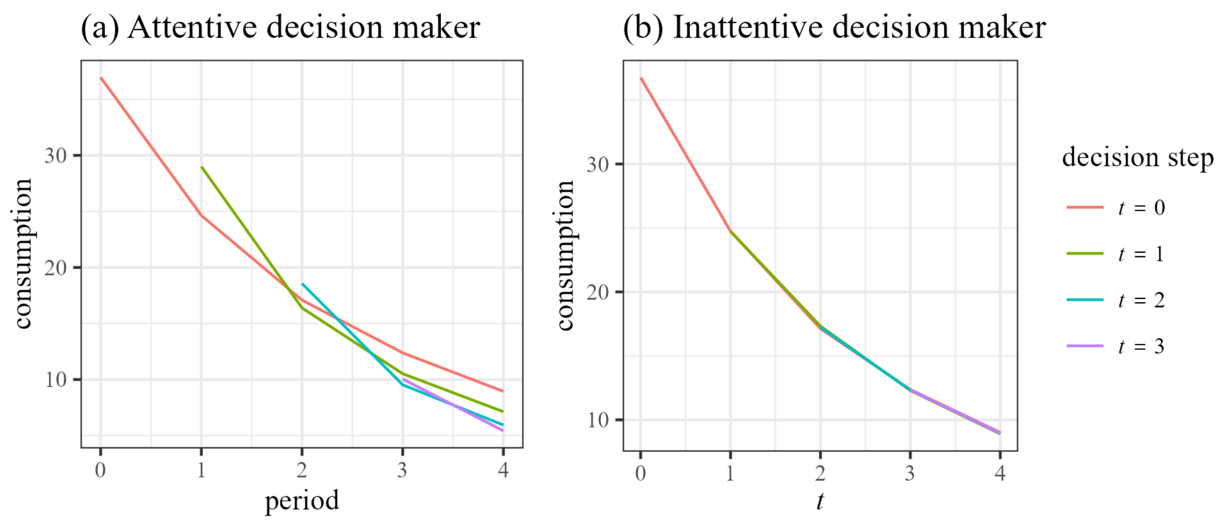


Figure 2: Simulation results for budget allocation. The decision maker allocations a budget of $B = 100$ across five periods, $\delta = 0.9$, $\lambda = 70$. For each period t, utility funtion $u(x_t) = x_t^{0.6}$.

stochastic reward sequence X_T, $X'_{T'}$, we have

$$X_T \succsim X'_{T'} \Longleftrightarrow U(X_T) \geq U(X'_{T'})$$

where $U(X_T) = \sum_{S_T \in \mathcal{S}(X_T)} \sum_{t=0}^{T} w(s_t)u(s_t)$, $U(X'_{T'}) = \sum_{S_{T'} \in \mathcal{S}(X'_{T'})} \sum_{t=0}^{T'} w'(s_t)u(s_t)$. S_T and $S'_{T'}$ are the potential realizations of sequence X_T and $X'_{T'}$, $w(.)$ and $w'(.)$ are the solutions to the constrained optimal discounting problems for X_T and $X'_{T'}$.

Axiom 1: (*state independence*) For any reward sequence X_T, X'_T, X''_T and $\alpha \in (0, 1)$, $X_T \succ X'_T$ implies $\alpha X_T + (1-\alpha)X''_T \succ \alpha X'_T + (1-\alpha)X''_T$.

Axiom 1 implies that the determination of attention weights in one potential realization of reward sequence will not interfere that in another. If Axiom 1 is satisfied, each state in the constrained optimal discounting problem has an independent solution.

Axiom 2: *(sequential outcome betweenness)* For any non-negative real number b and deterministic reward sequence S_T, let $S_T b$ denote $[s_0, s_1, ..., s_T, b]$, there always exists $\alpha \in (0, 1)$ such that $S_T b \sim \alpha S_T + (1-\alpha)b$.

Axiom 2 implies that if we add a new element to a given sequence, the value of the new sequence will lies between the value of the original sequence and the utility of the newly added element. The evidence of "violation of dominance" (Scholten and Read, 2014) may provide support for this axiom.

Axiom 3: (*sequential bracket independence*) For any non-negative real number b, c and deterministic sequence S_T, if there exist α_1, α_2, α_3, β_1, $\beta_2 \in \mathbb{R}_{>0}$ such that

$$S_T bc \sim \alpha_1 S_T + \alpha_2 b + \alpha_3 c \quad \text{and} \quad S_T bc \sim \beta_1 S_T + \beta_2 (bc)$$

where bc denotes a sequence with immediate reward b and period-1 reward c, then we must have $\alpha_1 = \beta_1$.

Axiom 3 implies that if we segment a given sequence into different elements, and find that the value of a linear combination of these elements is equivalent to the value of the overall sequence, then in this linear combination, the weight for any specific element can hold constant no matter how we segment or bracket the other elements.

Axiom 4: (*aggregate invariance of constant sequences*) For any deterministic sequences S_T, S'_T, given non-negative real number c', c and $\alpha \in (0, 1)$, if $\alpha s'_t + (1-\alpha)c' \succ \alpha s_t + (1-\alpha)c$ holds for every period t, then $\alpha S'_T + (1-\alpha)c' \succ \alpha S_T + (1-\alpha)c$.

Axiom 4 implies that in a given sequence, if the utility of every element plus an equal amount, then the overall utility of the sequence should plus the same amount. Note in conventional discounted utility models, sequences are typically assumed to be separable from each other. In that case, any sequence can be *aggregate invariant*. Thus, Axiom 4 can be viewed as a weaker version of "separability of sequences".

Theorem 1: $\succsim$ has an ADUS representation if and only if it has an ADU representation and satisfies Axiom 1-4.

3 Attention Allocation over Sequential Outcomes: An Experimental Test

3.1 Motivation

The second piece of work is an experimental test of the key assumptions in my proposed model. Suppose a decision maker faces a choice between receiving an immediate reward and receiving a sequence of rewards, and when evaluating the sequence, she has only limited attention to allocate across different time periods within the sequence. Then, if the length of sequence is substantially extended, on average the attention that can be allocated to each particular period would decrease. Under this circumstance, any changes in the amount of rewards offered in each period would have a smaller impact on her choice.

Moreover, suppose the decision maker tends to pay more attention to periods with larger rewards. If the amount of reward offered in one period is substantially increased, she would naturally shift more attention to this particular period. Thus, the attention available to be allocated to the rest of periods would decrease. Under this circumstance, any changes in reward amounts for those other periods would have a smaller impact on her choice.

3.2 Experimental Design

I designed a survey to test the above arguments. To illustrate how the arguments will be tested, imagine that there are two options:

A. "receive £M today"

B. "receive £X today and £Y in T months"

Note option A is an immediate reward and option B is a sequence of rewards. If people have to choose between option A and B, when either X or Y increases (and the others are equal), more people should prefer option B over A. Specifically, I make two predictions:

Prediction 1: If we increase T and keep Y unchanged, people should be less sensitive to changes in X as the sequence length is extended. In this case, increasing X by the same amount leads fewer people to shift from option A to option B.

Prediction 2: If we increase Y but keep T unchanged, people should also be less sensitive to changes in X as Y grabs a greater amount of attention. In this case, increasing X by the same amount leads fewer people to shift from option A to option B. The same is true for Y when we increase X instead.

I plan to recruit N=150 participants via Prolific. In this survey, participants need to answer 19 time preference questions and 1 risk preference question. All participant are presented with the same questions. Each time preference question consists of a list of choices. In each row of the list, the participants are required to choose between an immediate reward (the same as option A) and a sequence of rewards (the same as option B). Figure 3 presents an example question. The questions are designed with two conditions: in the first condition, X increases by rows while the other variables (M, Y, T) remain constant across the rows; in the second condition, it is Y that increases by rows while the other variables remain constant within a question. In the questions where X increases by rows, the value of X goes up by 10 with each row, starting from 10 and going up to 90. The same pattern is followed by Y as well.

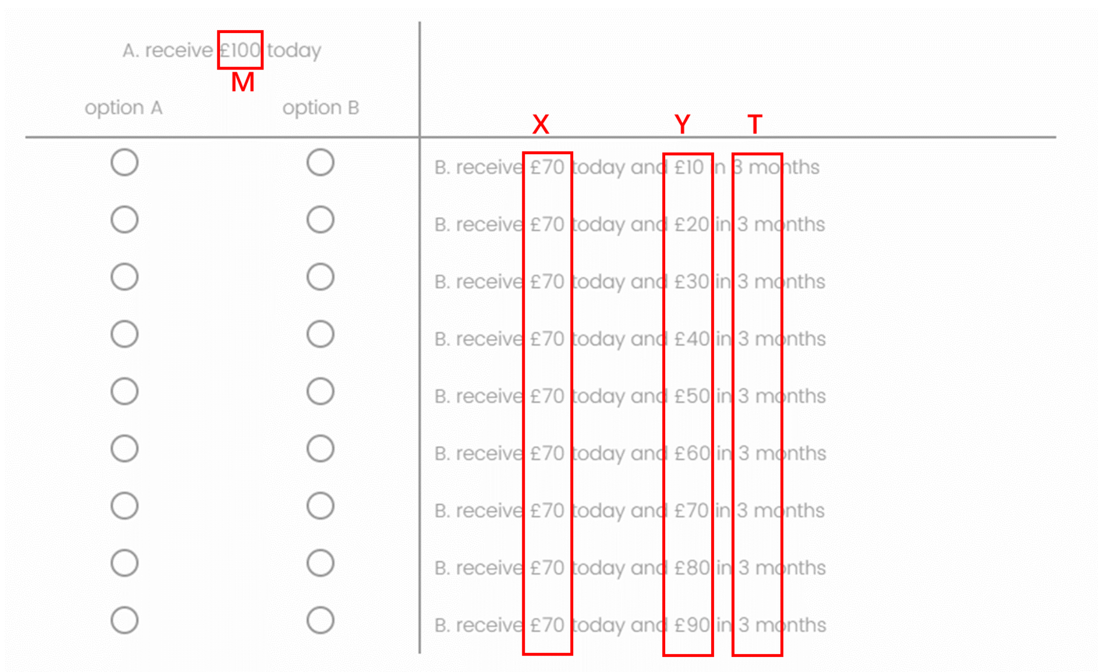


Figure 3: Example question.

One of the time preference questions is set for attention check. For attention check, M=50, X=100, T=3 and Y increases by rows. Given that X>M, participants should consistently prefer option B to A for every row in this question. For the rest of time preference questions, M is selected from {100,130}, X and Y are selected from {50, 70, 90}, T is selected from {3, 9, 18}. Table 1 shows how the values for M, X, Y and T are distributed across these questions. All the questions are presented in a random order.

At the end of the survey, there is a risk preference question. The risk preference question is also a choice list. In each row of the list, participants need to choose between a risky lottery ("win £100 with probability 50%") and a sure amount of reward. The risky lottery is constant across the rows while the sure reward increases by rows. Measuring the risk preference will help us fit the utility function and calibrate model parameters in statistical analysis.

Table 1: Time preference questions

X increases by rows			Y increases by rows		
M (£)	X (£)	T (months)	M (£)	Y (£)	T (months)
100	50	3	100	50	9
100	70	3	100	70	9
100	90	3	100	90	9
100	70	9	130	50	9
100	90	9	130	70	9
100	90	18	130	90	9
130	50	3			
130	70	3			
130	90	3			
130	70	9			
130	90	9			
130	90	18			

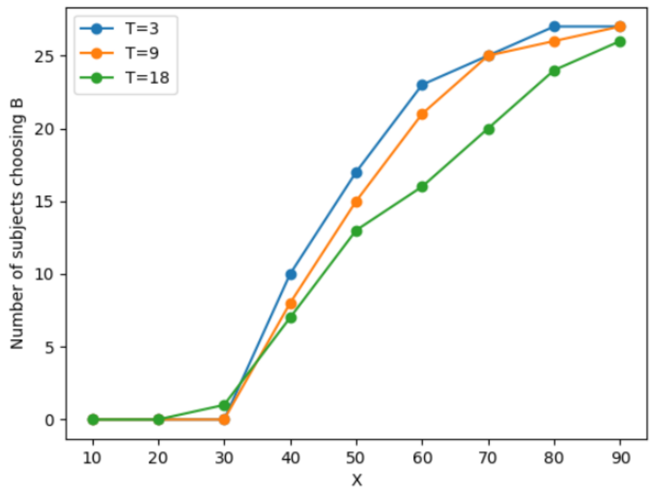
For statistical analysis, the key dependent variable will be the choice between option A and B in each row of time preference questions. I will run logistic regressions for two conditions respectively. In the condition that X increases by rows, the main independent variables will be M, X, and the interaction effects among Y, T and X. To validate Prediction 1, I will test whether for participants the likelihood of choosing B is significantly less sensitive to X while T being larger. To validate Prediction 2, I will test whether the likelihood of choosing B is significantly less sensitive to X while Y being larger. The condition that Y increases by rows is only used for validating Prediction 2. The main independent variables will be M, Y, and the interaction effects among X, T and Y. I will test whether the likelihood of choosing B is significantly less sensitive to Y while X being larger. For an additional analysis, I will fit multiple models to data. These models include exponential, hyperbolic, and quasi-hyperbolic discounting, and a variant of attentional discounted utility. The purpose of this additional analysis is to explore whether putting a non-linear structure that captures the key properties of attention in the discounted utility framework can improve predictive performance.

3.3 Pilot Study

I did two pilot studies in the last month. Each pilot study contains N=30 participants. Given the sample size is small, I do not perform a formal analysis here. However, some behavioral patterns observed in pilot studies can help illustrate how my predictions may be validated. See Figure 4 for details.

It can be observed that people are more likely to choose option B when X grows larger. However, the rate at which this likelihood of choosing B grows varies across settings. In Figure 4(a), the blue curve (T=3) has is the sharpest and the green curve (T=18) is the flattest. Thus, people may be less sensitive to the changes in X when T=18 than when T=3, which is in line with Prediction 1. In Figure 4(b), the blue curve (Y=70) is sharper than the orange curve (Y=90). Thus, people may be less sensitive to the changes in X when Y=90 than when Y=70, which is in line with Prediction 2.

(a) Y=70



(b) T=3

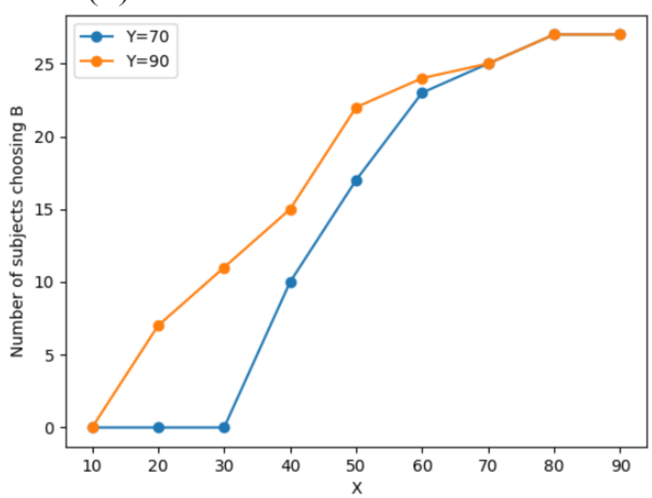


Figure 4: Behavioral patterns observed in a pilot study. X increases by rows, M=100.

4 Valuation of Risk and Effort

4.1 Experimental Procedure

The third piece of work (in design) aims to test how people jointly valuate risk and effort in an experiment. The experiment consists of multiple rounds of effort-exertion decision tasks. In each round, there are two types of balls on the screen – red balls and blue balls. Participants can change the color of each ball by clicking on it and completing a simple real-effort task (entering a CAPTCHA). If the original color is blue, then it will be changed to red, and vice versa. There is no limit on the times of color change. After they make such changes, the participants can click a button then a ball will be randomly drawn from all the balls presented on the screen. The participants can get different levels of reward when different colors of balls are drawn. Suppose the reward for a blue ball is greater than that for a red ball, in each round, the participants will be more inclined to change red balls to blue balls, by making efforts. They need to balance the benefit of increasing the probability of drawing a high-value ball, and the cost of making efforts to change the colors. Figure 5 presents a screenshot of this task.

4.2 Behavioral Implications

This experimental approach can help us examine how the cost of effort may impact decision making. While researchers today understand a lot about risk preferences, there is still limited evidence on the willingness to exert effort. By this approach, we can use our knowledge about risk preferences to elicit the preferences of effort exertion, and to characterize the effort cost function.

Moreover, although there has been some theories considering effort as a necessary input for decision making, the effort discussed in such theories, e.g. attention, is usually at the cognitive level; whereas, this experiment also involves physical effort or the effort to actually implement the decision. The setting in this experiment is consistent with that set up in such theories, therefore it allows a direct comparison between the two forms of effort. For

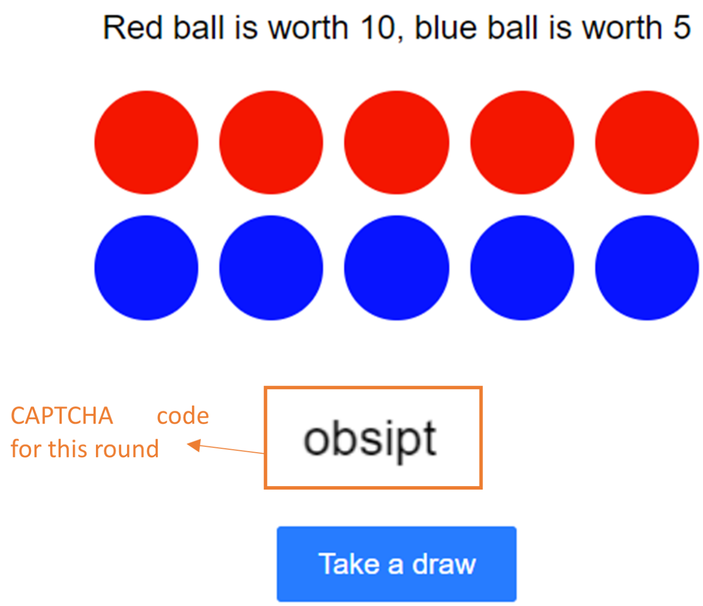


Figure 5: A screenshot of the experimental program (demo). After clicking on any of the balls, a pop-up window will appear and ask people to enter the CAPTCHA code, in order to change its color. The program is written in Javascript.

example, in rational inattention theories (Matějka and McKay, 2015), it is typically assumed that people should make a choice between different items and it costs some (cognitive) effort to acquire the value information of each item. When less effort is exerted, people may perform a higher degree of randomness in those choices (just like the red and blue balls have a nearly even chance to be drawn). Such a kind of comparisons may lead to a new theory of effort cost in future research.

This experiment also has a real-world implication. In experiment, participants are required to exert greater effort to increase their chances of drawing a high-value ball. In reality, when we work harder, it is often that we are more likely to achieve a good outcome. For instance, we may work hard on a paper to increase the probability that it is published on a top journal, or on researching and adjusting investment portfolio in order to get a higher return in financial markets. Thus, what we learn from the experiment may also shed light on these research fields.

5 Time Schedule

Table 2: Timeline

Time	Progress
Aug, 2023	1. Refine empirical analysis on the behavioral implications of ADU (Project 1) 2. Implement the experiment about the role of attention in intertemporal choice (Project 2)
Sept, 2023	1. Complete the relevant proofs (Project 1) 2. Analyze the experimental results and design a follow-up test (Project 2)
Oct, 2023	1. Finish the first draft of the paper (Project 1) 2. Complete the required procedures for a follow-up test (Project 2) 3. Complete the experimental design (Project 3)
Nov, 2023	1. Collect data from the follow-up test and conduct relevant statistical analyses (Project 2) 2. Complete the required procedures for running the experiment (Project 3)
Dec, 2023	Implement the experiment about valuation of risk and effort (Project 3)

Table 2: Timeline

Time	Progress
Jan-March, 2024	Draft research papers and consider additional analyses (Project 2&3)
Apr-July, 2024	Complete the thesis

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