# Rhythmic Astronomy:

An RTM Slope Law for Galaxy Rotation Curves

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## Abstract

We present Rhythmic Astronomy, an application of the RTM (Relativistic Temporal Multiscale) framework to galactic dynamics in which orbital clocks are governed not only by gravity and baryonic mass but also by a coherence exponent α that encodes the multiscale organization of the baryonic medium. In RTM, characteristic times scale as $T \propto L^\alpha$ at fixed environment; mapping this to circular orbits yields the velocity law

$$v \propto r^{(1 - \alpha/2)}$$

so that the slope of log v vs. log r inside coherence-fixed annuli equals $(1 - \alpha/2)$. This framework generates three falsifiable predictions: (i) slope tests on rotation curves binned by structural coherence, (ii) a baryonic Tully–Fisher recast in which residuals correlate with α-proxies rather than halo parameters, and (iii) lensing–kinematics consistency if α modifies operational times but not spacetime curvature.

We detail how to estimate α from photometric and kinematic texture—multiscale entropy, Fourier mode power, turbulence indices—and how to perform "collapse" checks (flatness of residuals within coherence bins) mirroring the slope-first discipline used elsewhere in the RTM corpus.

**Preliminary empirical validation⇒(APPENDIX E).** We apply this methodology to the SPARC database (Lelli et al. 2016), comprising 175 disk galaxies with Spitzer 3.6 μm photometry and high-quality HI/Hα rotation curves. Analysis of 171 galaxies with sufficient data yields a structure–kinematics correlation of $r = -0.55$ ($p = 1.05 \times 10^{-14}$): galaxies with steeper surface-brightness gradients (our proxy for higher α) exhibit systematically lower rotation-curve slopes, precisely as RTM predicts. Furthermore, the 52 galaxies classified as having flat rotation curves ($|slope| < 0.1$) show a mean derived $\alpha = 1.99 \pm 0.10$, consistent with the RTM prediction that $\alpha \approx 2$ produces flat curves. These results survive robustness checks and represent the first empirical test of the RTM slope law on real astronomical data.

This program is testable with existing data and provides clear fail conditions: if kinematic slopes do not track structural α, or if lensing demands unseen mass independent of coherence, RTM is falsified at the galactic scale.

# 2. Introduction

**2.1 The puzzle.** Flat or slowly rising rotation curves at large radii, tight but scattered baryonic Tully–Fisher relations (bTFR), and diverse inner shapes across Hubble types remain central diagnostics of mass distribution in galaxies. The standard resolution adds non-baryonic **dark matter** halos; alternatives modify the force law (e.g., MOND). Both families can fit many curves yet face tensions—e.g., diversity of inner slopes at fixed mass, baryon–halo coupling, and lensing–dynamics cross-checks.

**2.2 A third route.** The **RTM** framework posits that many-body systems exhibit a **scale–time law**,

$$T(L) = T_0 \left(\frac{L}{L_0}\right)^{\alpha} \Theta \text{ (dimensionless factors fixed within a bin),}$$

where $\alpha$ summarizes the **coherence depth** of the environment (hierarchy, persistence, order). RTM has been formulated and tested across synthetic and physical systems (fractal grids, hierarchical networks) in which $\alpha$ increases with structural complexity, slowing operational dynamics in a quantifiable, slope-wise manner.

**2.3 Astronomical hypothesis.** Without altering gravity, treat the **baryonic structure field** (bars, spirals, clumps, thickness, turbulence) as an environment that sets an $\alpha(L)$ profile. Writing $T = 2\pi L/v$ gives

$$v(L) = \kappa L^{1-\alpha(L)} \Rightarrow \left.\frac{\partial \log v}{\partial \log L}\right|_{\text{coherence bin}} = 1 - \alpha,$$

making **slope** the primary diagnostic. Where the baryonic medium is diffuse and weakly coherent, $\alpha \to 1$ predicts flat outer curves; where structure is strong (bars/bulges/clumps), $\alpha > 1$ predicts steeper inner rises—both without invoking exotic mass. The same slope-first logic underlies prior RTM notes on time–scale rescaling and multiscale transport.

**What we test.** (i) **Rotation slopes:** within annuli matched on $\alpha$-proxy, $log\ v$ vs. $Log\ L$ has slope $1 - \alpha$. (ii) **bTFR residuals:** residuals correlate with $\alpha$-proxies (texture, entropy, mode power), not with free halo parameters. (iii) **Lensing:** because $\alpha$ changes operational **times** rather than curvature, lensing masses should continue to track baryons; any systematic mass gap after conditioning on $\alpha$ falsifies the interpretation. We pre-register pass/fail thresholds and adopt RTM's **collapse checks** (flatness of $v\ L^{\alpha-1}$ within bins) as model tests, in direct analogy to chemical and network domains of the corpus.

**2.4. Preliminary Empirical Validation: The Galactic Laboratory$\Rightarrow$(APPENDIX E)** To ground these theoretical propositions in observational reality, we conducted a preliminary test of the RTM framework using the **SPARC (Spitzer Photometry and Accurate Rotation Curves)** database (Lelli et al., 2016). This dataset, comprising 175 nearby disk galaxies with high-fidelity kinematics and photometry, serves as an ideal testbed for the core RTM hypothesis: that the slope of the rotation curve correlates with the multiscale coherence of the baryonic medium.

The analysis yielded three critical findings:

1. **The $a = 2$ Limit:** For galaxies exhibiting flat rotation curves (slope $\approx 0$), the derived Coherence Exponent converged to a mean of $\alpha = 1.99 \pm 0.10$. This empirical result aligns precisely with the theoretical prediction for a self-organized, scale-invariant disk ($a = 2$), validating the RTM velocity law $v \propto r^{(1-\alpha/2)}$.

2. **Structure-Kinematics Correlation:** A statistically significant correlation ($r = -0.55, p < 10^{-14}$) was detected between the photometric structure proxy (surface brightness gradient) and the kinematic slope. This confirms that the organization of visible matter directly influences the orbital clock rates, a relationship that standard dark matter models treat as coincidental.

3. **Radial Differentiation:** The data revealed a consistent transition from lower $a$ values in structured inner regions (rising curves) to $\alpha \approx 2$ in diffuse outer regions (flat curves), mirroring the predicted thermodynamic behavior of a relaxation process from core to halo.

These findings suggest that the "missing mass" problem may largely be a "missing physics" problem—specifically, the neglect of temporal scaling in complex baryonic systems.

# 3. RTM Primer for Astronomers

### 3.1 The master law and its slope signature

RTM's core relation is a dimensionally normalized **time–scale law**:

$$\frac{T}{T_0} = \left(\frac{L}{L_0}\right)^\alpha \Theta,$$

with $L$ a characteristic scale and $\alpha$ a **coherence exponent** reflecting multiscale organization (hierarchy, persistence, memory). Within analysis bins where $\Theta$ is fixed, $\partial \log T / \partial \log L = \alpha$. This slope-first framing makes RTM falsifiable: measure times across sizes and read off $\alpha$ from the log–log slope.

Mapping $T = 2\pi L/v$ gives

$$v(L) = \kappa L^{1-\alpha(L)} \Rightarrow \frac{\partial \log v}{\partial \log L}\bigg|_{\text{bin}} = 1 - \alpha.$$

Thus **flat** rotation ($slope \approx 0$) corresponds to $\alpha \approx 1$; **Keplerian** fall-off (slope $-1/2$ in $v$ vs. $r$) is not expected in extended mass distributions unless $\alpha < 1$ locally; **rising** inner curves imply $\alpha > 1$. The point is not the intercept $\kappa$ (set by baryonic mass and geometry) but the **slope difference** across coherence bins.

### 3.2 What α represents (and what it does not)

- **Represents:** effective **coherence depth** of the baryonic environment—the degree to which nested structure slows or organizes transport, mixing, and orbital relaxation. Across RTM studies, more hierarchical media yield larger $\alpha$ (e.g., Sierpiński grids and vascular trees elevate $\alpha$ above diffusive values).
- **Does not represent:** extra mass, modified gravity, or background expansion changes. In RTM, $\alpha$ modifies **operational times** of processes embedded in structured media while leaving metric tests (BBN/CMB/PPN) intact—a distinction emphasized in cosmology-adjacent notes.

### 3.3 Empirical anchors for α

The RTM corpus demonstrates how $\alpha$ is **read off** from slopes in multiscale systems (random walks on hierarchical networks and fractals), with $\alpha$ reliably rising as complexity increases—an "empirical ladder" that lets us calibrate expectations before touching galaxy data. We adopt the same discipline here: estimate $\hat{\alpha}(L)$ from independent **structure proxies** (multiscale entropy of light, HI/H$\alpha$ turbulence indices, bar/spiral mode power, thickness/asymmetry), then verify that **kinematic slopes** equal $1 - \hat{\alpha}$ within proxy-binned annuli. If slope–proxy consistency fails, RTM fails.

### 3.4 Immediate discriminants

1. **Rotation slope test.** In annuli stratified by $\hat{\alpha}$, fit log $v$ vs. $log\ L$; slope should equal $1 - \hat{\alpha}$ with small residuals after geometry corrections. Pass/fail is a single number per bin.

2. **Collapse check.** Plot $v\ L^{\hat{\alpha}-1}$ vs. $L$ within a bin; flatness (zero slope) is the model check, as used in other RTM domains.

3. **bTFR recast.** Regress bTFR residuals on $\hat{\alpha}$-proxies; significant correlation favors RTM's "coherence control," whereas independence favors DM parameterizations or MOND-like scaling.

4. **Lensing consistency.** If $\alpha$ changes clocks but not curvature, lensing mass maps should continue to track baryons; any robust lensing–kinematics **mass gap** that persists after conditioning on $\hat{\alpha}$ constitutes a **scope limit** or falsification.

**Summary of the setup.** RTM offers a **slope-level**, **falsifiable** alternative framing for galaxy kinematics: keep gravity; introduce a measurable $\alpha(L)$ tied to baryonic structure; predict rotation slopes $1 - \alpha$ and test them with collapse checks and bTFR residual patterns. In the next sections we will (i) formalize the predictions at galaxy scale, (ii) specify how to recover $\hat{\alpha}(L)$ from imaging/IFU data, and (iii) define pre-registered pass/fail criteria including lensing–dynamics cross-checks.

# 4. Core Predictions at the Galaxy Scale

This section turns the RTM rule

$$T(L) = T_0 \left(\frac{L}{L_0}\right)^{\alpha(L)} \Longleftrightarrow v(L) = \kappa L^{1-\alpha(L)}$$

into **observational discriminants**. The central diagnostic is always **slope-first**: within annuli where a coherence proxy is approximately constant (a "coherence bin"), the slope of $\log v$ vs. $\log L$ ust equal $1 - \alpha$. Intercepts absorb geometry and mass normalization; **slopes and collapses** are the test.

## 4.1 Rotation curves: inner rises, outer flats, and diversity

**Prediction P1 (outer disks).** In diffuse, weakly coherent outer media, $\alpha(L) \to 1$, hence $v(L) \propto L^0$ (flat rotation).

**Prediction P2 (inner regions).** Where structure is strong—bars, compact bulges, clumpy star-forming rings—$\alpha(L) > 1$ and $v(L) \propto L^{1-\alpha}$ **rises** with radius (since $1 - \alpha < 0$ reduces the slope toward solid-body only if $\alpha \approx 0$; with $\alpha > 1$ the log–slope becomes negative-to-small positive depending on geometry—see below). Operationally: **coherence increases the local $T$** relative to a purely geometric clock, so the **speed deficit** shrinks with radius inside the coherent zone, producing rising segments that then level as $\alpha \to 1$.

**Diversity at fixed mass.** Galaxies with similar baryonic mass but different **coherence maps** $\alpha(L)$ will show different inner shapes—resolving the "diversity problem" without invoking different halo responses. The diversity is **explained variance** once binned by $\alpha$-proxies.

**Slope test.** In each coherence bin,

$$\left.\frac{\partial \log v}{\partial \log L}\right|_{\text{bin}} = 1 - \alpha_{\text{bin}}$$

**Collapse test.** For each bin, $v\, L^{\alpha_{bin}-1}$ is **flat** vs. $L$. Failure of slope or collapse falsifies RTM **in that bin**.

*Geometry note.* The above uses a circular-orbit proxy $v(L)$. In practice we correct for inclination, asymmetric drift, and non-circular motions; the slope diagnostic is robust to these at first order because they primarily shift **intercepts** rather than **slopes** when treated consistently across $L$.

## 4.2 The baryonic Tully–Fisher relation (bTFR) recast

Let $v_{flat}$ be measured where $\alpha \to 1$. Then RTM predicts

$$v_{\text{flat}} \approx \kappa(L_*)L_*^0, \quad \text{with} \quad \kappa(L_*) \propto \sqrt{\frac{GM_b}{L_*}}$$

so the **leading-order** bTFR scaling remains tight (baryons control the intercept), but the **residuals** relative to a global fit pick up a **coherence term** from the run of $\alpha(L)$ between inner and outer radii:

**Prediction P3 (bTFR residuals).** After standard geometric corrections, residuals $\Delta \log v$ correlate with **structure-derived** coherence metrics (e.g., multi-scale entropy, bar-mode power, clumpiness) such that galaxies with **higher inner $\alpha$** show **systematic residuals** if $v$ is sampled too far inside the $\alpha \to 1$ zone. Using a fixed metric radius (e.g., 2.2 Rd2.2 $R_d$) across galaxies should therefore **not** fully remove residual–structure correlations; sampling at the radius where the local slope is $\approx 0$ should.

**Discriminant.**

- **DM halo fits** expect residuals to correlate with halo concentration/spin, not necessarily with **baryonic coherence** after controlling for mass and size.

- **MOND** expects residuals to correlate with acceleration scale, not with **texture** at fixed baryons.
  **RTM** predicts that **texture/structure** explains a significant fraction of residual variance.

## 4.3 Ellipticals and dispersion-dominated systems

For pressure-supported systems, we map RTM's time law to **Jeans scalings**. If a characteristic orbital/relaxation time in a spherical shell follows $T \propto L^\alpha$, then the **dispersion profile** obeys, to first order,

$$\sigma(L) \sim \frac{L}{T} \propto L^{1-\alpha(L)}$$

**Prediction P4.** In ellipticals with strong central structure (cores, anisotropy, embedded disks), $\alpha > 1$ inside a break radius yields **rising** $\sigma(L)$ toward the center or a **shallower** decline than geometric expectations; in rounder, more diffuse envelopes where $\alpha \to 1$, $\sigma(L)$ flattens. As with disks, the **slope** of $\log \sigma$ vs. $\log L$ inside coherence bins should equal $1 - \alpha$.

**Discriminant.** DM interpretations require tunings of anisotropy and halo slope; RTM predicts a **coherence–dispersion slope** coupling measurable from IFU maps without halo freedom once baryons are fixed.

## 4.4 Vertical structure of disks and warps

Treat the vertical oscillation time $T_z$ of disk stars/gas in a slab as obeying $T_z \propto H^{\alpha_z}$, with $H$ a local thickness/scale height proxy and $\alpha_z$ a **vertical coherence** exponent (sensitive to stratification, turbulence, magnetic ordering).

**Prediction P5 (flaring).** In outer disks where the medium is less coherent vertically ($\alpha_z \to 1$), $T_z$ shortens relative to inner stratified regions, yielding a **gentle flaring** consistent with weaker vertical restoring forces but **coherent** oscillations; RTM expects the log–slope of vertical oscillation frequency with radius to approach 0 as $\alpha_z \to 1$.

**Prediction P6 (warps and $\nabla \alpha$).** Large-scale warps correlate with **gradients** in coherence, $\nabla \alpha$, across the disk—e.g., transitions from spiral/bar-ordered inner zones to more turbulent outer HI. RTM predicts systematic **phase lags** and **asymmetries** in vertical modes where $\nabla \alpha$ is largest (testable with HI tomography and Gaia DR kinematics).

## 4.5 Dwarfs and low-surface-brightness (LSB) galaxies

Dwarfs/LSBs have diffuse, weakly ordered baryons over most radii.

**Prediction P7.** Their $\alpha(L)$ profiles sit near **unity** across large radial ranges, so RTM expects:

- **Gently rising then flattening** rotation curves without a need for cuspy halos, consistent with α≈ 1
- **Small internal diversity** once binned by simple structure proxies (thickness, clumpiness), because $\alpha$ varies less across radius than in bar-dominated, high-surface-brightness disks.

**Discriminant.** Where DM fits invoke **cored** vs. **cuspy** halos to explain inner shapes, RTM predicts **measurable structure–slope coupling**: e.g., more clumpy star-forming dwarfs (slightly higher inner $\alpha$) show slightly steeper inner rises **at fixed mass profile**.

| Observable | Coherence dial (proxy) | RTM slope prediction | Collapse check | Distinctive discriminant |
|---|---|---|---|---|
| Disk rotation (inner) | Bar strength, bulge compactness, clumpiness | $\partial\,log\,v\,/\,\partial\,log\,L=1-\alpha>0$ or small; rises then levels as $\alpha\rightarrow1$ | $v\,L^{\alpha-1}$ flat within bin | Diversity at fixed mass explained by **structure**, not halo params |
| Disk rotation (outer) | Diffuse HI, low mode power | $\partial\,log\,v\,/\,\partial\,log\,L\rightarrow0$ | Flat within bin | Flatness without DM if $\alpha\approx1$ |
| bTFR residuals | Texture metrics, multi-scale entropy | Residuals correlate with coherence proxies | — | Residuals tied to **baryonic structure**, not halo concentration |
| Elliptical $\sigma(r)$ | Central anisotropy, embedded disks | $\partial\,log\,\sigma\,/\,\partial\,log\,L=1-\alpha$ in bins | $\sigma\,L^{\alpha-1}$ flat | Dispersion slopes predicted from structure maps alone |
| Vertical flaring | $\alpha_z$ (stratification, turbulence) | $\partial\,log\,v_z\,/\,\partial\,log\,R\rightarrow0$ as $\alpha_z\rightarrow1$ | $v_z\,H^{\alpha_z-1}$ flat | Phase/asymmetry of warps vs. $\nabla\alpha$ |
| Dwarfs/LSBs | Low-order baryons | Near-unity $\alpha\;\Rightarrow$ gentle rises, low diversity | Flat outer collapse | Structure–slope coupling at fixed mass profile |

**How these predictions are tested.** In Section 5 (Methods for $\alpha$ Estimation) we will define **structure$\rightarrow\alpha$** pipelines (multiscale entropy, bar/spiral mode power, turbulence indices), then enact **bin-by-bin slope and collapse tests** on rotation and dispersion profiles. In Section 6–7 (Comparisons & Consistency) we show how these RTM predictions separate from **dark-matter parameterizations** and **MOND-like scalings**, and we include **lensing–kinematics** cross-checks to enforce that altering clocks (via $\alpha$) does not smuggle in curvature changes.

# 5. From Light to $\alpha$: Structural Coherence Estimation

This section specifies **how** to build a radial field $\hat{\alpha}(L)$ from imaging and kinematics, with uncertainties and QA. The goal is an *operational $\alpha$* per annulus that (i) is derived from **independent structure proxies**, (ii) predicts the **slope** $1-\hat{\alpha}$ of $log\,v$ vs. $log\,L$, and (iii) passes **collapse** checks $v\,L^{\hat{\alpha}-1}\approx const$ inside coherence bins.

## 5.1 Data products and preprocessing

**Inputs (per galaxy):**

- Deep **broadband imaging** (e.g., *gri* or NIR) for stellar structure; PSF FWHM and variance maps.

- Spatially resolved **gas**: HI 21-cm (moment 0/1/2), and if available $H\alpha$ maps.

- **Kinematics**: rotation curves (HI or IFU), 2D velocity fields, and velocity dispersion maps.

- **Geometry**: distance, inclination iii, position angle (PA), disk scale length $R_d$, thickness indicators if available.

**Preprocessing:**

- PSF deconvolution (regularized; record effective resolution after deconvolution).

- Foreground/background mask; sky subtraction; isophotal ellipse fits to define **annuli**.

- Beam-smearing correction for velocity fields (forward modeling or standard recipes).

- Asymmetric-drift correction where needed (gas vs. stars).

- All maps resampled to a **common grid** with propagated uncertainty.

### 5.2 Structural proxies of coherence

We compute **multi-scale** descriptors in each annulus $A_j$ (width $\Delta \log L$ fixed). Each proxy is normalized to [0,1] and has an uncertainty.

1. **Multiscale entropy $E$.** Shannon entropy of image intensity after band-pass filtering (e.g., à trous wavelets) across spatial scales $s \in [s_{min}, s_{max}]$. Higher **order** (clear structure) → **lower** entropy → **higher** coherence. Define $E^\star = 1 - E_{norm}$.
2. **Fractal/turbulent index $D$.** 2-point structure function $S_2(\ell) \propto \ell^\zeta$ ($HI/H\alpha$ or stellar light). Map exponent $\zeta$ or fractal dimension $D$ to a **coherence score** $C_D$ (lower $D$ at large scales ⇒ higher coherence).
3. **Fourier mode power $P_m$.** Fractional power in $m = 2$ (bar), $m = 2 - 4$ (spiral), computed from deprojected surface brightness; normalize to $C_{mode} = \sum_{m \in M} P_m$.
4. **Clumpiness $S$ and smoothness $Q = 1 - S$.** High-$Q$ (smooth) suggests ordered structure; use standard CAS or Gini–$M_{20}$ family and convert to $C_{clump} = Q$.
5. **Thickness/asymmetry $T$.** From vertical proxies (when available) or minor/major axis ratios corrected for inclination; convert to $C_T$ (thinner, symmetric ⇒ higher coherence).
6. **Kinematic texture $K$.** Power in non-circular flows from residual velocity fields after subtracting axisymmetric model; invert to $C_K = 1 - NCF$.

Aggregate **feature vector** per annulus:

$$z_j = \left[E^*, C_D, C_{\text{mode}}, C_{\text{clump}}, C_T, C_K\right]_j, \quad \Sigma_j = \text{covariance of measurement errors.}$$

## 5.3 Proxy-to-$\alpha$ mapping

We map $z_j$ to a **provisional** coherence exponent $\bar{\alpha}_j$ via a monotone function $\mathcal{M}$. Two options (pre-registered; both allowed):

**(a) Parametric monotone map (transparent):**

$$\tilde{\alpha}_j = \alpha_0 + \sum_k w_k g_k(z_{jk}); \quad g_k \text{ monotone}, w_k \geq 0,$$

with $g_k$ chosen as identity or logistic transforms and $w_k$ fitted on **calibration subsets** (galaxies/annuli where the slope test already holds at high S/N). Impose priors $\alpha \in [0.8, 3.2]$ and $\|w\|_1 = 1$ for interpretability.

**(b) Rank-based ensemble (robust):**

$$\tilde{\alpha}_j = \alpha_0 + \lambda \, median_k \, rank(z_{jk}),$$

which reduces sensitivity to outliers and heterogeneous scales.

**Uncertainty.** Propagate $\Sigma_j$ to $\sigma_{\tilde{\alpha},j}$ via delta-method (option a) or bootstrap (option b).

## 5.4 Slope-check refinement ("closing the loop")

For each annulus $A_j$, we have local measurements $v(L)$. Within a **coherence bin** $B$ (collection of adjacent annuli with similar $\tilde{\alpha}$), fit

$$\log v = c_B + (1 - \hat{\alpha}_B) \log L$$

using Theil–Sen slope + Huber robust loss with **errors-in-variables** correction (SIMEX) for $L$ if deprojection uncertainties are non-negligible. Compare $\hat{\alpha}_B$ with the proxy-based $\tilde{\alpha}_j$ of its members.

**Acceptance rule (bin $B$):**

- **PASS:** $|\hat{\alpha}_B - median_{j \in B}\tilde{\alpha}_j| \leq 0.2$ and CI overlap;
- **TENTATIVE:** discrepancy 0.2 – 0.4 or wide CI;
- **FAIL:** >0.4 discrepancy or opposite slope sign.

We then define the **final** per-annulus estimate

$$\tilde{\alpha}_j = shrink(\tilde{\alpha}_j, \hat{\alpha}_B)$$

via a simple convex combination weighted by uncertainties.

## 5.5 Collapse check and residual diagnostics

Within each coherence bin $B$, compute

$$y(L) = v(L)L^{\hat{\alpha}_B - 1}$$

**Prediction:** $y(L)$ is **flat** vs. $L$. Regress $log\ y$ on $log\ L$; a residual slope with $|\ m\ |> 0.1$ (95% CI excluding 0) flags **model mis-specification** (e.g., variable $\alpha$ inside the bin, geometry systematics).

**Secondary residuals:** Examine $y(L)$ vs. (i) inclination error, (ii) beam-smear metric, (iii) asymmetric drift correction. Significant correlations indicate reduction pipelines need adjustment.

## 5.6 Binning strategy and sample size

- **Annuli:** logarithmic spacing with $\Delta\ log\ L = 0.08 - 0.12$, ensuring $\geq 5$ resolution elements across width.

- **Coherence bins:** cluster adjacent annuli by $\tilde{\alpha}$ using 1-D Ward clustering with constraint of **contiguity in radius**; target $\geq 5$ annuli per bin.

- **Cross-galaxy meta:** per bin type (low/mid/high coherence), pool slope estimates across galaxies using random-effects meta-analysis to report a population value of $1 - \alpha$.

## 5.7 Uncertainty, QA, and exclusions

- **Inclination/PA uncertainty:** propagate via Monte Carlo (draw $i$, PA from posteriors; refit slopes).

- **Distance uncertainty:** affects intercepts more than slopes; still propagated in the MC.

- **Resolution gate:** exclude annuli with fewer than 3 resolution elements across radial width or with PSF FWHM $> 0.5\ \Delta R$.

- **Beam smearing:** require correction factor < 20% or flag as TENTATIVE.

- **Asymmetric drift:** apply only when dispersion fraction > 0.15; otherwise gas rotation is used as is.

**Stop criteria (per galaxy):** mark galaxy **UNSUITABLE** if < 2 coherence bins pass both slope and collapse checks after QA.

## 5.8 Pseudocode (analysis contract)

```
for each galaxy G:

preprocess_images_and_kinematics(G)

annuli = make_log_annuli(G, dlogL=0.1)

for each annulus A_j in annuli:

z_j, Sigma_j = compute_structure_features(A_j)

talpha_j, sigma_talpha_j = map_features_to_alpha(z_j, Sigma_j)   # Sec. 5.3


# coherence binning with contiguity constraint

bins = cluster_adjacent_by_alpha(talpha_j, k_min=5 annuli)


results = []

for bin B in bins:

# Slope law

m, CI_m = robust_EIV_slope(log v vs. log L in B)

alpha_slope = 1 - m


# Compare with proxy alpha

alpha_proxy = median(talpha_j in B)

status = PASS if |alpha_slope - alpha_proxy| <= 0.2 and CI overlap else
TENTATIVE/FAIL
```

```
# Collapse

y = v * L**(alpha_slope - 1); m_c, CI_c = slope(log y vs. log L)

collapse_ok = (|m_c| <= 0.1 with CI including 0)



results.append({alpha_slope, CI_m, alpha_proxy, status, collapse_ok})



# Final per-annulus alpha by shrinkage to bin slope

for j in annuli:

alpha_final[j] = shrink(talpha_j, alpha_slope_of_bin(j), sigmas)



export(G, results, alpha_final, QA_flags)
```

### 5.9 Deliverables per Galaxy

- **Map:** $\hat{\alpha}(L)$ with $1\sigma$ band.
- **Plot:** $log\ v$ vs. $log\ L$ colored by coherence bins; slopes annotated with $1 - \hat{\alpha}$.
- **Collapse panel:** $v\ L^{\hat{\alpha}-1}$ vs. $L$ per bin.
- **Table:** for each bin$-\hat{\alpha}_{proxy}$, $\hat{\alpha}_{slope}$, CIs, collapse verdict, QA flags.

### 5.10 Interpretation rules (per bin)

1. **PASS (strong support):** slope $= 1 - \hat{\alpha}$ (CI overlap) and collapse flat; no strong QA flags.

2. **PARTIAL:** slope agrees but collapse weak (suggests mild α drift or geometry residuals).

3. **FAIL:** slope disagrees or collapse shows significant trend; check QA; if persistent, RTM not supported in that bin.

# 6. Comparison with Gravity-Only Expectations

This chapter turns RTM's slope law into **direct, falsifiable contrasts** with two baselines:

- **GR + baryons only (no DM):** classical dynamics with luminous mass distribution; rotation asymptotes depend on baryonic extent.

- **GR + DM halos (ΛCDM practice):** add a parametric halo (e.g., NFW, Burkert) and fit free parameters per galaxy.

RTM keeps gravity untouched but adds a **coherence field** $\alpha(L)$ that modifies **operational times**. The discriminants below are framed as **slope tests** and **collapse checks** that do not depend on absolute normalization.

## 6.1 Outer-disk asymptotes: flatness without halos vs. Keplerian falloff

**Gravity-only expectation.** For finite disks, beyond most baryons one expects $v(L) \propto L^{-1/2}$ (approach to Keplerian with geometric corrections). In practice, purely baryonic models struggle to keep $v$ **flat** over decades in $L$ without added mass.

**RTM prediction (P1 redux).** If the outer medium is **weakly coherent** ($\alpha \to 1$), then

$$\frac{\partial \log v}{\partial \log L} = 1 - \alpha \to 0 \Rightarrow v(L) \approx \text{const.}$$

**Discriminant D1 (slope audit).** In **outer annuli** selected by low coherence proxies, fit $\log v$ vs. $\log L$.

- **RTM PASS:** slope $m$ tightly clusters near 0 **and** the collapse $v\, L^{\alpha-1}$ is flat.
- **Baryons-only FAIL:** same data, same annuli, would require $m \approx -1/2$ unless hidden mass is added.
- **DM ambiguity:** halos can fit $m \approx 0$, but the **same annuli** must also pass D2–D4 below to distinguish RTM.

## 6.2 Inner-curve diversity: coherence vs. halo tuning

**Observed fact.** Galaxies with similar baryonic mass show **diverse inner shapes** (steep/slow rises). DM fits accommodate this with halo concentration/contracted profiles; MOND invokes local acceleration; **both** require per-galaxy *tuning*.

**RTM mechanism.** Inside bars/bulges/clumps, $\alpha(L) > 1$ elevates local orbital times, yielding

$$m = \frac{\partial \log v}{\partial \log L} = 1 - \alpha < 0 \quad \text{(steeper rises/shallower declines depend on geometry).}$$

The key point is **covariation**: the inner **slope** must track **structure-derived $\alpha$**, not a free halo parameter.

**Discriminant D2 (structure–slope coupling).** After controlling for mass and geometry, regress inner-slope residuals $\Delta m$ on coherence proxies (bar power $P_2$, multiscale entropy $E^\star$, clumpiness $Q$, etc.).

- **RTM PASS:** corr($\Delta m$, $\hat{\alpha}$) ⍰ is significant and positive in magnitude (more coherence $\rightarrow$ more negative $m$ or more pronounced rise/flattening, per geometry), and remains after partialing out size and surface density.
- **DM/MOND FAIL:** residuals align primarily with halo/acceleration parameters, and **not** with structure once baryons are fixed.

### 6.3 The baryonic Tully–Fisher relation (bTFR): residual anatomy

**Baseline behavior.** bTFR is tight but shows **residuals**. In DM fits, residuals correlate with **halo concentration/spin**; in MOND, with **interpolating function/acceleration** nuances.

**RTM recast.** If $v$ is sampled where $\alpha \rightarrow 1$, the **leading-order** bTFR holds with minimal residuals. If sampled further in (higher $\alpha$), the measured $v$ is **systematically biased** relative to the asymptotic value.

**Discriminant D3 (residual–coherence link).**

- Compute residuals $\Delta \log v$ from a galaxy-wide bTFR fit.
- Test $\Delta \log v$ vs. an $\alpha$ **mismatch index**, e.g., $\delta_\alpha \equiv \hat{\alpha}(R_{meas}) - 1$.
  - **RTM PASS:** $\Delta \log v$ correlates with $\delta_\alpha$ (sampling inside coherent zone depresses $v$, negative residual), and the correlation **vanishes** when measuring $v$ at the **slope-zero** radius in each galaxy.
  - **DM/MOND FAIL:** residual–$\delta_\alpha$ correlation is weak/absent once mass and size are controlled.

### 6.4 Cross-annulus collapse vs. parametric freedom

**RTM collapse.** Within any coherence bin $B$: $y(L) = v(L) \, L^{\hat{\alpha}_B - 1}$ must be **flat**. This is a **functional** constraint stronger than fitting an intercept.

**Discriminant D4 (bin-wise collapse).**

- **RTM PASS:** residual slopes $| mB | \leq 0.1$ (CI includes 0) across bins and galaxies; pooled random-effects meta-slope consistent with 0.
- **DM/MOND FAIL (as a mechanism test):** Although halos/acceleration laws can reproduce **a curve**, they do **not** predict per-bin collapses tied to **independently**

**measured** coherence. Failure to collapse after $\hat{\alpha}$ conditioning counts against RTM; success counts as a unique signature.

## 6.5 Ellipticals and dispersion profiles: Jeans vs. coherence

**Jeans baseline.** With anisotropy $\beta(r)$ and mass profile $M(r)$, $\sigma(r)$ follows from the Jeans equation; DM adds mass at large $r$, steepening/flattening profiles by choice of halo and $\beta$.

**RTM slope rule for dispersions.** In bins where $\alpha(r)$ is approximately constant,

$$\frac{\partial \log \sigma}{\partial \log r} = 1 - \alpha \quad \text{(up to anisotropy corrections)}$$

Discriminant D5 (dispersion slope vs. structure).

- **RTM PASS:** $\sigma$-slope tracks $\hat{\alpha}$ from photometric texture (cores/embedded disks $\rightarrow$ higher $\alpha$ $\rightarrow$ more positive/less negative slope), and bin-wise $\sigma\, r^{\hat{\alpha}-1}$ collapses.
- **DM/MOND FAIL:** Needed changes are absorbed into $M(r)$ or $\beta(r)$ with little/no link to **measured** structure.

## 6.6 Where baselines and RTM agree (sanity checks)

There are regimes where **all** models predict similar behavior; we use them as **null tests**:

- **Keplerian controls:** wide binaries, outer planetary systems, globulars at large rrr. Coherence is irrelevant; RTM must reduce to classical slopes.

- **Solid-body cores:** purely geometric effects in very central regions can mimic $m \approx +1$. RTM does **not** claim credit there; tests must avoid sub-resolution radii.

- **Ultra-diffuse outer HI:** if structure proxies confirm $\alpha \approx 1$, **all** models allow $m \approx 0$. The discriminants then shift to **bTFR residual anatomy** (Sec. 6.3) and **collapse** (Sec. 6.4).

## 6.7 Decision matrix (per galaxy, per bin)

| Test | Evidence for RTM | Evidence against RTM | What DM/MOND would say |
|---|---|---|---|
| **D1:** outer-slope | $m \approx 0$ in low-$\alpha$ bins **and** collapse | *m≈−1/2* or no collapse | DM can fit *m≈0* but doesn't predict collapse |

| Test | Evidence for RTM | Evidence against RTM | What DM/MOND would say |
|---|---|---|---|
| **D2:** inner diversity | $\Delta m$ correlates with $\hat{\alpha}$ (structure) | $\Delta m$ uncorrelated with structure | DM: halo params; MOND: acceleration scale |
| **D3:** bTFR residuals | $\Delta \log v \leftrightarrow \delta_\alpha$ vanishes at slope-zero radius | No relation to $\delta_\alpha$ | DM: residuals $\leftrightarrow$ halo concentration/spin |
| **D4:** collapse | $v\,L^{\hat{\alpha}-1}$ flat per bin | Residual slope ( | m_B |
| **D5:** dispersions | $\partial \log \sigma / \partial \log r = 1 - \hat{\alpha}$ & collapse | No slope–structure link | Tunable $M(r)$, $\beta(r)$ fit it post-hoc |

## 6.8 Pre-registered fail modes

RTM's galactic claims are **falsified** if, after QA (Sec. 5):

1. Outer low-α bins show **non-zero** slopes inconsistent with 0 (D1 fail) **and** do not collapse (D4 fail).

2. Inner-slope residuals do **not** correlate with structure-derived $\hat{\alpha}$ once mass/size are controlled (D2 fail).

3. bTFR residuals are **independent** of $\delta_\alpha$ ⬚ and remain so even when sampling at slope-zero radius (D3 fail).

4. Dispersion slopes in ellipticals show **no** relation to texture-based $\hat{\alpha}$ (D5 fail).

Any two independent fails under good QA mark RTM **not supported** on galaxy scales; passing D1–D4 across a diverse sample constitutes **strong support**.


# 7. Gravitational Lensing and Clusters: Consistency Checks

RTM claims to alter **operational times** (orbital clocks) via the coherence exponent $\alpha(L)$, not spacetime **curvature**. If true, **gravitational lensing**—which depends on curvature sourced by stress–energy—should continue to track the **baryonic mass distribution** (plus any genuinely non-baryonic mass, if present) independent of $\alpha$. This chapter lays out tests that compare **lensing-inferred mass** with **kinematics reinterpreted under RTM**, from galaxies to clusters. Any **persistent, coherent mass gap** after conditioning on $\hat{\alpha}(L)$ constitutes a **scope limit** or direct **falsification** on those scales.


## 7.1 Clocks vs. curvature: the guiding principle

- **What RTM changes:** the mapping $T \propto L^{\alpha(L)}$ that governs orbital/relaxation times. Kinematic observables that rely on periods or drift (rotation velocities, dispersions, epicyclic/vertical frequencies) are modified via $v \propto L^{1-\alpha}$ or $\sigma \propto L^{1-\alpha}$ **within coherence bins**.

- **What RTM does not change:** the Einstein field equations and the geodesics that set light bending and lensing. Thus, **lensing mass maps** should be consistent with **baryons** (to within known systematics) unless there exists actual unseen mass or RTM fails to describe the dynamics.

**Operational test.** Build, for each system, two mass inferences:

1. $M_{lens}(R)$ from strong/weak lensing (or dynamical+X-ray in clusters).
2. $M_{kin}^{RTM}(R)$ from observed velocities/dispersion **after** reinterpreting the kinematics with $\hat{\alpha}(R)$.

Consistency requires $M_{kin}^{RTM} \approx M_{lens}$ within uncertainties; a systematic bias that **survives** $\alpha$-conditioning signals a limit of RTM or genuine extra mass.

### 7.2 Strong-lensing galaxies (Einstein rings & quads)

**Set-up.** Choose lenses with high-quality Einstein rings/quads (precise $M_{lens}(R_E)$). Obtain IFU kinematics to construct $\hat{\alpha}(R)$ (Sec. 5).

RTM consistency test SL-1 (enclosed mass at $R_E$).

- Compute $M_{kin}^{RTM}(R_E)$ from the observed rotational/dispersion support using the **RTM velocity law** inside coherence bins intersecting $R_E$.
- **Pass:** $M_{kin}^{RTM} - M_{lens}$ $|/M_{lens} \leq \varepsilon$ (pre-registered $\varepsilon$, e.g., 15%)
- **Fail:** systematic over- or underestimates across the sample that cannot be traced to $\alpha$ calibration or anisotropy systematics.

**RTM discriminant SL-2 (annular collapse).**
Inside an annulus around $R_E$ with approximately constant $\hat{\alpha}$, the quantity

$$y(R) = v(R)R^{\hat{\alpha}-1}$$

should be **flat** vs. $R$. Failure to collapse while the lens mass is well constrained argues that RTM's kinematic reinterpretation is inadequate **at the lens scale**.

**Time-delay add-on SL-3.** For lensed quasars with measured time delays, check that cosmographic inferences (e.g., $H_0$) remain **unchanged** by switching the dynamical model to RTM, since delays depend on **curvature + potential differences**, not on orbital clocks. Any change indicates double-counting (incorrectly letting $\alpha$ leak into lensing).

### 7.3 Weak lensing in disk galaxies (stacked halos)

**Set-up.** Use stacked weak-lensing shear profiles of large disk samples binned by **structural coherence** (e.g., bar strength, texture metrics) to obtain $M_{lens}(R)$ at tens–hundreds of kpc.

**RTM consistency test WL-1 (outer bins).**
In **low-$\hat{\alpha}$** outer annuli (where rotation curves flatten), RTM predicts **flat kinematics** without extra curvature. Therefore, the **lensing** signal at large $R$ should be explainable by **baryons + known gas** alone if the curvature really traces mass only.

- **Pass:** stacked $M_{lens}(R)$ consistent with baryon maps and with $M_{kin}^{RTM}(R)$.
- **Scope/Fail:** a robust excess in shear **after** $\alpha$-conditioning indicates mass beyond baryons—either RTM's scope ends here or dark mass is needed.

**Internal cross-check WL-2 (structure split).**
Split disks at fixed stellar mass by coherence (high vs. low bar/texture).

- RTM expects **similar weak-lensing halos** (since lensing ignores $\alpha$) but **different inner kinematic slopes**.

- If the lensing profiles *also* split systematically with coherence at fixed baryon maps, that suggests a correlation between **structure** and **true mass** (not an $\alpha$-only effect), tightening the scope.

### 7.4 Clusters of galaxies: where RTM may (not) apply

**Reality check.** Rich clusters exhibit strong/weak lensing and X-ray hydrostatic masses that **exceed** baryons. If RTM only retimes **orbital clocks** inside structured baryons, it **should not** erase mass deficits in clusters—even if $\alpha$ affects some intracluster dynamics.

**Cluster test CL-1 (mass budget).**

- Build $M_{lens}(R)$ and $M_X(R)$ $(X-ray)$.
- Measure $\hat{\alpha}(R)$ fields from ICM texture (pressure/density fluctuations, power spectra) and galaxy substructure.
- Compute $M_{kin}^{RTM}(R)$ from galaxy dispersions using $\sigma \propto R^{1-\hat{\alpha}}$ in **coherence bins** (Jeans with RTM clocks).
- **Expected outcome:** even with RTM, a **significant residual mass** remains in clusters—the classic DM signal.
- **Interpretation:** RTM's **scope condition**: it is a **galaxy-scale** kinematic re-timing, not a replacement for DM on cluster scales. If, implausibly, RTM erased the cluster mass gap, the lensing–dynamics consistency would break (contradicting curvature-based mass).

**Bullet-like mergers CL-2.** In systems where gas–galaxy offsets occur, lensing peaks follow collisionless mass. RTM predicts **no shift** of lensing peaks with $\alpha$; any attempt to use $\alpha$ to mimic the offset would incorrectly let clocks alter curvature—**disallowed**.

### 7.5 Kinematics–lensing reconciliation algorithm (per system)

1. Measure $\hat{\alpha}(R)$: build coherence bins from structure proxies (Sec. 5).
2. **RTM-inferred dynamics:** within each bin, fit slopes $m = 1 - \hat{\alpha}$, check collapse $v\,R^{\hat{\alpha}-1}$, and recover $M_{kin}^{RTM}(R)$ with EIV corrections and anisotropy priors (for dispersions).
3. **Lensing mass:** obtain $M_{lens}(R)$ (strong/weak) with full covariances.
4. **Compare:** compute $\Delta(R) = M_{kin}^{RTM}(R) - M_{lens}(R)$ and its uncertainty; report **bin-wise** residuals rather than a single global number.
5. **Decision:**
   - **CONSISTENT:** $|\Delta|/M_{lens} \leq \varepsilon$ in most bins and no trend with $\hat{\alpha}$.
   - • **SCOPE LIMIT:** residuals concentrate at **cluster-scale radii** or in systems where $\hat{\alpha}$ cannot be stably estimated.
   - • **FALSIFIED:** coherent, significant residuals across many **galaxy-scale** bins where QA passes and $\hat{\alpha}$ is stable.

### 7.6 Time delays and relativistic tests (sanity)

- **Strong-lens time delays:** depend on the **Fermat potential** (curvature + geometry). RTM must **not** alter predicted delays when the mass map is fixed. We therefore re-fit delays under GR with the same mass and show **invariance** to replacing Newtonian dynamics with RTM for the **stellar/gas** motions.

- **PPN/solar system constraints:** in low-coherence regimes relevant to solar-system tests, $\alpha$ reduces to its classical baseline and lensing/deflection constraints remain **unchanged**—a built-in sanity check.

### 7.7 Pre-registered outcomes (pass/fail)

- **PASS (galaxy scale):**

  (i)   Outer low-$\hat{\alpha}$ bins show $m \approx 0$ **and** collapse;
  (ii)  $M_{kin}^{RTM}(R)$ agrees with $M_{lens}(R)$ in rings/quads within $\leq 15\%$;
  (iii) Weak-lensing stacks at fixed baryons do **not** split by coherence, while kinematic slopes **do**
- **SCOPE (clusters):**
  RTM does **not** remove the cluster mass gap; $M_{lens}(R)$ exceeds baryons + RTM-

kinematics. RTM is thereby bounded to **galaxy-scale** kinematics unless additional physics is introduced.

- **FAIL (galaxy scale):**
  Consistent, significant lensing–kinematics mass gaps **after** $\alpha$-conditioning, or bin-wise non-collapses coupled with stable $\hat{\alpha}$ estimates and good QA, falsify RTM as an explanatory mechanism for galaxy rotation/dispersion profiles.

**Bottom line.** Lensing is RTM's **guardrail**: by separating **clocks** from **curvature**, we can tell when coherence-driven re-timing suffices (galaxies) and where it cannot (clusters). Passing the lensing consistency checks makes RTM a credible, tightly scoped reinterpretation of galactic kinematics; failing them draws a clear boundary and preserves standard gravity where it must remain untouched.

# 8. Cosmic Structure Growth (Sketch)

This chapter sketches how an $\alpha$**-field**—a spatially varying coherence exponent tied to baryonic organization—could modulate **timescales** during the assembly of galaxies and their substructures without altering gravity. The stance remains **slope-first**: RTM predicts **how fast** processes unfold at a given scale, not **that** new forces appear. The section closes with **observables** and **fail tests** that keep the program falsifiable.

### 8.1 Collapse clocks under RTM

Let $t_{coll}(L)$ denote the characteristic time for a self-gravitating baryonic patch of size $L$ to proceed from linear growth to nonlinearity (fragmentation/condensation). Standard theory supplies a dynamical time $t_{dyn} \sim 1/\sqrt{G\rho}$ and additional delays from angular-momentum transport, cooling, turbulence. RTM treats the **operational time** as

$$t_{\text{coll}}(L) = t_{\text{dyn}}(L)\left(\frac{L}{L_0}\right)^{\alpha(L)-\alpha_0}\Theta$$

where $\alpha_0$ is a baseline (weakly coherent) band and $\Theta$ aggregates dimensionless microphysics held fixed **within** a coherence bin. Consequences:

- Regions with **higher coherence** ($\alpha > \alpha_0$) **lengthen** collapse clocks at that *same scale*, delaying bar/spiral growth or clump condensation relative to diffuse zones.
- **Gradients** $\nabla\alpha$ seed **differential timing** across radii, imprinting phase lags among bars, spirals, and warps.

## 8.2 Angular momentum transport and bar timelines

Bar formation requires angular-momentum redistribution. Let $t_J(L)$ be the characteristic timescale for $J$-transport in an annulus of width $\sim L$. With RTM:

$$t_J(L) \propto L^{\alpha(L)} \quad \Rightarrow \quad \frac{\partial \log t_J}{\partial \log L} = \alpha(L).$$

**Predictions.**

- **Inside–out sequencing.** If inner disks are more coherent ($\alpha_{in} > \alpha_{out}$), bars/inner spirals **lag** outer pattern growth; conversely, if feedback shreds inner coherence ($\alpha_{in} \to 1$), bars emerge **earlier** than standard secular times would suggest.

- **Bar length vs. $\alpha$-gradient.** Bar semi-major axes anticorrelate with $\nabla\alpha$: stronger outward **drops** in $\alpha$ (inner coherent $\to$ outer diffuse) cap bar growth earlier (outer disk outpaces inner in $J$-shed).

**Observables.** At fixed mass and gas fraction, **bar fraction** and **bar length** correlate with the **shape** of $\hat{\alpha}(R)$: long, strong bars prefer **flatter** $\alpha$-profiles; short/weak bars appear where $\alpha$ falls fast with radius.

## 8.3 Clump formation, migration, and thick disks

Massive star-forming clumps in high-$z$ disks migrate inward on a timescale $t_{mig}$ set by torques and dynamical friction.

**RTM modulation.**

$$t_{\text{mig}}(L_{\text{clump}}) \sim t_{\text{dyn}} \left(\frac{L_{\text{clump}}}{L_0}\right)^{\alpha-1}$$

so at fixed clump size, **higher local $\alpha$ slows migration**, allowing clumps to **live longer** and thicken disks via prolonged scattering.

**Predictions.**

- **Clump longevity vs. $\alpha$.** At fixed surface density, disks with higher $\hat{\alpha}$ sustain larger **clump lifetimes** and show **thicker** stellar layers earlier.

- **Age gradients.** If $\alpha$ declines with radius, inner clumps (higher $\alpha$) age **older** in situ than outer clumps (lower $\alpha$) for the same look-back time—an **inverted** age–radius trend relative to pure dynamical friction expectations.

**8.4 Satellite planes, warps, and phase lags**

Coherence gradients can **phase-lock** certain orbital families.

**Predictions.**

- **Satellite planes.** If a host's outer disk/CGM exhibits an **anisotropic $\alpha$** field (e.g., along filaments), satellite orbits preferentially **persist** in that plane (longer operational periods for out-of-plane diffusion), increasing the chance of **apparent planar alignments** without invoking special DM anisotropies.

- **Warp phasing.** Radial zones where $\nabla\alpha$ is largest should show **phase lags** between HI warps and stellar bends; the lag's sign flips with the sign of $\nabla\alpha$.

- **Lopsidedness.** Persistent $m = 1$ modes correlate with **azimuthal** variations in $\alpha$ (bars + clumps on one side), producing **kinematic asymmetries** that track structure maps.

**8.5 Star-formation histories (SFHs) and $\alpha$**

Because $t_{coll}$ and $t_J$ stretch with $\alpha$, **SFHs** inherit **coherence signatures**:

- **Inside–out vs. outside–in.** Disks with inner high $\alpha$ and outer low $\alpha$ trend **outside–in** in burst timing (outer rings ignite earlier); the reverse $\alpha$-shape flips the trend.
- **Burstiness.** Low-$\alpha$ patches (diffuse/turbulent) have **shorter cycles**, enhancing burstiness and driving larger HI/H$\alpha$ power at small scales; high-$\alpha$ patches smooth SFHs.
- **Metallicity spreads.** Prolonged migration under high $\alpha$ broadens metallicity distributions at given radius (longer phase-mixing times), testable with IFU metallicity maps.

**8.6 High-redshift trends**

At $z \gtrsim 1$, gas-rich disks are clumpy and turbulent. Two stylized scenarios:

- **Scenario A (low global $\alpha$).** If early disks are largely **diffuse** (feedback shreds coherence), $\alpha \approx 1$ over wide radii $\Rightarrow$ **rapid** pattern growth, **short** clump lifetimes, quicker approach to flat rotation beyond compact bulges.
- **Scenario B (hierarchical $\alpha$).** If nested structures (giant clumps, chains) increase coherence ($\alpha > 1$) locally, bars and long-lived clumps should **coexist** early; rotation

slopes exhibit strong **radial diversity** that **fades** as $\alpha \to 1$ with cosmic time (disk settling).

**Observable lever.** Compare the **evolution** of the **distribution of slopes** $m(R) = \partial \log v / \partial \log R$ across redshift after conditioning on $\boldsymbol{\alpha}$**-proxies**. RTM predicts that the **spread** in mmm at fixed mass narrows as $\alpha$ fields **flatten** with time.

### 8.7 Simulation sketch (how to test the above)

**Alpha-aware orbit integrator.** Take a standard N-body+gas code or a collisionless testbed; at each step, rescale **time advances** in a cell by $dt' = dt(L/L_0)^{\alpha(x)-\alpha_0}$. Keep forces **unchanged**. Feed $\alpha(x)$ from (i) analytic profiles (bar-centered high $\alpha$), (ii) light-derived proxy maps, or (iii) self-updating rules (coherence grows with sustained surface density). Read out:

- Rotation slopes and **collapse** $vR^{\alpha-1}$ within bins;
- Bar formation time vs. $\nabla\alpha$;
- Clump lifetimes and disk thickening vs. local $\alpha$;
- Phase lags of warps vs. $\nabla\alpha$.

**Falsification inside the sandbox.** If keeping forces fixed and only **retiming** cannot reproduce any of the observed sequencing (e.g., bar emergence patterns) when $\alpha$ fields are tuned to **measured** structure, the growth-level RTM story weakens.

### 8.8 Observable summary & fail conditions

| Phenomenon | RTM signature | How to measure | Fail if... |
|---|---|---|---|
| Bar emergence | Timing tracks $\nabla\alpha$; long bars need flat α(R) | Bar fraction/length vs. $\widehat{\alpha}$(R) shape | No correlation after mass/size control |
| Clump longevity | Higher local α ⇒ longer-lived, thicker disks | Clump ages, thickness vs. α | Lifetimes independent of $\widehat{\alpha}$ |
| Warps | Phase lags where $\nabla\alpha$ large | HI vs. stellar bends vs.$\nabla\alpha$ | No systematic lag–gradient link |
| Satellite planes | Alignment with anisotropic α in CGM | Plane orientation vs. α anisotropy | No alignment at fixed baryons |
| SFH timing | Outside–in or inside–out set by $(\alpha)$-shape | Resolved SFHs vs. α-shape | Trends vanish when conditioning on $\widehat{\alpha}$ |

### 8.9 Scope note

These sketches do **not** claim that RTM replaces detailed baryonic physics (cooling, feedback, turbulence). They assert that a **single exponent field** $\alpha(x)$ can **organize the timing** of otherwise standard processes. The pay-off is a portfolio of **slope-level** and **sequencing** tests—each with a clear **fail mode**—that connect growth histories to measurable **structure maps**. If those links do not materialize under good QA, RTM's role in cosmic growth is **bounded** or **falsified** for the regimes tested.


# 9. Data & Measurement Plan

This chapter turns the predictions into an **analysis contract**: datasets, selection, preprocessing, $\hat{\alpha}(L)$ construction (Sec. 5), slope/collapse tests, bTFR residual anatomy, and lensing–kinematics reconciliation (Sec. 7). Everything below is phrased so another group can reproduce the pipeline end-to-end.


### 9.1 Samples and inclusion criteria

**Disk galaxies (rotation focus):**

- Spatially resolved HI or Hα kinematics with ≥10 independent radial points beyond $2\,R_d$

- Deep optical/NIR imaging (PSF FWHM ≤ 0.5 of inner annulus width) for structure maps.

- Known distance, inclination $i \in [30°,80°]$, position angle (PA), and stellar/gas mass maps.

- Aim for **three cohorts** balanced in mass and morphology:
  C1: high-surface-brightness barred; C2: unbarred grand-design spirals; C3: dwarfs/LSBs.

**Ellipticals (dispersion focus):**

- IFU spectroscopy with radial $\sigma(R)$ profiles to $\geq 1.5 - 2\,R_e$

- High-S/N imaging (cores, embedded disks discernible)

**Strong-lens galaxies:**

- Einstein ring/quads with IFU kinematics intersecting $R_E$

- Public lens models with covariance (for $M_{lens}(R)$)

**Weak lensing stacks:**

- Large disk samples with shear catalogs and structural labels (bar strength, texture metrics).

## 9.2 Preprocessing & geometry

- **Imaging:** sky subtraction, masking, PSF characterization; deprojection using $i, PA$; regrid to common pixel scale.

- **Kinematics:** beam-smearing correction (forward model preferred); asymmetric drift applied where stellar dispersion fraction > 0.15; gas assumed cold.

- **Annuli:** logarithmic annuli with $\Delta \log L = 0.1$; require $\geq 3$ resolution elements per annulus.

All steps produce **per-annulus uncertainties** (covariant where relevant).

## 9.3 Building $\hat{\alpha}(L)$

Apply Sec. 5: compute structural features per annulus (multiscale entropy, mode power, clumpiness, fractal/turbulence indices, thickness, kinematic texture). Map features → provisional $\hat{\alpha}$ (parametric monotone or rank-ensemble), cluster adjacent annuli into **contiguous coherence bins**, fit slope $m = 1 - \hat{\alpha}_B$ in each bin (robust EIV), compare to proxy median, and **shrink** to obtain $\hat{\alpha}_j$, QA: collapse check $vL^{\hat{\alpha}_B - 1}$ slope $| mc | \leq 0.1$ with CI including 0.

## 9.4 Primary hypothesis tests (per galaxy)

**H-RC (Rotation slope):** In each coherence bin $B$:

- Estimate $m_B = \partial \log v / \partial \log L$
- Test $m_B = 1 - median(\hat{\alpha}_{j \in B})$ (CI overlap ±0.2).

**H-CL (Collapse):** Regress $\log [v L^{\hat{\alpha}_B - 1}]$ vs. $\log L$; require $| m_c | \leq 0.1$, CI includes 0.

**H-bTFR (Residual anatomy):**

- Global fit: $log\ v_{flat} = a + b\ log\ M_b$
- Residuals $\Delta\ log\ v$ regressed on $\delta_\alpha \equiv \hat{\alpha}(R_{meas}) - 1$, controlling for size and surface density.
- Recompute at **slope-zero radius**; correlation should vanish if RTM holds.

**H-Disp (Ellipticals):** In coherence bins, $\partial\ log\ \sigma / \partial\ log\ r = 1 - \hat{\alpha}$ (EIV-robust); collapse of $\sigma\ r^{\hat{\alpha}-1}$

**H-Lens (Lensing consistency):**

- **Strong lens:** compare $M_{kin}^{RTM}(R_E)$ to $M_{lens}(R_E)$; tolerance $\leq 15\%$.
- **Weak lens stacks:** at fixed baryons, shear profiles should **not** split by coherence; kinematic slopes **do**.

## 9.5 Statistical plan

- **Slopes:** Theil–Sen estimator with Huber loss; SIMEX for $L$ errors; bootstrap CIs (B=2000).

- **Meta-analysis:** Random-effects combine slopes across galaxies within the same bin type (low/mid/high coherence). Report pooled $m$, heterogeneity $I^2$

- **artial correlations:** For bTFR residuals, regress $\Delta\ log\ v$ on $\delta_\alpha$ while controlling for $log\ R_d, \Sigma_\star$

- **Multiple testing:** Benjamini–Hochberg FDR at 5% across bins and tests.

- **Pre-registration:** Freeze proxy-to-$\alpha$ maps and thresholds ($|\ m_c\ | \leq 0.1; |\ \hat{\alpha}_{slope} - \hat{\alpha}_{proxy}\ | \leq 0.2$) before looking at science targets.

## 9.6 Power expectations (order-of-magnitude)

- **Rotation slopes:** With 6–8 annuli per bin, $\sigma_{log\ v} \sim 0.04$, EIV-corrected slope $SE \sim$ 0.08 Differences of $\Delta(1 - \alpha) = 0.3$ between bins give $> 90\%$ power at $\alpha = 0.05$.
- **Collapse test:** Detect $|\ m_c\ |= 0.12$ with $\sim 80\%$ power per bin.
- **bTFR residual–$\delta_\alpha$:** With $N \sim 150$ disks and residual scatter 0.08 dex, correlation $|\ r\ | \geq 0.25$ is detectable at $> 90\%$ power.
- **Lensing (strong):** Ten high-quality rings with 10% lensing mass errors suffice to detect a systematic 15% bias at $> 80\%$ power.

### 9.7 QA, exclusions, and adversarial checks

- **Resolution gate:** drop annuli with PSF FWHM $>$ 0.5 of annulus width.
- **Beam smearing:** flag if correction $>$ 20%; exclude if $>$ 35%.
- **Inclination/PA:** Monte Carlo over $i, PA$ posteriors; bins failing stability (slope drift $>0.15$) are **TENTATIVE/FAIL**.
- **Proxy robustness:** recompute $\hat{\alpha}$ with (i) leave-one-proxy-out, (ii) rank-based map; require classification stability.
- **Negative control galaxies:** systems with extremely smooth structure (featureless S0) must yield $\alpha \to 1$ and outer $m \to 0$; failure triggers pipeline audit.

### 9.8 Deliverables

For each galaxy:

- **Maps:** $\hat{\alpha}(L)$ with uncertainties; mask of coherence bins.

- **Panels:** (i) $\log v$ vs. $\log L$ colored by bin with fitted slopes; (ii) collapse plots $vL^{\hat{\alpha}-1}$; (iii) residual diagnostics.

- **Tables:** per bin$-\hat{\alpha}_{proxy}$, $\hat{\alpha}_{slope}$, CI, collapse verdict, QA flags.

- **Lensing reconciliation (where available):** $M_{kin}^{RTM}(R_E)$ vs. $M_{lens}(R_E)$ with residuals.

For the sample:

- **Meta-slopes** (low/mid/high coherence), $I^2$, and pass/fail counts.
- **bTFR residual regressions** and "slope-zero radius" remeasurements.
- **Weak-lensing stack splits** (by coherence) and their null comparison.

### 9.9 Pass/fail ledger (pre-declared)
A galaxy contributes **support** if ≥2 coherence bins **PASS** both H-RC and H-CL, and (if applicable) H-Lens passes. A **partial** contribution requires PASS in either H-RC or H-CL with the other TENTATIVE and no QA red flags. **Fail** if all bins fail slope or collapse under good QA.

### 9.10 Reproducibility

- Release **analysis code** (proxy extraction, $\alpha$ mapping, EIV slopes, collapse checks) with version-locked environments.

- Provide **per-annulus catalogs** (features, $\hat{\alpha}$, kinematics, QA flags).

- Publish **pre-registration** (hypotheses, thresholds, exclusion rules) and **frozen** proxy maps before touching the main science sample.

**Outcome of this plan.** The data contract ensures that RTM's claims rise or fall on **bin-wise slopes and collapses** tied to independently measured **coherence**. Next (Sec. 10) we specify the **simulation suite** that stress-tests the pipeline, explores bias, and generates mock-observable benchmarks for $\alpha$-aware dynamics.

# 10. Simulations

This chapter specifies an **$\alpha$-aware simulation suite** to (i) test whether RTM's slope/collapse signatures are recoverable when forces are standard but clocks are retimed; (ii) quantify biases and failure modes of the pipeline in Sec. 5–9; and (iii) generate **mock surveys** with known truth ($\alpha_{true}(x)$, mass, geometry) for end-to-end validation.

## 10.1 Philosophy: keep forces, retime updates

We preserve Newtonian/GR forces (no modified gravity, no added mass). RTM enters **only** through a local **time-rescaling**:

$$dt'(x) = dt \left(\frac{L(x)}{L_0}\right)^{\alpha(x)-\alpha_0}$$

where $L(x)$ is a chosen structural scale (e.g., radial annulus scale, local disk thickness, smoothing length), $\alpha_0$ a baseline band ($\approx 1$), and $\alpha(x)$ the coherence field (fixed or evolving). All integrators below simply use $dt'$ for state updates while computing accelerations from the **unchanged** potential.

## 10.2 Simulation families

### S1. Collisionless testbed (orbits in fixed potentials).

- Potentials: Miyamoto–Nagai disks + Hernquist bulges + optional NFW halos (for baseline comparisons).

- Particles: $10^6$ tracers; integrator: leapfrog or 4th-order symplectic with **adaptive** $dt'$.

- $\alpha(x)$: analytic profiles (bar-centered bump, outer flat unity); or azimuthal anisotropy for warp experiments.

### S2. Thin-disk NNN-body with live bar/spiral response.

- Self-gravity on a 2D polar grid; softening chosen to resolve $< 0.5$ annulus width.

- Optional gas as inelastically colliding particles (sticky scheme) to emulate dissipation.

- $\alpha(x, t)$: (i) fixed; (ii) **structure-coupled** (see §10.5).

**S3. Mock-IFU cubes / HI moment maps.**

- Take S1/S2 snapshots; render **line-of-sight** velocity fields with beam, PSF, noise, and spectral resolution matched to real surveys.

- Generate rotation curves and dispersion profiles with the **same pipeline** as the data (Sec. 5 & 9).

**S4. Elliptical analogs (Jeans particles).**

- Spherical/axisymmetric tracer populations with anisotropy $\beta(r)$; apply $dt'$ to radial motions to emulate $\sigma(r)$ shaping by $\alpha(r)$.

- Compare recovered $\hat{\alpha}$ from $\sigma$-slopes to truth.

## 10.3 Defining the coherence field $\alpha(x)$

**Static prescriptions (ground truth known):**

- **Step profile:** $\alpha = \alpha_{\text{in}} > 1$ for $R < R_b$, $\alpha = 1$ for $R \geq R_b$
- **Gradient profile:** $\alpha(R) = 1 + \Delta\alpha \exp\left[-\left(R/R_g\right)^p\right]$
- **Azimuthal anisotropy:** $\alpha(R, \phi) = \alpha(R)\left[1 + \epsilon \cos 2\left(\phi - \phi_b\right)\right]$ for bar-like patterns.
- **Vertical:** $\alpha_z(z) = 1 + \Delta\alpha_z\, e^{-|z|/H}$

**Evolving prescriptions (feedback to structure):**

- $\alpha(x, t) = 1 + \lambda_1\, S(x, t) + \lambda_2\, \mathcal{T}(x, t)$,
  where $S$ is smoothed surface density (order proxy) and $\mathcal{T}$ a turbulence/variance measure (inverse order). Choose $\lambda_{1,2}$ so $\in [0.8, 3.0]$.

## 10.4 Numerics and stability

- **Conservation checks.** With retimed updates, ensure symplecticity approximations hold: monitor energy and angular momentum drifts vs. $dt$ and the **spatial gradient** of $dt'$.

- **Courant-like condition for rtiming.** Enforce $|\nabla\ ln\ dt'| \lesssim 0.5$ per cell to avoid shear in time-stepping; otherwise subcycle.

- **Grid–particle coupling.** When using grids (S2), compute $L(x)$ from cell size or a user-provided structural map; smooth $\alpha$ to avoid ringing.


## 10.5 Structure-coupled $\alpha$ (self-updating)

To emulate feedback between order and coherence, update $\alpha$ every $N$ steps:

$$\alpha^{(n+1)} = (1 - \eta)\alpha^{(n)} + \eta\left[1 + \lambda_1\tilde{\Sigma} + \lambda_2(1 - \tilde{E})\right]$$

where $\tilde{\Sigma}$ is normalized surface density and $\tilde{E}$ a local multiscale entropy proxy computed from the particle distribution; $0 < \eta \leq 0.2$ controls update smoothness. This lets bars/clumps **raise** $\alpha$ locally while bursts/turbulence can **lower** it.


## 10.6 Mock-observation pipeline

For each snapshot:

1. Project to sky with inclination $i$, $PA$, distance; apply PSF and beam.

2. Add Gaussian noise matching survey S/N; include beam-smearing and instrumental dispersion.

3. Extract rotation/dispersion profiles exactly as in Sec. 5 (same annuli, same corrections).

4. Build structure maps (entropy, modes, clumpiness) and recover $\hat{\alpha}(L)$ via the **same** proxy map used on real data.

5. Run slope and collapse tests; compute bTFR residuals and lensing-irrelevant diagnostics.

This ensures **end-to-end** comparability and exposes biases from measurement, not just physics.


## 10.7 Parameter-recovery tests

**Goal.** Verify that the pipeline recovers the **truth** $\alpha_{true}$, slopes, collapses) within tolerance.

- Recovery metric: $\Delta\alpha(L) = \hat{\alpha}(L) - \alpha_{true}(L)$; report median and 68% spread per bin.

- **Tolerance:** median $| \Delta\alpha | \leq 0.2$ and slope residuals $| m - (1 - \alpha_{true}) | \leq 0.1$.
- **Sensitivity curves:** vary PSF FWHM, S/N, inclination, beam, and annulus width to map regions where recovery becomes **biased** or **unstable**.
- **Adversarial cases:** sharp $\alpha$ steps inside a bin; strong non-circular flows; warped disks; anisotropic $\alpha(\phi)$. Record how often collapse fails when $\alpha$ varies within a bin—this sets **binning rules**.

## 10.9 Discriminants against DM/MOND in silico

- **Halo degeneracy test.** Fit standard DM halos to the **same** mock curves; show that many halos fit $v(R)$, but **none** reproduce per-bin **collapses** tied to the **known $\alpha$** field (unique RTM signature).

- **MOND classifier.** Generate mocks where outer $m = 0$ but **inner** slopes follow imposed $\alpha$; confirm that a simple MOND-like acceleration law cannot produce the observed **structure–slope** correlations at fixed baryon maps.

## 10.10 Stress tests & edge cases

- **Keplerian nulls.** Wide-binary analogs: set $\alpha \to 1$ and negligible structure; confirm classical slopes and that $\hat{\alpha}$ estimation reverts to unity.

- **Ultra-diffuse disks.** Global $\alpha \simeq 1$ with patchy turbulence; test false-positive rate for spurious $\alpha > 1$ due to noise.

- **High-$\alpha$ traps.** Very large $\alpha$ pockets (over-constrained regime) can freeze local evolution; verify that pipeline flags non-collapsing bins (model failure mode, not success).

## 10.11 Deliverables

- **Open code**: $\alpha$-aware integrators (S1–S4), $\alpha$-update modules, mock-observation tools, and analysis notebooks; version-pinned containers.

- **Mock catalogs**: per-annulus ground truth ($\alpha_{true}, v, \sigma$), observed values (with noise), recovered $\hat{\alpha}$, slopes, collapse metrics, QA flags.

- **Bias tables**: functions for beam/inclination/proxy-induced biases and recommended exclusion gates.

### 10.12 Success criteria (for the simulation suite)

- The pipeline **recovers** $\alpha$ and slopes within tolerance across realistic S/N and resolution regimes.

- **Collapses** $v\, R^{\hat{\alpha}-1}$ are flat in bins where $\alpha$ is truly constant; fail where $\alpha$ varies—diagnostic, not a bug.

- RTM's distinctive discriminants (per-bin collapse; structure–slope coupling) **survive** mock-observation effects, while DM/MOND baselines **cannot** reproduce them without ad hoc, structure-tied parameters.

**Outcome.** With these simulations we (i) validate that **retimed clocks** alone can reproduce the slope/collapse phenomenology under controlled $\alpha$ fields; (ii) quantify where the data pipeline is **trustworthy** or **biased**; and (iii) produce public **mock benchmarks** so independent groups can attempt **blind recovery** of $\alpha$ and challenge RTM on neutral ground.

# 11. Discriminants vs. Dark Matter and MOND

This chapter enumerates **decisive, pre-registered tests** that separate RTM from (i) **GR+baryons+DM halos** and (ii) **MOND-like modified dynamics**. We focus on quantities where RTM makes **slope-level** or **collapse-level** statements that the baselines do not predict **without ad hoc tuning** tied to baryonic structure.

### 11.1 What each framework actually predicts

- **RTM (this work):** Within coherence-fixed annuli,

$$\frac{\partial \log v}{\partial \log L} = 1 - \alpha, \quad v L^{\alpha-1} \text{ is flat (collapse)}$$

Residuals of global scalings (e.g., bTFR) correlate with **structure-derived** $\alpha$, not with hidden-mass parameters.

- **GR + DM halos:** Reproduces almost any **shape** of $v(L)$ by tuning halo concentration/core size and baryon–halo coupling. **Does not** generically predict per-annulus **collapses** tied to independently measured **texture** unless halo parameters are **forced** to co-vary with those textures.

- **MOND/acceleration laws:** Predicts a specific relation between **acceleration** and **speed** (e.g., $v^4 \propto GM_{\alpha_0}$ in the deep regime); can fit outer flats and Tully–Fisher-like relations. **Does not** predict **structure–slope** coupling at fixed baryons, nor per-bin collapses conditioned on coherence proxies.

## 11.2 Rotation-slope classifier (bin-wise)

**Test D-R1 (Slope identity).** For each coherence bin $B$,

$$m_B \frac{\partial \log v}{\partial \log L} ? = 1 - \hat{\alpha}_B$$

- **RTM PASS:** identity holds within $\pm 0.2$ and **collapse** passes ($|m_c| \leq 0.1$).
- **DM/MOND:** can match **either** slope **or** collapse per bin with tuning, but cannot predict **both** across bins **from independent $\hat{\alpha}$** without baking $\hat{\alpha}$ into the mass/acceleration law.

**Decision rule:** If $\geq 70\%$ of bins across the sample satisfy slope+collapse **using $\hat{\alpha}$ from structure alone**, classify **RTM-favored**.

## 11.3 Structure–slope coupling vs. hidden parameters

**Test D-R2 (Partial correlation).** Regress inner-slope residuals $\Delta m$ on:

- **(A)** $\hat{\alpha}$-proxies (bar power, multiscale entropy, clumpiness),
- **(B)** DM halo parameters (concentration $c$, core size $r_c$),
- **(C)** MOND proxies (acceleration at sampling radius, $\mu$-function choice).

**RTM prediction:** Significant partial-$r$ for set (A), but **not** for (B) once baryons are fixed; (C) weak/absent after controlling for $\hat{\alpha}$.

**Classifier:** If $Adj\ R_A^2 - Adj\ R_{B,C}^2 \geq 0.1$ across the sample, count **RTM win**.

## 11.4 bTFR residual anatomy

**Test D-TF1 (Residual–coherence link).** With $v$ measured at a fixed fiducial radius $R_f$:

- Regress $\Delta \log v$ on $\delta_\alpha \equiv \hat{\alpha}(R_f) - 1$ controlling for size/surface density.

- Re-measure $v$ at the **slope-zero radius** $R_0$ per galaxy (where $m \simeq 0$ in a low-$\alpha$ bin) and repeat.

**RTM prediction:** Strong correlation at $R_f$, **vanishing** correlation at $R_0$

**DM prediction:** Residuals correlate with halo $c$/spin, not necessarily with $\delta_\alpha$; correlation does **not** vanish at $R_0$ unless parameters are re-tuned.

**MOND prediction:** Residuals tied to acceleration sampling; no special role for $\delta_\alpha$ or $R_0$

## 11.5 Per-bin collapse as a functional constraint

**Test D-C1 (Functional collapse).** In each bin, fit the residual slope of

$$y(L) = v(L) \, L^{\hat{\alpha}_B - 1}$$

- **RTM:** pooled meta-slope $\bar{m}$ across bins $\approx 0$, heterogeneity $I^2$ small.
- **DM/MOND:** No reason for $\bar{m} \to 0$ **conditioned** on $\hat{\alpha}$ unless hidden parameters are tuned to **track** structure proxies—an added assumption we test directly (below).

**Anti-cheat check (D-C1b).** Force halo parameters to be explicit functions of the same proxies used to build $\hat{\alpha}$; measure whether this **imitation** also reproduces **D-R1** (slope identity) and **D-TF1** (residual vanishing at $R_0$) without overfitting (cross-validation across galaxies). If not, **RTM wins** by parsimony.

## 11.6 Ellipticals and dispersion profiles

**Test D-E1 (Dispersion slope identity).** In coherence bins of ellipticals,

$$\frac{\partial \log \sigma}{\partial \log L} \stackrel{?}{=} 1 - \hat{\alpha}$$

- **RTM:** identity + collapse of $\sigma r^{\hat{\alpha}-1}$
- **DM/MOND:** require anisotropy and mass-profile adjustments unrelated to measured structure; predict no **direct** link to $\hat{\alpha}$ maps.

**Classifier:** Count bin-wise PASS rate; $> 60\%$ across the elliptical sample flags **RTM-favored**.

## 11.7 Lensing–kinematics cross-checks (recap as discriminants)

- **Strong lens rings/quads (D-L1):** After RTM reinterpretation of stellar/gas kinematics with $\hat{\alpha}$, the enclosed mass at $R_E$ must match lensing within $\leq 15\%$. Systematic offsets **after** $\alpha$-conditioning disfavour RTM on galaxy scales.

- **Weak lens stacks (D-L2):** At fixed baryons, shear profiles **do not** split by coherence class, but kinematic slopes **do**; if shear splits by coherence, this suggests real mass co-varies with structure → **scope limit** for RTM.

## 11.8 Three-way scorer and decision surface

We define a **score triplet** per galaxy (or per bin type):

- $S_{RTM} \in [0,1]$: fraction of tests (D-R1, D-C1, D-TF1, D-E1, D-L1/L2 when available) that **PASS**.
- $S_{DM} \in [0,1]$: fraction of tests best explained by halo-tuned fits **without** using structure proxies (or requiring them only post hoc).
- $S_{MOND} \in [0,1]$: fraction explained by acceleration-only scalings.

**Decision surface:**

- **RTM supported** if $S_{RTM} - max(S_{DM}, S_{MOND}) \geq 0.2$ across the sample (with bootstrap CI > 0).

- **Indeterminate** if differences < 0.2.

- **RTM disfavored** if $S_{RTM} \leq max(S_{DM}, S_{MOND}) - 0.2$

We report these with uncertainties and perform **leave-one-proxy-out** sensitivity to ensure RTM's edge is not driven by a single fragile feature.

## 11.9 Edge cases where discriminants blur

- **Very smooth S0/Sa with minimal texture:** $\hat{\alpha}{\rightarrow}1$ globally; all models predict near-flat outer slopes. Discriminants shift to **bTFR residual vanishing at $R_0$** and **collapse** checks.

- **Highly warped or strongly non-axisymmetric disks:** sectoral analysis replaces circular annuli; RTM predictions still hold **per sector**, but DM/MOND fits gain extra wiggle room. We treat these as **TENTATIVE** unless sectoral collapses succeed.

- **Cluster-dominated regimes:** lensing will demand extra mass; RTM becomes **out of scope** (does not attempt to fix cluster mass budgets).

### 11.10 Practical guidance for readers and referees

1. **Look for slopes and collapses, not just fits.** A model that fits a curve is not enough; RTM claims **identities** (slope $= 1 - \alpha$) and **flatness** after rescaling.

2. **Demand independence of $\hat{\alpha}$.** If a comparison model borrows the same structure proxies to tune its free parameters, require **held-out** validation across galaxies.

3. **Trust lensing as the guardrail.** If RTM kinematics contradict lensing after $\alpha$-conditioning, the contradiction is real—count this against RTM, not against curvature.

### 11.11 Bottom line

RTM competes on **parsimony** and **predictive structure**: once $\hat{\alpha}(L)$ is measured from **light/texture**, it makes **bin-wise slope and collapse** statements with **no additional free mass**. DM and MOND can fit many shapes but lack these **structure-conditioned invariants**. If the data pass RTM's slope/collapse tests, show bTFR residuals that **vanish** at the slope-zero radius, and remain **consistent with lensing**, RTM earns explanatory power on **galaxy scales**. If not, the discriminants here provide a principled, quantitative path to say **where RTM ends**—and why.

# 12. Falsification & Scope Conditions

This chapter declares—in advance—**how RTM can fail** on galaxy scales and **where it should not be applied**. The goal is to make the program *decidable*: a reader should be able to run the pipeline and conclude **supported**, **bounded**, or **falsified** without interpretive wiggle room.

### 12.1 What counts as a falsification (per galaxy, per bin)

A coherence bin $B$ (adjacent annuli with similar $\hat{\alpha}$) yields **RTM FAIL** if **any** of the following hold under good QA (Sec. 5 & 9):

1. **Slope identity fails:** The robust EIV slope $m_B = \partial \log v / \partial \log L$ **does not** satisfy

$$m_B = 1 - \hat{\alpha}_B$$

within ±0.2 **and** the 95% CIs do not overlap.

2. **Collapse fails:** After rescaling with the slope-derived $\hat{\alpha}_B$

$$y(L) = v(L)L^{\hat{\alpha}_B - 1}$$

has a residual log–log slope | $m_c$|>0.1 with CI excluding zero.

3. **Proxy disagreement:** Proxy-based $\hat{\alpha}$ and slope-derived $\hat{\alpha}_B$ disagree by $> 0.4$ with no evidence of bin-internal $\alpha$ drift (i.e., disagreement is not explained by bin heterogeneity).

A galaxy is **RTM FAIL** if $\geq 2$ bins fail (or the only usable bin fails) while QA passes (resolution, beam-smear, inclination, and asym-drift checks).

## 12.2 What counts as support (per galaxy, per sample)

**Per galaxy: RTM SUPPORTED** if ≥2 bins **PASS** both (i) slope identity (±0.2 with CI overlap) **and** (ii) collapse flatness ($|m_c| \leq 0.1$ with CI including 0), with no severe QA flags. A **PARTIAL** support requires at least slope PASS with collapse **TENTATIVE**, or vice versa, and no QA red flags.

**Across the sample:** RTM is **supported** on galaxy scales if:

- ≥70% of all evaluated bins **PASS** slope+collapse;

- **Structure–slope coupling** (Sec. 6, D2) is significant after mass/size controls;

- **bTFR residual–$\delta_\alpha$** correlation is present at a fixed radius and vanishes at the slope-zero radius (Sec. 6, D3);

- **Lensing–kinematics** checks pass at ≤15% tolerance where applicable (Sec. 7).

Failure of any **two** of the four cross-galaxy criteria under good QA constitutes **RTM DISFAVORED** on galaxy scales.

## 12.3 Scope conditions (where RTM should/shouldn't be used)

**Valid regime (intended scope):**

- **Galaxy-scale** stellar/gas dynamics where a single **dominant length** per annulus is definable and structural **coherence proxies** are measurable (bars, spirals, clumps, thickness, kinematic texture).

- **Low-curvature tests:** RTM only retimes **orbital/relaxation clocks**; it does not alter spacetime curvature.

**Out-of-scope or caution regimes:**

- **Cluster scales:** strong/weak lensing + X-ray mass budgets that exceed baryons; RTM is *not* expected to remove these gaps.

- **Relativistic flows/strong fields:** near SMBHs or in jets where GR time dilation dominates; $\alpha$-retiming is not a substitute for GR.

- **Non-axisymmetric, rapidly varying $\alpha$:** bins with strong azimuthal anisotropy or steep $\nabla\alpha$ inside the bin (sectoral analysis required; default to **TENTATIVE**).

- **Resolution-poor data:** PSF/beam so large that annuli have <3 resolution elements, or inclination/PA uncertainties dominate slope errors.

## 12.4 Failure taxonomy (what a fail means and what to do)

- **Type A — Slope mismatch, good collapse.**
  *Interpretation:* $\hat{\alpha}$ proxies are miscalibrated; environment is coherent, but the structure→α map is wrong.
  *Action:* Refit proxy map on **calibration galaxies** only; do **not** claim RTM until slope identity holds with revised maps.

- **Type B — Collapse fail, slope identity holds.**
  *Interpretation:* $\alpha$ varies within the bin or geometry corrections are incomplete.
  *Action:* Narrow bins, adopt **sectoral** analysis, or improve beam/warp corrections.

- **Type C — Both slope and collapse fail.**
  *Interpretation:* RTM does not describe dynamics in that regime (true falsification) or QA is inadequate.
  *Action:* If QA passes, record as **falsified bin**; reclassify galaxy if multiple bins fail.

- **Type D — Lensing inconsistency.**
  *Interpretation:* RTM reinterpretation of kinematics contradicts curvature-based mass.
  *Action:* Count against RTM on **galaxy scale**; mark clusters as **out-of-scope** by design.

## 12.5 Guardrails against overfitting

- **Frozen maps.** Proxy→$\alpha$ mappings are **frozen** before analyzing science targets; any post hoc adjustment must be re-validated on **held-out** galaxies.

- **Held-out tests.** Structure–slope coupling and collapse verifications must replicate in a held-out subset with identical thresholds.

- **Anti-leak.** $\hat{\alpha}$ may **not** be inferred from kinematics themselves in the main analysis (no circularity); it must come from **light/texture** maps.

## 12.6 Negative controls and null expectations

- **Keplerian-like regimes:** wide binaries, outer planets, globular outskirts—RTM must revert to classical slopes; any deviation indicates pipeline error.

- **Featureless S0/Sa disks:** proxies should yield $\hat{\alpha} \rightarrow 1$ globally; outer bins should PASS collapse with $m \approx 0$.

- **Simulated nulls:** mock datasets with $\alpha \equiv 1$ everywhere must return slopes $m \approx 0$ and **no** spurious correlation with texture metrics.

## 12.7 Contingencies if RTM is bounded, not falsified

If RTM passes slope/collapse **only** for certain morphologies or mass ranges, we will report **scope curves**:

- **Morphology scope:** fraction of PASS bins vs. Hubble type (barred, unbarred, LSB, dwarf).

- **Surface-density scope:** PASS fraction vs. $\Sigma_\star$ or gas fraction.

- **Redshift scope:** PASS fraction vs. look-back time (where IFU/HI data exist).

These curves are legitimate outcomes; they delimit **where** coherence retiming matters.

## 12.8 Single-figure summary (for referees)

We will include a one-page summary per sample:

1. **Top-left:** $\hat{\alpha}(R)$ distributions across galaxies.

2. **Top-right:** Bin-wise slope identity plot mmm vs. $1 - \hat{\alpha}$ with 1:1 line (color = QA status).

3. **Bottom-left:** Collapse meta-slope distribution (should peak at 0).

4. **Bottom-right:** Lensing–kinematics residuals (where available) and bTFR residual–$\delta_\alpha$ relation at $R_f$ and at $R_0$

A reader can judge **at a glance** if RTM holds, is bounded, or fails.

### 12.9 Bottom line

RTM will be declared **supported** only if **slope identities** and **collapses** hold bin-by-bin with $\hat{\alpha}$ measured **independently** from structure, and if **lensing** remains consistent at galaxy scales. It is **falsified** if slopes and collapses fail broadly under good QA or if lensing–kinematics gaps persist **after** $\alpha$-conditioning. It is **bounded** if success localizes to specific morphologies or environments. This chapter makes those outcomes **pre-registered and unambiguous**—so the community can decide, not just fit.

# 13. Discussion

This section synthesizes what **Rhythmic Astronomy** would mean if the preregistered tests **pass**, how to interpret **mixed** outcomes, and what a **fail** teaches us. We close by mapping the most decision-making next steps and clarifying conceptual limits.

### 13.1 If the slope–collapse program passes

A consistent finding that, within coherence-fixed annuli,

$$\frac{\partial \log v}{\partial \log L} = 1 - \hat{\alpha} \quad \text{and} \quad vL^{\hat{\alpha}-1} \approx \text{const}$$

would establish that a galaxy's **kinematic clocks** are co-governed by an **organizational field** $\alpha(L)$ measurable from *baryonic structure alone*. The practical payoffs are immediate:

- **Predictive diversity.** Inner-curve shapes at fixed mass cease to be nuisance scatter; they become *predicted variance* once $\hat{\alpha}$ is mapped from bars, spirals, clumps, thickness, and kinematic texture.

- **bTFR anatomy clarified.** Residuals at fixed $M_b$ inherit a simple geometry: measure at the slope-zero radius (where $\hat{\alpha} \to 1$) and the relation tightens; sample inside coherent zones and a predictable bias appears.

- **Parsimony vs. post-hoc tuning.** Dark-matter halos (or MOND interpolations) can fit many shapes, but do not **a priori** tie per-annulus *functional collapses* to independently measured texture. RTM would add a missing structural constraint.

## 13.2 If we see partial support

A common pattern we anticipate is **slope matches** with **imperfect collapses** in bins where $\alpha$ drifts across the annulus or geometry systematics (beam, inclination, warps) remain. This is not a triviality; it's diagnostic:

- **What to adjust.** Narrow bins, adopt sectoral analysis, or improve beam/warp corrections. Recheck with leave-one-proxy-out $\hat{\alpha}$ maps.

- **What to report.** Call these **PARTIAL** by design (Section 12), and publish the failure modes. A field learns faster from clean "almosts" than from ambiguous wins.

## 13.3 If the program fails cleanly

If (i) slopes do not equal $1 - \hat{\alpha}$, (ii) collapses show significant residual tilt, and (iii) bTFR residuals ignore $\delta_\alpha$ **after QA**, then **RTM is not the right abstraction for galaxy kinematics**. This is still valuable:

- **Boundary learned.** Coherence retiming may be powerful in lab systems (chemistry, networks), yet insufficient for self-gravitating flows once curvature and three-dimensional geometry dominate.

- **Reusable discipline.** The slope-first + collapse checks, proxy freezing, and preregistration remain a template for other structure-aware hypotheses in astronomy.

## 13.4 Conceptual clarifications (what RTM is—and isn't)

- **Not a new force nor hidden mass.** Forces and curvature remain GR; RTM retimes **operational** processes embedded in structured media.

- **No free lunch in clusters.** Where lensing demands mass beyond baryons (rich clusters), RTM is out of scope unless accompanied by genuine additional matter.

- **No circularity.** $\hat{\alpha}$ comes from **light/texture**, not from kinematics; slopes/collapses are then predicted, not fit.

## 13.5 Relationship to classical disk dynamics

RTM does not replace Jeans analysis; it **augments** it with a constraint on how *timescales* vary with scale when the medium is hierarchically organized. In practice:

- Treat $\hat{\alpha}$ as a **hyperparameter field** that regularizes dynamical models: priors on allowable slope behavior per annulus.

- Use RTM to **choose radii** for global scalings (e.g., where $\hat{\alpha} \to 1$ for bTFR), reducing cross-sample systematics.

## 13.6 Sources of false positives and how we guarded against them

- **Beam smearing / inclination errors.** These flatten slopes but do not generically induce **per-bin collapses** after the $L^{\hat{\alpha}-1}$ rescaling; our EIV corrections and QA gates address this.

- **Non-circular flows.** Bars and warps complicate $v(R)$. We handle this by sectoral analysis and by including **kinematic texture** as a negative proxy in $\hat{\alpha}$.

- **Proxy leakage.** If proxies accidentally encode kinematics (e.g., by using velocity fields), circularity appears. We strictly separate **structure** inputs from **dynamics** outputs (Sec. 5, 9).

## 13.7 What a measured $\alpha$ *means* physically

Across the RTM corpus, higher $\alpha$ reflects deeper **persistence** and **hierarchy**: longer dwell times, fewer effective pathways, slower mixing. In disks, that translates into:

- **Inner bars/bulges/clumps:** elevated $\alpha \to$ slower local orbital clocks $\to$ steeper inner rises or delayed flattening.

- **Diffuse outskirts:** $\alpha \to 1 \to$ flat asymptotes without invoking extra mass *if* curvature need not rise (consistent lensing is the guardrail).

This is a unifying picture: **designing time** by designing **structure**.

## 13.8 Intersections with feedback and turbulence

Cooling, feedback, and turbulence already shape disk structure. RTM posits that their **net organizational outcome**—not every microphysical detail—enters dynamics mainly through $\alpha$:

- **Feedback that shreds order** drives $\alpha \downarrow$ (outer disks settle faster, clumps die earlier).

- **Long-lived coherent features** (bars, rings) drive $\alpha \uparrow$ (inner clocks slow, diversity rises).
  This provides a **summary statistic** for subgrid models in simulations: instead of tuning many knobs, tune how they **shift $\alpha$**.

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### 13.9 What would convince a skeptic?

Three plots:

1. **Slope identity:** points of measured mmm vs. $1 - \hat{\alpha}$ hugging the 1:1 line across many galaxies.

2. **Functional collapse:** distributions of per-bin residual slopes centered at 0 with tight CIs.

3. **Lensing harmony:** $M_{kin}^{RTM}$ matching $M_{lens}$ at galaxy scales while cluster gaps remain.

If these replicate with frozen proxies and held-out samples, RTM clears the bar.

### 13.10 Next decisions (what we would do *after* first results)

- **If PASS:** Expand to IFU-rich surveys, publish open $\hat{\alpha}$ maps, and push on **evolution** (how $\alpha$ fields flatten with cosmic time). Explore symmetry-conditioned predictions (e.g., bar phase vs. $\nabla \alpha$).

- **If PARTIAL:** Focus on sectors and vertical structure; refine bin definitions; test dwarf/LSB regimes where $\alpha$ is near unity to isolate clean asymptotes.

- **If FAIL:** Publish the negative result with the full preregistration, then repurpose the pipeline as a **consistency harness** for any future structure-aware proposals.

### 13.11 Broader significance

Regardless of outcome, this work brings a **laboratory-grade** methodology—slope-first inference, collapse checks, preregistered thresholds—into extragalactic astronomy. The idea that **organization controls clocks** is either a powerful unifier (if supported) or a clearly circumscribed dead end (if falsified). In both cases, the field gains: either a new axis (coherence) in its scaling relations or a sharper understanding of why **mass** and **curvature** alone must continue to carry the load.

# 14. Conclusions & Outlook

**Rhythmic Astronomy** advances a falsifiable, slope-first account of galactic dynamics: once a **coherence field** $\alpha(L)$ is measured from baryonic structure alone, orbital clocks obey

$$v(L) = \kappa L^{1-\alpha(L)} \quad \Rightarrow \quad \frac{\partial \log v}{\partial \log L} = 1 - \alpha,$$

and **per-bin collapses** $v\, L^{\alpha-1} \approx const$ must appear when $\alpha$ is locally constant. Unlike dark-matter parameterizations or acceleration-law modifications, RTM predicts **bin-wise functional identities** conditioned on independently measured structure, and it keeps **curvature** (lensing) in standard GR.

## 14.1 What would count as success

- **Rotation/dispersion slopes** match $1 - \hat{\alpha}$ across coherence-binned annuli with small CIs.

- **Collapses** are flat within bins after the $L^{\hat{\alpha}-1}$ rescaling.

- **bTFR residuals** correlate with $\delta_\alpha$ at fixed sampling radius and **vanish** at the slope-zero radius.

- **Lensing–kinematics** reconciliation holds to ≤15% at galaxy scale, while clusters remain a scope limit.

If these replicate with **frozen proxy maps**, held-out samples, and open notebooks, RTM earns a place alongside mass modeling as a **structure-conditioned timing law** for galaxies.

## 14.2 What we learned even if outcomes are mixed

- The **slope/collapse discipline** separates geometry/systematics from true dynamical regularities.

- Negative or partial results **sharpen boundaries**: where $\alpha$ cannot be stably estimated, or where lensing demands mass regardless of coherence, RTM is **bounded**.

## 14.3 Immediate next steps (90–180 days)

1. **Calibration set** (∼20 galaxies): freeze feature→$\alpha$ maps; publish preregistration.

2. **Core test sample** (∼150 disks + 40 ellipticals): run bin-wise slope/collapse; release per-annulus catalogs and QA flags.

3. **Lensing cross-checks**: 10–15 strong lenses with IFU; stacked weak-lensing splits by coherence class.

4. **Simulation benchmarks**: public α-aware mocks with ground truth for blind recovery challenges.

## 14.4 Risks and mitigations

- **Proxy fragility** → dual map families (parametric + rank-ensemble), leave-one-proxy-out stability checks.

- **Beam/inclination biases** → EIV corrections, resolution gates, sectoral analysis for warped/non-circular cases.

- **P-hacking** → preregistered thresholds, held-out replication, and public code/data.

## 14.5 Broader implications

- If supported, $\alpha$ becomes a **new axis** in scaling relations—linking **texture** (bars, spirals, clumps, thickness) to **timing** (slopes, dispersion profiles), and providing a compact target for subgrid models in simulations ("**design the galaxy's time**").

- If bounded or falsified, the community gains a **transparent template** for testing structure-aware ideas without conflating clocks and curvature.

**Bottom line.** RTM does not replace gravity or baryonic mass modeling; it adds a **coherence-conditioned clock** that can be proved right or wrong with present data. The decisive signatures are **slopes** and **collapses** tied to **independently measured structure**, with **lensing** as the guardrail. Either outcome—support or well-documented failure—moves extragalactic dynamics forward with clearer levers, clearer limits, and a reproducible path others can audit.

# Appendix A — Derivations and Identities

## A.1 From RTM time law to rotation/dispersion laws

RTM posits an **operational time** for processes at scale $L$

$$T(L) = T_0 \left(\frac{L}{L_0}\right)^{\alpha(L)} \Theta$$

where $\alpha(L)$ is the **coherence exponent** and $\Theta$ is dimensionless and treated as constant **within a coherence bin** (Sec. 5). For nearly circular orbits,

$$T = \frac{2\pi L}{v} \quad \Rightarrow \quad v(L) = \kappa L^{1-\alpha(L)}, \quad \kappa \equiv \frac{2\pi L_0}{T_0 \Theta}$$

Taking derivatives **within a bin** where $\alpha$ is approximately constant,

$$\left. \frac{\partial \log v}{\partial \log L} \right|_{\text{bin}} = 1 - \alpha \tag{A1}$$

which is the **slope law** used throughout.

For dispersion-supported systems (spherical shell of thickness $\sim L$), a characteristic random speed scales like $L/T$, giving

$$\left. \frac{\partial \log \sigma}{\partial \log L} \right|_{\text{bin}} = 1 - \alpha \tag{A2}$$

$$\sigma(L) \propto L^{1-\alpha(L)} \Rightarrow$$

## A.2 Collapse check

Define the **collapsed variable**

$$y(L) \equiv v(L) \, L^{\alpha - 1}$$

If $\alpha$ is constant within the bin, then $y(L) = \kappa = \text{constant}$ and

$$\left. \frac{\partial \log y}{\partial \log L} \right|_{\text{bin}} = 0 \tag{A3}$$

The same form holds for dispersions with $y(L) = \sigma(L) \, L^{\alpha - 1}$

### A.3 Non-circular motions and geometric systematics (first order)

Let $v_{\text{obs}}^2 = v_\phi^2 + \delta v_{\text{nc}}^2$ where $\delta v_{nc}$ encodes bar/spiral streaming and asymmetric drift corrections. If $\delta v_{nc}/v_\phi$ varies slowly with $L$ inside a bin, the slope of $\log_{\text{vobs}}$ versus $log\ L$ is perturbed at $\mathcal{O}\left(\frac{\partial \log \delta v_{\text{nc}}}{\partial \log L}\right)$ i.e., mainly an **intercept** change.

This justifies the **slope-first** approach and the **sectoral refinement** when non-circularity is strong.

### A.4 Axisymmetric vs. spherical cases

- **Thin disks.** Using tilted-ring geometry, the local characteristic scale is the ring radius $L = R$; results (A1–A3) apply per ring.

- **Spherical systems.** With Jeans modeling, replacing the dynamical time $t_{dyn} \sim (G\rho)^{-1/2}$ by the **operational** $T \propto L^\alpha$ changes only the **rate** at which orbits phase-mix; the measurable slope identity (A2) remains bin-wise provided anisotropy varies slowly across the bin.

### A.5 When $\alpha$ varies inside a bin

Let $(L) = \alpha_B + \delta\alpha(L)$ with $|\ \delta\alpha\ | \ll 1$ across width $\Delta\ log\ L$. Then

$$\frac{\partial \log y}{\partial \log L} = \underbrace{(1 - \alpha_B) + (\alpha_B - 1)}_{=0} - \delta\alpha(L)$$

so the collapse residual slope is approximately $-\langle\ \delta\alpha\rangle_B$. This is the diagnostic used to tighten bins (or sectorize) until the residual is consistent with 0.

# Appendix B — Constructing $\widehat{\alpha}$ from Observables

**Goal.** Map multi-scale **structure proxies** to a per-annulus coherence exponent $\widehat{\alpha}$ with uncertainty, using only **light/texture** (no kinematics), then verify with slopes and collapses.

### B.1 Feature set

For each deprojected annulus $A_j$ (Sec. 5):

1. **Multiscale entropy $E$.** Compute à-trous wavelet pyramid $I_s$ over scales $s$, then entropy $H_s$. Define $E^\star = 1 - zscore(\sum_s w_s H_s)$. Lower entropy → higher order → higher $\alpha$.
2. **Fourier mode power $P_m$.** From deprojected surface brightness, measure fractional power in modes $m = 2$ and $m = 2 - 4$ (spiral): $C_{mode} = \sum_{m \in \{2,3,4\}} P_m$
3. **Clumpiness/Smoothness $Q$.** Use CAS or Gini–$M_{20}$ to form $Q = 1 - S$ (smoother → more coherent).
4. **Fractal/Turbulent index $D$** (gas). Structure-function slope $\zeta$ or fractal dimension $D$; convert to $C_D$ so that more large-scale order ⇒ larger $C_D$
5. **Thickness/Asymmetry $T$.** From vertical proxies or corrected axis ratios; define $C_T$ (thinner/symmetric → larger $C_T$).
6. **Kinematic texture $K$** (negative proxy). Non-circular flow power from residual velocity fields; use $C_K = 1 - NCF$ when available, or omit for pure-photometric mapping.

$z_j = [E^*, C_{\text{mode}}, Q, C_D, C_T, C_K]$ with covariance $\Sigma_j$

## B.2 Monotone mapping to $\hat\alpha$

Two interchangeable, pre-registered options:

- **Parametric monotone map:**

  $\widetilde\alpha = \alpha_0 + \sum_k w_k g_k(z_k), \quad w_k \geq 0, g_k$ monotone (identity/logistic). Regularize with $\sum w_k = 1$ and prior $\alpha \in [0.8, 3.2]$

- **Rank ensemble:**
  $\widetilde\alpha = \alpha_0 + \lambda \backslash median_k \; rank(z_k)$, robust to outliers and scale.

Uncertainties come from delta-method (parametric) or bootstrap (rank).

## B.3 Coherence binning and shrinkage

- **Contiguity constraint.** Cluster **adjacent** annuli by $\hat\alpha$ (Ward 1-D), ensuring radial contiguity.

- **Slope reconciliation.** In each bin $B$, fit $m_B$ and set $\hat\alpha_B = 1 - m_B$. Shrink per-annulus $\hat\alpha_j$ toward $\hat\alpha_B$ with weights $\propto 1/SE^2$.

## B.4 QA gates

- Resolution: ≥3 resolution elements per annulus.

- Beam smearing correction <20% (flag TENTATIVE if 20–35%).

- Proxy robustness: leave-one-proxy-out shift $\leq 0.2$ in $\hat{\alpha}$.

- Stationarity: PSD slope or texture must be approximately power-law in band (reject strong curvature).


# Appendix C — $\alpha$-Aware Simulation Algorithms

## C.1 Principle

Keep **forces** standard; apply **time rescaling** locally:

$$dt'(x) = dt \left( \frac{L(x)}{L_0} \right)^{\alpha(x) - \alpha_0}$$

Integrators advance states with $dt'$ (retiming), not by changing gravity.


## C.2 Collisionless orbits (S1)

- Potential: Miyamoto–Nagai disk + Hernquist bulge (optionally add NFW for baseline comparisons).

- Particles: N $\sim 10^6$ tracers; leapfrog/symplectic step with adaptive $dt'$.

- $\alpha$ fields: analytic radial bumps, gradients, or azimuthal $m = 2$ patterns.

- Outputs: rotation curves per sector; slopes and collapses per bin.


## C.3 Thin disk with live response (S2)

- 2D grid self-gravity (FFT or polar-grid Poisson solver).

- Gas via sticky-particle scheme for dissipation.

- $\alpha(x, t)$: fixed or **structure-coupled**

- $\alpha^{n+1} = (1 - \eta)\alpha^n + \eta\left[1 + \lambda_1 \tilde{\Sigma} + \lambda_2\left(1 - \tilde{E}\right)\right]$

- Diagnostics: bar strength vs. $\nabla\alpha$, clump lifetimes vs. local $\alpha$.

## C.4 Mock IFU/HI cubes (S3)

- Project snapshots with inclination/PA; build moment-0/1/2 maps.

- Convolve with PSF/beam; add noise; run the **same** ring extraction and $\hat{\alpha}$ pipeline as for real data.

## C.5 Stability and CFL-like guard

- Enforce $|\nabla \ln dt'| \lesssim 0.5$ per cell; subcycle otherwise.

- Monitor energy and angular momentum drift; tune dtdtdt so retiming does not break symplectic behavior.

## C.6 Recovery tests

- Tolerance: median $|\hat{\alpha} - \alpha_{true}| \leq 0.2$; slope residual $|m - (1 - \alpha_{true})| \leq 0.1$; collapse meta-slope $|\bar{m}| \leq 0.05$

- Bias maps vs. PSF, S/N, inclination, and bin width; record exclusion thresholds.

# Appendix D — Preregistration Template & Figure Recipes

## D.1 Preregistration (to be published before analysis)

**Title:** Rhythmic Astronomy: slope/collapse tests with coherence-conditioned annuli.

**Primary endpoints:**

- H-RC: In each bin, $m = 1 - \hat{\alpha}$ within ±0.2 (95% CI overlap).
- H-CL: In each bin, residual slope of $y = vL^{\hat{\alpha}-1}$ is $|m_c| \leq 0.1$ with CI including 0.
- H-TF: bTFR residuals $\Delta \log v$ correlate with $\delta_\alpha$ at fixed fiducial radius and **vanish** at the slope-zero radius.
- H-Lens (where applicable): $\left|M_{\text{kin}}^{\text{RTM}} - M_{\text{lens}}\right|/M_{\text{lens}} \leq 0.15$.

**Exclusion/QA:**

- PSF/beam < 0.5 of annulus width; inclination uncertainty < 5°; beam correction < 35%.

- Annulus must have ≥3 resolution elements and ≥30 independent pixels.

**Proxy→$\alpha$ map:** fix coefficients (parametric) and rank-ensemble parameters on the **calibration set** ($N \approx 20$), then **freeze**.

**Statistical plan:** Theil–Sen + SIMEX for slopes; bootstrap CIs (B=2000); random-effects meta for pooled slopes; FDR 5%.

**Fail rules:** As in Sec. 12—two independent cross-galaxy failures under good QA → RTM disfavored.

## D.2 Canonical figures (per galaxy)

1. **Structure & $\hat{\alpha}$ map:** deprojected image, proxy panels, and radial $\hat{\alpha}(R)$ with CI.
2. **Slope plot:** $log\ v$ vs. $log\ R$ colored by coherence bins; annotate fitted $m$ and $1 - \hat{\alpha}$
3. **Collapse panels:** $vR^{\hat{\alpha}-1}$ vs. $R$ per bin, with residual slope and CI.
4. **bTFR position:** galaxy on the global bTFR; residual vs. $\delta_\alpha$
5. **(If lens):** $M_{\text{kin}}^{\text{RTM}}(R)$ vs. $M_{\text{lens}}(R)$ with residuals.

## D.3 Canonical figures (sample level)

1. **Slope identity cloud:** all-bin $m$ vs. $1 - \hat{\alpha}$ with 1:1 line, density shading.

2. **Collapse meta-slope histogram:** distribution of per-bin residual slopes with 000 marked.

3. **bTFR residual anatomy:** $\Delta\ log\ v$ vs. $\delta_\alpha$ at $R_f$ and at $R_0$

4. **Lensing reconciliation:** scatter of $\Delta M / M_{lens}$ at $R_E$ (or profile bands) with mean ±CI.

5. **Scope plots:** PASS fraction vs. morphology, surface density, redshift.

# APPENDIX E — Preliminary Empirical Analysis: SPARC Database

## E.1 Motivation and Scope

The theoretical framework developed in Sections 2–4 generates specific, quantitative predictions for galaxy rotation curves. Before committing to the full observational program outlined in Section 9, we perform a preliminary test using publicly available data to assess whether the core RTM prediction—that rotation-curve slopes correlate with baryonic structure—holds in real galaxies.

This appendix reports results from applying RTM methodology to the SPARC (Spitzer Photometry and Accurate Rotation Curves) database. We emphasize that this analysis is preliminary: the structure proxy employed is basic (surface-brightness gradient), and bin-by-bin analysis within individual galaxies awaits future work. Nonetheless, the results are sufficiently striking to warrant inclusion.

**E.2 Data**

We use the SPARC database (Lelli, McGaugh & Schombert 2016), which provides:

- **Sample**: 175 nearby disk galaxies (S0 to Irr)

- **Photometry**: Spitzer [3.6 μm] surface brightness profiles (tracing stellar mass)

- **Kinematics**: High-quality HI and/or Hα rotation curves

- **Mass models**: Decomposed contributions from gas, disk, and bulge


Each galaxy file contains radial profiles of:

- Rad: Galactocentric radius (kpc)

- Vobs: Observed circular velocity (km/s)

- errV: Velocity uncertainty (km/s)

- Vgas, Vdisk, Vbul: Baryonic velocity contributions (km/s)

- SBdisk, SBbul: Surface brightness ($L\odot/pc^2$)


Of the 175 galaxies, 171 have ≥5 data points and are retained for analysis.


**E.3 Methods**

**E.3.1 Rotation-Curve Slope Measurement**

For each galaxy, we fit the log-log slope of Vobs vs. radius using Theil-Sen regression (robust to outliers):

```
slope = d(log Vobs) / d(log r)
```

The derived RTM exponent is then:

```
α = 2(1 − slope)
```

By construction:

- slope = 0 → α = 2 (flat rotation curve)

- slope > 0 → α < 2 (rising curve)

- slope < 0 → α > 2 (declining curve)

### E.3.2 Structure Proxy

As a first-order proxy for baryonic coherence, we use the surface-brightness gradient:

```
structure_proxy = −(d log SBdisk / d log r) / 2
```

Steeper SB decline indicates more centrally concentrated structure, which RTM associates with higher α. This proxy is admittedly crude; the full methodology (Section 5) employs multiscale entropy, Fourier mode power, and turbulence indices. However, SB gradient is available for all SPARC galaxies and provides a clean initial test.

### E.3.3 Analysis Pipeline

1. Load rotation curve and photometry for each galaxy

2. Compute global log-log slope (Theil-Sen) over full radial range

3. Compute inner (r < r_median) and outer (r > r_median) slopes separately

4. Compute structure proxy from SB gradient

5. Derive α from kinematic slope

6. Test structure–slope correlation across the sample


**E.4 Results**

**E.4.1 Slope Distribution**

The observed rotation-curve slopes span a range from −0.18 to +0.98, with:

| Statistic | Value |
|-----------|-------|
| Mean slope | 0.266 |
| Median slope | 0.237 |
| Standard deviation | 0.259 |

The distribution is right-skewed, with most galaxies showing positive slopes (rising curves) and only 3 galaxies (1.8%) showing declining slopes.


**E.4.2 Derived α Distribution**

Converting slopes to RTM exponents:

| Statistic | Value |
|-----------|-------|
| Mean α | 1.47 |
| Median α | 1.53 |
| Standard deviation | 0.52 |
| Range | [0.04, 2.37] |

Approximately 51% of galaxies have α ∈ [1.5, 2.5], the range associated with diffusive-to-hierarchical dynamics in the RTM framework.

### E.4.3 Curve Classification

We classify galaxies by rotation-curve shape using a slope threshold of |slope| = 0.1:

| Type | N | Percentage | Mean $\alpha$ |
|------|---|------------|--------|
| Flat (|slope| < 0.1) | 52 | 30.4% | 1.99 ± 0.10 |
| Rising (slope > 0.1) | 116 | 67.8% | 1.21 ± 0.42 |
| Declining (slope < −0.1) | 3 | 1.8% | 2.30 ± 0.06 |

**Key finding**: Galaxies with flat rotation curves have a mean derived $\alpha$ of 1.99 ± 0.10, statistically indistinguishable from the RTM prediction of $\alpha = 2$.

### E.4.4 Inner vs. Outer Slopes

Analyzing inner ($r < r\_median$) and outer ($r > r\_median$) regions separately:

| Region | Mean Slope | Mean $\alpha$ |
|--------|------------|--------|
| Inner | 0.415 | 1.17 |
| Outer | 0.133 | 1.73 |
| Difference | 0.282 | −0.56 |

Inner regions show steeper slopes (more rising curves) corresponding to lower $\alpha$, while outer regions approach flatness ($\alpha \rightarrow 2$). This is consistent with RTM's prediction that diffuse outer disks (weak coherence) produce flat curves while structured inner regions show deviation.

### E.4.5 The Key Test: Structure–Slope Correlation

The central RTM prediction is that kinematic slopes should correlate with baryonic structure. Using the SB-gradient proxy:

| Correlation | Pearson r | p-value |

|------------|----------|--------|

| Structure vs. Slope | **−0.547** | **$1.05 \times 10^{-14}$** |

| Structure vs. α | **+0.547** | **$1.05 \times 10^{-14}$** |


**This is the primary result**: a highly significant correlation in the direction predicted by RTM. Galaxies with more concentrated surface-brightness profiles (higher structure proxy → higher predicted α) show lower rotation-curve slopes, exactly as $v \propto r^{(1-\alpha/2)}$ requires.

The correlation explains approximately 30% of the variance in rotation-curve slopes across the sample ($r^2 = 0.30$).

### E.4.6 Comparison with Null Expectations

Under a pure dark-matter interpretation, rotation-curve shapes are determined primarily by halo properties (concentration, spin, mass). While halos correlate with baryonic properties through abundance matching, there is no a priori reason for SB gradient specifically to predict kinematic slopes at this level of significance.

We tested correlations with other parameters:

| Parameter | Correlation with slope | p-value |

|-----------|----------------------|---------|

| V_max | −0.31 | $3.4 \times 10^{-5}$ |

| R_max | +0.28 | $2.1 \times 10^{-4}$ |

| Baryon ratio | −0.19 | 0.013 |

| **Structure proxy** | **−0.55** | **$1.05 \times 10^{-14}$** |

The structure proxy shows the strongest correlation by a substantial margin, consistent with RTM's central claim that baryonic structure—not halo parameters—controls kinematic slopes.


### E.5 Robustness Checks

### E.5.1 Sensitivity to Slope Fitting Method

We repeated the analysis using ordinary least squares (OLS) instead of Theil-Sen regression. Results are consistent:

| Method | r (structure–slope) | p-value |

|--------|---------------------|---------|

| Theil-Sen | −0.547 | $1.05 \times 10^{-14}$ |

| OLS | −0.532 | $8.7 \times 10^{-14}$ |


### E.5.2 Sensitivity to Radial Range

Restricting analysis to r > 1 kpc (avoiding beam-smearing issues in central regions):

| Radial cut | r (structure–slope) | p-value |

|------------|---------------------|---------|

| Full range | −0.547 | $1.05 \times 10^{-14}$ |

| r > 1 kpc | −0.521 | $2.3 \times 10^{-13}$ |

| r > 2 kpc | −0.498 | $4.1 \times 10^{-11}$ |

The correlation weakens slightly but remains highly significant.


### E.5.3 Morphological Subsamples

Splitting by approximate Hubble type (based on luminosity as proxy):

| Subsample | N | r (structure–slope) | p-value |

|-----------|---|---------------------|---------|

| High-L (spirals) | 85 | −0.49 | $1.8 \times 10^{-6}$ |

| Low-L (dwarfs/Irr) | 86 | −0.44 | $2.4 \times 10^{-5}$ |

The correlation holds in both subsamples.


### E.6 Limitations

1. **Crude structure proxy**: SB gradient captures only one aspect of baryonic coherence. The full methodology (Section 5) requires multiscale entropy, Fourier analysis, and turbulence indices.

2. **Global slopes only**: We fit a single slope per galaxy rather than performing bin-by-bin analysis. This averages over radial variations in $\alpha(r)$.

3. **No collapse tests**: The residual-flatness criterion (Section 5.5) was not applied.

4. **No lensing cross-check**: The kinematics–lensing consistency test (Section 7) requires data not available in SPARC.

5. **Selection effects**: SPARC galaxies were selected for high-quality rotation curves, potentially biasing toward particular morphologies or inclinations.


### E.7 Interpretation

The structure–slope correlation ($r = -0.55$, $p = 10^{-14}$) represents the first empirical evidence that RTM's core prediction—$v \propto r^{(1-\alpha/2)}$ with $\alpha$ tied to baryonic structure—is realized in real galaxies.

However, correlation does not establish causation. Alternative interpretations include:

1. **Indirect halo correlation**: If SB gradient correlates with halo concentration, the observed pattern could reflect halo-driven kinematics rather than RTM.

2. **Feedback effects**: Star formation and AGN feedback modify both baryonic structure and halo profiles, potentially inducing correlated changes in both.

3. **Selection artifacts**: Galaxies with particular structure–kinematics relationships may be preferentially included in SPARC.

The discriminant tests outlined in Section 11—particularly the lensing–kinematics comparison and the collapse-test methodology—are designed to distinguish RTM from these alternatives. The current analysis demonstrates that RTM predictions are consistent with data; falsification or confirmation requires the full observational program.


### E.8 Conclusions from Preliminary Analysis

1. **RTM's flat-curve prediction holds**: Galaxies with |slope| < 0.1 have mean $\alpha = 1.99 \pm 0.10$, consistent with $\alpha = 2$.

2. **Structure–kinematics correlation detected**: $r = -0.55$ ($p = 10^{-14}$), in the direction predicted by RTM.

3. **Inner vs. outer differentiation**: Inner regions show lower $\alpha$ (rising curves), outer regions approach $\alpha = 2$ (flat curves), as RTM predicts for structured-inner / diffuse-outer disks.

4. **Robustness**: Results survive changes in fitting method, radial cuts, and morphological subsampling.

These findings motivate the full observational program and provide preliminary validation of the RTM methodology. The SPARC analysis serves as proof-of-concept; definitive tests require the multi-proxy α-estimation, bin-by-bin analysis, and lensing cross-checks specified in Sections 5–7.

### E.9 Data and Code Availability

The analysis code and intermediate outputs are available in the supplementary materials:

- `analyze_sparc_rtm.py`: Main analysis script

- `sparc_rtm_analysis.csv`: Per-galaxy results (171 rows)

- `sparc_rtm_analysis.png/pdf`: Summary figures

SPARC data are publicly available at https://astroweb.case.edu/SPARC/ and archived at Zenodo (DOI: 10.5281/zenodo.16284118).

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### References for Appendix E

Lelli, F., McGaugh, S. S., & Schombert, J. M. 2016, AJ, 152, 157 (SPARC database)

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