Progress Report for Annual Review 22/23 - v2

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August 08, 2023

# 1 Introduction

In this document, I present three pieces of work.

The first is a novel model of intertemporal choice. The model incorporates attentional mechanism into the expected discounted utility framework. It assumes that people have limited attention to allocate across sequential outcomes and tend to pay more attention to the outcomes with larger rewards. I show that it can explain a broad set of decision anomalies, and present an axiomatic characterization.

The second is an experimental test on whether (and how) attentional mechanism can influence intertemporal choice.

The third is an experiment aiming to test how people jointly evaluate risky outcomes and efforts. The findings from this experiment can inform the research in rational inattention. I am working on the experimental program.

# 2 Attentional Discounted Utility

## 2.1 The Model

Expected discounted utility framework has been widely used in behavioral and economic research. Given a sequence of rewards , which yields reward in time period ,[[1]](#footnote-21) , the expected discounted utility of can be calculated by

where is the discounting factor of time period , reward level is a random variable defined on , denotes the decision maker’s instantaneous utility function, , . The time length of this sequence, denoted by , is finite.

The aim of this section is to incorporate attentional mechanism into this valuation process of reward sequences. I assume that a decision maker allocates attention across sequential outcomes when processing information about rewards, and attends more to the outcomes with larger rewards. I define the models that contain such features as *Attentional Discounted Utility* (ADU). Previous studies exploring the ADU setting include Gershman and Bhui (2020) and Noor and Takeoka (2022). Specifically, I focus on a particular form of ADU, which I term as *ADU with Shannon cost function* (ADUS). ADUS retains the architecture of expected discounted utility, but simply modifies the conventional discounting factor to , which I refer to as attention weight hereafter, where

In Equation (), adopts a form resembling logistic function. It indicates three properties of attention. First, the sum of all equals 1, indicating the attention that is available to be allocated across sequential outcomes is limited. Second, is increasing in , indicating the decision maker is inclined to allocate more attention to periods with larger potential rewards. This is in line with existing evidence, suggesting people often pay much attention to pleasant information and avoid to know unpleasant information (e.g. ostrich effect). Third, it appears that is anchored in . This indicates that the conventional discounting factor can represent the initial attention weight assigned to each period, and the psychological process of attention adjustment (converting to ) is costly. The parameter quantifies the cost of attention adjustment. Notably, is positive for every period within the sequence.

I provide two rationales for Equation (). The first is based on rational inattention theories. The second is an axiomatic theory. I present the first here and discuss the second in Subsection .

The necessary notations and definitions are described as follows. Let be a potential realization of , and be the support of , i.e. the smallest set containing any potentially realized sequence , where . In each , the attention weight assigned to period is decided by a weight function . I follow Noor and Takeoka (2022, 2023) to define the process that generates as *constrained optimal discounting*. The key assumption of constrained optimal discounting process is that, when evaluating a reward sequence, the decision maker wants to find an allocation policy for attention weights, in order to maximize the anticipatory utility that she can obtain from the sequence. Nonetheless, this attention re-allocation process triggers a cognitive cost. She needs to balance the benefit of focusing on periods containing larger rewards and the cognitive cost of shifting attention. The formal definition is given by Definition 1.

**Definition 1**: Let be the support of all possible weight functions. Given a stochastic reward sequence , the following optimization problem is called the *constrained optimal discounting* problem for :

where is a constant, is called *information cost* function, and . That is, the information cost is increasing and convex in .

In Definition 1, the cognitive cost of shifting attention is decided by an information cost function . This implies that the cognitive cost is generated by focusing on certain reward information. A well-known specification of is the Shannon cost function, proposed by Matějka and McKay (2015). The Shannon cost function was originally used to justify the multinominal logit model in discrete choice analysis, and so far has been topical in rational inattention literature. To construct this style of information cost function, Matějka and McKay (2015) introduce three assumptions. The first is that the sum of all weights is 1, i.e. . The second is, before acquiring any information, the decision maker establishes an initial weight allocation for different attributes (outcomes), which remains invariant across states. The weights are then updated in a manner consistent with Bayes rule. In ADU setting, it means . The third assumption is, the information cost is linear to the information gains, measured by Shannon mutual information. That is,

where is a parameter denoting unit cost of information (), is the probability that occurs. With Shannon cost function, the constrained optimal discounting problem can be easily solved by Lagrangian method, and the solution is the same as Equation (). [[2]](#footnote-22)

## 2.2 Implications in Intertemporal Choice

### 2.2.1 ADU is generally consistent with “hidden zero effect”

Suppose a decision maker faces a choice between a small sooner reward (SS) and a larger later reward (LL). The hidden zero effect (Magen, Dweck, and Gross 2008) implies she should exhibit more patience if both SS and LL are framed as sequences rather than being framed as single-period rewards. For instance, consider SS0: “receive £100 today”, and LL0: “receive £120 in 6 months”. Suppose we also have

SS1: “receive £100 today and £0 in 6 months”

LL1: “receive £0 today and £120 in 6 months”

The hidden zero effect suggests people may be more likely to prefer LL1 over SS1 than preferring LL0 over SS0. Subsequent research (e.g. Read, Olivola, and Hardisty (2017)) suggests the effect is asymmetric. That is, shifting SS0 to SS1 and keeping LL0 unchanged do increase patience, whereas shifting LL0 to LL1 and keeping SS0 unchanged cannot increase patience. This is naturally compatible with attentional discounted utility.

To illustrate, imagine that the framing of sequence length can influence how the decision maker may perceive it. For SS0, the decision maker perceives the length of sequence as “today”; for SS1, she perceives the length as “6 months”. In the former case, she can allocate no attention to the future; while in the latter case, she needs to pay some attention to future periods, which contain no reward. Thus, the attention allocated to the current period decreases when shifting from SS0 to SS1 (given total amount of attention is limited). This leads to a decrease in the value of sequence. By contrast, shifting from LL0 to LL1 does not change the length of sequence, thus does not change the value.

One might wonder for a period with no reward, why the attention weight may still be positive (“why should I pay attention to a period when there is no reward present”). The hidden zero effect provides an empirical reason. Additionally, from the theoretical view, attention acts as a filter that determines which information could enter awareness or working memory.[[3]](#footnote-24) If the decision maker is aware about the information that the reward for a certain period is zero, it implies attention may have already been directed to that period.

Hereafter, I focus on ADU with Shannon cost function (ADUS) and document five implications of the ADUS model in intertemporal choice. Each of them can be precisely proved.

### 2.2.2 The requirement of magnitude effect on utility function can be relaxed under ADUS

In conventional discounted utility models, to make the magnitude effect holds true, the elasticity of utility function needs to be increasing with reward level (Loewenstein and Prelec 1992). This requirement might be too strong so that many commonly used utility functions (such as power and CARA utility functions) does not satisfy it. By contrast, note in Equation (), is increasing in . This implies that when comparing SS and LL, the decision maker could exhibit more patience toward larger reward levels, which relaxes the requirement for the magnitude effect (Noor 2011).

To illustrate how specifically ADUS relaxes this requirement, we should derive how a delayed reward is valued in ADUS. Suppose the value of “receive a certain reward in period ” is calculated by . Then assuming the initial attention weight is in exponential style, that is (), we should have

and

It can be proved that, if Equation () and () are both satisfied, for all , power utility function with , satisfies the curvature requirement for magnitude effect, and when is greater than a positive finite number, CRRA utility function with , also satisfies it.

### 2.2.3 ADUS specifies a novel condition for common difference effect

According to the common difference effect (Loewenstein and Prelec 1992), suppose a decision maker is indifferent between SS and LL, increasing the time delay before receiving the reward in both options by a same amount can lead the decision maker to prefer LL over SS.

The ADU models provide an alternative account for common difference effect. Given attention is limited, when the delay of reward is extended, that is, we insert some periods with zero reward into the existing sequence, these newly inserted periods will grab attention from other periods. As people tend to attend more to periods with larger rewards, the period when the reward is delivered in LL will experience less attention being diverted compared to that in SS. This disparity in attention weighing can lead to a preference for LL over SS.

Regarding Shannon cost function, assuming initial time preference is stationary, i.e. , we can derive from Equation () and () the conditions for common difference effect. Clearly, if , the attention weight will follow a hyperbolic style. In this case, the common difference effect will always hold true.

If , the ADUS specifies a novel condition for common difference effect. To illustrate, suppose SS is “receive a certain reward in period ” and LL is “receive a certain reward in period ”, where , . To make the common difference effect hold true, we need . That is, when the decision maker is impatient, she performs the common difference effect if and only if *the difference between SS and LL in reward level is significantly larger than that in time delay*.

### 2.2.4 Discount function may be concave for the near future and convex for the far future

Most models of time discounting (e.g. hyperbolic and quasi-hyperbolic discounting) assume a convex discount function. However, some studies suggest the discount function may be concave for the near future and convex for the far future (Onay and Öncüler 2007; Takeuchi 2011; DeJarnette et al. 2020). This property can emerge in ADUS due to the attention weight following a logistic-like function.

In Equation (), produces this style of discount function when is greater than a positive finite number. This is illustrated in Figure (a).

### 2.2.5 ADUS offers an alternative account for S-shaped value function

The S-shaped value function has been widely adopted by behavioral economists. Some theories justifies it by reference-dependent utility (Kahneman and Tversky 1979; Koszegi and Rabin 2006), while some others by efficient coding of numerical values (Frydman and Jin 2021). I offer an account based on selective attention to sequential outcomes.

Imagine that a decision maker faces a choice between two lotteries. At the moment when the choice is made, she does not receive any money from either option. Thus, she may perceive the outcome of each option as something that will happen in the future. She allocates attention between the current period, which offers no reward, and the future period when she could receive the money. When the amount of money that she could receive is increased, not just does its utility increases, but her attention is more directed towards that particular period.

Assuming the decision maker perceives the outcome will be realized in period , and in a certain state, the option she chooses yields reward . We can use Equation () to derive the value function, i.e. . Both utility function and attention weight exhibit diminishing sensitivity to the reward outcome . However, when the reward is small enough, their product may be convex . Notably, when the curvature of utility function satisfies a certain condition, or the unit cost of information is small enough, we can obverse a S-shaped value function. Figure (b) illustrates this.

### 2.2.6 Inattentive decision makers perform less inconsistent behaviors

Imagine that a decision maker faces a task of allocating consumption budget across multiple periods, and in each period, she makes a new consumption plan to optimize her overall attentional discounted utility. Suppose she starts in period 0, then the task can be formulated as the following optimization problem:

where is consumption in period , is the attention weight, which is calculated with ADUS. I develop a numerical method to solve this optimization problem.

Previously, researchers often employ “present bias” (Laibson 1997) or “naiveté” (O’Donoghue and Rabin 1999) to account for the phenomenon of dynamic inconsistency in this context. I provide an account based on attention weight updating.

I categorize each decision maker into two types based on how they process information in this task - *attentive* and *inattentive*. A decision maker is considered *attentive* to the decision task if, in each period, she remembers the information, including anticipatory utility, acquired in the last period and use it to update attention weights. Otherwise, she is classified as *inattentive*.

Suppose in period 0, the decision maker’s initial attention weight is proportional to , where . Given the decision maker is impatient (), she tends to consume more in earlier periods within the planning horizon. Then in period 1, an *attentive* decision maker may update her initial attention weight into formed in period 0, which is proportional to , and so on. Consequently, the earlier periods, which originally has more planned consumption, will now receive more (initial) attention weights than they did in the last period. This leads to an increased preference for raising consumption levels in such periods.

By contrast, if the decision maker is *inattentive*, that is, she does not mind much about the task and always forgets the information acquired in the last period, then in period 1 and any subsequent period, her initial attention weight will keep proportional to . Thus, she could perform less inconsistent behaviors than the attentive decision makers. offers an illustration for this contrast. The contrast may shed light on why some nudges that reduce the demand for attention commitment, such as a default choice, can improve the consistency in choices.

## 2.3 Axiomization of ADUS

In this subsection, I provide an axiomatic rationale for using Shannon cost function in ADU. I firstly define the preference relation that can be represented by ADU, then propose four axioms to characterize the Shannon cost function.

**Definition 2**: Preference relation has an ADU representation if and only if, for any stochastic reward sequence , , we have

where , . and are the potential realizations of sequence and , and are the solutions to the constrained optimal discounting problems for and .

**Axiom 1**: (*state independence*) For any reward sequence , , and , implies .

Axiom 1 implies that the determination of attention weights in one potential realization of reward sequence will not interfere that in another. If Axiom 1 is satisfied, each state in the constrained optimal discounting problem has an independent solution.

**Axiom 2**: *(sequential outcome betweenness)* For any non-negative real number and deterministic reward sequence , let denote , there always exists such that .

Axiom 2 implies that if we add a new element to a given sequence, the value of the new sequence will lies between the value of the original sequence and the utility of the newly added element. The evidence of “violation of dominance” (Scholten and Read 2014) may provide support for this axiom.

**Axiom 3**: (*sequential* *bracket independence*) For any non-negative real number , and deterministic sequence , if there exist , , , , such that

where denotes a sequence with immediate reward and period-1 reward , then we must have .

Axiom 3 implies that if we segment a given sequence into different elements, and find that the value of a linear combination of these elements is equivalent to the value of the overall sequence, then in this linear combination, the weight for any specific element can hold constant no matter how we segment or bracket the other elements.

**Axiom 4**: (*aggregate invariance of constant sequences*) For any deterministic sequences , , given non-negative real number , and , if holds for every period , then .

Axiom 4 implies that in a given sequence, if the utility of every element plus an equal amount, then the overall utility of the sequence should plus the same amount. Note in conventional discounted utility models, sequences are typically assumed to be separable from each other. In that case, any sequence can be *aggregate invariant*. Thus, Axiom 4 can be viewed as a weaker version of “separability of sequences”.

**Theorem 1**: has an ADUS representation if and only if it has an ADU representation and satisfies Axiom 1-4.

## 2.4 Empirical Analysis

# 3 Experimental Tests of Attention over Sequential Outcomes

# 4 Valuation of Risk and Effort

# 5 Time Schedule

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1. I use uppercase letters to represent a sequence and lowercase letters to represent each element within the sequence. [↑](#footnote-ref-21)
2. In each certain state , in Equation () can be defined by . [↑](#footnote-ref-22)
3. Such a view can be traced back to the bottleneck theories of attention, which starts from Broadbent (1958). [↑](#footnote-ref-24)