A Lifecycle Model for Chinese Urban Households

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1 Introduction

We develop a representative agent model of Chinese urban households' consumption and investment choices over the lifecycle. Our model framework is similar to other models of this kind based on US households, such as Yogo (2016) and Duarte et al. (2021). We differ from them by considering key features of China's pension and housing system compared to the US system.

China's pension system heavily relies on mandatory public pension, whereas occupational and private pensions (e.g. 401(k) and IRA) account for a large proportion in the US pension system. In China, employers that are willing to offer occupational pensions are usually large state-owned enterprises (especially those in railway, electricity or communication industries) and public sector. Most firms do not have enough incentives to offer occupational pensions. In 2021, the net mandatory pension replacement rate to labor income is 92% in China and 51% in the US.[1] Therefore, in our model, we examine how the public pension benefit is calculated in detail. Meanwhile, China just launched its private pension scheme in 2022. We believe that the size and importance of private pension in its pension market will keep growing in the near future. So, we also introduce private pension in our model.

Another noticeable part in China's social security system is the housing provident fund (hereafter referred to as "housing fund"). An employee and her employer need to jointly contribute a total of no less than 10% of her pre-tax labor income to her housing fund account every month. The fund can only be used by her to pay housing expenditure (including rent and mortgage repayment), or enable local governments to issue low-interest mortgages. We will also examine the role of housing fund in household financial decisions.

[1] https://www.oecd-ilibrary.org/sites/75fed0dc-en/index.html?itemId=/content/component/75fed0dc-en

2 Model Framework

2.1 Objective Function

We set time is discrete and the agent's preferences are time-separable. The agent's instantaneous utility at age t, denoted by u_t, is a weighted sum of three separate components: the utility from the agent's own consumption u_t^1, the utility from the consumption of her dependent children u_t^2, and the utility from leaving bequests u_t^3. Following Palumbo (1999), we also consider health status as a complement to her (and her dependent children) consumption. Therefore, we have:

$$u_t = \Psi(e_t)(u_t^1 + \kappa_2 u_t^2) + \kappa_3 u_t^3 \tag{1}$$

where κ_2 and κ_3 measure the relative importance of supporting children and leaving bequests, e_t is the agent's health status at age t (greater e_t for better health status). Keeping in line with De Nardi, French, and Jones (2010), we set $\Psi(e_t) = 1 - \kappa_1 \cdot e_t$.

In each utility component, we use a CRRA-style utility function. For the first component u_t^1, we set:

$$u_t^1 = \frac{(c_t^\rho h_t^{1-\rho})^{1-\gamma}}{1-\gamma} \tag{2}$$

where c_t and h_t are the agent's non-housing consumption (by herself) and housing consumption; we follow Cocco (2005) and Yao and Zhang (2005) to use the Cobb-Douglas product in this utility component. γ is the coefficient of relative risk aversion, ρ can be interpreted as the optimal proportion of non-housing consumption in the agent's own consumption in a single-period consumption decision problem. Hereafter, we assume the price per unit non-housing consumption and per unit housing consumption is 1 and v_t, and let h_t also represent the size of house where the agent lives.

For the second utility component u_t^2, if the agent has to support more than one child at the same time, we assume each child consumes equally. Let n_t be the number of children she has to support, F_t be the consumption of each child, then keeping our setting in line with Barro and Becker (1989), we have:

$$u_t^2 = n_t^\alpha \frac{F_t^{1-\gamma}}{1-\gamma} \tag{3}$$

where α determines the impact of the number of children to support on the overall utility.[2]

[2] One puzzle about China that has received considerable attention is why its household savings rate rose

For the third utility component u_t^3, we refer to Hubbard, Skinner, and Zeldes (1994) that assumes the agent's life span is risky. Let W_t be her net wealth at the end of age t, ψ_t be the probability that she can survive at that moment (given that she is alive in the previous year). we have:

$$u_t^3 = (1 - \psi_t)\frac{W_t^{1-\gamma}}{1-\gamma} \tag{4}$$

Let β be the agent's subjective discounting factor, V_t be the optimal state-value function for the agent. We can construct the following Bellman equation:

$$V_t(s) = \max_a \{u_t(a, s) + \beta E_t[V_{t+1}(s)]|\Theta\} \tag{5}$$

where a and s denote action variables and state variables, Θ denotes the parameters in the model.

There are three types of financial assets: short-term bond, long-term bond, and equity. The agent can allocate assets across them via two investment accounts: The first is a liquid savings account. She can deposit or withdraw money to or from this account at any time. The second is a tax-deferred defined-contribution pension account (hereafter referred to as "private pension account"). She can invest in this account each year prior to retirement, and withdraws funds from it since retirement. Meanwhile, the agent can invest in housing. We assume she can and can only either rent or own one house at a time.

Throughout the working life, the agent experiences the following six steps each year: (i) earning labor income and asset returns; (ii) making mandatory contribution to social insurances and housing fund; (iii) making voluntary contribution to private pension; (iv) paying income tax; (v) paying for her own and her children's consumption, paying supplementary health insurance, and making investment choices. If she meets certain conditions (such as having an outstanding mortgage), she may receive some tax rebates after step (iv). When she retires, she redeems the assets from the housing fund. Since then, the agent's labor income will be replaced by pension benefit and she will no longer need to implement step (ii)-(iv) or pay supplementary health insurance. Instead, she can withdraw money from private pension account, and will pay medical expenditure according to her health status. We assume the longest time she can live is T, i.e. $\psi_T = 0$.

rapidly from the 1980s to the 2000s and has remained consistently at a high level. This cannot be simply explained by the lack of social security or precautionary saving motive. Some influential explanations focus on inter-generation transfer. For example, Curtis, Lugauer, and Mark (2015, 2017) propose that the one-child policy, starting from 1980 and ending in 2016, led to a decline in family size, thus the Chinese parents can spend less in their children and have more to save. To capture this insight, we introduce the spending on children to the agent's objective function.

2.2 Income Process

2.2.1 Labor Income

Let y_t denote the agent's (real) pre-tax labor income.[3] Let random variables z_t^u, z_t^p, z_t^q be the employment status, permanent income shock, and transitory income shock. The deterministic part of labor income is represented by function $f_y(t, X_t)$, where X_t is the agent's demographic features (sex and education).[4] Employment status z_t^u can only be 0 (employed) or 1 (unemployed), $P\{z_t^u = 1\}$ denotes the probability of being unemployed. Transitory shock z_t^q follows an i.i.d. normal distribution. We assume permanent shock z_t^p follows a AR(1) process with innovations drawn from a mixture of two normal distributions - this allows the distribution of income shock to have a negative skewness and high kurtosis, as is shown in Guvenen, Ozkan, and Song (2014) and Guvenen et al. (2021). Therefore, the agent's labor income is determined by

$$
\begin{aligned}
& y_t = (1 - z_t^u) \cdot e^{f_y(t, X_t) + z_t^p + z_t^q} \\
& z_t^q \sim N(0, \sigma_q^2) \\
& z_t^p = \delta^p z_{t-1}^p + \eta_t \\
& \eta_t \sim \begin{cases} N(\mu_{\eta_1}, \sigma_{\eta_1}) \text{ with prob. } p_\eta \\ N(\mu_{\eta_2}, \sigma_{\eta_2}) \text{ with prob. } 1 - p_\eta \end{cases} \\
& P\{z_t^u = 1\} = 1/(1 + e^{-f_u(t, z_t^p)})
\end{aligned}
\tag{6}
$$

where $\mu_{\eta_1} p_\eta + \mu_{\eta_2}(1 - p_\eta) = 0$, and $\mu_{\eta,1} < 0$.

2.2.2 Pension Benefit

China's social security system consists of five social insurances and a housing fund. In every month during the working life, the agent and her employer make a mandatory contribution to these social security schemes, so long as she is employed. This contribution is tax-deductible. Normally, the annual contribution made by the agent herself is a fixed rate ξ_c multiplied by a fraction (or all) of her labor income for the previous year - the former (ξ_c) is termed as

[3] Given the agent's nominal pre-tax income $\tilde{Y}_t$ and price level P_t, we can calculate y_t by $y_t = \tilde{Y}_t / P_t$. In estimation, we set $P_t = 1$ in the first year of the panel data, and for the subsequent years, P_t grows at the observed inflation rate. In simulation, we replace the observed inflation rate by a fixed rate i_c.

[4] For how to estimation $f_y(t, X_t)$, see Campbell et al. (2001) and Cocco, Gomes, and Maenhout (2005).

"employee contribution rate",[5] the latter is termed as "social security contribution base". We represent the agent's nominal income by $\tilde{Y}_t$, and her contribution base by Y_t^B. If she is unemployed in the previous year, that is, she is a new employee to the current company, then Y_t^B will refer to her salary for the current year.

Legally, the social security contribution base cannot be less than 0.6 times the province-wide average wage, or more than 3 times the province-wide average wage. Let Y_t^* be the province-wide average wage, we can calculate Y_t^B via:

$$Y_t^B = \max\{\min\{\xi_b[\tilde{Y}_{t-1}z_{t-1}^u + \tilde{Y}_t(1 - z_{t-1}^u)], 3Y_t^*\}, 0.6Y_t^*\} \times z_t^u \tag{7}$$

where ξ_b is a parameter between 0 and 1. When the agent is unemployed at age t, we set $Y_t^B = 0$. For simplicity, we assume Y_t^* is a AR(1) process.

In China's social security schemes, the public pension scheme, which is also termed as "basic pension", accounts for the largest portion of the contribution. Normally, an employee contributes 8% of her contribution base to the basic pension every month. Her employer makes a match contribute equal to 16% of her contribution base to the basic pension.[6] Since retirement, the agent receives pension benefit B_t each year, which replaces her labor income. This benefit is constituted by two parts: the first part, which we term as B_t^1, comes from a scheme called "fundamental pension"; the second part, which we term B_t^2, comes from the agent's "individual account pension". The amount of fundamental pension benefit is dependent on the agent's previous contribution bases as well as province-wide average wage, whereas individual account pension benefit is dependent on the balance of her basic pension account.[7]

To calculate pension benefit, we suppose the agent starts working at age t_0, retires at t_{ret}. The employee contribution rate to basic pension is ξ_m ($\xi_m < \xi_c$), the employer's match contribution rate is ξ_n, the annuity rate for basic pension fund is i_b, the life expectancy in China is t_{exp}, the balance of the agent's basic pension account is A_t^B, where $A_{t_0}^B = 0$.

[5] China's five social insurances are public pension, health insurance, unemployment insurance, maternity insurance, work injury insurance. When calculate the employee contribution rate, we mainly consider basic pension (8%) and health insurance (2%), for the contribution rates for other insurances are trivial ($\leq 0.5\%$) compared to the two.

[6] Before 2019, the employer's contribution rate to basic pension is 20%.

[7] Readers interested in China's pension system can find useful information in Fang and Feng (2020), but please note this reference does not include any pension reform after 2019.

According to the official calculation rules, we define B_t by

$$
\begin{aligned}
B_t^1 &= \frac{1}{2}\left[\sum_{\tau=t_0}^{t_{ret}-1}\left(\frac{Y_\tau^B}{Y_\tau^*}\right) + \sum_{\tau=t_0}^{t_{ret}-1} z_\tau^u\right] \times Y_t^* \times 1\% \\
A_{t_{ret}}^B &= \sum_{\tau=t_0}^{t_{ret}-1}\left(A_\tau^B \cdot (1+i_b) + Y_\tau^B \cdot \xi_m\right) \\
B_t^2 &= A_{t_{ret}}^B / \left(\frac{1-(1+i_b)^{-(t_{exp}-t_{ret})}}{1-(1+i_b)^{-1}}\right) \\
B_t &= B_t^1 + B_t^2
\end{aligned}
\tag{8}
$$

We explain equation (8) briefly. B_t^1 equals to the average of the agent's indexed average wage and province-wide average wage, multiplied by the number of years in which she makes a contribution, then by 1%. The agent's indexed average wage refers to the average ratio of her contribution base to province-wide average wage across the working life. After simplification, we obtain the first line of equation (8). For the second line, $A_{t_{ret}}^B$ is the balance of the agent's individual pension account at retirement. For the third line, we choose B_t^2 to make the annual withdrawal from the individual pension account remain constant from the retirement age t_{ret} to the expected lifespan t_{exp}. Finally, we sum up fundamental pension and individual account pension benefit, then obtain B_t.

2.3 Financial Assets

2.3.1 Asset Markets

We assume the short-term bond return rate equals to a fixed risk-free rate, i_f. For the other assets, we assume their return rates follow a log-normal distribution. Let $\tilde{R}_{j,t}$ be the return of asset type j, we set $r_t^j = \ln(\tilde{R}_{j,t})$, where $j \in \{l, e, h, m\}$: r_t^l denotes log long-term bond interest rate, r_t^e denotes log equity return rate, r_t^h denote log housing price growth rate, r_t^m denote log commercial mortgage rate. We use a VAR(1) model to simulate the fluctuation of each r_t^j:

$$
\mathbf{r}_t = \delta_0^r + \delta_1^r * \mathbf{r}_{t-1} + \epsilon_t \tag{9}
$$

where $\mathbf{r}_t = [r_t^l, r_t^e, r_t^h, r_t^m]'$ is a four-dimension vector, ϵ_t follows a normal distribution with a mean of 0.[8]

[8] One way to construct Chinese housing price indices is to divide Chinese urban area to Tier-1, Tier-2, Tier-3 and Tier-4 cities, then constructing housing price index for each tier. See Fang et al. (2016) for details.

2.3.2 Investment Decisions

The agent has two investment accounts: a liquid savings account, and a private pension account. We use A_t^L and A_t^P to represent the balance of each account at the end of age t, $S_{j,t}^L$ and $S_{j,t}^P$ to represent the net flow into asset type j in each account.

The balance of liquid savings account is determined by

$$A_{t+1}^L = \sum_{j \in J} (\tilde{R}_{j,t+1} A_{j,t}^L + S_{j,t+1}^L) \tag{10}$$

where $A_{t+1}^L \geq -\xi_a \tilde{Y}$, ξ_a denotes the ratio of overdraft limit to labor income. The total saving in this account is $S_t^L = \sum_{j \in J} S_{j,t}^L$. We assume the agent can borrow some short-term bonds, but cannot short on long-term bond and equity. Therefore, $A_{j,t}^L \geq 0$ for any $j \in \{l, e\}$.

Chinese authority started implementing private pension scheme in 2022. It is notable that private pension differs from "individual account pension" we mentioned in subsection 2.2.2 in three significant ways: First, individual account pension belongs to basic pension, which is mandatory, whereas the participation in private pension scheme is voluntary. Second, the so-called "individual account" is managed by National Council for Social Security Fund, so people have no control over how the assets are allocated in their individual accounts. By contrast they can decide how much to invest and how to allocate the funds in their private pension accounts. Third, basic pension benefit is tax-free, while any withdrawal from the private pension account is taxed at 3%, i.e. the lowest rate of individual income tax.

The balance of private pension account is determined by

$$A_{t+1}^P = \sum_{j \in J} (\tilde{R}_{j,t} A_{j,t}^P + S_{j,t+1}^P) \tag{11}$$

Each year, net flow into the private pension account cannot exceed a certain limit. Hence, we set $S_{t+1}^P \equiv \sum_{j \in J} S_{j,t+1}^P \leq S_{\max}$, where $S_{\max}$ is the private pension saving limit. For all $j \in J$, $A_{j,t}^P \geq 0$. The domain for S_t^P is $[0, \infty)$ when $t < t_{ret}$ and is $(-\infty, 0]$ when $t \geq t_{ret}$. A negative S_t^P implies the agent withdrawing money from the account at age t.

2.4 Housing

2.4.1 Homeownership

Our settings about housing are similar to Cocco (2005) and Yao and Zhang (2005). The main difference is that when the agent borrows mortgage, in our model, she can borrow both a commercial loan from bank, and a housing fund loan from a government-run institute. This setting is based on China's housing provident fund system, a social security scheme that began to be implemented in 1999. Chinese authority referred to Singapore's Central Provident Fund when designing the system. According to this system, an employee is required to contribute a certain portion of her pre-tax labor income to a housing fund account every month. This contribution is normally 5%-12% of her social security contribution base. Her employer is also required to contribute an equal amount. These funds are managed by Housing Provident Fund Management Center run by each local authority, and can only be used for purchasing, decorating, repairing or renting houses. Individuals who have contributed to the housing fund can apply for a subsidized loan (under a certain limit) from the management center on their first or second home purchase. When the employee retires and has already repaid the loan, she can redeem the remaining funds from her housing fund account.

Let v_t be housing price, o_t be the agent's homeonwership choice, K_t be her housing expenditure (including renting and buying). The rent-to-price ratio is ϕ_{rp}, the transaction cost for home buying is ϕ_{buy} of the house value. We set $v_{t+1} = R_t^h v_t$. If the agent is a homeowner at age t, then $o_t = 1$; otherwise, $o_t = 0$. Negative K_t implies a selling of home. If the agent borrows a mortgage, the loan-to-value ratio she faces is ϕ_{lv} and the mortgage term is m. As we mentioned, h_t denotes the quantity of housing service she consumes at age t. We assume the agent either rents or owns only one home at a time.[9] For simplicity, we do not consider either the cost of moving and maintaining home or home value deprecation, and we assume the agent keeps living in her own home until selling it.

When the agent keeps to be a pure renter at $t + 1$, we set

$$K_{t+1}\{o_t = 0, o_{t+1} = 0\} = \phi_{rp} v_{t+1} h_{t+1} \tag{12}$$

[9]This setting is plausible because of the restrictive home purchase policies in many Chinese cities. Normally, individuals face higher downpayments and higher mortgage rates for their second and third home purchase. Third-time home buyers cannot apply for housing fund loans. In Tier-1 and Tier-2 cities, the eligibility to purchase newly-built housing is usually determined by a lottery system. People not owning a house have a higher chance of winning the lottery.

When the agent buys a home at $t+1$, i.e. becoming a homeowner,

$$
\begin{aligned}
K_{t+1}\{o_t = 0, o_{t+1} = 1\} &\leq (1+\phi_{buy})v_{t+1}h_{t+1} \\
K_{t+1}\{o_t = 0, o_{t+1} = 1\} &\geq (1-\phi_{lv}+\phi_{buy})v_{t+1}h_{t+1}
\end{aligned}
\tag{13}
$$

The agent's mortgage debt consists of both commercial loan and housing fund loan. Let D_t be the agent's mortgage debt ($D_t \geq 0$), D_t^H denote the agent's outstanding housing fund loan ($0 \leq D_t^H \leq D_t$) . The interest rate for housing fund loan is i_d lower than the commercial mortgage rate $\tilde{R}_{m,t}$. If she borrows at $t+1$, she has to repay the debt in each subsequent year.

$$
D_{t+1} = \begin{cases} v_{t+1}h_{t+1} - K_{t+1}, & \text{if } o_t = 0, o_{t+1} = 1 \\ (\tilde{R}_{m,t} - i_d)D_t^H + \tilde{R}_{m,t}(D_t - D_t^H) - K_{t+1}, & \text{if } o_t = 1, o_{t+1} = 1, D_t > 0 \\ 0, & \text{else} \end{cases}
\tag{14}
$$

Before paying off the mortgage, i.e. $D_t > 0$, the agent is not allowed to change or sell the home, or borrow a new mortgage. Under this circumstance, the domain for o_{t+1} is $\{1\}$ and the domain for h_{t+1} is $\{h_t\}$ (otherwise they will be $\{0,1\}$ and $[0,\infty)$).

In each installment, the repayment she made should be no less than a minimum limit. Therefore,

$$
\begin{aligned}
&K_{t+1}\{o_t = 1, o_{t+1} = 1, D_t > 0\} \geq \Phi_{t+1}^{HFL}D_t^H + \Phi_{t+1}^{CL}(D_t - D_t^H) \\
&K_{t+1}\{o_t = 1, o_{t+1} = 1, D_t > 0\} \leq D_t \\
&K_{t+1}\{o_t = 1, o_{t+1} = 1, D_t = 0\} = 0
\end{aligned}
\tag{15}
$$

where Φ_t^k ($k \in \{CL, HFL\}$) denotes the ratio of minimum repayment to debt for each loan type (CL denotes commercial loan, HFL denotes housing fund loan). We set that, for the remaining mortgage term, if she repays equal amount each year , then this amount is the minimum repayment. Therefore,

$$
\Phi_t^k = {i_{k,t}}^{-1} / \left(\frac{1 - {i_{k,t}}^{-(m_t^k+1)}}{1 - {i_{k,t}}^{-1}} \right)
\tag{16}
$$

where $k \in \{CL, HFL\}$, $i_{CL,t} = \tilde{R}_{m,t}$, $i_{HFL,t} = \tilde{R}_{m,t} - i_b$. m_t^k denotes the length of remaining mortgage term for loan type k, and

$$
\begin{aligned}
m_{t+1}^{CL} &= \begin{cases} m, & \text{if } o_t = 0, o_{t+1} = 1 \\ m_t^{CL} - 1, & \text{if } o_t = 1, o_{t+1} = 1, D_t - D_t^H > 0 \\ 0, & \text{else} \end{cases} \\
m_{t+1}^{HFL} &= \begin{cases} \min\{m, t_{ret} - t\}, & \text{if } o_t = 0, o_{t+1} = 1 \\ m_t^{HFL} - 1, & \text{if } o_t = 1, o_{t+1} = 1, D_t^H > 0 \\ 0, & \text{else} \end{cases}
\end{aligned} \tag{17}
$$

where the maximum mortgage term is $\min\{m, t_{ret} - t\}$, implying that the housing fund loan should be paid off before retirement.

When the agent sells a house at $t + 1$,

$$
K_{t+1}\{o_t = 1, o_{t+1} = 0\} = -v_{t+1} h_{t+1} \tag{18}
$$

2.4.2 Housing Fund

During the working life, the agent's annual contribution to the housing fund account is a fixed rate ξ_h ($\xi_h < \xi_c$) multiplied by her contribution base. Her employer's match contribution is at the same amount. The funds in housing fund account grow at risk-free rate. At retirement, the agent redeems the remaining funds from the account. Let A_t^H denote the balance of housing fund account, S_t^H denote the money withdrawn from the account, K_t^H denote the repayment for housing fund loan, then before retirement,

$$
A_{t+1}^H = \begin{cases} (1 + i_f) A_t^H + 2\xi_h Y_{t+1}^B - S_{t+1}^H, & \text{if } t + 1 < t_{ret} \\ 0, & \text{if } t + 1 \geq t_{ret} \end{cases} \tag{19}
$$

where $0 \leq S_{t+1}^H \leq K_{t+1}$, $A_{t+1}^H \geq 0$.

The agent's debt on housing fund loan is

$$
D_{t+1}^H = \begin{cases} \min\{\phi_h A_t^H, \bar{D}_H, D_{t+1}\}, & \text{if } o_t = 0, o_{t+1} = 1 \\ (\tilde{R}_{m,t} - i_d) D_t^H - K_{t+1}^H, & \text{if } o_t = 1, o_{t+1} = 1, D_t^H > 0 \\ 0, & \text{else} \end{cases} \tag{20}
$$

where $0 \leq K_{t+1}^H \leq K_{t+1}$, ϕ_h denotes the ratio of housing fund loan the agent's can borrow to her housing fund account balance in the previous year, $\bar{D}_H$ denotes the ceiling of housing fund loan. When the agent is a pure renter, i.e. $o_{t+1} = 0$, she can also use housing fund to pay her rent. In this case, $K^H_{t+1} = 0$, $0 \leq S^H_{t+1} \leq \phi_{rp} v_{t+1} h_{t+1}$.

When solving our model, we assume the agent's homeownership o_t, house size h_t, housing expenditure K_t, withdrawl from the housing fund account S^H_t, and housing fund loan repayment K^H_t are her action variables. She control these variables and use equation (14)(19)(20) to decide the total mortgage debt D_t, housing fund loan debt D^H_t, and the balance of housing fund account A^H_t. Moreover, the range of K_t is constrained by o_t and h_t, and the ranges of K^H_t and S^H_t are constrained by K_t.

2.5 Supplementary Health Insurance

Like Duarte et al. (2021), we consider the agent's working life and retirement life separately. Throughout the working life, the agent keeps at the good health status and does not need to pay medical expenditure. Since retirement, her health status shifts between several values randomly and she spends in medical care each year, according to her age, health status, and income.

Following Koijen, Van Nieuwerburgh, and Yogo (2016), we assume the agent's health status e_t is one of $\{0, \underline{e}, 1\}$, where $e_t = 0$ indicates death, $e_t = \underline{e}$ indicates bad status, $e_t = 1$ indicates good status ($0 < \underline{e} < 1$). Before retirement, we set $e_t = 1$. Since retirement, we denote the transition probability between the health status by $P\{e_{t+1}|e_t\}$ and set $e_t = 0$ (death) as an absorbing status. Thus, $P\{0|0\} = 1$, the agent's survival probability at the end of $t+1$ conditional on e_t is:

$$\psi_{t+1}|e_t = 1 - P\{0|e_t\} \tag{21}$$

Let M_t denote the agent's out-of-pocket medical expenditure at age t. Before retirement and at the time of death, $M_t = 0$. The deterministic part of log medical expenditure is given by $f_m(t, y_t, e_t)$. Following French and Jones (2004) and De Nardi, French, and Jones (2010), we have:

$$\begin{aligned}
&\ln(M_t) = f_m(t, y_t, e_t) + \zeta^p_t + \zeta^q_t \\
&\zeta^p_t = \delta^\zeta \zeta^p_{t-1} + \omega_t \\
&\omega_t \sim N(0, \sigma_\omega) \\
&\zeta^q_t \sim N(0, \sigma_\zeta)
\end{aligned} \tag{22}$$

where ζ_t^p is the permanent shock to medical expenditure, ζ_t^q is the transitory shock to medical expenditure. ζ_t^p follows a AR(1) process with a normally-distributed innovation, ζ_t^q follows a normal distribution.

Our setting about supplementary health insurance is similar to Koijen, Van Nieuwerburgh, and Yogo (2016). The agent can buy insurances at different ages during her working life (from t_0 to $t_{ret}-1$), and these insurances remains valid until her death. The compensation of each supplementary health insurance can cover the difference in medical expenditure between good bad health status, and the insurance company makes zero profit. Let I_t denote the price of insurance that the agent buys at age t. Then,

$$\begin{aligned} I_t &= \sum_{\tau=t_{ret}}^{T} \frac{\psi_\tau^e \cdot E[\Delta M_\tau]}{(1+i_f)^{\tau-t}} \\ \Delta M_\tau &= (f_m(\tau, y_\tau, \underline{e}) - f_m(t, y_\tau, 1)) \cdot e^{\zeta_\tau^p+\zeta_\tau^q} \\ \psi_\tau^e &= P\{e_\tau = \underline{e}\} \end{aligned} \tag{23}$$

where ψ_τ^e is the unconditional probability of $e_\tau = \underline{e}$, ΔM_τ is the medical expenditure under bad health status minus that under good health at age τ. We normalize the amount of insurance the agent purchases at age t to g_t, where $g_t \in [0,1]$ and $\sum_{t=t_0}^{t_{ret}} g_t \leq 1$. Therefore, the premium that she pays at age t is $g_t \cdot I_t$.

2.6 Budget Constraints

The agent's income tax $L(Y_t^{tax})$ is a function to of the taxable income Y_t^{Tax}. In China, taxable income is the agent's nominal income minus social security contribution and special additional deductions, then minus tax-exempt income threshold. As is mentioned above, the agent's social security contribution is $\xi_c Y_t^B$. Three elements in our model can be counted as special additional deductions: private pension savings, spending on each child, housing expenditure. Expenses in these fields can be deducted from pre-tax income before tax is calculated.

The special additional deduction has two modes: one is to deduct a fixed amount Γ from pre-tax income, the other is to deduct actual expenses. The deduction for every child supported by the agent, housing rent, and mortgage repayment applies for the former. Private pension savings and medical expenditure applies to the latter. Let $\underline{Y}$ denote the tax-exempt income threshold. We can calculate the agent's taxable income by

$$Y_t^{Tax} = \max\{0, \tilde{Y}_t - \xi_c Y_t^B - \Gamma(n_t + \mathbf{1}\{o_t = 0\} + \mathbf{1}\{o_t = 1, D_t > 0\}) - S_t^P - \underline{Y}\} \tag{24}$$

China's income tax system was reformed in 2008, 2011 and 2018. To approximate the statutory tax progressivity, we follow Heathcote, Storesletten, and Violante (2017) to define the income tax function by:

$$L(Y_t^{Tax}) = Y_t^{Tax} - \delta_0^l (Y_t^{Tax})^{\delta_1^l} \tag{25}$$

where δ_0^l determines the average taxation level of the economy, δ_1^l determines the elasticity of post-tax to pre-tax income.

The agent's net wealth is the sum of her basic pension account balance, private pension account balance, liquid savings account balance, housing fund account balance, housing value minus mortgage debt (if she is a homeowner):

$$W_t = A_t^B + A_t^P + A_t^L + A_t^H + o_t v_t h_t - D_t \tag{26}$$

Before retirement, the agent's non-housing consumption is her labor income minus social security contribution, total spending on her children, income tax, housing expenditure that is not paid by withdrawals from housing fund account, private pension and liquid savings, and supplementary health insurance premium. To include government transfer to low-income households in our model, we set a non-housing consumption floor, $\underline{c}$. Hence,

$$c_t = \max\{\tilde{Y}_t - \xi_c Y_t^B - n_t F_t - L(Y_t^{Tax}) - (K_t - S_t^H) - (S_t^L + S_t^P) - g_t I_t, \underline{c}\} \tag{27}$$

Since retirement, the agent's non-housing consumption is her pension benefit and the lump-sum redemption from housing fund, and withdrawals from private pension account, minus liquid savings, total spending on her children, housing expenditure, medical expenditure that is uncovered by health insurance. Also, we consider a consumption floor:

$$c_t = \max\{B_t + A_t^H \mathbf{1}\{t = t_{ret}\} - (1 - \xi_p) S_t^P - S_t^L - n_t F_t - K_t - (1 - \sum_{t=t_0}^{t_{ret}} g_t) M_t, \underline{c}\} \tag{28}$$

where ξ_p is the tax rate for withdrawals from private pension account.

In summary, the agent's action variables are $a = \{F_t, h_t, o_t, g_t, K_t, K_t^H, S_t^H, S_{j,t}^P, S_{j,t}^L, \phi_e\}$, the state variables are $s = \{e_t, c_t, M_t, \tilde{Y}_t, Y_t^B, B_t, A_t^B, A_t^L, A_t^P, A_t^H, D_t, D_t^H, W_t\}$, where $j \in \{l, e\}$.

3 Parametrization

We estimate the parameters in income process and tax function using longitudinal household survey data (e.g. CFPS, CHFS). The number of dependent children n_t is also estimated on the same dataset.

Health status transition probabilities and medical expenditure are estimated using China Health and Retirement Longitudinal Study (CHARLS) data.

For long-term bond return, we can refer to 30Y Government Bond and 30Y Financial Bond of Commercial Bank (AAA). For equity, we can use CSI 300 or a capitalization-weighted index for all stocks listed on Shanghai Stock Exchange and Shenzhen Stock Exchange. For housing price, we can refer to the sales prices of residential buildings in 70 medium and large-sized cities, reported by National Bureau of Statistics of China.

Preference-related parameters, esp. the impact of children number on overall utility κ_2 and bequest motive κ_3, are better estimated with method of simulated moment after we solve the model.

The other parameters are listed in the following table:

Parameter	Description	Value	Source
Age			
t_0	age of start working	20	Calibration
t_{ret}	age of retirement	60	Calibration
t_{exp}	life expectancy	75	Calibration
T	maximum lifespan	99	Calibration
Preferences			
α	concavity for the impact of the number of children on utility	0.76	Curtis, Lugauer, and Mark (2015)
β	discounting factor	0.99	Calvet et al. (2021)
γ	relative risk aversion	5.24	Calvet et al. (2021)
$\underline{e}$	bad health status	0.74	Koijen, Van Nieuwerburgh, and Yogo (2016)

Parameter	Description	Value	Source
κ_1	the impact of health status on overall utility	0.3	De Nardi, French, and Jones (2010)
Asset Markets			
i_b	annuity rate	4%	Calibration
i_c	inflation rate	2%	Calibration
i_d	spread between commercial mortgage and housing fund loan	2%	Calibration
i_f	risk-free rate	2%	Calibration
Social Security			
ξ_b	ratio of contribution base to labor income	100%	Calibration
ξ_c	employee contribution rate to all social security schemes	15%	Chinese gov.
ξ_m	employee contribution rate to basic pension	8%	Chinese gov.
ξ_h	employee contribution rate to housing provident fund	5%	Chinese gov.
ξ_n	employer's match contribution rate to basic pension	16%	Chinese gov.
ξ_p	tax rate for withdrawals from private pension	3%	Chinese gov.
Housing and Investment			
S_{max}	private pension saving limit	12,000	Chinese gov.
ξ_a	ratio of overdraft limit to labor income	50%	calibration

Parameter	Description	Value	Source
$\bar{D}_H$	housing fund loan ceiling	600,000	calibration
ϕ_{rp}	rent-to-price ratio	1/600	calibration
ϕ_{buy}	transaction-cost-to-home-value ratio in home buying	5%	calibration
ϕ_{lv}	loan-to-value ratio	0.7	calibration
ϕ_h	maximum ratio of housing fund loan to housing fund account balance	15	calibration
m	mortgage term	20	calibration
Tax and Government Transfer			
Γ	fixed special additional deduction	12,000	Chinese gov.
$\underline{Y}$	tax-exempt income threshold	60,000	Chinese gov.
$\underline{c}$	consumption floor	3,6000	calibration
δ_0^l	average taxation level	0.15	estimation
δ_1^l	elasticity of post-tax income to pre-tax income	0.98	estimation

Reference

Barro, Robert J., and Gary S. Becker. 1989. "Fertility Choice in a Model of Economic Growth." *Econometrica* 57 (2): 481–501. https://doi.org/10.2307/1912563.

Calvet, Laurent, John Campbell, Francisco Gomes, and Paolo Sodini. 2021. "The Cross-Section of Household Preferences." w28788. Cambridge, MA: National Bureau of Economic Research. https://doi.org/10.3386/w28788.

Campbell, John, Joao Cocco, Francisco Gomes, and Pascal Maenhout. 2001. "Investing

Retirement Wealth: A Life-Cycle Model." In *Risk Aspects of Investment-Based Social Security Reform*, 439–82. National Bureau of Economic Research Conference Report. Chicago: University of Chicago Press. https://doi.org/10.3386/w7029.

Cocco, João F. 2005. "Portfolio Choice in the Presence of Housing." *Review of Financial Studies* 18 (2): 535–67. https://doi.org/10.1093/rfs/hhi006.

Cocco, João F., Francisco J. Gomes, and Pascal J. Maenhout. 2005. "Consumption and Portfolio Choice over the Life Cycle." *Review of Financial Studies* 18 (2): 491–533. https://doi.org/10.1093/rfs/hhi017.

Curtis, Chadwick C., Steven Lugauer, and Nelson C. Mark. 2015. "Demographic Patterns and Household Saving in China." *American Economic Journal: Macroeconomics* 7 (2): 58–94. https://doi.org/10.1257/mac.20130105.

———. 2017. "Demographics and Aggregate Household Saving in Japan, China, and India." *Journal of Macroeconomics* 51 (March): 175–91. https://doi.org/10.1016/j.jmacro.2017.01.002.

De Nardi, Mariacristina, Eric French, and John B. Jones. 2010. "Why Do the Elderly Save? The Role of Medical Expenses." *Journal of Political Economy* 118 (1): 39–75. https://doi.org/10.1086/651674.

Duarte, Victor, Julia Fonseca, Aaron Goodman, and Jonathan Parker. 2021. "Simple Allocation Rules and Optimal Portfolio Choice over the Lifecycle." w29559. Cambridge, MA: National Bureau of Economic Research. https://doi.org/10.3386/w29559.

Fang, Hanming, and Jin Feng. 2020. "The Chinese Pension System." In *The Handbook of China's Financial System*, 421–43. Princeton: Princeton University Press. https://lccn.loc.gov/2020005409.

Fang, Hanming, Quanlin Gu, Wei Xiong, and Li-An Zhou. 2016. "Demystifying the Chinese Housing Boom." *NBER Macroeconomics Annual* 30 (1): 105–66. https://doi.org/10.1086/685953.

French, Eric, and John Bailey Jones. 2004. "On the Distribution and Dynamics of Health Care Costs." *Journal of Applied Econometrics* 19 (6): 705–21. https://doi.org/10.1002/jae.790.

Guvenen, Fatih, Fatih Karahan, Serdar Ozkan, and Jae Song. 2021. "What Do Data on Millions of u.s. Workers Reveal about Lifecycle Earnings Dynamics?" *Econometrica* 89 (5): 2303–39. https://doi.org/10.3982/ECTA14603.

Guvenen, Fatih, Serdar Ozkan, and Jae Song. 2014. "The Nature of Countercyclical Income Risk." *Journal of Political Economy* 122 (3): 621–60. https://doi.org/10.1086/675535.

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante. 2017. "Optimal Tax Progressivity: An Analytical Framework." *The Quarterly Journal of Economics* 132 (4):

1693–1754. https://doi.org/10.1093/qje/qjx018.

Hubbard, R. Glenn, Jonathan Skinner, and Stephen P. Zeldes. 1994. "The Importance of Precautionary Motives in Explaining Individual and Aggregate Saving." *Carnegie-Rochester Conference Series on Public Policy* 40 (June): 59–125. https://doi.org/10.1016/0167-2231(94)90004-3.

Koijen, Ralph S. J., Stijn Van Nieuwerburgh, and Motohiro Yogo. 2016. "Health and Mortality Delta: Assessing the Welfare Cost of Household Insurance Choice: Health and Mortality Delta." *The Journal of Finance* 71 (2): 957–1010. https://doi.org/10.1111/jofi.12273.

Palumbo, Michael G. 1999. "Uncertain Medical Expenses and Precautionary Saving Near the End of the Life Cycle." *Review of Economic Studies* 66 (2): 395–421. https://doi.org/10.1111/1467-937X.00092.

Yao, Rui, and Harold H. Zhang. 2005. "Optimal Consumption and Portfolio Choices with Risky Housing and Borrowing Constraints." *Review of Financial Studies* 18 (1): 197–239. https://doi.org/10.1093/rfs/hhh007.

Yogo, Motohiro. 2016. "Portfolio Choice in Retirement: Health Risk and the Demand for Annuities, Housing, and Risky Assets." *Journal of Monetary Economics* 80 (June): 17–34. https://doi.org/10.1016/j.jmoneco.2016.04.008.