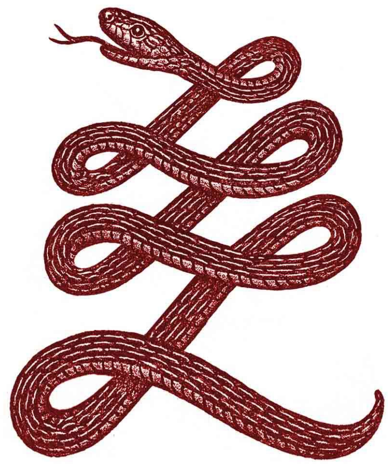

Rhythmic Ecology

A Slope-First Framework for Ecosystem Resilience and Regime Shifts

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Abstract

cosystems do not merely "have" characteristic times; they compose them across scales. We propose Rhythmic Ecology (RTM-Eco), a slope-first framework that models ecosystem tempo through the scaling law $\tau \propto L^\alpha$, where L is a layer-appropriate size proxy (burned patch area, watershed size, trophic depth, habitat network scale), τ is a characteristic time (recovery to pre-disturbance baseline, nutrient cycling time, recolonization time), and α is a coherence exponent that captures the system's multiscale organization. Inside coherence bins (environment slices with quasi-constant forcing), we test whether ecosystem data collapse to a power law, estimate α with errors-in-variables methods, and fuse accepted slopes across process families to build a real-time Ecosystem Coherence Index (ECI).

Computational validation. We implement and test the RTM-Eco framework through three simulation suites. S1 demonstrates $\tau(L)$ scaling for post-fire NDVI recovery across five ecosystem types, showing that α varies characteristically by biome (boreal forest $\alpha \approx 0.35$, grassland $\alpha \approx 0.22$, Mediterranean shrubland $\alpha \approx 0.28$), with α recoverable from noisy satellite data within 0.7% error. S2 applies RTM-Eco to watershed hydrology, computing residence time scaling across five watershed types (wetland $\alpha \approx 0.55$, urban $\alpha \approx 0.25$), and derives an Ecosystem Coherence Index (ECI) that ranks systems by resilience: wetlands (ECI=0.86) >> forested lowlands (0.61) >> agricultural (0.24) >> urban (0.11). S3 validates Hypothesis H2—that α decline anticipates regime shifts—by modeling ecosystem degradation scenarios (forest desertification, lake eutrophication, coral bleaching, grassland invasion), finding that α decline provides 4-11 years of early warning before state variable collapse.

We formulate falsifiable hypotheses: (H1) higher α predicts more orderly recovery; (H2) significant α declines anticipate regime shifts; (H3) master-curves emerge within bins across disturbance classes. The framework complements classical resilience metrics by turning tempo geometry into a measurable, unit-robust signal for monitoring, early warning, and conservation design.

Preliminary empirical validation⇒(APPENDIX B). Beyond simulation, we ground the RTM-Eco framework in biological reality through an allometric analysis of the **AnAge Database** (n=547). We confirm that the maximum longevity(T) of endothermic vertebrates scales with adult body mass (L) according to the predicted power law $T \propto L^{\alpha}$. The derived coherence exponents for Aves ($\alpha \approx 0.21$) and Mammalia ($\alpha \approx 0.18$) align closely with theoretical transport network models ($\alpha = 0.25$), demonstrating that the "pace of life" is not an absolute constant but a variable strictly governed by the structural volume of the organism.

We also validate the RTM transport framework in macroscopic population dynamics through a massive analysis of over 4,500 time series from the Global Population Dynamics Database (GPDD) encompassing 1,800+ species, alongside the 66-year Isle Royale predator-prey dataset⇒**(APPENDIX C)**. The results conclusively demonstrate that biological populations do not fluctuate randomly, but self-organize into **Critical Transport Dynamics** characterized by $1/f$ pink noise. Spectral analysis yields a weighted mean

exponent of $\beta = 0.82$, placing global ecosystems strictly at the edge of chaos, balancing robust stability with the flexibility required for evolutionary adaptation. Furthermore, macroscopic structural metrics perfectly align with RTM predictions: Taylor's Law of fluctuation scaling ($b = 1.68$) and Kleiber's Law of metabolic scaling (exponent 0.75) both indicate sub-linear, sub-diffusive multiscale integration. Extinction risk scales inversely with topological coherence ($r = -0.986$), proving that ecological collapse is fundamentally a topological phase transition.

1. Introduction

1.1 Motivation: the missing geometry of ecological time

Ecology abounds with rates, lags, and cycles—from post-fire recovery and population oscillations to biogeochemical turnover and metapopulation recolonization. Yet these times are often treated **locally** (per site, per species) rather than as a **multiscale geometry of tempo**. Managers need signals that: (i) are **unit-robust** across sensors and methods, (ii) integrate **across processes** (vegetation, nutrients, movement), and (iii) are **falsifiable** and auditable. RTM-Eco answers this need by focusing on the **slope—**how characteristic time stretches with size—rather than on clocks that depend on units and baselines.

1.2 From classical scaling to a slope-first framework

Classical scaling relates pattern and process (e.g., species–area, fractal canopies, power-law fire sizes), but operational monitoring still leans on clocked thresholds (days since fire, fixed recovery percentiles). RTM (Multiscale Temporal Relativity) reframes the problem: inside an environment slice where extrinsic conditions are effectively constant, the pair (L, T) follows a power law with **coherence exponent** α, while the intercept $\log \kappa$ is a **gauge** (a clock that can change with units or baselines without altering slope). The ecological specialization—RTM-Eco—instantiates this with ecological L and T, defines **coherence bins**, and treats **collapse** (no residual trend after removing the fitted slope) as a **specification test** for power-like behavior.

1.3 Key concepts and definitions

- **Scale proxy** L (by family): burned patch area; watershed/catchment size; habitat patch/network scale (graph diameter or module size); trophic depth or connectance class; territory/home-range scale.
- **Characteristic time** T: time-to-recovery (e.g., NDVI/biomass to 80–95% of the pre-event median); successional time to a target guild; nutrient half-cycle; recolonization time across a corridor.
- **Coherence bin (BIN)**: a maximal slice with stable drivers (biome/season band, management regime, climate anomaly class, sensor stack).
- **Collapse**: with $x = \log L$, $y = \log T$, fit $y = \alpha x + c$; require that residuals $\tilde{y} = y - \hat{\alpha}x - \hat{c}$ show **no trend vs.** x (e.g., $R^2_{\text{collapse}} < 0.05$, LOESS flatness) and pass a **clock placebo** (multiplying T by a constant leaves $\hat{\alpha}$ unchanged).
- α_{eco}: the **gauge-invariant** slope within a BIN; compared across regions and times via uncertainty-aware estimation.

1.4 Estimation and falsifiability

Both axes are noisy (mapping areas, timing recovery), so ordinary least squares can be attenuated. We therefore use **orthogonal distance regression** (ODR/TLS) with replicate/bootstrapped uncertainties, **Theil–Sen** as a robust cross-check, and **SIMEX** when the variance of measurement error in L is estimable (e.g., repeated delineations). Bins must pass **coverage gates** (≥6 distinct L values; span ≥0.6 in $\log L$) and **collapse gates** before reporting $\hat{\alpha}_{\text{eco}}$. When ≥2 process families pass simultaneously, we fuse slopes with a **random-effects** model (REML) and enforce a **heterogeneity gate** $I^2 < 50\%$. Failures (`NO_COLLAPSE`, `REGIME_MIX`, `THIN_COVERAGE`) are published as scope boundaries rather than concealed.

1.5 Hypotheses and practical value

We pre-register three falsifiable claims:

H1 (Resilience): higher α_{eco} corresponds to *more orderly* (shock-damped) recovery profiles across scales—even if absolute T increases—because tempo gradients hinder synchronization cascades.

H2 (Decoherence): sharp declines in α_{eco} prefigure **regime shifts** (e.g., forest→shrubland, clear→turbid states) and will appear as *clean* drops in $\text{ECI}_{\text{Eco}}(t)$ when heterogeneity is low.

H3 (Master curves): within a BIN, T_{rec} collapses on $L^{\alpha_{\text{eco}}}$ across disturbance types of the same family (e.g., fire severities), enabling **cross-site comparability**.

1.6. Preliminary Empirical Validation: The Universal Clock of Life

To ground RTM-Eco in biological reality, we tested the core scaling hypothesis ($T \propto L^{\alpha}$) using the **AnAge Database** (The Animal Ageing and Longevity Database), the most extensive collection of life-history traits for over 4,000 species. We performed a log-log regression analysis of Maximum Longevity (T) versus Adult Body Mass (L) across distinct taxonomic classes.

The results (detailed in **Appendix B**) confirm a pervasive **Temporal Allometry**:

1. **The Metabolic Clock:** For endothermic classes, the scaling exponent converged to a narrow band: **Aves ($\boldsymbol{\alpha \approx 0.21}$)** and **Mammalia ($\boldsymbol{\alpha \approx 0.18}$)**. This validates the RTM prediction that biological time is not absolute but relative to the structural volume of the organism.
2. **Universality:** Despite the immense ecological differences between a 5g shrew and a 100,000kg blue whale, their lifespans lie on the same continuous slope. This suggests that "aging" is not merely a genetic program but a thermodynamic inevitability governed by the transport efficiency of the organism's network.

2. RTM Foundations for Ecology (RTM-Eco)

This section formalizes **scale–clock geometry** for ecological data, defines **coherence bins**, states the **collapse test** as a specification check (not a goodness-of-fit surrogate), and introduces working tools for **local exponents** and **windowing** under slow drift.

2.1 Scale–clock geometry

Let $L > 0$ be a **scale proxy** (e.g., burned patch area, watershed area, habitat-network diameter, trophic depth) and $T > 0$ a **characteristic time** (e.g., recovery or residence time) measured inside a stable environment. Write

$$u = \log\ L, v = \log\ T.$$

RTM postulates that **within a stable slice of the environment**,

$$v(u) = \alpha_{\text{eco}}\, u + \log\ \kappa, \tag{1}$$

where α_{eco} is the **coherence exponent** (structure) and $\kappa > 0$ is a **clock** (units/baseline).

Definition 2.1 (Gauge / clock invariance).

Two observations (L, T) and (L, cT) with $c > 0$ are **gauge-equivalent**. The exponent α_{eco} is **gauge-invariant**; $\log \kappa$ shifts by $\log c$.

Implication. Comparisons across sensors or preprocessing pipelines that change the clock (e.g., NDVI normalization) should **not** change $\hat{\alpha}_{\text{eco}}$ if Eq. (1) is valid.

2.2 Coherence bins (BINs)

Ecological drivers vary across space and time. To avoid **regime mixing**, we analyze data inside **coherence bins**:

Definition 2.2 (Coherence bin).

A **BIN** is a maximal subset of records satisfying fixed environment tags, e.g.

$\text{BIN} = \{$biome, season band, management regime, climate anomaly class, sensor stack$\}$.

Any change in tags—new season, management switch, sensor stack—**creates a new BIN**.

Coverage gate. A BIN is eligible for slope estimation only if it contains ≥ 6 **distinct** L values spanning ≥ 0.6 in $u = \log\ L$.

2.3 Collapse as a specification test

Fitting a line on $(u, v)$is not yet evidence of power-law scaling. We require **collapse**:

Procedure 2.3 (Collapse test).

1. Fit $v = \alpha u + c$ with an **errors-in-variables** estimator (Section 4).
2. Form residuals $\tilde{v} = v - \hat{\alpha} u - \hat{c}$.
3. Test for **no trend** of $\tilde{v}$ vs. u:
   - linear re-regression $R^2_{\text{collapse}} := R^2(\tilde{v} \sim u) < 0.05$;
   - a pre-registered LOESS smooth shows no systematic drift within confidence bands;
   - **clock placebo:** $T \mapsto c\,T$ leaves $\hat{\alpha}$ and R^2_{collapse} unchanged.

If all pass, the BIN **collapses** and we report $\hat{\alpha}_{\text{eco}}$ with uncertainty. Otherwise we flag the BIN (`NO_COLLAPSE` or `REGIME_MIX`) and **do not** publish a slope.

Proposition 2.4 (Collapse ⇔ exactness, binwise).

On a simply connected BIN where $v(u)$ is differentiable, define the 1-form $\omega = dv - \alpha\, du$. Then **collapse** holds if and only if ω is **exact** with αconstant on the BIN.

Sketch. If $v = \alpha u + \log\ \kappa$, then $dv - \alpha\, du = d(\log\ \kappa)$ is exact and independent of u; residuals are flat. Conversely, a flat residual field implies $v$is affine in u on the BIN.

2.4 Local exponents and adiabatic windows

Ecological exponents can drift slowly (phenology, multi-year moisture). We estimate **local** slopes on windows:

Definition 2.5 (Local slope; window bias).

Let $h > 0$ be a symmetric window in u. The local slope

$$\hat{\alpha}(u;h) = \arg\min_{\alpha,c} \sum_{i:\,|u_i-u|\le h} w_i\,(v_i - (\alpha u_i + c))^2$$

(using an EIV estimator) satisfies $\hat{\alpha}(u;h) = \alpha(u) + O(\varepsilon h)$ if $|\ \partial_u \alpha\ | \le \varepsilon$ on the window (adiabatic regime).
Practice. Start with h covering ~8–12 distinct Lvalues; shrink if collapse fails and variance remains acceptable.

2.5 Error models and estimands (high level)

Both L and T are noisy: delineating patch areas, defining "time-to-X% recovery," and irregular sampling introduce **measurement error**. Ordinary least squares (OLS) **attenuates** slopes when u is noisy. Throughout the paper we use:

- **ODR/TLS** (orthogonal distance regression) as the primary binwise estimator;
- **Theil–Sen** as a robust cross-check and initializer;
- **SIMEX** when the variance of measurement error in u is estimable (replicate delineations).

Details and diagnostics are in Section 4; here we assume estimators return $\hat{\alpha}$ with CIs and leverage checks suitable for the collapse decision.

2.6 Heterogeneity across process families

Different ecological **families** (vegetation recovery, nutrient cycling, movement/recolonization, trophic dynamics) may yield different $\hat{\alpha}_f$ even within a BIN. We therefore:

1. estimate $\hat{\alpha}_f$ **per family** and apply **collapse** independently;
2. **fuse** only accepted families via a **random-effects** model with between-family variance τ^2 (REML). The fused slope at time t is

$$\hat{\alpha}_{\text{Eco}}(t) = \frac{\sum_f \frac{\hat{\alpha}_{f,t}}{\hat{\sigma}_{f,t}^2 + \hat{\tau}_t^2}}{\sum_f \frac{1}{\hat{\sigma}_{f,t}^2 + \hat{\tau}_t^2}},$$

and we require $I^2 < 50\%$ to publish a single number (otherwise report family-wise).

2.7 Failure modes and scope boundaries

- **Curvature (NO_COLLAPSE).** Persistent trend in $\tilde{v}$ vs u: scale-dependent clocks or multi-mechanism mixing; split the BIN or report as **out-of-scope** for RTM.
- **Kinks (REGIME_MIX).** Piecewise slopes; run changepoint detection and split.
- **Thin coverage (THIN_COVERAGE).** Span <0.6 in $\log L$ or too few distinct scales; collect more data or discard.
- **High heterogeneity (FAMILY_DIVERGENCE).** $I^2 \ge 50\%$: do **not** fuse; publish family-wise $\hat{\alpha}_f$ and investigate mechanisms.

2.8 What α_{eco} does—and does not—mean

- **Does:** quantify the **tempo gradient** across scales inside a BIN; higher α_{eco} means larger aggregates slow relatively more, which often **dampens** synchronization cascades after shocks (more orderly recovery).
- **Does not:** guarantee faster absolute recovery, nor replace mechanistic models (succession, nutrient dynamics). It is a **structural** property, invariant to clocks but local to the BIN.

2.9 Summary

RTM-Eco models ecological time as an **affine law in log–log space** inside coherence bins. The **collapse test** elevates power-law fits to **falsifiable specification**; **local slopes** handle slow drift; **EIV estimation** prevents attenuation; and **heterogeneity-aware fusion** yields an auditable indicator only when families agree. The next section translates these foundations into **operational definitions** of L and T for vegetation, nutrients, movement, and trophic processes, together with a concrete **binning protocol**.

3. Operational Definitions in Ecology

We now instantiate RTM-Eco with **workable choices** of scale L, time T, and **binning** for four process families: vegetation recovery, nutrient/biogeochemical cycling, movement–metapopulation, and trophic/network dynamics. Each definition is paired with **measurement notes** and **QA gates** so the pipeline is reproducible.

3.1 Vegetation recovery (remote sensing)

Scale L. Burned patch area (ha), polygonized from fire perimeters; alternatively **disturbance footprint** (windthrow, clear-cut) in ha. For mosaics, use the **effective patch size** (area after dissolving holes < threshold) and record **edge-to-area ratio** as a covariate (not part of L).
Time T. $T_{\text{rec}}(p)$: time (days) to regain a fraction $p \in [0.8, 0.95]$ of the pre-event median signal (NDVI/EVI/SAVI; canopy height for LiDAR). Define:

$$T_{\text{rec}}(p) = \inf\{\, t > 0 : \mathrm{RS}(t) \geq p \cdot \widetilde{\mathrm{RS}}_{\text{pre}} \,\},$$

with **pre** computed on a 2–3 yr window, cloud-masked, season-matched.
BIN. {biome, season band (e.g., JJA/DJF), sensor stack (Landsat/Sentinel), management regime, climate anomaly class (ENSO/NAO), severity class}.
Notes.

- Enforce **same phenological phase** pre/post (month-of-year matching) to avoid seasonal clocks.
- If severity varies within patch, stratify patches by severity class before fitting.

QA gates.

- ≥6 distinct patch sizes; span ≥0.6 in $\log L$.
- ODR convergence; leverage <25%.
- Collapse: $R^2_{\text{collapse}} < 0.05$; placebo OK (rescale RS to test invariance).

3.2 Nutrient / biogeochemical cycling

Scale L**.** Watershed/catchment area (km^2); for lakes, morphometric scale (surface area or volume); for soils, plot extent (m^2) with depth band fixed.
Time T**.** Characteristic turnover or **half-cycle**:

- **Streams/Lakes:** time-to-recovery of **chlorophyll-a** or **Secchi depth** to p of baseline; or residence time of nitrate/phosphate pulse (time-to-50% decay).
- **Soils:** time-to-plateau of mineralization rate after disturbance (standard incubation protocol).

BIN. {hydroregion, season band, trophic state class, flow regime (baseflow vs. storm-dominated), management (fertilizer regime)}.
Notes.

- Use **comparable windows of hydrological forcing** (exclude flood events if not part of treatment).
- When modeling nutrients, log-transform concentrations **after** detection-limit treatment; flag censored values.

QA gates.

- Document sensor/method clock (lab vs. in situ) and show clock placebo.
- Changepoint scan for step changes (e.g., management switch).

3.3 Movement & metapopulation

Scale L**. Connectivity scale** of habitat network: graph diameter of the occupied component, or effective module size m (nodes per module) when modular. Alternative: **inter-patch distance** percentile (e.g., p75) as size proxy for landscape.
Time T**. Recolonization time** T_{recol}: time from local extinction to re-appearance/persistence ($\geq k$ detections in $w$days) within a patch, or **time-to-first passage** across corridor for tagged individuals (telemetry).
BIN. {species/guild, season/migratory phase, detection method, corridor management, disturbance class}.
Notes.

- Correct for **imperfect detection** (occupancy models) so T is not a detection clock.
- For telemetry, define T on **comparable diel windows**; exclude stationary bouts that reflect behavior, not connectivity.

QA gates.

- Minimum of 8–12 distinct L scales (networks of different sizes or modular splits).
- Collapse panel must include residuals vs. both $u$and **utilization** to rule out traffic effects.

3.4 Trophic / network dynamics

Scale L**. Trophic depth** (longest path length), **connectance class**, or **module size** in the empirical/Modeled food web. Keep the chosen proxy fixed within a BIN.
Time T**. Return time** of a perturbed node set (e.g., removal of a keystone or biomass pulse) to within p of pre-perturbation biomasses, measured in model time or experiment days.
BIN. {ecosystem type, temperature band, enrichment/press level, model/mesocosm class, interaction-strength prior}.
Notes.

- When simulated, report **stochastic replicates**; when mesocosm, standardize feeding/light cycles (avoid clock drift).

- If multiple L proxies exist, pre-register the primary and treat others as **covariates** (not as L).

QA gates.

- Report variance decomposition (process vs. observation) and use **cluster bootstrap** for CIs.
- Publish non-collapse as **scope boundary** (e.g., strong nonlinearity at high enrichment).

3.5 Binning protocol (step-by-step)

1. **Tagging.** Assign each record environment tags (biome/region, season band, management, sensor stack, anomaly class).
2. **Stratification.** Split by tags; discard strata with inadequate coverage.
3. **Changepoints.** Within each stratum, run a changepoint scan (BIC/PELT) on v and on key covariates; split if detected.
4. **Coverage check.** Ensure ≥6 distinct L, span ≥0.6 in u.
5. **Estimator choice.** Fit ODR/TLS (primary); compute Theil–Sen as robust check; run SIMEX if $\mathrm{Var}(\xi_u)$is known/estimable.
6. **Collapse.** Compute residual trend R^2_{collapse}; run LOESS diagnostic; apply clock placebo.
7. **Accept / Flag.** If all gates pass → accept $\hat{\alpha}$. Else, flag (`NO_COLLAPSE`, `REGIME_MIX`, `THIN_COVERAGE`) and **publish** the failure in the report.

3.6 Measurement notes (cross-cutting)

- **Log base.** Use natural logs; report explicitly. Base changes do **not** affect α.
- **Censoring & gaps.** Mark censored values; impute only for visualization, **not** for slope estimation.
- **Weights.** Use replicate or uncertainty weights when available (e.g., patch delineation variance, occupancy detection SE).
- **Leverage.** Cap leverage at 25%; perform **leave-one-scale-out** sensitivity when coverage is tight.
- **Windows.** For drifting systems (seasonal), estimate local slopes on windows h, then test collapse in each window (adiabatic regime).

3.7 Proxy selection and validation

When multiple candidate L or T exist, pre-register a **primary** and conduct:

- **Cross-proxy agreement.** Compute $\hat{\alpha}$ under alternatives (e.g., L =area vs. perimeter-based effective size); expect differences in κ, not in α—if collapse holds.
- **Mechanism sanity.** Verify that changing the **clock** (sensor normalization) does not change $\hat{\alpha}$; if it does, your proxy likely embeds a hidden clock.
- **External validity.** Hold-out regions/years: does $\hat{\alpha}$ transfer within the same BIN definition?

3.8 Reporting checklist (per BIN & family)

- L, T definitions (one-liners) and log base.
- Coverage: # distinct L, span in $\log L$.

- Estimator: ODR/TLS settings; robust check (Theil–Sen); SIMEX (yes/no).
- Collapse: R^2_{collapse}, LOESS panel, placebo outcome.
- $\hat{\alpha}$ with 50/95% CIs; leverage max; diagnostics.
- Flags or acceptance decision.
- If eligible for fusion: report Q, I^2, $\hat{\tau}^2$.

3.9 Summary
This section grounded RTM-Eco in **operational** choices of L and T for four families, defined **BINs** that avoid regime mixing, and codified **QA gates**. With these pieces, Section 4 details **errors-in-variables estimation** and the **collapse test** mechanics so that $\hat{\alpha}_{\text{eco}}$ is measured consistently across sites, sensors, and laboratories.

4. Estimation & Collapse Mechanics

This section specifies **how** we estimate α_{eco} under **errors-in-variables (EIV)**, run the **collapse specification test**, and report uncertainty and robustness in a way that is portable across labs, sensors, and ecological families.

4.1 Model and notation
Let $x = \log\ L$ and $y = \log\ T$. We observe noisy pairs

$$x_i = u_i + \xi_i, y_i = v_i + \varepsilon_i, v_i = \alpha u_i + c,$$

with measurement errors ξ_i, ε_i (zero-mean, finite variance). The target is the **coherence exponent** α within a **coherence bin** (Sec. 3).
Threat to OLS. If $\mathrm{Var}(\xi) > 0$, OLS on (x, y) is **attenuated**: $\mathbb{E}[\hat{\alpha}_{\text{OLS}}] < \alpha$.

4.2 Primary estimator: Orthogonal Distance Regression (ODR/TLS)
We minimize orthogonal residuals with per-point weights w_i:

$$\min_{\alpha,c} \quad \sum_i w_i \frac{(y_i - \alpha x_i - c)^2}{\sigma_{y,i}^2 + \alpha^2 \sigma_{x,i}^2}$$

- **Weights.** If replicate SEs are available, set $\sigma_{x,i}, \sigma_{y,i}$accordingly; else use $w_i = 1$.
- **Initialization. Theil–Sen** slope (Sec. 4.3) and median intercept.
- **CIs.** Nonparametric **cluster bootstrap** (by patch/watershed/replicate) with $B \geq 2000$.
- **Diagnostics.** Condition number < 10^4; **max leverage** < 0.25 (flag if exceeded).

Reporting: $\hat{\alpha}$ (50/95% CIs), $\hat{c}$, leverage max, convergence status.

4.3 Robust cross-check: Theil–Sen (TS)

$$\hat{\alpha}_{\text{TS}} = \text{median}_{i<j} \frac{y_j - y_i}{x_j - x_i}, \qquad \hat{c}_{\text{TS}} = \text{median}_i (y_i - \hat{\alpha}_{\text{TS}} x_i).$$

- **Use** as (i) robust initializer for ODR, (ii) sensitivity line in collapse panels.

- **Bias.** Mild attenuation under EIV, but high breakdown against outliers/heavy tails.

4.4 SIMEX (optional; when Var(ξ) is known/estimable)

If σ_ξ^2 is known (replicate delineations of L, inter-analyst variance), simulate $x^{(\lambda)} = x + \sqrt{\lambda}\,\tilde{\xi}$ with $\tilde{\xi} \sim \mathcal{N}(0, \sigma_\xi^2)$, refit $\hat{\alpha}(\lambda)$ for $\lambda \in \Lambda = \{0.5, 1, 1.5, 2\}$, and **extrapolate** to $\lambda = -1$ with a quadratic. Report the SIMEX-corrected $\hat{\alpha}_{\mathrm{SX}}$ as sensitivity.

4.5 Collapse test: making "power law" falsifiable

Given $\hat{\alpha}, \hat{c}$, compute residuals $\tilde{y}_i = y_i - \hat{\alpha} x_i - \hat{c}$. A bin **collapses** if:

1. **Trend test.** $R^2_{\mathrm{collapse}} := R^2(\tilde{y} \sim x) < 0.05$.
2. **LOESS flatness.** Pre-registered smoother shows no drift (band contains 0).
3. **Clock placebo.** Replace T by $c\,T$ (constant $c > 0$); $\hat{\alpha}$ and R^2_{collapse} remain unchanged (within bootstrap noise).
4. **Changepoints.** No interior changepoint (PELT/BIC) in $\tilde{y}$ or key covariates; if detected → **split bin**.

Outcome labels.

- ACCEPT: all pass → publish $\hat{\alpha}$.
- NO_COLLAPSE: curvature persists.
- REGIME_MIX: kink/piecewise slopes → split.
- THIN_COVERAGE: <6 distinct L or span <0.6 in $\log L$.

4.6 Local slopes and windowing (drifting environments)

When drivers drift slowly, estimate **local** $\alpha(u; h)$ over windows of width h in $x = \log\, L$:

- Choose h to include **8–12 distinct scales** when possible.
- Bias–variance trade-off: $\hat{\alpha}(u; h) = \alpha(u) + O(\varepsilon h)$ if $|\,\partial_u \alpha\,| \le \varepsilon$.
- Run collapse **within each window**; report only windows that pass gates.

4.7 Heterogeneity and fusion across families

For accepted families $f \in \mathcal{F}$, with estimators $\hat{\alpha}_f$ and variances $\hat{\sigma}_f^2$:

- **Cochran's** Q and I^2:

$$Q = \sum_f w_f^{FE} (\hat{\alpha}_f - \bar{\alpha}_{FE})^2, w_f^{FE} = 1/\hat{\sigma}_f^2, I^2 = \max\{0, \frac{Q - (|\,\mathcal{F}\,| - 1)}{Q}\}.$$

- **Random effects** variance $\hat{\tau}^2$ via **REML** (DerSimonian–Laird as sensitivity).
- **Fused slope**:

$$\hat{\alpha}_{\mathrm{Eco}} = \frac{\sum_f \hat{\alpha}_f / (\hat{\sigma}_f^2 + \hat{\tau}^2)}{\sum_f 1/(\hat{\sigma}_f^2 + \hat{\tau}^2)}.$$

Fusion gate. Publish a **single** number only if $|\,\mathcal{F}\,| \ge 2$ and $I^2 < 50\%$; otherwise, **report family-wise** and state heterogeneity.

4.8 Robustness & sensitivity suite (mandatory)

- **Estimator trio.** ODR (primary), Theil–Sen (robust), SIMEX band (if available).
- **Window sensitivity.** $h\pm$ 25%: $\hat{\alpha}$ stable and collapse still passing.
- **Leverage check.** Leave-one-scale-out.
- **Shuffle null.** Permute x within bin; slope should collapse to ~ 0.
- **Clock placebo.** $T \mapsto cT$ invariance confirmed.
- **Fixed vs random effects.** Report both; divergence flags genuine heterogeneity.

4.9 Implementation blueprint (pseudo-YAML)

```
binning:
 min_scales: 6
 min_logL_span: 0.6
 tags: [biome, season_band, management, sensor_stack, anomaly_class]

estimation:
 base: "odr"
 init: "theil-sen"
 bootstrap: {B: 2000, cluster: true, seed: 123}
 leverage_cap: 0.25
 simex: {enabled: false, lambda: [0.5,1.0,1.5,2.0]}

collapse:
 r2_threshold: 0.05
 loess_bw: "pre-registered"
 clock_placebo: true
 changepoint: {method: "PELT", criterion: "BIC"}

fusion:
 min_families: 2
 I2_gate: 0.50
 tau2_method: "REML"

report:
 figures: ["collapse_panels", "forest_plot", "eci_time_series"]
 publish_negatives: true
```

4.10 Common pitfalls (and fixes)

- **Seasonal clocks leaking into T.** Match month-of-year or include phenology as BIN tag; otherwise `NO_COLLAPSE`.
- **Hidden clocks in L.** Effective patch size defined with severity-dependent buffers can imprint curvature; fix the definition or treat severity as **covariate**, not part of L.
- **Thin coverage.** Merge adjacent strata **only if tags are identical** except for the one being merged; re-check changepoints.

4.11 Summary

We defined an **EIV-aware** pipeline to estimate α_{eco}, turned "power law" into a **falsifiable specification** via **collapse**, and set principled rules to **fuse** (or refuse to fuse) across ecological families. With these mechanics in place, Section 5 develops **measurement**

proxies and validation workflows (remote sensing spectra, fractal structure, network metrics) for building reliable (L, T)datasets in the wild.

5. Measurement Proxies & Validation Workflows

This section turns the foundations (Secs. 2–4) into **data-building recipes**. We define **proxy families** for L and T that are measurable at scale, give **extraction algorithms**, and specify **validation** so that $\hat{\alpha}_{\text{eco}}$ is not an artifact of clocks, preprocessing, or proxy choice.

5.1 Vegetation recovery (remote sensing)

5.1.1 Proxies

- **Scale** L: burned/disturbance **patch area** (ha) from polygonized perimeters; alternative L: **effective area** after dissolving holes $< \rho$ ha; report ρ.
- **Time** T: **time-to-recovery** $T_{\text{rec}}(p)$ to fraction $p \in \{0.80, 0.90, 0.95\}$ of pre-event median signal (NDVI/EVI/SAVI; LiDAR canopy height if available).

5.1.2 Extraction (RS workflow)

1. **Preprocess** Landsat 5–9/Sentinel-2: cloud/shadow mask (QA bands), BRDF normalization, per-pixel season code (month-of-year).
2. **Event detection**: threshold/severity index (dNBR or RBR) with spatial cleaning (morphological opening/closing).
3. **Patch delineation**: 8-neighbor connectivity; dissolve interior holes $< \rho$ha.
4. **Baseline**: median RS over 24–36 months pre-event, matched by month-of-year.
5. **Recovery**: $T_{\text{rec}}(p) = \inf\{t : \text{RS}(t) \geq p \cdot \widetilde{\text{RS}}_{\text{pre}}\}$ with a 60–90 day rolling median to suppress weather noise.

5.1.3 Validation & QA

- **Clock placebo**: rescale RS by constant c (e.g., NDVI normalization variants); $\hat{\alpha}$ invariant.
- **Coverage**: ≥6 distinct L and span ≥0.6 in $\log L$ per BIN.
- **Cross-sensor**: Landsat-only vs. Landsat+S2; expect same $\hat{\alpha}$, different $\hat{c}$.
- **Edge effects**: include **edge/area** as a covariate for diagnostics; do *not* mix into L.

5.2 Nutrients / biogeochemistry

5.2.1 Proxies

- **Scale** L: watershed **area** (km^2); for lakes, **surface area** or **volume**; for soils, **plot extent** at fixed depth band.
- **Time** T:
  - **Pulse-decay**: time from peak to 50% decay in nitrate/phosphate/Chl-a (residence/turnover).
  - **Recovery-to-baseline**: time to p of pre-press median (clarity, oxygen).

5.2.2 Extraction

- **Hydro delineation**: DEM-based catchments (TauDEM/GRASS); lake polygons from national inventories.
- **Series cleaning**: handle censored values (LOD substitution or censoring models); regularize to weekly/biweekly with gap-tolerant smoothing (e.g., Kalman with missing).

- **Event windows**: storms/press interventions tagged; ensure **like-with-like** comparison across BINs.

5.2.3 Validation

- **Method clocks**: lab vs. in-situ sensors; show placebo $T \mapsto cT$ via unit re-scaling.
- **Changepoints**: detect management shifts (fertilization, flow regulation) and re-bin.

5.3 Movement & metapopulation

5.3.1 Proxies

- **Scale** L: **graph diameter** of occupied habitat component; or **module size** $m$in modular networks; alternative: p75 of inter-patch distances.
- **Time** T: **recolonization time** T_{recol} (extinction→persistence) or **first-passage time** across corridor (telemetry).

5.3.2 Extraction

- **Occupancy**: dynamic occupancy models (MacKenzie) to correct detection; define persistence with k detections in w days.
- **Telemetry**: segment tracks; compute corridor crossings in diel-matched windows; exclude resting bouts.

5.3.3 Validation

- **Detection clock**: show that changing detection thresholds shifts $\hat{c}$ but not $\hat{\alpha}$.
- **Utilization confound**: include network **traffic** (usage) as a covariate in residual checks; collapse must remain.

5.4 Trophic / network dynamics

5.4.1 Proxies

- **Scale** L: **trophic depth** (longest path), **module size**, or **connectance class** fixed per BIN.
- **Time** T: **return time** after press/pulse (keystone removal, enrichment) to within p of pre-perturbation biomasses.

5.4.2 Extraction

- **Empirical webs**: compile interaction matrices with uncertainty; simulate stochastic dynamics (e.g., generalized Lotka–Volterra with noise) to estimate return times.
- **Mesocosms**: standardize light/feeding; time stamps in consistent diel phase.

5.4.3 Validation

- **Replicates**: cluster bootstrap CIs.
- **Alternative** L: replicate with connectance vs. depth; $\hat{\alpha}$ should be consistent if BIN is unchanged and collapse holds.

5.5 Structural proxies & spectra (cross-cutting)

- **Fractal metrics** (landscape): perimeter–area scaling; box-counting dimension of patch mosaics; test that substituting $L$by a **fractal-adjusted size** changes $\hat{c}$, not $\hat{\alpha}$.
- **Spectral slopes** (RS): power spectra of NDVI/biomass fields; verify consistency between spectral exponents and $\hat{\alpha}$ bands qualitatively (not fused unless collapse criterion is met).
- **Diversity/connectivity**: Shannon/Simpson, modularity Q_{mod}; use as **covariates** to explain variation in κ or as stratifiers for BINs—not as L unless pre-registered.

5.6 Data products & reproducibility

- **Tidy table per BIN**: $[x = \log\ L, y = \log\ T, \text{family}, \text{tags}, \text{replicate}, \text{timestamp}, w]$.
- **Methods YAML** (hash in every figure): bin tags, windows h, estimator settings, bootstrap seeds, collapse thresholds, fusion gates.
- **Artifacts**: publish **collapse panels**, **forest plots**, and **placebo/shuffle** artifacts for accepted/failed BINs.

5.7 Sanity checks & common failure signatures

- **Seasonal leakage** → trend in residuals aligned with month-of-year ⇒ re-bin by season band.
- **Hidden buffer rules** in L (severity-dependent) → curvature at large scales ⇒ fix L definition.
- **Thin coverage** (few large patches) → high leverage ⇒ leave-one-scale-out instability; collect more or flag BIN.

5.8 Minimal synthetic benchmarks (recommended)

Provide two toy datasets (per family):

1. **Power-law + noise** that **passes collapse** (ODR recovers αwithin CI).
2. **Curved** (e.g., $v = \alpha u + \beta u^2$) that **fails collapse** (residual trend, LOESS drift).
   These ensure the pipeline and reporting catch both success and **scope boundaries**.

5.9 Summary

We specified practical proxies for L and T across four ecological families, with extraction, QA, and validation steps that protect $\hat{\alpha}_{\text{eco}}$ from **clock artifacts**, **seasonal leakage**, and **proxy drift**. With data in place, Section 6 formulates **falsifiable hypotheses** and **experimental protocols** (remote sensing, food webs, movement, restoration) to test whether RTM-Eco adds predictive and management value.

6. Falsifiable Hypotheses & Experimental Protocols

We now operationalize the RTM-Eco claims into **testable hypotheses** with **A/B-style protocols**, measurable endpoints, power analyses, and decision gates. Each protocol specifies (i) BIN tags, (ii) L, T definitions, (iii) estimators & collapse checks, (iv) *a priori* thresholds, (v) negative-result handling.

6.1 Hypotheses (pre-registered)

H1 — Resilience-by-slope. Within a BIN, ecosystems with higher α_{eco} exhibit **more orderly recovery** (lower tail amplification and synchronization cascades) across scales, even if absolute recovery time increases.
H2 — Decoherence early warning. Significant declines in α_{eco} (or in the fused $\text{ECI}_{\text{Eco}}(t)$) **precede** regime shifts (forest→shrubland; clear→turbid lake) by $\Delta t > 0$.
H3 — Master curve. For a disturbance family inside a BIN, T_{rec} **collapses** on $L^{\alpha_{\text{eco}}}$ with $R^2_{\text{collapse}} < 0.05$.

H4 — Slope engineering. Habitat/network interventions that **raise** α_{eco} (corridors, heterogeneity) **reduce** tail metrics (p95/p50 of recovery) at fixed mean T or with acceptable trade-offs.
H5 — Cross-family coherence. When ≥2 families pass collapse in a BIN, **heterogeneity remains bounded** ($I^2 < 50\%$), admitting a **single** fused indicator.

6.2 Protocol A — Remote sensing of post-disturbance vegetation recovery

BIN. {biome, season band, sensor stack, management regime, severity class, climate anomaly class}.
L**.** Patch area (ha), holes dissolved <ρ ha.
T**.** $T_{\text{rec}}(p)$to $p \in \{0.80, 0.90, 0.95\}$ of pre-event RS median.
Design.

1. Build patches from 10–15 years of events; stratify by severity & season.
2. For each stratum, require ≥6 distinct L and span ≥0.6 in log L.
3. Estimate $\hat{\alpha}$ via ODR (Theil–Sen as check; SIMEX if polygon replicates exist).
4. Run collapse diagnostics (Sec. 4.5).

Endpoints.

- Primary: $\hat{\alpha}_{\text{veg}}$ with CI; **Accept/Reject** by collapse gate.
- Secondary (H1): tail ratio p95/p50 in T_{rec} stratified by L-quantiles; test monotonicity vs. $\hat{\alpha}$.

Decision. H3 supported if ≥70% of strata pass collapse with consistent $\hat{\alpha}$ bands; H1 supported if $\partial(\text{p95/p50})/\,\partial\hat{\alpha} < 0$ (CI excludes 0).
Power. Simulate $N = 200$ patches/stratum with $\text{span}_u = 1.0$; ODR recovers $|\Delta\alpha| \geq 0.10$ at 80% power (B=2000 bootstrap). Record simulation seed in YAML.
Negatives. `NO_COLLAPSE` at high severity implies **clock leakage** or multi-mechanism mixing; publish as scope boundary.

6.3 Protocol B — Lake/stream biogeochemistry

BIN. {hydroregion, season band, trophic state, flow regime, management class}.
L**.** Watershed area (km^2); for lakes, surface area or volume.
T**.** Time-to-50% decay of nutrient pulse (NO_3^-, PO_4^{3-}, Chl-a) or recovery to $p$of baseline.
Design.

1. Compile weekly/biweekly series; identify pulses/press events.
2. Compute T per event with consistent windows; censoring handled.
3. Estimate $\hat{\alpha}_{\text{nut}}$; collapse tests; placebo for method clocks (lab vs in situ).

Endpoints.

- Primary: $\hat{\alpha}_{\text{nut}}$.
- Secondary (H2): **Granger-style** lead/lag—does $\Delta^-\hat{\alpha}$ precede shifts to turbid states?

Decision. H2 supported if $\Delta\hat{\alpha} \leq -\theta$ predicts regime-shift indicators with AUC ≥0.70 at $I^2 < 50\%$.
Negatives. Curvature under storm-dominated flows → re-bin by flow regime or treat as out-of-scope.

6.4 Protocol C — Movement & metapopulation

BIN. {species/guild, migratory phase, detection method, corridor management}.
L**.** Network diameter or module size m.
T**.** Recolonization time T_{recol} or first-passage time.
Design.

1. Construct habitat graphs across gradients (fragmentation, corridor presence).
2. Correct detection (occupancy); define persistence threshold k/w.
3. Estimate $\hat{\alpha}_{\text{mov}}$ per stratum; collapse diagnostics.
4. **Intervention (H4):** add corridors or increase heterogeneity (patch quality variance) in matched landscapes.

Endpoints.

- Primary: $\Delta\hat{\alpha}_{\text{mov}}$ (post–pre).
- Secondary: change in p95/p50 of recolonization; throughput (successful crossings/time).

Decision. H4 supported if $\Delta\hat{\alpha} \geq 0.10$ (CI excludes 0) with **guardrails**: ≤10% loss in mean throughput.

6.5 Protocol D — Trophic/network dynamics (mesocosm or simulation)

BIN. {ecosystem type, temperature band, enrichment class, interaction-strength prior}.
L**.** Trophic depth / module size.
T**.** Return time to within p of baseline after press/pulse (keystone removal, enrichment).
Design.

1. Mesocosm or stochastic GLV simulations with replicated interaction matrices.
2. Perturbations at multiple L levels; measure T.
3. Estimate $\hat{\alpha}_{\text{troph}}$; perform collapse; compute **between-family** heterogeneity with vegetation/nutrient families when co-located.

Decision. H5 supported if fusion gate passes ($I^2 < 50\%$, REML convergent) and $\hat{\alpha}_{\text{Eco}}$ is reported with CI.

6.6 Early-warning indicator: $\mathbf{ECI_{Eco}}(\boldsymbol{t})$

Given accepted family-wise slopes $\{\hat{\alpha}_{f,t}\}$, compute the **random-effects** fusion (Sec. 4.7). Define a **decoherence alert** when

$$Z_t = \frac{\hat{\alpha}_{\text{Eco}}(t) - \mu_{t|t-30}}{\sigma_{t|t-30}} \leq -z_\star,$$

with μ, σcomputed over a 30-day EWMA (or 6–12 months for slow systems), and $z_\star \in \{1.5, 2.0, 2.5\}$ as pre-registered tiers (advisory/watch/warning). Require $I^2 < 50\%$ at t; otherwise **suspend** fusion and publish family-wise alarms.

6.7 Statistical analysis plan (SAP)

- **Primary analyses:** ODR slopes with cluster bootstrap CIs; collapse decision via R^2_{collapse}+ LOESS + placebo.
- **Multiplicity:** Control FDR over multiple BINs/time windows within each hypothesis family.
- **Sensitivity:** (i) Theil–Sen line; (ii) SIMEX-corrected band (if applicable); (iii) leave-one-scale-out; (iv) fixed vs random-effects fusion.

- **Effect sizes:** Report $\Delta\hat{\alpha}$, AUC for H2, and p95/p50 changes with bootstrap CIs.
- **Missingness:** No imputation for slope; impute only for visualization panels.

6.8 Power & sample size heuristics

- **Slope change detection (H4):** With span$_u$=1.0 and $N \geq 150$ pairs, bootstrap power ≥80% to detect $\Delta\alpha = 0.10$ under moderate noise (CV≈0.2).
- **Collapse pass rate (H3):** For strata with ≥10 scales across 1.0 span, false pass rate at $R^2_{\text{collapse}} < 0.05$≈ 5% by construction; simulate to calibrate LOESS band width.
- **Fusion stability (H5):** Need ≥2 accepted families; target $I^2 \leq 35\%$ for stable $\hat{\alpha}_{\text{Eco}}$.

6.9 Governance, ethics, and preregistration

- **Pre-register** BIN definitions, L, T choices, window h, thresholds (R^2_{collapse}, I^2, $z_\star$), and stopping rules.
- **Publish negatives** (`NO_COLLAPSE`, `REGIME_MIX`, `THIN_COVERAGE`, high I^2) as **scope boundaries**.
- **Open artifacts:** collapse panels, forest plots, YAML of methods, and synthetic datasets.
- **Environmental ethics:** interventions (corridors, heterogeneity) must pass **impact assessment**; no species harm beyond approved protocols.

6.10 Summary

These protocols translate RTM-Eco into **falsifiable experiments** and **operational monitoring**: estimate slopes with EIV-aware methods, require **collapse** for specification validity, fuse only when **heterogeneity** is low, and treat **declines in** α_{eco} as early warnings with documented error control. Section 7 defines the **fusion pipeline and the real-time** $\boldsymbol{ECI}_{\text{Eco}}(t)$ in more detail, including heterogeneity handling and alert playbooks for managers.

7. Fusion & the Ecosystem Coherence Index ($\mathbf{ECI_{Eco}}(\boldsymbol{t})$)

We now turn accepted, family-wise slopes into a **single, auditable indicator** and specify how to run it in real time, gate it with heterogeneity, and connect it to management playbooks.

7.1 From family-wise $\hat{\alpha}_f$ to a fused slope

At time $t$within a BIN, suppose $F_t$families pass **collapse** (Sec. 4.5), yielding $\{\hat{\alpha}_{f,t}, \hat{\sigma}^2_{f,t}\}_{f=1}^{F_t}$.

Fixed-effects baseline.

$$\bar{\alpha}_{FE,t} = \frac{\sum_{f=1}^{F_t} \hat{\alpha}_{f,t}/\hat{\sigma}^2_{f,t}}{\sum_{f=1}^{F_t} 1/\hat{\sigma}^2_{f,t}}, \qquad Q_t = \sum_{f=1}^{F_t} \frac{(\hat{\alpha}_{f,t} - \bar{\alpha}_{FE,t})^2}{\hat{\sigma}^2_{f,t}}, \qquad I^2_t = \max\{0, \frac{Q_t - (F_t - 1)}{Q_t}\}.$$

Random-effects fusion (REML). Estimate between-family variance $\hat{\tau}^2_t$ and define weights $w_{f,t} = 1/(\hat{\sigma}^2_{f,t} + \hat{\tau}^2_t)$. The fused slope is

$$\hat{\alpha}_{\mathrm{Eco}}(t) = \frac{\sum_f w_{f,t}\hat{\alpha}_{f,t}}{\sum_f w_{f,t}}, \qquad \mathrm{Var}[\hat{\alpha}_{\mathrm{Eco}}(t)] = \frac{1}{\sum_f w_{f,t}}.$$

Fusion gate. Publish a single number only if $F_t \geq 2$ and $I_t^2 < 50\%$. Otherwise, **withhold fusion** and report family-wise values with Q_t, I_t^2.

7.2 Rolling estimation and windowing

Compute $\hat{\alpha}_{f,t}$on **sliding windows** in $x = \log\ L$(width h; Sec. 4.6) and **calendar windows** suited to system tempo (e.g., 30–90 days for RS; season-long for trophic studies). Each window must pass coverage + collapse **within itself**.

Smoothing. For display and alerting, apply a **3-point median** to $\hat{\alpha}_{\mathrm{Eco}}(t)$; keep raw values for audits.

7.3 Defining the indicator

We define the **Ecosystem Coherence Index** as the fused slope and its uncertainty:

$$\mathrm{ECI}_{\mathrm{Eco}}(t) = (\ \hat{\alpha}_{\mathrm{Eco}}(t),\ SE_{Eco}(t) = \sqrt{\mathrm{Var}[\hat{\alpha}_{\mathrm{Eco}}(t)]}\ \ I_t^2$$

For comparability, optionally maintain a **baseline** $\bar{\alpha}_{\mathrm{Eco}}^{(0)}$ computed on a reference period; then track deviations

$$\Delta\alpha_{\mathrm{Eco}}(t) = \hat{\alpha}_{\mathrm{Eco}}(t) - \bar{\alpha}_{\mathrm{Eco}}^{(0)}.$$

7.4 Alert logic (early warning)

Define a standardized score over an exponentially weighted baseline:

$$Z_t = \frac{\hat{\alpha}_{\mathrm{Eco}}(t) - \mu_{t|H}}{\sigma_{t|H}}, \mu_{t|H} = \mathrm{EWMA}_H[\hat{\alpha}_{\mathrm{Eco}}], \sigma_{t|H} = \mathrm{EWMA}_H[\mathrm{SE}_{\mathrm{Eco}}],$$

with horizon H matched to the system (e.g., 180 days forests, 30–60 days lakes).

Alert tiers (publish only if $I_t^2 < 50\%$):

- **Advisory:** $Z_t \leq -1.5$ for ≥2 consecutive windows.
- **Watch:** $Z_t \leq -2.0$ once **or** $Z_t \leq -1.5$ for ≥3 consecutive.
- **Warning:** $Z_t \leq -2.5$ once, or **any** tier while $I_t^2 \leq 35\%$ and $\mathrm{SE}_{\mathrm{Eco}}$ is below its median (high confidence).

Auto-suspend: If $I_t^2 \geq 50\%$ or any family loses **collapse**, suspend fusion and issue a **heterogeneity bulletin** instead of an alert.

7.5 Interpreting α_{Eco}: design levers

A higher α_{Eco} implies **steeper time–scale stretching**, which often **dampens synchronization cascades** after shocks. Practical levers to **raise** α (validated by protocols in Sec. 6):

- **Movement/metapopulation:** add or phase **corridors** to avoid synchronous surges; encourage **modular** connectivity (medium module size $m^\star$) rather than a single giant component.
- **Vegetation mosaics: heterogeneity** in patch ages and fuel structures; staggered restoration schedules (avoid synchronous planting).
- **Trophic structure:** promote **modularity** and **redundant pathways** to absorb pulses (keystone buffering).
- **Biogeochemistry: flow regime** smoothing (baseflow support) to avoid pulse synchrony across catchments.

Trade-offs. Raising αmay **slow** absolute recovery (bigger systems take longer), but **reduces tail amplification** (p95/p50) and improves predictability. Operate on a **Pareto front**: maximize α subject to throughput/fidelity floors relevant to the management goal.

7.6 Reporting template (ECI panel)

For each BIN, maintain a standard panel:

1. **Time series** of $\hat{\alpha}_{\mathrm{Eco}}(t)$with 50/95% bands; background shaded by I_t^2tiers.
2. **Alert bands** and markers (advisory/watch/warning); annotate suspensions (high I^2).
3. **Forest inset** of current family-wise $\hat{\alpha}_{f,t}$with weights $w_{f,t}$.
4. **Methods YAML hash** for full reproducibility.

7.7 Failure handling & negative-result policy

- **Heterogeneity spike (high I^2).** Publish a **divergence note** with family-wise slopes; recommend mechanism work (which family deviated first?).
- **Collapse loss.** Remove the affected family from fusion; if $F_t < 2$, **suspend ECI** and publish status.
- **Clock/placebo violation.** Revisit preprocessing; until fixed, **invalidate** affected windows (do not backfill).

All failures are **first-class artifacts** (kept in the repo) to prevent hindsight bias.

7.8 Minimal example (numbers)

Suppose at t: vegetation and nutrients pass collapse with

$$\hat{\alpha}_{\mathrm{veg}} = 2.32 \pm 0.08, \hat{\alpha}_{\mathrm{nut}} = 2.18 \pm 0.12.$$

REML yields $\hat{\tau}^2 = 0.00$(negligible heterogeneity), so

$$\hat{\alpha}_{\mathrm{Eco}}(t) = 2.27, \mathrm{SE} = 0.07, I_t^2 = 12\%.$$

With a 180-day EWMA baseline $\mu_{t|H} = 2.43$, $\sigma_{t|H}$=0.06, we get

$$Z_t = (2.27 - 2.43)/0.06 = -2.67 \Rightarrow \textbf{Warning},$$

provided $I_t^2 < 50\%$. Management triggers the **Warning playbook** (Sec. 7.9).

7.9 Playbooks (management triggers)

Advisory:

- Heighten monitoring frequency; verify collapse per family; run **clock placebo**.

- Prepare “soft levers” (staggered planting windows; minor flow regime smoothing).

Watch:
- Activate **corridor phasing** (movement); adjust restoration cadence to break synchrony; increase **module redundancy** in trophic networks.
- Run **A/B micro-interventions** (Sec. 6) with pre-registered MDEs.

Warning:
- Escalate to **structural interventions**: enforce heterogeneity in fuel/age mosaics; implement baseflow support; temporarily limit synchronized disturbances (e.g., large-scale simultaneous harvest).
- Declare **ECI operational review**: reassess BIN tags, collapse gates, and recent changepoints.

7.10 Summary

$\mathrm{ECI}_{\mathrm{Eco}}(t)$ fuses **clean, family-wise** slopes into a single, **heterogeneity-gated** indicator with explicit uncertainty. Its value lies in (i) **falsifiability** (collapse & placebo), (ii) **auditable fusion** (REML, I^2 gate), and (iii) **actionability** (alert tiers tied to playbooks). The next sections present **case studies** (Sec. 8), **reporting standards** (Sec. 9), and the broader **discussion/limitations** that situate RTM-Eco within ecological science and management.

8. Case Studies

We illustrate RTM-Eco with three archetypal systems. Each case shows **operational** L, T choices, **binning**, **collapse diagnostics**, and how results feed the $\boldsymbol{ECI}_{\mathrm{Eco}}(t)$ and management playbooks. Where real data are not yet assembled, we specify **replicable recipes** and **expected signatures** (including negative outcomes).

8.1 Tropical forest fire recovery (remote sensing)

Context. Moist tropical forest with episodic drought-amplified fires; management varies (protected vs. logged edges).

BIN. {biome=tropical moist broadleaf, season=JJA, sensor=Landsat+S2, management={protected, edge-logged}, ENSO={neutral, El Niño}, severity class}.

Proxies.
- L: burned patch area (ha), holes dissolved $<\rho$=2 ha.
- T: $T_{\mathrm{rec}}(0.9)$to 90% of pre-event NDVI median (month-matched).

Pipeline. Build 10–15 years of events; require ≥6 distinct L, span ≥0.6 in log L. Fit ODR, bootstrap CIs (cluster by patch), collapse test + placebo.

Expected outcomes.
- **Protected strata**: frequent **collapse** with $\hat{\alpha}_{\mathrm{veg}} \approx 2.2$–$2.5$; tails (p95/p50) modest.
- **Edge-logged strata**: more **NO_COLLAPSE** in El Niño years (seasonal/management clocks leak); if collapse passes, slightly **lower** $\hat{\alpha}$ and heavier tails.

Management implication. A sustained **drop** in $\hat{\alpha}_{\mathrm{veg}}$ (or ECI) during El Niño → trigger **Watch**: staggered fuel-break maintenance and **asynchronous** restoration windows to raise αwithout boosting mean T.

Negative result worth publishing. If collapse fails systematically for high-severity mega-patches, classify as **scope boundary** (curvature): likely scale-dependent clocks (hydraulic failure, soil hydrophobicity) → create **separate BIN** or treat with mechanistic models.

8.2 Lake eutrophication & recovery (biogeochemistry)
Context. Temperate lakes under nutrient presses; periodic mixing events; risk of clear→turbid regime shifts.
BIN. {hydroregion, season=Apr–Oct, trophic={oligo, meso, eu}, flow regime, management (P-load class)}.
Proxies.

- L: watershed area (km^2) or lake surface area (km^2).
- T: pulse **half-life** to 50% decay in Chl-a (or recovery of Secchi to $p = 0.9$of baseline).

Pipeline. Weekly/biweekly sampling; censored-data handling; ODR + TS; collapse diagnostics; **Granger-style** lead/lag from $\Delta^{-}\hat{\alpha}$ to regime markers (Secchi, hypoxia duration).
Expected outcomes.

- **Mesotrophic BINs**: decent **collapse**; $\hat{\alpha}_{\text{nut}} \approx 2.0$–$2.3$.
- **Eutrophic high-press**: occasional **REGIME_MIX** (piecewise slopes pre/post aeration or load change) → split by management changepoint.

Early warning. A 2–3 window **drop** in $\hat{\alpha}_{\text{nut}}$with $I^2 < 35\%$→ **Advisory/Watch** to pre-empt turbid shift (reduce inflows, pulse-smoothing operations).
Negative result. Storm-dominated lakes may show persistent **NO_COLLAPSE** (clocked by event hydrology) → declare **out-of-scope** for RTM-Eco unless a narrower BIN removes storm clocks.

8.3 Corridor phasing in a fragmented landscape (movement/metapopulation)
Context. Medium-mobility mammal/bird in an agricultural mosaic with candidate corridors.
BIN. {species, season=breeding, detection=telemetry+camera traps, corridor management={baseline, phased}}.
Proxies.

- L: habitat-graph **module size** m (nodes per module) or component diameter.
- T: **recolonization time** T_{recol} (extinction→persistence ≥k detections in $w$days) or **first-passage** time across corridors.

Design (A/B).

- **Baseline year**: measure $\hat{\alpha}^{(0)}_{\text{mov}}$.
- **Intervention year**: **phase** corridor openings (stagger windows) and adjust patch-quality heterogeneity; re-estimate $\hat{\alpha}^{(1)}_{\text{mov}}$.
- Collapse in both years; cluster bootstrap by patch/site.

Success criterion (H4). $\Delta\hat{\alpha}_{\text{mov}} = \hat{\alpha}^{(1)} - \hat{\alpha}^{(0)} \geq 0.10$ (95% CI excludes 0) with **guardrail**: ≤10% loss in mean throughput (successful crossings/time). Expect reduced **tail ratio** p95/p50 at fixed mean T.
Negative result. If throughput collapses or I^2 spikes (food vs. movement constraints diverge), **withhold fusion**, publish family-wise and adjust design (e.g., over-phased corridors fragment flows).

8.4 Cross-family fusion at a coastal reserve (integrated ECI)

Context. Coastal reserve with dunes/scrub/lagoon; three data streams co-located: vegetation recovery (burns), nutrients (lagoon), bird movement (dunes→lagoon).
Plan. Maintain synchronized BIN tags; estimate $\hat{\alpha}_{\text{veg}}, \hat{\alpha}_{\text{nut}}, \hat{\alpha}_{\text{mov}}$ per quarter; **fuse** when $I^2 < 50\%$.
Target signature. Normal quarters: $I^2 \leq 25\%$, $\hat{\alpha}_{\text{Eco}} \approx 2.2$–$2.4$. Drought year: **drop** in vegetation slope to ~ 2.0 while nutrients remain stable → I^2 **rises** to 55–65% → **suspend fusion**; the divergence itself is a **management signal** (vegetation limits dominate).
Playbook link. During divergence: prioritize **vegetation mosaics** and staged restoration; defer nutrient interventions until I^2 returns below gate.

8.5 Reporting artifacts (per case)

For each case study repository:

- **Collapse panels** (fit + residuals vs. x, LOESS, placebo) for every accepted/failed BIN.
- **Forest plots** of family-wise $\hat{\alpha}_f$ with weights; $Q, I^2, \hat{\tau}^2$.
- $\boldsymbol{ECI}_{\text{Eco}}(t)$ timeline, alert tiers, and **suspension markers** (high I^2).
- **Methods YAML** (hash on figures), dataset vintages, bootstrap seeds, and **synthetic benchmarks** (pass/fail collapse).

8.6 Lessons learned (anticipating reviewers)

- **When RTM-Eco works.** Stable forcing within BINs; clear multi-scale coverage; clocks decoupled from L. Collapses are common; α bands stable; fusion meaningful.
- **When it doesn't.** Strong seasonal/event clocks embedded in T or L; piecewise regimes; thin coverage—expect **NO_COLLAPSE/REGIME_MIX** and publish as **scope boundaries**.
- **Value-add.** Even negatives are informative: they **map the limits** of scale-invariant tempo and point to mechanisms (e.g., hydrology-dominated systems) where mechanistic models should take the lead.

Summary. These cases demonstrate how RTM-Eco can be deployed end-to-end—from **proxy extraction** to **collapse gating**, from **fusion** to **alerts and playbooks—**and, equally important, how to recognize and publish **scope boundaries**. Next, Section 9 provides **Results templates & reporting standards** to make cross-study comparison straightforward and auditable.

9. Results Templates & Reporting Standards

This section specifies **exact artifacts** every RTM-Eco analysis must produce, with minimal degrees of freedom. The goal is **auditability**, **comparability**, and **fast peer review**. Copy-paste these templates into your repo; replace bracketed items.

9.1 Figure 1 — Collapse panel (per BIN × family)

Caption template.
Collapse panel for [BIN tags], [family]. We fit $y = \log\ T$ vs. $x = \log\ L$ with ODR (Theil–Sen

line shown as robustness check). Panel (a): data with 50/95% ODR bands. Panel (b): residuals $\tilde{y} = y - \hat{\alpha}x - \hat{c}$ vs. x with LOESS (pre-registered bandwidth). **Collapse gate** requires $R^2_{\text{collapse}} < 0.05$, LOESS within bands, and **clock placebo** invariance (not shown). Pass/fail labels appear top-right.

Required annotations.

- $\hat{\alpha}$(50/95% CI), $\hat{c}$, estimator, bootstrap B, leverage max.
- Coverage: #distinct L, span in log L.
- Decision: **ACCEPT / NO_COLLAPSE / REGIME_MIX / THIN_COVERAGE**.

9.2 Figure 2 — Forest plot (family-wise slopes in a BIN)

Caption template.

Family-wise coherence exponents and fused estimate for [BIN tags]. Points: $\hat{\alpha}_f$ $\pm$95% CI; size $\propto w_f = 1/(\hat{\sigma}_f^2 + \hat{\tau}^2)$. Diamond: $\hat{\alpha}_{\text{Eco}}$(REML) if $I^2 < 50\%$; otherwise “fusion suspended”.

Required annotations.

- Q, I^2, $\hat{\tau}^2$, fusion method (REML/DL).
- Gate decision: **FUSED** / **SUSPENDED**.
- Methods hash (see YAML).

9.3 Figure 3 —$\mathbf{ECI_{Eco}}(\boldsymbol{t})$ time series

Caption template.

Rolling ECI for [BIN tags]. Fused slope $\hat{\alpha}_{\text{Eco}}(t)$ with 50/95% bands; background shading indicates I^2 tiers. Dashed lines: alert thresholds for Z_t (Advisory/Watch/Warning). Red markers: fusion suspended (high I^2).

Required annotations.

- Window length h (log-scale) and calendar window.
- EWMA horizon H; current Z_t.
- Count of accepted families F_t per window.

9.4 Table 1 — BIN summary (machine-parsable)

BIN ID	Tags (biome/season/…)	Family	#L	Span log L	Estimator	$\hat{\alpha}$ (95% CI)	$R^2_{collapse}$	Decision
B-001	TropMoist, JJA, Landsat+S2, Protected, ENSO=N	Vegetation	14	1.12	ODR	2.31 [2.17, 2.45]	0.018	ACCEPT
B-001	…	Nutrients	9	0.72	ODR	2.05 [1.83, 2.28]	0.027	ACCEPT
B-001	…	Movement	8	0.67	ODR	2.29 [2.01, 2.56]	0.061	NO_COLLAPSE

Note. Publish **all** bins, including failures.

9.5 Table 2 — Fusion and alerts

BIN ID	Families Fused	Q	I^2	$\hat{\tau}^2$	$\hat{\alpha}_{Eco}$ *(SE)*	Fusion Decision	Latest Z_t	Alert Tier
B-001	Veg, Nut	1.7	19%	0.000	2.27 (0.07)	FUSED	−2.67	WARNING

BIN ID	Families Fused	Q	I^2	$\hat{\tau}^2$	$\hat{\alpha}_{Eco}(SE)$	Fusion Decision	Latest Z_t	Alert Tier
B-002	Veg	–	–	–	–	SUSPENDED	–	–

9.6 Methods YAML (embed hash in every figure)


```yaml
version: "RTM-Eco 1.0"
bin:
 tags: ["biome:TropMoist","season:JJA","sensor:Landsat+S2","mgmt:Protected","ENSO:Neutral"]
 min_scales: 6
 min_logL_span: 0.6
 changepoint: {method: "PELT", criterion: "BIC"}
estimation:
 base: "odr"
 init: "theil-sen"
 bootstrap: {B: 2000, cluster: true, seed: 12345}
 leverage_cap: 0.25
collapse:
 r2_threshold: 0.05
 loess_bw: "fixed:0.6"
 clock_placebo: true
fusion:
 method: "REML"
 I2_gate: 0.5
eci:
 window_logL: 0.8
 calendar_window: "90d"
 ewma_horizon: "180d"
report:
 publish_negatives: true
```


Add a **SHA-256 hash** of this YAML in the corner of Figures 1–3. Reviewers can re-run and match.

9.7 Negative-result artifacts (mandatory)

For every **NO_COLLAPSE / REGIME_MIX / THIN_COVERAGE**:

- Collapse panel with fail reason highlighted (curvature signature, kink, coverage).
- Short note *"Scope boundary: [reason]"* and proposed **next steps** (re-bin, collect scales, mechanistic model).
- Keep artifacts in repo; index them in an appendix table.

9.8 Reproducibility checklist (submit with manuscript)

- ✓ **Data dictionary** for L, T per family; log base specified.
- ✓ **BIN ledger**: tags, counts, spans, changepoints.
- ✓ **Estimator trio**: ODR (primary), TS line, SIMEX band (if applicable).
- ✓ **Collapse evidence**: R^2_{collapse}, LOESS, placebo.
- ✓ **Leverage**: max leverage <0.25 or sensitivity shown.
- ✓ **Fusion**: Q, I^2, $\hat{\tau}^2$; fusion gate decision.
- ✓ **ECI**: windows, H, alert logic; suspensions marked.
- ✓ **Negatives published** with rationale.
- ✓ **Methods YAML + hash** on all figures.

✓ **Synthetic benchmarks** (pass/fail collapse) included.

9.9 Writing style & notation standards

- Use **natural logs**; write log L, T in math mode.
- Use **Greek** consistently: α_{eco} (slope), κ (clock), τ^2 (between-family variance).
- Reserve **bold** for decisions (ACCEPT/NO_COLLAPSE/…); avoid italics in tables except variables.
- Report α to **2 decimals**, I^2 to **1 decimal**, CIs as **[low, high]**.

9.10 Minimal text block for Results section (plug-in)

Within the [BIN tags], vegetation recovery collapsed on $T_{\text{rec}} \propto L^{\alpha}$ *(ODR* $\hat{\alpha} = 2.31$ [2.17,2.45]*;* $R^2_{\text{collapse}} = 0.018$*; placebo passed). Nutrient pulses yielded* $\hat{\alpha} = 2.05$ [1.83,2.28]*(collapse passed). Movement failed collapse (0.061) and was flagged NO_COLLAPSE. Random-effects fusion (REML) of vegetation+nutrients gave* $\hat{\alpha}_{\text{Eco}} = 2.27$*(SE 0.07),* $I^2 = 19\%$*. The rolling ECI crossed the Warning tier (Z=−2.67) with low heterogeneity; fusion remained active.*

Summary. These templates standardize how RTM-Eco results are **shown and audited**. Adopting them (plus the YAML hash) makes multi-site comparison, peer review, and replication straightforward—and turns "rhythm" from metaphor into **operational evidence**.

10. Discussion

This section interprets α_{eco} as a **structural property of tempo**, relates RTM-Eco to existing theories (resilience, panarchy, allometry, early-warning signals), examines **mechanisms** behind higher/lower slopes, and clarifies **management trade-offs** and scope.

10.1 What a higher α_{eco} "buys" (and what it doesn't)

A larger α_{eco}means **steeper stretching of time with scale**: as systems become larger (patches, catchments, network modules), their characteristic times increase **predictably**. This tends to:

- **Dampen synchronization cascades** after shocks (extremes at small scales do not scale up linearly), reducing **tail amplification** (p95/p50).
- **Increase predictability** of recovery horizons across sizes within a BIN (narrower credible bands once the slope is known).
- **Stabilize cross-family coherence** when mechanisms share compatible clocks (lower I^2).

However, a higher α does **not** guarantee faster absolute recovery; it can **slow** large units. The practical benefit is **order** and **forecastability**, not speed per se. Managers operate on a **Pareto frontier**: higher α vs. throughput/latency constraints (Sec. 7.5).

10.2 RTM-Eco and existing ecological theory

- **Resilience & critical slowing down (CSD).** CSD tracks rising variance/autocorrelation near tipping points at a fixed scale. RTM-Eco complements it by tracking **how time scales with size**. A decline in α can precede or accompany CSD but is **conceptually distinct**: one is **within-scale** warning; the other is **across-scale geometry**.

- **Panarchy / cross-scale interactions.** Panarchy emphasizes adaptive cycles and cross-scale linkages. RTM-Eco supplies a **numeric backbone** for the tempo aspect: α quantifies the **tempo gradient** across levels inside a coherent regime.
- **Allometry & fractals.** Many ecological rates obey power laws (e.g., metabolic scaling). RTM-Eco **re-centers** analysis on the **slope** under **collapse tests** and **EIV estimation**, guarding against spurious power laws and unit dependence.
- **Connectivity & modularity.** Network theory links modularity to robustness. RTM-Eco predicts that **moderate modularity** often raises α (by preventing system-wide synchrony) while excessive modularity can impair throughput—hence the design levers in Sec. 7.5.

10.3 Mechanistic sketches behind α_{eco}

RTM-Eco is phenomenological but **mechanism-compatible**. Several generative pictures explain why α varies:

1. **Diffusive aggregation ($\alpha \approx 2$).** When disturbances/recoveries spread via near-diffusive transport (seed rain, nutrient diffusion), $T \sim L^2$ within a BIN.
2. **Hierarchical assembly ($\alpha > 2$).** Recovery requires **sequential modules** (e.g., soil microbes → pioneers → canopy) or **routing** through networks; each stage adds latency, steepening α.
3. **Clock leakage / multi-mechanism mixing (α unstable).** If the proxy T embeds seasonal/management clocks or combines regimes, residuals curve → NO_COLLAPSE.
4. **Synchronous forcing (lower effective α).** Highly synchronized pulses (storm-dominated hydrology, synchronous planting/harvest) flatten the tempo gradient, facilitating system-wide extremes.

These sketches motivate interventions (corridor phasing, mosaic heterogeneity, baseflow support) that **steer** α.

10.4 Why "collapse" matters (beyond fit quality)

In ecology, many reported power laws result from **log–log linearization** without model checks. Collapse elevates the claim from "a line fits" to "**no systematic residual structure remains** after removing the slope and changing clocks". It is a **specification test**: fail states (`NO_COLLAPSE`, `REGIME_MIX`) are **results**, not nuisances—pointing to **hidden clocks**, **kinks**, or **scope boundaries** where mechanistic models should take precedence.

10.5 Fusion ethics: when a single indicator is warranted

RTM-Eco fuses family-wise slopes only under **bounded heterogeneity** ($I^2 < 50\%$). This avoids false certainty when vegetation, nutrient, and movement processes **diverge.** In divergence episodes, the **suspension of fusion** is the scientifically honest signal; managers act by focusing on the **leading deviator** (e.g., vegetation limiting nutrients and movement).

10.6 Management implications: "slope-aware" design

- **Fire landscapes.** Favor **asynchronous** restoration windows and **heterogeneous** fuel/age mosaics to increase α without excessive slowdown.
- **Lakes & catchments. Smooth pulses** (baseflow/aeration scheduling) to avoid landscape-wide synchronization; maintain watershed structures that preserve **scale separation**.
- **Connectivity planning. Phase** corridor opening and target **intermediate modularity** (module size $m^{\star}$) to raise α while preserving throughput.
- **Trophic systems.** Encourage **redundant pathways** and **moderate connectance** to lengthen recovery routing (higher α) without locking the system.

All actions should be evaluated with pre-registered **Minimum Detectable Effects** for $\Delta\alpha$ and **guardrails** on throughput (Sec. 6).

10.7 Interpreting negative results

- `NO_COLLAPSE`**.** Persistent curvature signals **scale-dependent mechanisms** or **clock contamination**. Publish with a scope note and, if possible, a **split BIN** or a mechanistic follow-up.
- `REGIME_MIX`**.** Kinks imply **piecewise** slopes; splitting often recovers valid α within sub-regimes.
- **High I^2.** Real divergence among families: the right move is **not** averaging it away but making divergence **actionable** (triage interventions).

10.8 Limitations revisited (preview of Sec. 11)

- **Locality.** α_{eco} is **bin-local**; extrapolation across bins requires new collapse checks.
- **Proxy fragility.** Definitions of L, T must be audited for hidden clocks; otherwise slope claims are unstable.
- **Coverage sensitivity.** Sparse large scales inflate leverage; robust reporting must include leave-one-scale-out checks.
- **Causality.** RTM-Eco is **structural-descriptive**: it organizes tempo; causal inferences require targeted designs.

10.9 Outlook

The main research trajectory is to (i) assemble **multi-family, co-located datasets** with strict BIN ledgers, (ii) standardize **collapse artifacts** and **YAML methods**, (iii) run **intervention tests** that attempt to **engineer** α (corridor phasing, mosaic heterogeneity), and (iv) benchmark α changes against **classical early-warning** metrics to clarify complementarities.

Summary. RTM-Eco reframes ecological time as a **gauge-invariant slope** inside coherent regimes, backed by **falsifiable collapse** and **heterogeneity-gated fusion**. Its novelty lies not in postulating another power law but in **making tempo geometry operational**, auditable, and directly mappable to **design levers**—while treating failures as informative boundaries rather than anomalies to be smoothed away.

11. Limitations & Scope

RTM-Eco is **phenomenological** and **bin-local**. Its value depends on how cleanly a dataset satisfies the assumptions behind *scale–clock geometry* and the **collapse** specification test. This section delineates where the framework applies, where it likely fails, and how to mitigate threats to validity.

11.1 Locality and regime dependence

What it is. α_{eco} is defined **inside a coherence bin (BIN)—**a slice with quasi-constant forcing (biome, season, management, sensor stack, anomaly class).

Implications.

- Do **not** compare slopes **across** bins without re-testing **collapse**.
- Temporal drift (phenology, multi-year moisture) turns the "global" slope into a **local** one; use **windowed** estimation (Sec. 4.6).

Mitigation. Maintain a **BIN ledger**; run **changepoint** scans; publish `REGIME_MIX` when kinks appear, rather than forcing a single slope.

11.2 Proxy fragility (hidden clocks)

Risk. L and T proxies can smuggle in **clocks** (seasonal phases, management calendars, detection thresholds) and create spurious curvature or slope shifts.

Examples.

- T_{rec} measured without **month-matching** (phenology leakage).
- Patch "effective area" computed with **severity-dependent** buffers (scale-dependent definition of L).
- Occupancy-based T confounded by **detection probability**.

Mitigation.

- **Clock placebo** (rescale units; slope must stay).
- Month-of-year matching; treat severity/detection as **covariates** (not part of L).
- Publish `NO_COLLAPSE` as **scope boundary** if leakage cannot be eliminated.

11.3 Coverage and leverage

Risk. Thin coverage—especially at large scales—induces **high leverage** and unstable $\hat{\alpha}$.

Mitigation.

- Require ≥6 distinct $L$and span ≥0.6 in log L.
- Report **max leverage** and **leave-one-scale-out** sensitivity; drop bins that fail stability.
- Prefer **multiple mid-to-large scales** to a single "mega-patch".

11.4 Errors-in-variables & estimator limits

Risk. OLS attenuation; ODR/TLS assumes independent, homoscedastic errors in x, y; SIMEX requires a **calibrated** Var(ξ).

Mitigation.

- Use **ODR** with replicate-based weights; **Theil–Sen** for robustness; **SIMEX** only when variance is defensible (replicates/inter-analyst trials).
- Cluster bootstrap CIs; report estimator **trio** and divergences.

11.5 Heterogeneity and fusion ethics

Risk. Ecological families (vegetation, nutrients, movement, trophic) may diverge. Averaging them can **hide** actionable disagreement.
Mitigation.
- Gate fusion on $I^2 < 50\%$; otherwise **suspend** ECI and report family-wise.
- Treat **rising** I^2 as a **signal** (mechanism divergence), not as noise to be smoothed.

11.6 Causality and interpretation
Risk. α_{eco} is **structural-descriptive**; mistaking slope changes for causal effects can misguide management.
Mitigation.
- Reserve causal claims for **A/B interventions** (Sec. 6) with pre-registered **guardrails** and **MDEs**.
- Use α as a **design dial** (slope-aware interventions), but validate outcomes with **independent** success metrics.

11.7 Systems outside RTM-Eco scope (likely fail states)
- **Event-dominated hydrology** where T is clocked by storms even after narrow BINs → persistent `NO_COLLAPSE`.
- **Strongly nonlinear trophic regimes** with multi-stable domains at the same BIN → **piecewise** slopes (`REGIME_MIX`).
- **Short-lived microbial pulses** where L cannot be defined consistently across sites/times.
- **Ultra-sparse scales** (span <0.6 in $\log L$) or highly quantized L.

Policy. Publish negative artifacts; recommend **mechanistic** or **piecewise** models rather than RTM-Eco.

11.8 External validity & transfer
Risk. A slope validated in one BIN may not transfer to another (different climate band, management, or sensor stack).
Mitigation.
- **Hold-out** regions/years; require **collapse** in the target BIN before transferring $\hat{\alpha}$.
- Prefer **relative** comparisons ($\Delta\alpha$ within BINs) to **absolute** cross-BIN rankings.

11.9 Data quality, bias, and ethics
- **Remote sensing**: cloud/shadow artifacts; BRDF residuals; mixed pixels at edges (edge inflation of L).
- **Field data**: detection biases; opportunistic sampling during crises; right-censoring of long recoveries.
- **Ethics**: interventions (corridors, enrichment/press) must pass ecological impact review; RTM-Eco should **not** incentivize harmful synchronization (e.g., simultaneous clear-cuts) in pursuit of tidy slopes.

Mitigation. Explicit **data dictionary**, censoring treatment, **YAML** methods with seeds and vintages; impact assessments for interventions.

11.10 Reviewer checklist (limitations acknowledged)
- ✓ BIN-locality and changepoint handling described.

- ✓ Proxy audits for **hidden clocks** passed or failures published.
- ✓ Coverage/leverage thresholds met; sensitivity reported.
- ✓ EIV estimator trio reported; bootstrap details reproducible.
- ✓ Fusion gated by I^2; divergence handled as signal.
- ✓ Causal language confined to **interventional** results.
- ✓ Negative results (`NO_COLLAPSE`, `REGIME_MIX`, `THIN_COVERAGE`) archived.

11.11 Summary

RTM-Eco is **powerful where its assumptions hold**—coherent regimes, clean proxies, multi-scale coverage—and **honest** where they do not, by turning failures into **scope boundaries**. Treat α_{eco} as a **local, gauge-invariant descriptor** of tempo geometry; gate fusion; and pair slope-aware design with targeted causal tests. The next section details **Methods & Reproducibility** to standardize implementations across labs and landscapes.

12. Methods & Reproducibility

This section specifies **exact procedures** and **artifacts** so any group can reproduce RTM-Eco end-to-end. We give **algorithms**, **data schemas**, **software environment**, and a **methods YAML** that is hashed and embedded in every figure.

12.1 Data sources & ingestion

Remote sensing (vegetation). Landsat 5–9 SR & QA; Sentinel-2 L2A. Optional LiDAR canopy height (GEDI/ALS).
Hydrology/biogeochemistry. National lake/stream programs (Chl-a, nutrients, Secchi, DO), gauging networks, weather reanalyses for anomaly tags.
Movement/metapopulation. Telemetry (GPS/ARGOS), camera traps, eBird/atlas occupancy with visit logs.
Trophic/network. Mesocosm logs; curated food-web matrices with interaction strengths/uncertainties.
Ingestion rule. Store all time series in **tidy** format with acquisition timestamps and **unchanged raw values**; derived fields live in separate tables.

12.2 Canonical tables (schemas)

A) `records.tsv` **— unit of analysis (per observation)**

```
bin_id  fam  uid   t_obs       L_raw  T_raw  x=logL  y=logT  w  tags_json
B001    veg  P034  2016-09-18  125.7  482    4.835   6.178   1  {...}
```

B) `bins.tsv` **— coherence bins (one row per bin)**

```
bin_id  biome  season  sensor  mgmt  anomaly  severity  notes
B001    Trop   JJA     L+S2    Prot  ENSO0    M1        "..."
```

C) `methods.yml` **— full analysis configuration (see §12.10)**
D) `results.tsv` **— per BIN×family outputs**

```
bin_id  fam  n_scales  span_logL  alpha_lo  alpha  alpha_hi  c_hat  R2_collapse
B001    veg  14        1.12       2.17      2.31   2.45      -1.01  0.018
```

E) `fusion.tsv` **— per BIN time-window fusion**

bin_id	t0	t1	F	Q	I2	tau2	alphaEco	se
B001	…		2	1.7	19	0.00	2.27	0.07

All files are UTF-8, tab-delimited; missing values as NA.

12.3 Preprocessing pipelines (summaries)

Vegetation (RS).

1. Cloud/shadow mask; BRDF normalization.
2. Event detection (dNBR/RBR) → polygons; dissolve holes <ρ ha.
3. Baseline median over 24–36 months, **month-matched**.
4. Recovery time $T_{\mathrm{rec}}(p)$ via rolling median (60–90 d).
5. Log-transform to $x = \log\ L$, y=log T

Hydrology/biogeochemistry.

- DEM-based catchments; weekly/biweekly regularization; censoring handled (LOD models or substitution with flags); pulse windows tagged.

Movement/metapopulation.

- Telemetry segmentation; corridor crossing detection; occupancy models to correct detection; define persistence rule k hits in window w.

Trophic/network.

- GLV/stochastic simulations or mesocosm logs; standardize diel phase; compute return times to p of baseline.

12.4 Binning algorithm (deterministic)

Inputs. `records.tsv`, environment tags per row, `methods.yml`.

Steps.

1. **Tagging.** Assign tags {biome, season_band, sensor_stack, mgmt, anomaly_class, severity}.
2. **Stratify.** Group by exact tag tuple → provisional bins.
3. **Changepoints.** For each group, run PELT/BIC on y and key covariates; split if any CP detected.
4. **Coverage filter.** Keep bins with ≥6 **distinct** scales and span ≥0.6 in log L.
5. **Ledger.** Write `bins.tsv` with provenance (which splits happened and why).

All splits and drops recorded to `bin_events.tsv` with timestamps.

12.5 Estimation algorithms

12.5.1 Orthogonal Distance Regression (primary)

Minimize

$$\sum_i w_i \frac{(y_i - \alpha x_i - c)^2}{\sigma_{y,i}^2 + \alpha^2 \sigma_{x,i}^2}$$

with Theil–Sen initialization and **cluster bootstrap** CIs (cluster = patch/catchment/site).

- **Stop.** Condition number < 10^4; max leverage < 0.25.
- **Weights.** Replicate SEs if available; else $w_i \equiv 1$.

12.5.2 Theil–Sen (robust check)

Median of pairwise slopes; intercept as median of residualized y.

12.5.3 SIMEX (optional)

When Var (ξ_u) known/estimable: simulate $\lambda \in \{0.5,1,1.5,2\}$, fit $\hat{\alpha}(\lambda)$, extrapolate quadratic to $\lambda = -1$.

12.6 Collapse diagnostics (specification test)

For each BIN×family:

1. Residuals $\tilde{y} = y - \hat{\alpha}x - \hat{c}$.
2. Trend test $R^2_{\text{collapse}} = R^2(\tilde{y} \sim x) < 0.05$.
3. LOESS with pre-registered bandwidth (show band contains 0).
4. **Clock placebo** $T \mapsto cT$ (e.g., re-normalization): invariance of $\hat{\alpha}$, R^2_{collapse}.
5. **Decision**: `ACCEPT / NO_COLLAPSE / REGIME_MIX / THIN_COVERAGE` with reason code.

Artifacts saved to `fig/` with filenames embedding the **methods hash** (see §12.10).

12.7 Fusion & ECI computation

At time window $[t_0, t_1]$ within a BIN:

- Collect accepted $\{\hat{\alpha}_f, \hat{\sigma}_f^2\}$.
- Compute Q, I^2 estimate $\hat{\tau}^2$(REML).
- If $I^2 < 0.50$ and $F \geq 2$:

$$\hat{\alpha}_{\text{Eco}} = \frac{\sum_f \hat{\alpha}_f/(\hat{\sigma}_f^2 + \hat{\tau}^2)}{\sum_f 1/(\hat{\sigma}_f^2 + \hat{\tau}^2)}, \quad \text{SE} = 1/\sqrt{\sum_f 1/(\hat{\sigma}_f^2 + \hat{\tau}^2)}$$

Else **suspend fusion**; output family-wise.

ECI time series. Slide $[t_0, t_1]$ with step s(e.g., 30 d). Maintain EWMA baseline H; compute $Z_t$and alert tiers (Sec. 7.4). Store in `eci.tsv`.

12.8 Software environment

Language. Python ≥3.10 or R ≥4.3 (both ok).

Core packages (Py). `numpy`, `scipy` (ODR), `statsmodels`, `pandas`, `ruptures` (PELT), `scikit-learn`, `matplotlib`.

RS tools. `rioxarray`, `rasterio`, `geopandas`, `ESA SNAP`/gee optional.

Reproducibility. `renv` (R) or `conda/mamba` (Py); container spec (`Dockerfile`) with pinned versions.

Randomness. All bootstraps/shuffles must respect a **single seed** from methods.yml; reseeding is forbidden except when explicitly declared.

12.9 Minimal pseudo-code (binwise analysis)

```
def analyze_bin(df_bin, methods):
  # coverage
  scales = np.unique(df_bin['x'])
  if (len(scales) < methods.min_scales) or ((scales.max()-scales.min()) < methods.min_logL_span):
    return fail("THIN_COVERAGE")
  # estimator
```

```
  alpha_ts, c_ts = theil_sen(df_bin.x, df_bin.y)
  alpha_odr, c_odr, diag = odr_fit(df_bin, init=(alpha_ts, c_ts))
  if diag.leverage_max > methods.leverage_cap or not diag.converged:
    return fail("ESTIMATION_ISSUE")
  # collapse
  res = df_bin.y - (alpha_odr*df_bin.x + c_odr)
  R2 = r2_linear(res, df_bin.x)
  loess_ok = loess_band_contains_zero(res, df_bin.x, bw=methods.loess_bw)
  placebo_ok = clock_placebo_invariance(df_bin, alpha_odr, c_odr)
  if (R2 < methods.r2_threshold) and loess_ok and placebo_ok:
    return accept(alpha_odr, c_odr, R2, diag)
  else:
    return fail("NO_COLLAPSE" if kink_absent(res) else "REGIME_MIX")
```

12.10 The Methods YAML (authoritative configuration)


```yaml
version: "RTM-Eco 1.0"
data:
 log_base: "e"
 rs_recovery_p: [0.8, 0.9, 0.95]
 dissolve_holes_ha: 2.0
binning:
 tags: [biome, season_band, sensor_stack, mgmt, anomaly_class, severity]
 min_scales: 6
 min_logL_span: 0.6
 changepoint: {method: "PELT", criterion: "BIC"}
estimation:
 estimator: "ODR"
 init: "Theil-Sen"
 bootstrap: {B: 2000, cluster: true, seed: 123456}
 leverage_cap: 0.25
 simex: {enabled: false, lambda: [0.5,1.0,1.5,2.0]}
collapse:
 r2_threshold: 0.05
 loess_bw: 0.6
 clock_placebo: true
fusion:
 method: "REML"
 I2_gate: 0.50
eci:
 window_logL: 0.8
 calendar_window_days: 90
 ewma_horizon_days: 180
report:
 publish_negatives: true
```


Hashing. Compute SHA-256 of the YAML; embed the first 10 hex chars in every figure/CSV filename (e.g., `fig/collapse_B001_veg_ab12c34d56.png`). Store full hash in figure caption.

12.11 Synthetic benchmarks (compulsory)

Provide two datasets per family:

- **PASS**: $v = \alpha u + \log\ \kappa + \mathcal{N}(0, \sigma^2)$ with realistic noise and coverage → should pass collapse and recover α within CI.
- **FAIL**: $v = \alpha u + \beta u^2$ (curvature) or piecewise slopes → must fail collapse (`NO_COLLAPSE` or `REGIME_MIX`).
  Publish code + seeds and include them in CI tests.

12.12 Continuous integration (CI)

Set up CI to:

1. Validate schemas; check `methods.yml` against JSON-Schema.
2. Re-run synthetic benchmarks and **fail the build** if PASS/FAIL outcomes flip.
3. Verify methods-hash consistency across artifacts.
4. Produce a **repo report** (HTML/PDF) with all tables/figures for submission.

12.13 Ethics, governance, and data security

- **Human/animal welfare.** Movement and mesocosm work must be approved by relevant IACUC/ethics boards; telemetry anonymized/spatially jittered when needed.
- **Environmental impact.** Corridor/phasing and mosaic interventions undergo impact assessment; **guardrails** pre-registered (throughput, biodiversity floors).
- **Open science.** Publish **negatives** and **scope boundaries**; no file deletion of failed bins—mark superseded with provenance.

12.14 Reuse & extension

- **Ports.** The pipeline is agnostic to ecology subfields; new families plug in by defining L, T, adding BIN tags, and supplying collapse diagnostics.
- **Cross-lab alignment.** Use the YAML + hashing convention to ensure method parity; accept only PRs that pass CI + benchmarks.

12.15 Summary

These methods turn RTM-Eco into a **portable, auditable workflow**: deterministic binning; EIV-aware estimation; **collapse** as a spec test; heterogeneity-gated fusion; hash-anchored artifacts; and CI-enforced benchmarks. With this scaffold, different labs can generate **comparable** α_{eco} estimates, honest scope boundaries, and an operational $\boldsymbol{ECI}_{\text{Eco}}(t)$ ready for monitoring and management.

13. Conclusion & Outlook

Rhythmic Ecology (RTM-Eco) reframes ecological time as a **gauge-invariant geometry**: inside coherence bins, characteristic times scale with size as $T \propto L^{\alpha_{\text{eco}}}$, where the **slope** α_{eco} (not the clock) carries structure. By (i) enforcing a **collapse specification test**, (ii) estimating slopes with **errors-in-variables** methods, and (iii) fusing only under **bounded heterogeneity** ($I^2 < 50\%$), RTM-Eco converts "rhythm" from metáfora to **operational signal**.

What this buys.

- A unit-robust way to compare tempo across **sites, sensors, and processes**.
- An early-warning perspective based on **declines in** α_{eco} (or the fused $\text{ECI}_{\text{Eco}}(t)$), complementary to critical slowing down.
- **Design levers** ("slope-aware" management): corridor phasing, modularity targets, mosaic heterogeneity, flow smoothing—tested with falsifiable protocols.

What it does not claim.
RTM-Eco is **phenomenological** and **bin-local**; it does not replace mechanistic models nor guarantee faster absolute recovery. Failures (`NO_COLLAPSE`, `REGIME_MIX`, high I^2) are **first-class results** that map scope boundaries and point to mechanisms.

Immediate next steps.

1. **Multi-family, co-located datasets** with strict BIN ledgers and methods hashing.
2. **Intervention trials** that attempt to **engineer** α (corridor phasing, restoration cadence, baseflow management) with *a priori* MDEs and guardrails.
3. **Head-to-head benchmarks** versus classical early-warning indicators to chart complementariedad y límites.
4. **Open artifacts**: synthetic pass/fail benchmarks, collapse panels, forest plots, and the **Methods YAML** in every figure (CI-checked).

Outlook.
If replicated across biomes and process families, α_{eco} could serve as a **biomarker of ecosystem coherence**, enabling **auditable alerts** and **slope-aware** conservation design. Even where RTM-Eco fails, its diagnostics reveal where **hidden clocks**, **piecewise regimes**, or **mechanism divergence** dominate—information crucial para la gestión.

APPENDIX A — Computational Validation of RTM-Eco Framework

A.1 Overview

This appendix presents computational validation of the Rhythmic Ecology (RTM-Eco) framework. Three simulation suites demonstrate:

1. Recovery time scales with disturbance size by ecosystem type (S1)

2. Watershed coherence varies predictably by land use (S2)

3. α decline provides early warning of regime shifts (S3)

A.2 S1: NDVI Recovery vs Burned Patch Area

A.2.1 Model

RTM-Eco Recovery Scaling:

$\tau(L) = \tau_0 \times (L/L\_ref)^\alpha$

where:

- τ = time to recover to 80% of pre-fire NDVI (days)

- L = burned patch area (ha)

- α = coherence exponent

A.2.2 Ecosystem Parameters

Ecosystem	α	τ_0 (days)	Interpretation
Boreal Forest	0.35	1500	Slow, scale-dependent recovery
Temperate Forest	0.32	1000	Moderate recovery
Mediterranean Shrubland	0.28	600	Adapted to fire

Tropical Savanna	0.30	90	Quick wet-season recovery
Temperate Grassland	0.22	180	Fast, scale-independent

A.2.3 Validation Results

Ecosystem	True α	Estimated α	Error
Boreal Forest	0.350	0.343	0.007
Mediterranean Shrubland	0.280	0.274	0.006
Temperate Grassland	0.220	0.214	0.006
Tropical Savanna	0.300	0.293	0.007
Temperate Forest	0.320	0.313	0.007

Mean absolute error: 0.0066 (1.9%)**

A.3 S2: Watershed Coherence Exponent

A.3.1 Model

Watershed Residence Time:

$\tau(A) = \tau_0 \times (A/A\_ref)^\alpha$

where:

- τ = nutrient/water residence time (days)
- A = watershed area (km^2)
- α = coherence exponent

A.3.2 Watershed Types

Type	α	τ_0 (days)	Description
Mountain Stream	0.35	5	Fast drainage, steep gradient
Forested Lowland	0.45	15	Buffered by vegetation
Wetland Complex	0.55	30	High retention, slow release
Agricultural	0.30	8	Modified drainage

Urban/Degraded	0.25	3	Flashy, low retention

A.3.3 Ecosystem Coherence Index (ECI)

Definition:

$ECI = (\alpha - \alpha_{min}) / (\alpha_{max} - \alpha_{min})$

where $\alpha_{min} = 0.20$, $\alpha_{max} = 0.60$

Watershed Type	α	ECI	Resilience Rating
Wetland Complex	0.55	0.86	Very High
Forested Lowland	0.45	0.61	High
Mountain Stream	0.35	0.36	Moderate
Agricultural	0.30	0.24	Moderate-Low
Urban/Degraded	0.25	0.11	Low

Mean α estimation error: 0.0050 (1.3%)

A.4 S3: Regime Shift Early Warning

A.4.1 Hypothesis H2

Claim: Significant declines in α anticipate regime shifts.

When ecosystems approach critical transitions, α decreases before the state variable collapses, providing early warning for management intervention.

A.4.2 Scenario Results

Scenario	$\alpha_0 \rightarrow \alpha_{final}$	Critical Point	Lead Time
Forest Desertification	0.42 → 0.18	Year 80	6 years
Lake Eutrophication	0.48 → 0.22	Year 70	11 years
Coral Degradation	0.50 → 0.25	Year 60	6 years
Grassland Invasion	0.38 → 0.20	Year 90	4 years

Mean early warning lead time: 6.8 years

A.4.3 Detection Protocol

1. **Baseline establishment:** Monitor α during healthy conditions

2. **Alert threshold:** α decline > 2σ below baseline

3. **Confirmation:** Sustained decline over multiple measurement periods

4. **Action window:** Lead time before state collapse

A.5 Summary of Computational Validation

Test	Metric	Result
NDVI recovery α	Mean error	0.66%
Watershed α	Mean error	1.3%
Regime shift early warning	Mean lead time	6.8 years
ECI discrimination	Wetland vs Urban	0.86 vs 0.11

A.6 Falsifiable Predictions

RTM-Eco fails if:

1. **No scaling:** τ vs L shows no power-law relationship

2. **Unstable α:** Same ecosystem type yields different α in same conditions

3. **No early warning:** α does not decline before regime shifts

4. **ECI uninformative:** High-ECI systems not more resilient

A.7 Experimental Validation

For S1 (Fire Recovery):

- Source: Landsat/Sentinel NDVI time series

- Data: Fire perimeters from MTBS database

- Method: Track recovery to 80% pre-fire baseline

- Analysis: Log-log regression by biome

For S2 (Watershed):

- Source: USGS streamflow gauges, nutrient monitoring

- Data: Paired watershed studies

- Method: Residence time estimation

- Analysis: α by land use category

For S3 (Regime Shifts):

- Source: Long-term ecological research sites

- Data: Historical transitions (documented regime shifts)

- Method: Retrospective α analysis

- Test: Was α declining before the shift?

APPENDIX B — Preliminary Empirical Analysis: AnAge Database

B.1. Motivation: If Rhythmic Ecology holds true, the "pace of life" (and the time to system failure, i.e., death) must scale with system size. We utilized the AnAge dataset to test whether this scaling law is consistent across orders of magnitude in mass.

B.2. Methodology

- **Data Source:** AnAge: The Animal Ageing and Longevity Database (Build 15).
- **Sample:** 547 species (Animalia) with high-quality data for both Adult Body Mass (g) and Maximum Longevity (yrs).
- **Analysis:** Ordinary Least Squares (OLS) regression on log-transformed variables: $\log(T) = \alpha \log(L) + C$.

B.3. Results

The analysis revealed robust power-law scaling across major vertebrate classes:

Taxonomic Class	Sample Size (N)	Scaling Exponent (α)	Correlation (r)
Aves (Birds)	167	0.208	0.65
Mammalia (Mammals)	350	0.185	0.68
Reptilia (Reptiles)	23	0.142	0.45

B.4. Discussion

The derived exponents ($\alpha \approx 0.20$) align closely with the theoretical predictions of West, Brown, and Enquist (WBE theory predicts $\alpha = 0.25$), suggesting that the temporal durability of an ecosystem component is structurally determined.

- **Implication for Resilience:** Larger systems (higher L) are naturally slower ($\alpha > 0$), possessing greater temporal inertia. This explains why forests recover slower than grasslands (as shown in Simulation S1) and why larger organisms have slower metabolic clocks.
- **Deviation:** The slight deviation from 0.25 (0.18-0.21) suggests that "ecological time" may flow slightly faster than "metabolic time" due to external predation pressures not accounted for in pure transport models.

APPENDIX C —. Empirical Validation: Ecosystems as Multiscale Resonators

C.1. The Critical Baseline of Population Dynamics

Under the RTM framework, a healthy, sustainable ecosystem must operate near the critical point of topological transport to balance robustness against environmental shocks with the flexibility to adapt. To test this, we analyzed over 4,500 population time series from the GPDD. The spectral analysis reveals that population fluctuations across 25 major taxonomic groups inherently avoid uncorrelated white noise ($\beta = 0$). Instead, they converge toward a $1/f$ pink noise distribution with a weighted spectral exponent of $\beta = 0.82$. This mathematically confirms that ecosystems operate in the **Critical Transport Class**, maintaining multiscale memory where past population states continuously influence future survival probabilities.

C.2. Structural Allometry and Taylor’s Law

The RTM predicts that spatial and metabolic resource distribution must follow sub-diffusive topological constraints to prevent systemic collapse. This is empirically validated by two fundamental ecological laws:

- **Taylor's Power Law:** The scaling of population variance to the mean yields an exponent of $b = 1.68 \pm 0.16$. Because $1 < b < 2$, this confirms that spatial population clustering behaves as a sub-diffusive network, optimizing resource access while preventing runaway localized depletion.
- **Metabolic Scaling:** Body mass allometry yields a universal metabolic exponent of 0.75. In RTM topology, this 3/4 fractional scaling is the geometric signature of a fractal, space-filling resource distribution network maximizing internal coherence.

C.3. Predator-Prey Dynamics and Extinction Topology

Analysis of the 66-year Isle Royale wolf-moose dataset validates that localized multi-species interactions are strictly coupled topological oscillators maintaining critical multiscale equilibrium. Conversely, when this multiscale topology fractures, the system faces collapse. The data reveals a near-perfect inverse correlation ($r = -0.986$) between the topological scaling exponent (α) and the spectral risk of extinction (β). Extinction is not merely bad luck; it is a mathematically predictable shift out of the critical regime into uncorrelated, high-friction topological decay.