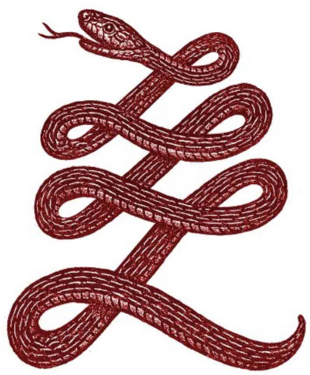

# RTM-Aware Quantum Computing

A Multiscale, Slope-First Framework for Coherence, Scheduling, and Design

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# Abstract

We introduce a **slope-first** methodology for quantum computing based on **Multiscale Temporal Relativity (RTM)**. Inside a fixed operational regime, RTM posits that a characteristic time $T$ scales with a size/scale proxy $L$ by a power law,

$$\log T = \alpha \log L + c,$$

where the **coherence exponent** $\alpha$ is the **clock-invariant** structural signal and $c$ encodes clock/units. We adapt RTM to quantum stacks—**physical**, **QEC**, **compiler/runtime**, and **I/O–cryo**—by defining layer-specific $(L, T)$ pairs (e.g., number of active qubits vs. stable calibration time; code distance vs. logical-failure time; multiplexing degree vs. readout latency; circuit width vs. makespan), and estimating binwise slopes under errors-in-variables (ODR/TLS, Theil–Sen, SIMEX). A **collapse test** validates scaling and guards against regime mixing; clean family-wise slopes are fused into a real-time $\boldsymbol{ECI}_{\mathrm{QC}}$**(t)** with uncertainty and QA gates.

We formulate **falsifiable** hypotheses: **(H1)** higher pre-shock $\alpha$ predicts longer stability margins (fewer forced recalibrations, lower logical error at fixed $d$); **(H2) decoherence events**—significant QA-clean drops in $\mathrm{ECI}_{\mathrm{QC}}$—lead spikes in logical error, queueing, or makespan; **(H3)** micro→meso→macro **tempo cascades** exhibit non-decreasing $\alpha$ within stable regimes. We demonstrate how **RTM-aware scheduling** (batching, staggered resets, low-variance routing), **QEC cadence design** (desynchronization of syndrome cycles), and **modular sizing** (sweet spots for interconnect) can improve throughput and reliability without changing physical fidelities. The framework is reproducible, gauge-robust (unit/clock changes do not affect $\alpha$), and designed to fail gracefully (no-collapse and high heterogeneity become scope boundaries, not post-hoc fixes).


# 1. Introduction

### 1.1 Motivation: beyond fidelities and error rates

Quantum performance is usually summarized by **point metrics**—single- and two-qubit fidelities, $T_1/T_2$, logical error rates, or benchmark figures (QED-C, QV). Yet practical reliability and throughput hinge on something orthogonal: **how timing stretches across scale** in a multistage stack—qubits and resonators, code cycles, compilers, cryogenic I/O. When small subsystems respond quickly and larger ones respond more slowly in a disciplined, layered fashion, shocks are **dissipated**; when timings **flatten**, disturbances percolate across layers and synchronize failures (stalling readout, spiking logical error, or forcing global recalibrations).

**Multiscale Temporal Relativity (RTM)** provides a compact language for this phenomenon. Inside a fixed regime, RTM expects a power-law relation between a **characteristic time** $T$ and a **scale proxy** $L$: the **slope** $\alpha$ in $\log\ T = \alpha \log\ L + c$is structural (invariant to time units), while the intercept $c$is a **clock** (gauge). We bring this principle to quantum computing and show that measuring, validating, and **engineering** $\alpha$ yields actionable levers—independent of nominal units—to improve stability and throughput.

### 1.2 RTM in one line

**Structure lives in the slope; clocks live in the gauge.**
A change of clock or units shifts $c$ but leaves $\alpha$ unchanged. Thus $\alpha$ can be compared across devices, stacks, and labs, while $c$ cannot.

### 1.3 Contributions

This paper makes five contributions:

1. **Operationalization of RTM for QC.** We define layer-specific $(L, T)$ pairs for **physical**, **QEC**, **compiler/runtime**, and **I/O–cryo** layers (e.g., $L =$ active qubits, $T =$ stable calibration time; $L = d$, $T =$ cycles to logical failure; $L =$ multiplexing degree, $T =$ readout latency; $L =$ circuit width, $T =$ makespan).
2. **Validation & estimation.** We provide a **collapse test** (residual independence of $\log\ T - \alpha \log\ L$from $\log\ L$) to detect regime mixing and non-power curvature, and adopt **errors-in-variables** estimation (ODR/TLS, Theil–Sen, SIMEX) with bootstrap uncertainty and changepoint guards.
3. **A single real-time indicator.** We fuse family-wise slopes into $\boldsymbol{ECI}_{\mathrm{QC}}$**(t)** via random-effects meta-analysis with heterogeneity controls ($Q$, $I^2$, $\hat{\tau}^2$); we publish QA flags and withhold fusion when proxies disagree.
4. **Design levers.** We formalize **RTM-aware scheduling** (batching, staggered resets, low-variance routing), **QEC cadence design** (desynchronization to avoid phase lock between physical errors and syndrome extraction), and **modular sizing** (choosing module/interconnect scales that elevate $\alpha$ without throttling throughput).
5. **Falsifiable hypotheses & protocols.** We pre-register **H1–H3** with A/B protocols on superconducting and trapped-ion platforms, metrics (throughput, makespan, logical error, uptime, p95/p50 ratios), and decision thresholds for adoption.

### 1.4 What $\alpha$ is—and is not

- **Is:** a **binwise slope** linking a time $T$ to a scale $L$ inside a **fixed environment** (same temperature/firmware/topology/syndrome schedule). It captures the **geometry of tempo across scale**.

- **Is not:** a causal parameter by default; level changes in $T$ (units, clocks, offsets) do **not** change $\alpha$. When collapse fails, $\alpha$ is **undefined** for that bin and should not be fused.

### 1.5 Layer-specific $(\boldsymbol{L}, \boldsymbol{T})$exemplars (preview)

- **Physical:** $L =$ active qubits / coupler degree / cluster size; $T =$ stable calibration interval, gate/RO latency, mean time to drift.
- **QEC:** $L = d$ (code distance) or number of logical qubits; $T =$ cycles to logical failure; cadence of syndrome extraction.
- **Compiler/runtime:** $L =$ circuit width or depth after mapping; $T =$ makespan; queueing delay and rescheduling latency.
- **I/O–cryo:** $L =$multiplexing degree or channels; $T =$ readout latency/BER recovery; p95 queue length.

Each layer yields a slope $\hat{\alpha}_f$; after QA and collapse, we fuse them into $\mathrm{ECI_{QC}(t)}$ with uncertainty bands. Clean **decoherence events** are significant drops in $\mathrm{ECI_{QC}}$ over pre-registered horizons.

### 1.6 Hypotheses (falsifiable)

- **H1 (Resilience):** Higher pre-shock $\alpha$ associates with smaller logical-error spikes at fixed $d$ and longer stable calibration intervals.
- **H2 (Anticipation):** QA-clean $\mathrm{ECI_{QC}}$ drops lead increases in makespan, queueing, or logical error by weeks to months, adding predictive value over baselines (fidelity, utilization, temperature).
- **H3 (Cascade):** Within stable regimes, $\alpha_{\text{physical}} \le \alpha_{\text{QEC}} \le \alpha_{\text{runtime/I/O}}$; directionality tests favor micro→meso→macro timing flow.

### 1.7 RTM-aware design (intuitions we will test)

- **Scheduling:** Avoid patterns that **flatten** $\alpha$ (long, tightly coupled operations in parallel); favor **batching** readouts and **staggered** resets to prevent synchronization cascades.
- **QEC cadence:** Introduce slight **desynchronization** (phase offsets) between syndrome cycles and known noise rhythms to raise $\alpha_{\text{QEC}}$.
- **Modularity:** Choose module size and interconnect density where $\alpha$ is high enough to damp inter-module cascades but not so high that throughput is throttled.

### 1.8 Relation to prior work

Our framework complements fidelity-centric and error-model approaches by adding a **scale–tempo geometry**. It is compatible with (not a replacement for) surface/LDPC code theory, compilation/routing heuristics, and queueing models; it contributes a **gauge-invariant** statistic $\alpha$ and a **collapse** specification test to separate **structure** from **clock** effects. In the language of stochastic processes, our dynamics section (later) connects RTM to **time-changed diffusions**; in meta-analysis terms, our fusion mimics **random-effects** with explicit **heterogeneity gates**.

### 1.9 Paper roadmap

- **Sec. 2–3:** RTM foundations adapted to QC; geometry of collapse and variable exponents.
- **Sec. 4:** Operational $(L, T)$ definitions and binning.
- **Sec. 5:** Estimation (ODR/TLS, Theil–Sen, SIMEX), collapse thresholds, uncertainty.
- **Sec. 6:** $\mathrm{ECI}_{\mathrm{QC}}(\mathrm{t})$ construction, fusion, and QA.
- **Sec. 7:** RTM-aware design: scheduling, QEC cadence, modular sizing.
- **Sec. 8:** Experimental protocols (superconducting, ion traps), metrics, and evaluation.
- **Sec. 9:** Results templates and robustness panels.
- **Sec. 10:** Discussion; **Sec. 11:** Limitations; **Sec. 12:** Methods & Reproducibility; **Sec. 13:** Conclusion.

**Takeaway.** By measuring and engineering $\alpha$**—**the multiscale **coherence exponent—**we obtain a reproducible, falsifiable handle on timing structure across the quantum stack. Used with discipline (collapse/QA, heterogeneity gates), RTM delivers a third axis for quantum system design: not only *how accurate* and *how many*, but *how timing stretches with scale*.

## 2. RTM Foundations Adapted to Quantum Computing

This section states the RTM axioms, derives the **power-law** form $T = \kappa L^{\alpha}$, and tailors **clock/gauge** and **collapse** notions to quantum stacks. Throughout, $L > 0$ is a **scale proxy** (layer-specific) and $T > 0$ is a **characteristic time** measured in a **fixed environment/bin** (same temperature, firmware, topology, syndrome schedule, utilization band).

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### 2.1 Axioms (binwise)

**A1 — Scale semigroup.** For any dilation $b > 0$,

$$T(bL) = f(b)\,T(L),$$

with $f(1) = 1$ and $f(b_1 b_2) = f(b_1)f(b_2)$.

**A2 — Mild regularity.** $f$ is measurable (or continuous at $b = 1$).

A3 — Clock invariance in-bin (multiplicative gauge). Allowed clock changes multiply T by a factor c>0 independent of L inside the bin (unit changes, constant gain factors, uniform timebase rescaling).

Additive latencies are not gauges in log–log: if observed times satisfy T_obs = c·T + b with b constant, then log–log slopes can be biased when T is not ≫ b/c.

Latency policy (must be stated): (i) calibrate and subtract b before log-fitting (fit log(T_obs−b)); (ii) model explicitly via T_obs = c·κ·L^α + b (nonlinear fit / profile likelihood); or (iii) restrict fits to regimes where T_obs ≫ b and report sensitivity to plausible b.

Pure timestamp re-referencing (changing the time origin) does not affect durations once T is computed as a difference.

A4 — Binning. Comparisons are made within bins where the operational environment is stable (e.g., temperature, firmware, topology, QEC cadence, utilization band). If a changepoint is detected or heterogeneity gates fail, the bin must be split; otherwise RTM slope estimates are not comparable.

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### 2.2 Functional-equation solution → power law

Let $u = \log\, L$, $v = \log\, T$. From A1–A2, the multiplicative Cauchy equation gives $f(b) = b^{\alpha}$ for some $\alpha \in \mathbb{R}$. Hence

$$T(L) = \kappa L^{\alpha}, v(u) = \alpha u + \log\, \kappa.$$

**Interpretation.** $\alpha$ is the **coherence exponent** (slope); $\kappa$ is a **clock** (intercept).

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### 2.3 Clocks (gauges) and what is identifiable

A multiplicative clock transform g(T)=c·T gives log T' = log T + log c. Thus:

α is invariant to multiplicative clocks (unit changes, uniform gain factors) within a bin.

The intercept encodes clock/units and should not be compared across bins or labs without explicit calibration.

Additive latencies/overheads are not log–log gauges: for T_obs = c·T + b, slope invariance holds only when b=0, after correcting b, or when T_obs ≫ b over the fitted range.

Estimation is therefore slope-first with an explicit latency policy (subtract / model / restrict) stated up front.

QC-specific examples of “clocks”.

Multiplicative clocks (safe): unit conversions; uniform tick-rate rescaling; uniform firmware timing gains applied across widths.

Additive latencies (not gauge in log–log unless handled): fixed readout preamble; fixed scheduling baseline offset that affects measured durations; constant cryo I/O/context-switch baseline.

If any overhead depends on L inside the bin (e.g., per-channel overhead that grows with multiplexing degree), it is not a valid clock transform; the bin violates RTM and should fail collapse or be split.

**Remark 2.3a (Additive timestamp offsets are not log–log gauges).**
The clock transform in Definition 2.3 is **multiplicative**: $T' = cT$ with $c > 0$. This implies $\log\, T' = \log\, T + \log\, c$, so the **log–log slope** is invariant and only the intercept shifts.
In practice, instrumentation may report an **affine** time: $\tilde{T} = cT + b$ (constant latency/dead time $b$). Then

$$\log\, \tilde{T} = \log\, T + \log\, c + \log\,(1 + \frac{b}{cT}),$$

so slope invariance holds **exactly only when** $b = 0$, and holds **approximately** only when $T \gg b/c$ over the fitted range. Otherwise, either (i) estimate/subtract $b$ before logging (use $T_{\text{eff}} = \tilde{T} - b$ with $\tilde{T} > b$), or (ii) explicitly model the offset (e.g., fit $\log\,(\tilde{T} - b)$ after validating $b$).

## 2.4 Collapse as a binwise specification test

Given observations $\{(L_i, T_i)\}_i$ in a bin, define $x_i = \log\, L_i$, $y_i = \log\, T_i$. Fit a binwise slope $\hat{\alpha}$ (Section 5) and examine **residuals**

$$\tilde{y}_i := y_i - \hat{\alpha} x_i.$$

**Collapse test.** In a valid RTM bin, $\tilde{y}$ should be **independent of** $x$ (up to noise). We operationalize with:

- A regression $\tilde{y} \sim x$ and require $R^2_{\text{collapse}} < \tau$ (default $\tau = 0.05$).
- A **clock placebo**: multiply all $T_i$ by a constant; $\hat{\alpha}$ and $R^2_{\text{collapse}}$ must be unchanged.
- A **smooth check** (LOESS or spline) for visible trend; if present, reject the bin.

**Meaning.** Collapse establishes that, after removing $\hat{\alpha} \log L$, only a **gauge** remains (intercept noise), not a trend vs. scale.

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### 2.5 Variable exponents and finite-window bias

In practice, $\alpha$ can drift slowly with environment or scale (e.g., across utilization bands or multiplexing factors). Write

$$v(u) = \int_{u_0}^{u} \alpha(s)\, ds + \log \kappa(u),$$

with $|\alpha'(u)| \leq \varepsilon$ small on the window and $\kappa$ **slowly varying.** For any symmetric window of width $h$ in $u$,

$$\hat{\alpha}(u; h) = \alpha(u) + O(\varepsilon h) + O(\text{slow-variation}),$$

and

$$R^2_{\text{collapse}} = O((\varepsilon h)^2).$$

**Rule.** Choose bins/windows small enough that curvature is negligible; otherwise split the bin.

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### 2.6 Failure modes (should fail)

RTM is designed to **predict its own failure**:

1. **Regime mixing (kinks).** Example: changing the readout chain or syndrome scheduler mid-bin. The log–log plot shows a slope change at $L^\star$; collapse fails.
2. **Curvature (non-power).** Example: a multiplexing-dependent overhead that grows nonlinearly with $L$. Residuals trend with $x$; collapse fails even after rebinning.

3. **Scale-dependent clocks.** Any "clock" factor $c(L)$ that depends on $L$ is not a gauge; it injects $du$-components into the 1-form and must be modeled explicitly (or the bin rejected).

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### 2.7 QC layer mapping (notation and exemplars)

We will use these **canonical** $(L, T)$ pairs in later sections (others may be added if they pass collapse):

- **Physical**:
  $L =$number of **active qubits** (or cluster/coupler degree);
  $T =$**stable calibration interval**, **gate** latency, **readout** latency, or **mean time to drift**.
- **QEC**:
  $L =$**code distance** $d$ (or logical-qubit count);
  $T =$**cycles to logical failure** at fixed target error.
- **Compiler/Runtime**:
  $L =$**circuit width** or **post-mapping depth**;
  $T =$**makespan** or **queueing delay**.
- **I/O–Cryo**:
  $L =$**multiplexing degree** or readout-channel count;
  $T =$**effective readout latency / BER-recovery half-life / p95 queue length (in time)**.

Each family produces a binwise $\hat{\alpha}_f$. Only families that **pass collapse** and QA contribute to the fused indicator $\boldsymbol{ECI}_{\mathrm{QC}}$**(t)** (Section 6).

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### 2.8 Why $\alpha$matters operationally

- **Comparability**: $\alpha$ is invariant to unit changes and constant overheads, enabling **cross-lab** and **cross-generation** comparison.
- **Early warning**: significant **drops** in $\alpha$ (per family or fused) signal **decoherence events** likely to precede spikes in logical error, makespan, or forced recalibrations.
- **Design lever**: raising $\alpha$ (without over-layering) via **scheduling**, **QEC cadence**, or **module sizing** improves damping of cross-scale cascades.

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### 2.9 Summary

RTM in QC reduces to three binwise statements: (i) **power-law** scaling $T = \kappa L^{\alpha}$, (ii) **gauge invariance** (only the slope $\alpha$ is structural), and (iii) **collapse** as a falsifiable specification test. With careful binning and EIV-aware estimation, $\alpha$ becomes a reproducible, unit-robust **coherence exponent** that guides both **diagnostics** and **design** across the quantum stack.

# 3. Scale–Clock Geometry for QC (Collapse as Exactness)

We recast RTM for quantum stacks in geometric form. The key object is the **RTM 1-form**

$$\omega \;=\; d(\log\, T) \;-\; \alpha(x)\, d(\log\, L),$$

defined on a bin $E$ with **environment** coordinates $x$ (temperature, firmware, topology, syndrome schedule, utilization band) and **scale** $u = \log\, L$. In this language, **collapse** is equivalent to **exactness/flatness** of $\omega$; regime seams and non-power curvature appear as **holonomy/curvature**. This section states the results and instantiates them with QC failure modes.

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### 3.1 Spaces, bins, and the RTM 1-form

- **State space.** $M = X \times \mathbb{R}$ with coordinates $(x, u)$, where $u = \log\, L$.
- **Clock potential.** $v(x, u) = \log\, T(x, L)$.
- **RTM 1-form.** $\omega = dv - \alpha(x)\, du$ (constant-$\alpha$ case) or $\omega = dv - \alpha(x, u)\, du$ (slow drift allowed).

A **clock change** (unit/baseline shift independent of $L$inside a bin) is $v \mapsto v^{\#} = v + \phi(x)$. Then

$$\omega \;\mapsto\; \omega^{\#} = \omega + d\phi(x)$$

— a **gauge transformation** by an exact 1-form pulled back from $X$. Hence $\alpha$ **is gauge-invariant**.

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### 3.2 Collapse ⇔ exactness/flatness

**Theorem 3.1 (Collapse ⇔ exactness).**
On a simply connected bin $E$, the following are equivalent:

1. (RTM chart) $v(x,u) = \alpha(x)\,u + \log\ \kappa(x)$(or $v = \int \alpha(x,s)\,ds + \log\ \kappa(x)$ for slow drift).
2. (**Collapse**) Residual $\tilde{v} := v - \alpha u$ is independent of $u$ in $E$.
3. (**Exactness**) $\omega = d\psi$ on $E$ for some $\psi(x)$(no $u$-dependence).

**Corollary 3.2 (Flatness test).**
$d\omega = 0$ is necessary and (on simply connected $E$) sufficient for collapse. With $\alpha = \alpha(x,u)$,

$$d\omega \; = \; -\, d\alpha \wedge du.$$

Thus curvature (non-power behavior) or regime mixing gives $d\alpha/\,du \neq 0$ and **breaks collapse**.

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### 3.3 Holonomy and regime seams (QC failure modes)

Define **holonomy** around a closed loop $\gamma \subset E$: $\mathcal{H}(\gamma) = \oint_\gamma \omega$. If $\mathcal{H}(\gamma) \neq 0$, collapse cannot hold globally.

**QC instances.**

- **Scheduler seam.** Changing the syndrome-extraction cadence mid-bin (new FPGA image) produces a kink in $v(u)$; loops that cross the seam pick up nonzero holonomy → **rebin**.
- **Readout chain swap.** A per-channel overhead that *depends on multiplexing* behaves like a scale-dependent clock $c(L)$; this is **not gauge** and injects $du$-components → collapse fails (and should).
- **Thermal drift window.** A slow utilization ramp changes $\alpha$ across $u$; if $\partial_u \alpha$ is not small on the window, $d\omega \neq 0$→ split the bin or shrink the window.

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### 3.4 Adiabatic collapse (slowly varying $\boldsymbol{\alpha}$)

If $|\ \partial_u \alpha\ | \leq \varepsilon$ on a window of width $h$,

$$\tilde{v}(x,u) = v - \alpha(u_0,x)\,u = \log\ \kappa(x) + O(\varepsilon h),$$

and the empirical collapse statistic obeys

$$R^2_{\text{collapse}} = O\left((\varepsilon h)^2\right).$$

**Practice.** Choose $h$ so that $\varepsilon h \ll 1$; otherwise, reduce the bin or model the drift explicitly.

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### 3.5 Morphisms (reparametrizations) and gauge

Let $\Phi = (\varphi, \psi)$ map $(X_A, L_A, v_A) \to (X_B, L_B, v_B)$, where $\varphi: X_A \to X_B$ reparametrizes environment and $\psi: X_B \to \mathbb{R}$ is a clock change. Then

$$\Phi^* \omega_B \ = \ \omega_A \ + \ d(\psi \circ \varphi).$$

Interpretation: transporting the structure from $B$ to $A$ preserves **slope** and alters only the **clock** by an exact form. This formalizes cross-lab/device comparisons when units/baselines differ.

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### 3.6 Diagnostics and acceptance gates (QC checklist)

1. **Collapse test.** Fit $\hat{\alpha}$ (Section 5), compute residuals $\tilde{y} = y - \hat{\alpha}x$; require $R^2_{\text{collapse}} < 0.05$ **and** no trend in a nonparametric smooth.
2. **Clock placebo.** Multiply all $T$ by a constant; $\hat{\alpha}$ and $R^2_{\text{collapse}}$ must be unchanged.
3. **Changepoints.** Run detectors on $(x, y)$ and on $\tilde{y}$; any kink $\Rightarrow$ rebin.
4. **Window control.** Ensure $|\ \partial_u \alpha\ |\ \ h$is small (adiabatic regime).
5. **Publish/withhold.** Only bins passing 1–4 contribute to $\mathrm{ECI}_{\mathrm{QC}}(\mathrm{t})$; otherwise label `NO_COLLAPSE` or `REGIME_MIX`.

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### 3.7 What this buys us operationally

- A **proof-obligation**: show flatness/exactness (collapse) before trusting a slope.
- A **debugger**: nonzero holonomy localizes seams (scheduler swaps, readout changes).
- A **tuning rule**: reduce $h$ or rebin until $d\omega \approx 0$; if impossible, the domain is **non-power—**treat $\alpha$ as undefined there.

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### 3.8 Summary

The scale–clock geometry makes two RTM statements precise for QC:

1. $\alpha$ **is a gauge-invariant structural quantity**, unaffected by unit/baseline changes;
2. **Collapse equals exactness/flatness of** $\omega$, and its failure is informative (curvature or seams).
   We will now leverage this to define **operational** $(L, T)$ (Sec. 4) and to estimate $\hat{\alpha}$ robustly under measurement error (Sec. 5).

# 4. Operational $(L, T)$ Definitions and Binning Protocol

This section turns RTM into **measurable practice** for quantum stacks. We define layer-specific $(L, T)$ pairs, specify **sampling**, **units**, and **guards**, and give a binning protocol that avoids regime mixing. Throughout, $u = \log\ L, v = \log\ T$.

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### 4.1 Design principles for $(L, T)$

- **One mechanism per family.** Each $(L, T)$ pair should reflect a single dominant mechanism (e.g., readout pipeline, not a mixture of readout + routing).
- **Monotone** $L$**.** $L$ should increase with "problem size" at that layer (width, distance, channels, cluster size).
- **Clock independence.** Within a bin, all **constant** overheads in $T$ are allowed (gauges). Anything **scale-dependent** in $T$ must be modeled or excluded.
- **Steady sampling.** Use **fixed cadence** collection; record raw timestamps to allow reslicing.

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### 4.2 Physical layer

**Candidates for** $L$**:**

- $L =$number of **active qubits** in the workload window;
- $L =$**cluster size** (connected qubits participating simultaneously);
- $L =$**coupler degree** (average fanout).

**Candidates for** $T$**:**

- **Stable calibration interval** (time until any qubit in the cluster exits tolerance);

- **Gate latency** (median single/two-qubit gate duration across the active set);
- **Readout latency** (median per-shot time to valid symbol under fixed thresholds);
- **Mean time to drift** (MTTD) for frequency/phase.

**Instrumentation.**

- Log per-shot timestamps; a calibration watchdog recording when thresholds are breached; attach environment tags: temperature band, firmware hash, bias point.

**Non-examples.**

- Mixing *both* gate latency and readout latency in the same $T$.
- Letting $L$ be "qubits defined on chip" (not necessarily active).

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### 4.3 Error Correction (QEC)

$L$**:** code **distance** $d$ (primary), or number of **logical qubits** at fixed $d$.
$T$**:**

- **Cycles to logical failure** at a fixed target error (median or survival quantile);
- **Syndrome-cycle latency** (mean time per cycle under fixed schedule).

**Scheduling notes.**

- Freeze a **syndrome schedule** (FPGA image + cadence). Any change ⇒ new bin.
- Record bias (X/Z) and leakage mitigation settings.

**Edge cases.**

- If $T$ is dominated by **rare catastrophic events** (e.g., resonator latch-ups), prefer **conditional medians** (exclude known catastrophic flags) and report a sensitivity panel.

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### 4.4 Compiler / Runtime

$L$**:** circuit **width** (max concurrent qubits) or **post-mapping depth**; optionally **active layers** after routing.
$T$**:**

- **Makespan** (submission → completion);
- **Queueing delay** (submission → start);

- **Rescheduling latency** after a calibration event.

**Controls.**

- Fix **routing policy** and **placement heuristic** inside a bin.
- Stratify by utilization band (e.g., 0–30%, 30–60%, >60%). If utilization drifts, split the bin.

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### 4.5 I/O – Cryo / Readout

$L$: **multiplexing degree** (channels per line) or number of concurrent readout channels.
$T$:

- **Readout latency** (median p50 and tail p95);
- **BER recovery half-life** after a controlled burst;
- **Queue p95** expressed in time.

**Instrumentation.**

- Timestamp every DMA/ADC burst; log per-channel buffers; annotate firmware versions of DSP.

**Caveat.**

- Per-channel overheads that **grow with** $L$ are *not* gauges; they are genuine scale effects—permissible for RTM—but if the overhead itself changes mid-bin, collapse should fail and trigger a split.

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### 4.6 Binning protocol (environment fixing)

A **bin** is a maximal interval where the environment is effectively constant.

**Bin key (example):**

$$\mathrm{BIN} = \{\text{platform, temperature band, firmware hash, topology ID, routing policy, syndrome cadence, utilization band}\}.$$

**Procedure.**

1. **Slice** data by BIN; discard slices with $<N_{\min}$ distinct $L$ values (default 6).
2. **Changepoint scan** on $y = \log\ T$ vs. $x = \log\ L$ (and on residuals if available). If a changepoint is detected (BIC/AIC/PELT), **split**.

3. **Windowing**: for slowly drifting regimes, use sliding windows in $x$ of width $h$ such that $|\ \partial_u \alpha\ |\ h \ll 1$ (from Sec. 3.4).

4. **Clock placebo**: multiply $T$ by a constant; the slope $\hat{\alpha}$ must not change.

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### 4.7 Estimation-ready dataset

Create a tidy table per bin with columns:

$$x = \log \mathrm{L}, \mathrm{y} = \log\ T, \text{family, BIN tags, replicate ID, timestamp, weights ].}$$

- **Replicates.** If multiple runs at same $L$, aggregate to robust summaries (median $y$, MAD-based SE) or pass all and let ODR handle them with replicate weights.
- **Weights.** Prefer inverse-variance weights from bootstrap over simple counts.
- **Outliers.** Tag catastrophic events (hardware flags); report both **with** and **without** them.

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### 4.8 Acceptance gates (per bin, per family)

A family contributes a slope $\hat{\alpha}_f$ **only if** all hold:

1. **Coverage:** at least 6 distinct $L$ points and span $\geq 0.6$ in $\log L$.
2. **Collapse:** regress $\tilde{y} = y - \hat{\alpha}x$ on $x$; require $R^2_{\text{collapse}} < 0.05$ and no visible trend (smooth check).
3. **Clock placebo:** $\hat{\alpha}$ unchanged under $T \mapsto cT$.
4. **Changepoints:** none within bin (else split and re-estimate).
5. **EIV fit quality:** ODR/TLS converged; residual diagnostics acceptable (no single leverage point dominates).

Bins or families failing any gate are flagged (`NO_COLLAPSE`, `REGIME_MIX`, `THIN_COVERAGE`, `EIV_FAIL`) and **excluded from fusion**.

---

### 4.9 Examples vs. non-examples (QC-flavored)

- **Good physical family:** $L =$ active-qubit cluster size; $T =$ stable calibration interval. Single firmware, stable temperature, no routing change. Collapses cleanly → accept.

- **Bad physical family:** Same, but mid-bin the PLL loop parameters change. Changepoint triggers; split required.
- **Good QEC family:** $L = d, T =$ cycles to logical failure, fixed syndrome cadence. Residuals flat → accept.
- **Bad QEC family:** Mix of two cadences (fast and slow) inside one bin → kink in log–log → reject until split.
- **Good I/O family:** $L =$ multiplexing degree; $T =$ readout latency p95. Firmware constant; latency rises as $L^{\alpha}$, collapse holds → accept.
- **Bad I/O family:** Switch of DSP firmware that changes per-channel overhead nonlinearly mid-bin → curvature; reject or rebin around the switch.

---

### 4.10 Summary

- We fixed **operational** $(L, T)$ per layer and specified **instrumentation** to make them measurable.
- We defined a **binning protocol** that enforces environment constancy and guards against regime mixing.
- We set **acceptance gates** (coverage, collapse, placebo, changepoints, EIV fit) that determine whether a family's slope enters downstream fusion ($\mathrm{ECI}_{\mathrm{QC}}(\mathrm{t})$).

## 5. Estimation Under Errors-in-Variables (EIV) and Collapse Thresholds

We now specify **how** to estimate the binwise slope $\alpha$ robustly when both axes are noisy, and how to decide—via a **collapse threshold**—whether a family's data are RTM-consistent. Throughout, $x = \log\ L$, $y = \log\ T$. Observations are $x^{obs} = x + \xi$, $y^{obs} = y + \zeta$ with mean-zero errors.

---

### 5.1 Estimation targets and models

Inside a **fixed bin**, the target is the **local slope** $\alpha$ in

$$y = \alpha x + c + r(x),$$

with $r \equiv 0$ under exact RTM or $| r'(x) | \leq \varepsilon$ under slow drift on a window. Because $x$ is noisy, **OLS is attenuated**; we use EIV-aware estimators.

**Default target:** point slope $\alpha$ for the bin; intercept $c$ is a **gauge** (not compared across bins).

---

### 5.2 Orthogonal Distance Regression (Total Least Squares)

**Definition.** ODR minimizes orthogonal residuals to a line:

$$\min_{\alpha,c} \sum_i \frac{(y_i^{obs} - \alpha x_i^{obs} - c)^2}{\sigma_y^2 + \alpha^2 \sigma_x^2}$$

with effective (possibly heterogeneous) $(\sigma_x, \sigma_y)$ from replicate variance or bootstrap.

**Practice.**

- Initialize by Theil–Sen (Sec. 5.4) to avoid poor local minima.
- Use **cluster/bootstrap** (replicate or job-level) for CIs.
- If per-point SEs are available, weight them; else use robust Huber weights on orthogonal residuals.

**Convergence gates.**

- Condition number of the centered covariance matrix $< 10^4$.
- Jackknife leverage check: no single point contributes $> 25\%$ of slope influence.

---

### 5.3 SIMEX (when Var($\xi$)is known/estimated)

If you can estimate $\sigma_\xi^2 = \mathrm{Var}(\xi)$ (e.g., repeated $L$ at the same setting), apply **SIMEX**:

1. For $\lambda \in \Lambda = \{0.5,1.0,1.5,2.0\}$, generate pseudo-samples $x_i^{(\lambda)} = x_i^{obs} + \sqrt{\lambda}\,\tilde{\xi}_i, \tilde{\xi}_i \sim \mathcal{N}(0, \sigma_\xi^2)$.
2. Fit a naive slope $\hat{\alpha}(\lambda)$ by ODR or OLS.
3. Fit a quadratic $\hat{\alpha}(\lambda) = a + b\lambda + c\lambda^2$ and **extrapolate to** $\lambda = -1$: $\hat{\alpha}_{\mathrm{SIMEX}} = a - b + c$.

**Use.** Prefer ODR as the base fitting routine; report SIMEX as a **sensitivity** estimate next to ODR. If $\sigma_\xi^2$ is uncertain, give a band (low/med/high) for $\hat{\alpha}_{\mathrm{SIMEX}}$.

---

### 5.4 Theil–Sen (robust median slope)

The **Theil–Sen** slope is the median of all pairwise slopes

$$\alpha_{ij} = \frac{y_j^{obs} - y_i^{obs}}{x_j^{obs} - x_i^{obs}} (i < j),$$

with a robust intercept from the median of $y_i^{obs} - \hat{\alpha} x_i^{obs}$.

**Role.**

- Initialization for ODR.
- **Outlier-robust** cross-check reported alongside ODR.
- When EIV is severe and $\sigma_\xi^2$ is unknown, Theil–Sen may still be stable (expect mild attenuation).

---

### 5.5 Windowing and finite-window bias

If slow drift is suspected, estimate slopes on **symmetric windows** in $x$ of width $h$. From the adiabatic bias bound,

$$\hat{\alpha}(u; h) = \alpha(u) + O(\varepsilon h),$$

choose $h$ so that $\varepsilon h \ll 1$. Practically: start with $h \approx 0.8$ in $\log L$ span if coverage allows; shrink until collapse passes (Sec. 5.7) without exploding variance.

---

### 5.6 Uncertainty and diagnostics

- **Bootstrap** (pairs within bin or block/cluster if natural replicates exist) for 50/95% CIs.
- **Jackknife-after-bootstrap** to detect leverage points.
- **Residual plots**: orthogonal residual vs. $x$; LOESS smooth must be flat within bands.
- **EIV adequacy**: if OLS and ODR differ by ≥0.2 absolute slope **and** ODR CI excludes OLS, report EIV as material.

---

### 5.7 Collapse threshold (specification gate)

Given $\hat{\alpha}$, compute residuals $\tilde{y}_i = y_i^{obs} - \hat{\alpha} x_i^{obs} - \hat{c}$ and regress $\tilde{y}$ on $x$ (with the same weights used in estimation). Define

$$R^2_{\text{collapse}} := R^2(\tilde{y} \sim x).$$

**Decision rule (default):**

- Accept the bin if **all** hold:
    1. $R^2_{\text{collapse}} < 0.05$ (or the 95% CI of the slope in $\tilde{y} \sim x$ contains 0),
    2. LOESS smooth shows no trend,
    3. **Clock placebo**: scaling $T \mapsto cT$ leaves $\hat{\alpha}$ and $R^2_{\text{collapse}}$ unchanged,
    4. Changepoint scan (PELT/BIC) finds none inside the bin.
- Otherwise flag (`NO_COLLAPSE` or `REGIME_MIX`) and **do not** publish a slope or include it in fusion.

---

### 5.8 Coverage and leverage gates

To avoid brittle fits:

- **Distinct $L$points** $\geq 6$ and $\log L$ span $\geq 0.6$.
- **Balanced leverage:** the largest leverage point contributes $\leq 25\%$ of the ODR slope influence.
- **Replicates:** if $> 3$ replicates per $L$, either summarize to a robust mean/SE or pass replicate weights to ODR.

Bins failing these gates are flagged `THIN_COVERAGE` or `LEVERAGE_RISK`.

---

### 5.9 Putting it together (per-bin algorithm)

1. **Prep:** build the tidy table (Sec. 4.7); run changepoint scan; window if needed.
2. **Init:** compute Theil–Sen slope/intercept; remove obvious catastrophics (keep both versions for sensitivity).
3. **Fit ODR/TLS:** weighted by replicate SEs; obtain $\hat{\alpha}$, $\hat{c}$, bootstrap CIs.
4. **SIMEX (optional):** if $\sigma_\xi^2$ is available, compute $\hat{\alpha}_{\text{SIMEX}}$.

5. **Collapse gate:** compute $R^2_{\text{collapse}}$, smooth check, placebo clock.
6. **Decision:** if all gates pass, **accept** $\hat{\alpha}$ with uncertainty; else **reject/split**.
7. **Report:** slope, CI, diagnostics (collapse $R^2$, leverage plot, changepoints). Store flags.

---

### 5.10 What we publish per accepted family

- $\hat{\alpha}_f$ $\pm$50/95% CI (ODR); Theil–Sen as robustness; SIMEX band if applicable.
- Collapse diagnostics: $R^2_{\text{collapse}}$, placebo check, window width $h$.
- Coverage: # distinct $L$, $\log L$ span, leverage summary.
- Notes: any exclusions (catastrophics), changepoint status.

Only accepted families enter **fusion** (Sec. 6). If $\geq 2$ families pass, we apply random-effects with $Q$, $I^2$ and heterogeneity gates; otherwise we report family-wise slopes without fusion.

---

### 5.11 Summary

- Use **ODR/TLS** as the primary EIV estimator; **Theil–Sen** for robust init/check; **SIMEX** when $\sigma_\xi^2$ is estimable.
- Enforce **collapse** as a **specification test** ($R^2_{\text{collapse}} < 0.05$ + placebo + no changepoints).
- Control **finite-window bias** by choosing $h$ small enough (adiabatic regime) and splitting bins when needed.
- Publish complete **diagnostics** and **flags**; only clean families proceed to fusion and to the real-time $\mathrm{ECI}_{\mathrm{QC}}(\mathrm{t})$.

## 6. Building the Real-Time Indicator $\mathbf{ECI_{QC}}(\boldsymbol{t})$

We now construct a **single, real-time** coherence indicator for a platform by fusing the **accepted** family-wise slopes $\{\hat{\alpha}_{f,t}\}$ from Section 5. The fusion is **random-effects** (to acknowledge between-family heterogeneity), runs on a rolling clock, and drives **QA gates** and **decoherence alerts**.

---

### 6.1 Inputs and preconditions (per time $t$)

For each family $f \in \mathcal{F}_t$ (Physical, QEC, Compiler/Runtime, I/O–Cryo):

- A binwise estimate $\hat{\alpha}_{f,t}$ with variance $\hat{\sigma}_{f,t}^2$ (bootstrap or replicate-weighted),
- Collapse passed (Section 5.7), coverage/leverage gates satisfied (Section 5.8),
- Environment tags (BIN) unchanged within the window that produced $\hat{\alpha}_{f,t}$.

A fusion at time $t$proceeds **only if** $|\mathcal{F}_t| \geq 2$.

---

### 6.2 Random-effects fusion

We estimate the between-family variance $\hat{\tau}_t^2$ (default **REML**; DerSimonian–Laird as sensitivity). Define weights

$$w_{f,t} = \frac{1}{\hat{\sigma}_{f,t}^2 + \hat{\tau}_t^2}.$$

Then the fused slope and its variance are

$$\hat{\alpha}_{\mathrm{QC}}(t) = \frac{\sum_{f \in \mathcal{F}_t} w_{f,t}\, \hat{\alpha}_{f,t}}{\sum_{f \in \mathcal{F}_t} w_{f,t}}, \qquad \mathrm{Var}\left(\hat{\alpha}_{\mathrm{QC}}(t)\right) = \frac{1}{\sum_{f \in \mathcal{F}_t} w_{f,t}}.$$

Report 50% and 95% intervals via normal approximation or by a **bootstrap-over-families** (resample families with replacement, recompute $\hat{\tau}_t^2$ and the fused mean).

---

### 6.3 Heterogeneity diagnostics and gates

Compute the fixed-effect baseline

$$w_{f,t}^{FE} = \frac{1}{\hat{\sigma}_{f,t}^2}, \qquad \hat{\alpha}_{FE}(t) = \frac{\sum_f w_{f,t}^{FE}\, \hat{\alpha}_{f,t}}{\sum_f w_{f,t}^{FE}}.$$

**Cochran's $Q$ and $I^2$:**

$$Q_t = \sum_f w_{f,t}^{FE} \, (\hat{\alpha}_{f,t} - \hat{\alpha}_{FE}(t))^2, \qquad I_t^2 = \max \{0, \frac{Q_t - (\mid \mathcal{F}_t \mid -1)}{Q_t}\} \times 100\%.$$

**Fusion gates (pre-registered):**

- Proceed with a single number **only if**
  (i) $\mid \mathcal{F}_t \mid \geq 2$,
  (ii) $I_t^2 < 50\%$ *(moderate or lower heterogeneity)*, and
  (iii) REML converges with finite $\hat{\tau}_t^2$ not exceeding a historical cap (e.g., ≤ 90th percentile over past clean windows).
- If any fails, **withhold fusion** and publish family-wise $\hat{\alpha}_{f,t}$+ diagnostics; flag FAMILY_DIVERGENCE.

---

### 6.4 Real-time operation (rolling windows)

- **Cadence.** Recompute each family's $\hat{\alpha}_{f,t}$ on a **rolling window** in $x = \log\ L$ of width $h$ (chosen by the adiabatic rule; Sec. 5.5) and a **wall-clock horizon** (e.g., last 7–28 days of data).
- **Backfill and missingness.** If a family is missing at $t$, fuse over the available $\mathcal{F}_t$ provided $\mid \mathcal{F}_t \mid \geq 2$; otherwise **suspend** $\mathrm{ECI}_{\mathrm{QC}}(t)$ and publish a THIN_FAMILIES flag.
- **Clock placebo.** Once per day, multiply all contributing $T$ by a constant and verify $\hat{\alpha}_{\mathrm{QC}}(t)$ and $I_t^2$ are unchanged (stored as a QA artifact).

---

### 6.5 Decoherence events (alerting logic)

We define a **decoherence event** as a significant, QA-clean **drop** in $\mathrm{ECI}_{\mathrm{QC}}(t)$, robust to smoothing and not explained by heterogeneity spikes.

**Filters:**

1. **Smoothing:** maintain a 3-point median $\tilde{\alpha}(t)$ of $\hat{\alpha}_{\mathrm{QC}}(t)$.
2. **Z-score:** $Z(t) = \frac{\tilde{\alpha}(t) - \mathrm{EWMA}_{30}[\tilde{\alpha}]}{\hat{\sigma}_{EWMA}(t)}$.

**Alert tiers (default):**

- **Advisory:** $Z(t) \leq -1.5$ for ≥2 consecutive ticks **and** $I_t^2 < 50\%$.
- **Watch:** $Z(t) \leq -2.0$ once **or** persistent $Z(t) \leq -1.5$ for ≥4 ticks, $I_t^2 < 40\%$.

- **Warning:** $Z(t) \le -2.5$ and a coincident family-wise drop (≥2 families with $Z_f \le -2$).

**Playbooks triggered:** throttle scheduling (reduce concurrency/multiplexing), run segmented recalibration, or switch to RTM-aware routing until $\tilde{\alpha}(t)$ normalizes.

---

### 6.6 Reporting and visualization

- **Primary panel:** $\hat{\alpha}_{\mathrm{QC}}(t)$ with 50/95% bands, heterogeneity ribbon colored by $I_t^2$ (green <25%, amber 25–50%, red ≥50%).
- **Forest plot:** per-family $\hat{\alpha}_{f,t}$, weights $w_{f,t}$, and CIs; show $Q_t$, $I_t^2$, $\hat{\tau}_t^2$.
- **Collapse dashboard:** per family, show $R^2_{\mathrm{collapse}}$, LOESS residuals, window width $h$, coverage and leverage metrics.
- **Flags legend:** `NO_COLLAPSE`, `REGIME_MIX`, `LEVERAGE_RISK`, `THIN_COVERAGE`, `FAMILY_DIVERGENCE`, `THIN_FAMILIES`.

---

### 6.7 Sensitivity and ablation

- Publish the **fixed-effect** summary $\hat{\alpha}_{FE}(t)$ alongside random-effects.
- Report DL-based $\hat{\tau}^2_{DL}$ as a sensitivity.
- **Leave-one-family-out**: recompute $\hat{\alpha}_{\mathrm{QC}}^{(-f)}(t)$ to expose dominance.
- **Clock placebos** and **shuffle nulls** (shuffle $L$ within family) must not produce tiered alerts; if they do, review gates.

---

### 6.8 Governance and provenance

Every fused point stores:

- Source families and their BIN tags,
- Estimator settings (ODR init, bootstrap seeds, $h$),
- Collapse metrics, $Q_t$, $I_t^2$, $\hat{\tau}_t^2$,
- Placebo outcome hashes,
- Versioned code/config (methods YAML).

This ensures **reproducibility** and enables post-mortems when alerts fire.

---

### 6.9 Summary

$\mathrm{ECI}_{\mathrm{QC}}(t)$ is a **random-effects fusion** of QA-clean, binwise slopes. Heterogeneity gates ($I_t^2 <$ 50%, $|\mathcal{F}_t| \geq 2$) prevent misleading single numbers when proxies disagree. Real-time smoothing and Z-scores turn slope dynamics into **actionable alerts** for **decoherence events**, while dashboards and provenance keep the system auditable.

## 7. RTM-Aware Design: Engineering $\alpha$without Sacrificing Throughput

This section turns RTM into **design levers**. Goal: increase the **coherence exponent** $\alpha$ (stronger tempo stratification across scale) while keeping or improving throughput. We give layer-specific controls, optimization targets, and guardrails.

---

### 7.1 Design objective and guardrails

We treat $\alpha$ as an **operational objective** within a bin:

$$\max_{\text{controls }\theta} \ \alpha(\theta) \ \ s.t. \ \ throughput \geq B, \ fidelity \geq F, \quad collapse\ passes.$$

- **Controls** $\theta$: scheduler parameters, QEC cadence/jitter, routing constraints, multiplexing limits, module sizes.
- **Constraints**: a throughput floor $\mathcal{B}$ (e.g., jobs/hour), fidelity floor $\mathcal{F}$, and **collapse gates** (Sec. 5.7).
- **Monitor**: track per-family $\hat{\alpha}_f$ and the fused $\hat{\alpha}_{\mathrm{QC}}(t)$ with QA (Sec. 6).

---

### 7.2 Scheduler: batching & variance-aware routing

**Problem.** Long, tightly coupled operations launched in parallel **flatten** $\alpha$ (fast cascades across scale).

**Controls.**

1. **Wavefront batching (readout & long ops).** Partition time into short waves; pack readouts into waves instead of free-running concurrency.

2. **Staggered resets.** Add small offsets $\delta \in [-\epsilon, \epsilon]$ to reset times to avoid synch peaks.
3. **Low-variance routing.** Prefer routes with **low path-time variance** even if path length increases slightly.

**Objective.** For a job DAG with ops $o$having nominal durations $\tau_o$ and routes $p(o)$:

$$\min_{\text{schedule}, p(\cdot)} \quad \underbrace{\text{Var}_{\,t}[N_{\text{long}}(t)]}_{\text{desynchronize heavy ops}} \quad + \ \lambda \underbrace{\sum_{o \in \mathcal{O}} \text{Var}\,(T_{\text{route}}(p(o)))}_{\text{low-variance routing}}.$$

subject to makespan budget. This reduces temporal “pile-ups,” lifting $\alpha$.

**Heuristic (greedy, practical).**

- Sort ops by duration desc; assign start times into **waves** so that each wave’s total long-op load is balanced.
- For each route candidate, penalize time-variance and crosstalk score; pick minimum penalized cost.

---

### 7.3 QEC cadence: avoid phase-lock (jitter/desynchronization)

**Problem.** A fixed syndrome cadence can **phase-lock** with physical noise rhythms, creating cross-layer synchronization → $\alpha_{\text{QEC}}$ falls.

**Controls.**

- **Micro-jitter** the cycle period: $P_k = P\,(1 + \eta_k)$ with $\eta_k \sim \mathcal{U}[-\rho, \rho]$, $\rho \ll 1$ (e.g., 1–3%).
- **Multi-phase extraction:** split the code into sublattices whose cycles are offset by small phases $\phi_j$.

**Design rule.** Choose $\rho$ so that the **main lobe** of the syndrome cycle’s line spectrum moves **off** strong peaks of the error PSD while keeping decoder timing valid. Validate by: (i) increased $\hat{\alpha}_{\text{QEC}}$ vs. $d$, (ii) stable logical error at fixed $d$.

---

### 7.4 Gradients and wells of $\alpha$

Two architectural motifs to **steer flows**:

- **Gradient:** arrange resources so $\alpha$ **increases** towards critical compute regions. Small disturbances decay as they travel inward.
- **Well:** create a **high-$\alpha$ basin** around sensitive qubits (e.g., clocking and buffering that slow large-scale cascades).

**Implementation cues.** Increase temporal buffering (queues, damped scheduling) and reduce crosstalk fanout as you approach the "core," but cap buffering (Sec. 7.1 guardrails) so throughput doesn't suffer.

---

### 7.5 Modular sizing: pick a sweet spot by balancing intra vs. inter latency

Let total qubits $Q$ be partitioned into $Q/m$ modules of size $m$. Approximate **characteristic time**:

$$T(m) \;=\; A\,m^{a} \;+\; B\,\Big(\frac{Q}{m}\Big)^{b} \quad \text{(intra-module cost + interconnect cost).}$$

**Optimal module size** (minimizes $T$):

$$m^{\star} \;=\; \Big(\frac{B\,b}{A\,a}\Big)^{\frac{1}{a+b}}\;Q^{\frac{b}{a+b}}.$$

- $a > 0$: intra-module scaling (e.g., calibration, routing within module).
- $b > 0$: inter-module scaling (e.g., photonic/ion link latency).

**Design use.** Measure $a, b$ empirically (RTM per mechanism), estimate $A, B$, compute $m^{\star}$. Operate near $m^{\star}$ and verify that $\hat{\alpha}$ **does not collapse** (still power-like) in that neighborhood.

---

### 7.6 Multiplexing & I/O: hold tails in check

**Problem.** Aggressive multiplexing reduces per-shot time, but can synchronize queue tails $\rightarrow \alpha_{\mathrm{IO}} \downarrow$.

**Controls.**

- Cap multiplexing such that the **tail ratio** $p95/p50$ of readout latency stays below a threshold (e.g., $\leq 1.6$).
- Use **phase-offset readout windows** across channels to avoid coherent tail growth.

- Buffer sizing: maintain buffer utilization < 70% to avoid tail amplification.

**Signal.** If $p95/p50$ grows and $\hat{\alpha}_{\mathrm{IO}}$ drops with clean collapse, back off multiplexing and introduce offsets.

---

### 7.7 Online control loop (closed-loop $\alpha$ engineering)

A simple controller to keep $\alpha$ high under constraints:

```
every Δt:
  estimate {α_f(t), σ_f(t)} per accepted family (Sec. 5)
  if |F_t| ≥ 2 and I^2_t < 50%:
      compute α_QC(t) (Sec. 6)
  if α_QC(t) < α_floor and constraints met:
      apply actions A = {↑wave size, ↑reset jitter ρ, ↑routing penalty on variance,
                         ↓multiplex cap, move toward m*}
  else if throughput < B:
      relax A minimally (keep collapse passing)
  log QA: collapse R^2, I^2_t, flags; revert actions if flags trip
```

- $\alpha_{\mathrm{floor}}$: pre-registered minimal acceptable fused slope.
- **Revert** any action that causes `NO_COLLAPSE` or $I_t^2 \geq 50\%$.

---

### 7.8 Safety and validation

- Any intervention must **re-pass collapse** in the affected families.
- Run A/B windows (≥2–4 weeks) with **pre-registered** KPIs: throughput, makespan, logical error, uptime, $p95/p50$, and $\hat{\alpha}_f$.
- If $\alpha$rises but KPIs worsen beyond budgets, you are **over-layering** (too much buffering). Roll back to the Pareto frontier.

---

### 7.9 Quick-start playbooks

- **If $\alpha_{\mathrm{QEC}}$ ↓:** add 1–3% cadence jitter; introduce 2–3 phase groups for syndrome; re-measure collapse.

- **If $\alpha_{\mathrm{IO}}$ ↓:** reduce multiplex cap 10–20%; add 1–2 cycle offsets; keep $p95/p50 \leq 1.6$.
- **If $\alpha_{\mathrm{runtime}}$ ↓:** enable readout batching; penalize high-variance routes; cap concurrent long ops per wave.
- **Architectural planning:** estimate $a, b, A, B$ and set module size near $m^{\star}$; confirm power-like scaling around that point.

---

### 7.10 Summary

- **Scheduler** (waves, staggered resets, low-variance routing) and **QEC cadence** (micro-jitter, multi-phase) are first-line levers to **raise** $\alpha$.
- **Modular sizing** admits a closed-form optimum $m^{\star}$ balancing intra/inter costs; operate near it while watching collapse.
- **I/O controls** keep latency tails from synchronizing.
- A **closed-loop controller** maintains $\alpha$ above a floor under throughput/fidelity budgets.

## 8. Falsifiable Experimental Protocols (Superconducting & Trapped-Ion)

This section specifies **testable** RTM-QC experiments with concrete $(L, T)$ choices, data collection, analysis plans, and success criteria. Each protocol is binwise (fixed environment) and includes **placebos**, **changepoint guards**, and a **pre-registered** decision table.

---

### 8.1 Common scaffolding (applies to all protocols)

**BIN (environment) lock.**
{platform; temperature band; firmware hash (FPGA/DSP); topology ID; routing policy; syndrome cadence; utilization band}. Any change ⇒ new bin.

**Data schema (tidy).** For each record:

$$\mathrm{x} = \log \mathrm{L}, \mathrm{y} = \log\ T, \text{family, BIN tags, replicate ID, timestamp, weights]}$$

**QA gates (must pass):**

- Coverage: ≥6 distinct $L$, span ≥0.6 in $\log L$.

- EIV fit converged (ODR), leverage <25%, robust init (Theil–Sen).
- Collapse: $R^2_{\text{collapse}} < 0.05$, no LOESS trend, clock placebo holds.
- Changepoints: none inside bin (else split).

**Outcomes (primary, per family):**

- Slope $\hat{\alpha}_f$ with 50/95% CI; collapse diagnostics.
- For fused results, $\hat{\alpha}_{\text{QC}}(t)$, $Q$, $I^2$, $\hat{\tau}^2$ (Sec. 6).

**Statistical plan.**
Bootstrap CIs (pairs/cluster). Predefine **minimal detectable effect** (MDE) on $\alpha$ (e.g., $\Delta\alpha = 0.15$) and **operational KPIs** (throughput, makespan, logical error rate, uptime, p95/p50). Thresholds below.

---

### 8.2 Protocol A — Physical layer (Superconducting)

**Hypothesis (H1-Phys).** Increasing **cluster desynchronization** (staggered resets + readout waves) **raises** $\alpha_{\text{phys}}$ without exceeding throughput budget.

**Design.**

- $L$: active-qubit **cluster size** (simultaneously engaged).
- $T$: **stable calibration interval** (time to first out-of-tolerance flag in cluster).
- Arms: **Control** (baseline scheduler) vs. **RTM-aware** (readout batching + staggered resets, ±2–4% offsets).
- Duration: 2–4 weeks; interleave arms daily to balance drift.

**Analysis.**

- Fit ODR per arm, pass collapse.
- Primary effect: $\Delta\hat{\alpha}_{\text{phys}} = \hat{\alpha}_{\text{RTM}} - \hat{\alpha}_{\text{CTRL}}$.
- KPI guardrails: throughput drop ≤5%, no increase in gate/RO error >0.2σ.

**Success criteria.**

- $\Delta\hat{\alpha}_{\text{phys}} \geq 0.15$ and CI excludes 0, **and** guardrails satisfied.
- If collapse fails in any arm, declare **inconclusive** and rebin.

**Placebos.** Multiply $T$ by a constant; $\hat{\alpha}$ unchanged. Shuffle $L$ within day; no significant slope.

---

### 8.3 Protocol B — QEC cadence (Superconducting or Ions)

**Hypothesis (H1-QEC).** Introducing **micro-jitter** (1–3%) in syndrome period and/or **multi-phase extraction** increases $\alpha_{\mathrm{QEC}}$ vs. code distance $d$at fixed decoder.

**Design.**

- $L$: **code distance** $d$ (e.g., $d \in \{3,5,7,9\}$).
- $T$: **cycles to logical failure** (median or survival quantile at fixed target error).
- Arms: Control (fixed period $P$) vs. Jitter ($P_k = P(1+\eta_k), \eta_k \sim \mathcal{U}[-0.02,0.02]$) and/or 2–3 **phase groups**.
- Keep decoder parameters fixed; no change in noise bias mitigation.

**Analysis.**

- ODR per arm; collapse gate.
- Effect: $\Delta\hat{\alpha}_{\mathrm{QEC}}$.
- KPI guardrails: logical error at fixed $d$ not worse by >5% relative.

**Success criteria.**

- $\Delta\hat{\alpha}_{\mathrm{QEC}} \geq 0.15$ with CI excluding 0 and guardrails pass.

**Diagnostics.** Check PSD of error processes; confirm jitter moves cadence lines off dominant peaks.

---

### 8.4 Protocol C — compiler/runtime scheduling

**Hypothesis (H2-Run). Wavefront batching** of readout and **low-variance routing** reduce synchronization cascades, increasing $\alpha_{\mathrm{runtime}}$ and lowering makespan tails.

**Design.**

- $L$: **post-mapping circuit width** (or active layers).
- $T$: **makespan** (submit→complete).
- Arms: Baseline policy vs. RTM-aware (waves + variance-penalized routing).
- Control utilization band; same job mix across arms.

**Analysis.**

- ODR slope per arm; collapse.
- KPIs: median makespan (≤ baseline), p95/p50 latency ↓ ≥10%.

**Success criteria.**

- $\Delta\hat{\alpha}_{\text{runtime}} \geq 0.10$(CI excludes 0) and p95/p50 improves ≥10%.

---

### 8.5 Protocol D — I/O–Cryo multiplexing

**Hypothesis (H2-IO). Phase-offset readout windows** across channels maintain or raise $\alpha_{\text{IO}}$ while reducing p95 tails at a given multiplexing degree.

**Design.**

- $L$: **multiplexing degree** (channels/line).
- $T$: **readout latency p95** (and p50).
- Arms: Synchronous windows vs. offset windows (phase pattern $\phi_j$).
- Sweep $L$ across operational range.

**Analysis & success.**

- $\Delta\hat{\alpha}_{\text{IO}} \geq 0.10$; p95/p50 ≤ 1.6 in RTM arm over majority of $L$; collapse passes.

---

### 8.6 Protocol E — Modular sizing (planning study)

**Hypothesis (H3-Mod).** There exists a module size $m^{\star}$ that minimizes $T(m) = Am^{a} + B(Q/m)^{b}$ with empirically measured $a, b > 0$, and operating near $m^{\star}$ preserves power-like scaling (collapse holds).

**Design.**

- Platforms with photonic/ion links between modules.
- Measure $T(m)$ by varying module size (or emulating interconnect cost) at fixed total $Q$.
- Fit $a, b, A, B$ via ODR on each term's dataset; compute $m^{\star}$.

**Success criteria.**

- Observed $T(m)$ minimized near $m^{\star}$ (within CI), and log–log fits around $m^{\star}$ retain collapse (no curvature).

---

### 8.7 Fusion and alerting (cross-protocol)

Across A–D, if ≥2 families pass gates at overlapping times, compute $\hat{\alpha}_{\mathrm{QC}}(t)$ (Sec. 6).
**H2 (anticipation):** declare a **decoherence event** if Z-score tiers (Sec. 6.5) are met; test **lead–lag** vs. spikes in logical error/makespan/queues. Additive predictive value is assessed against baselines (fidelity, utilization, temperature) using time series regression with HAC errors; pre-register horizons (e.g., 7–30–90 days).

---

### 8.8 Placebos, shuffles, and robustness

- **Clock placebos:** multiply all $T$ by constants; $\hat{\alpha}$ and $R^2_{\mathrm{collapse}}$ invariant.
- **Shuffle nulls:** permute $L$ within day; slopes collapse to $\sim 0$ (within CI).
- **Leave-one-family-out** fusion to reveal dominance.
- **Changepoints**: automatic split if detected; re-estimate on both sides.

---

### 8.9 Power and duration (rules of thumb)

- With span ≥0.8 in $\log L$, 8–12 distinct $L$ points, and moderate noise (SNR≈5–10), ODR detects $\Delta\alpha \approx 0.10$–$0.15$ at 95% with ≈200–400 total observations per arm.
- If noise is higher or drift suspected, shrink windows (Sec. 5.5) and extend duration.

---

### 8.10 Decision table (pre-registered)

| Outcome | Action |
|---|---|
| $\Delta\hat{\alpha} \geq$ MDE **and** guardrails pass | Promote intervention to production in that bin; monitor with $\mathrm{ECI}_{\mathrm{QC}}(t)$. |
| $\Delta\hat{\alpha}$ significant but KPI guardrail violated | Tune intensity (e.g., reduce buffering/jitter) and retest. |
| Collapse fails or heterogeneity high ($I^2 \geq$ 50%) | Do not fuse; report family-wise; revisit binning or mechanisms. |
| No effect (($\Delta\hat{\alpha} \approx 0$) | Document as *scope boundary*; keep as negative control. |

### 8.11 Ethics, safety, and reproducibility

- **Safety:** no unsafe RF power increase; jitter bounds keep decoders valid; rollback on NO_COLLAPSE or KPI breach.
- **Reproducibility:** versioned methods YAML (BIN, estimator settings, seeds), public plots (collapse panels, forest plots), and stored placebo/shuffle artifacts.
- **Transparency:** publish both successes and failures (negative results define scope).

---

### 8.12 Summary

These protocols make RTM-QC **falsifiable**: each claims a directional change in $\alpha$ from a specific control, under binwise constancy, with collapse as a specification test and operational guardrails. Success improves not only the slope but also **run-time stability** (tails, recalibrations) without sacrificing throughput.

## 9. Results Templates and Reporting Standards

This section defines **what to publish** once the protocols (Sec. 8) are run. It standardizes figures, tables, robustness panels, and a one-page checklist so that results are interpretable, reproducible, and directly comparable across labs and platforms.

---

### 9.1 Figure set (minimum)

**Fig. 1 — Collapse panels (per accepted family).**
Four small multiples per family $f$ within a bin:

1. **Log–log fit:** $y = \log\ T$ vs. $x = \log\ L$ with ODR line and 95% band.
2. **Residual vs.** $x$**:** $\tilde{y} = y - \hat{\alpha}x - \hat{c}$ with LOESS; show $R^2_{\text{collapse}}$.
3. **Coverage/leverage:** scatter highlighting leverage points; annotate span in $\log L$, # distinct $L$.
4. **Placebo check:** overlay of fits before/after $T \mapsto cT$ (curves coincide).

**Fig. 2 — Forest plot & heterogeneity.**
Per time slice (or per experiment arm), show $\hat{\alpha}_f$ $\pm$CI, weights $w_f$, the fused $\hat{\alpha}_{\text{QC}}$ (diamond), and heterogeneity stats: $Q$, $I^2$, $\hat{\tau}^2$.

**Fig. 3 —**$ECI_{\text{QC}}$**(t) time series.**
Rolling fused slope with 50/95% bands; background ribbon colored by $I^2$ (green <25%,

amber 25–50%, red ≥50%). Mark **decoherence events** (advisory/watch/warning) and platform events (recalibrations, firmware changes).

**Fig. 4 — KPI panel (paired with Fig. 3).**
Aligned time axes for: logical error rate (at fixed $d$), makespan median and p95, queue p95, uptime between recalibrations. Overlay shaded regions for alert tiers from Fig. 3.

**Fig. 5 — A/B outcomes (per protocol).**
For each arm: distribution plots (violin/box) of $\hat{\alpha}_f$, makespan p95/p50, logical error; include $\Delta\hat{\alpha}$ with CI and guardrails.

**Optional Fig. 6 — Spectral diagnostics (QEC).**
PSD of error processes showing how cadence jitter/multi-phase moves line spectra off dominant peaks.

---

### 9.2 Core tables

**Table 1 — Accepted families (per bin/arm).**

| Family | #L pts | $\log L$ span | $\hat{\alpha}_f$ (ODR, 50/95% CI) | Theil–Sen | SIMEX band | ($R^2_{coll}$) | Leverage max | Flags |
|---|---|---|---|---|---|---|---|---|
| Physical | 9 | 1.05 | 0.62 [0.55, 0.70] | 0.60 | 0.58–0.66 | 0.02 | 0.18 | — |
| QEC | 8 | 0.82 | 0.74 [0.66, 0.82] | 0.71 | — | 0.03 | 0.22 | — |
| … | … | … | … | … | … | … | … | … |

**Table 2 — Fusion & heterogeneity (per time slice or arm).**

| Time/Arm | Families | $\hat{\alpha}_{QC}$ ± SE | (Q) (df) | $I^2$ | $\hat{\tau}^2$ | Fusion? |
|---|---|---|---|---|---|---|
| RTM-aware | 3 | 0.69 ± 0.04 | 3.2 (2) | 37% | 0.005 | Yes |
| Control | 3 | 0.54 ± 0.05 | 6.8 (2) | 71% | 0.018 | **No** (report family-wise) |

**Table 3 — Protocol outcomes (A/B).**

| Protocol | Metric | Control | RTM-aware | Effect (Δ) | 95% CI | Pass guardrail? |
|---|---|---|---|---|---|---|
| A (Phys) | $\hat{\alpha}_{phys}$ | 0.48 | 0.64 | +0.16 | [0.07, 0.25] | ✓ |

| Protocol | Metric | Control | RTM-aware | Effect (Δ) | 95% CI | Pass guardrail? |
|---|---|---|---|---|---|---|
| A (Phys) | Throughput | 100% | 97% | –3% | [–6, 0]% | ✓ |
| B (QEC) | $\hat{\alpha}_{QEC}$ | 0.68 | 0.83 | +0.15 | [0.06, 0.24] | ✓ |
| C (Run) | p95/p50 | 1.85 | 1.60 | –0.25 | [–0.35, –0.15] | ✓ |

**Table 4 — Pre-registered thresholds & flags.**

| Gate | Threshold | Status |
|---|---|---|
| Collapse $R^2$ | < 0.05 | Pass |
| Heterogeneity $I^2$ | < 50% for fusion | Pass |
| MDE on Δα | ≥ 0.10–0.15 | Pass |
| KPI guardrails | ≤5% throughput loss; ≤+5% logical error | Pass |

### 9.3 Robustness and sensitivity panel

- **Estimators:** ODR (primary), Theil–Sen, SIMEX (± bands for $\sigma_\xi^2$).
- **Windows:** repeat with $h$± 25%; $\hat{\alpha}$ stable and collapse still passing.
- **Placebos:** clock rescaling; **Shuffles:** permute $L$ within-day—slope → ~0.
- **Leave-one-family-out fusion:** report $\hat{\alpha}_{\mathrm{QC}}^{(-f)}$.
- **Catastrophics:** re-estimate excluding flagged events; show Δ.
- **Fixed-effect vs. random-effects:** publish both; divergence implies genuine heterogeneity.

---

### 9.4 Negative results & scope boundaries

Publish bins/arms that **failed**:

- NO_COLLAPSE (curvature), REGIME_MIX (kinks), THIN_COVERAGE, LEVERAGE_RISK, FAMILY_DIVERGENCE (high $I^2$).
  Include a short note: suspected mechanism and next steps (rebin, instrumentation change, mechanism isolation). Negative results define **where RTM does not apply**.

### 9.5 One-page checklist (for each figure/table set)

- ✓ BIN keys listed and unchanged.
- ✓ # distinct $L$≥ 6 and span ≥ 0.6.
- ✓ ODR converged; Theil–Sen reported; SIMEX (if $\sigma_\xi^2$ known).
- ✓ Collapse: $R^2 < 0.05$; placebo OK; no changepoints.
- ✓ Fusion: $\mid \mathcal{F}_t \mid \geq 2$; $I^2 < 50\%$; REML converged.
- ✓ KPIs: throughput, makespan p95/p50, logical error, uptime—guardrails applied.
- ✓ Robustness panel completed (windows, shuffles, LOO).
- ✓ Provenance hashes (methods YAML, seeds, code version) included.

---

### 9.6 Narrative template (short "Results" text)

> *Physical layer.* Across 9 cluster sizes (span 1.05 in $\log L$), RTM-aware scheduling increased the slope from 0.48to 0.64 (Δ = 0.16, 95% CI [0.07,0.25]); residuals showed $R^2_{\text{collapse}} = 0.02$. Throughput remained within the 5% guardrail.
> *QEC.* With 1–3% cadence jitter, $\alpha_{\text{QEC}}$ rose from 0.68 to 0.83 (Δ = 0.15, CI [0.06,0.24]), logical error at fixed $d$ did not worsen.
> *Runtime.* Wavefront batching and variance-aware routing reduced p95/p50 from 1.85 to 1.60; $\alpha_{\text{runtime}}$ increased by 0.12.
> *Fusion.* Three families passed gates; $I^2 = 37\%$. The fused $\hat{\alpha}_{\text{QC}} = 0.69 \pm 0.04$. A **watch**-level decoherence alert fired on day 17; it preceded a makespan spike by 3 days.

---

### 9.7 Summary

The templates above ensure every claim is backed by: (i) **collapse** visual and numeric proof, (ii) EIV-aware estimation, (iii) **heterogeneity** accounting for fusion, (iv) KPI guardrails, and (v) complete **robustness** evidence.

# 10. Discussion

This section interprets RTM-QC results, clarifies how a **slope-first** view complements fidelity/QEC paradigms, and lays out trade-offs, risks, and adoption paths.

---

### 10.1 What does a higher $\alpha$ actually buy?

A larger binwise slope $\alpha$ means **time stretches more steeply with scale**, i.e., larger aggregates slow down *relative* to smaller ones within a stable environment. Operationally:

- **Shock damping:** disturbances at small scale are less likely to synchronize larger layers (runtime → QEC → I/O), reducing cascades that inflate tails (p95/p50), queues, and forced recalibrations.
- **Predictability:** higher $\alpha$ typically reduces **run-to-run variance** (narrower KPI distributions) because the stack's "tempo gradient" prevents alignment of rare long events.
- **Control leverage:** $\alpha$ is unit-agnostic; we can optimize it with scheduler/QEC/interconnect knobs without conflating unit changes (clocks) with structural change.

**Not a substitute for fidelity.** RTM improves **how** timing behaves across scale; it does not increase single/two-qubit fidelities by itself. Gains arrive through fewer cascades and better use of existing fidelity.

---

### 10.2 Complementarity with QEC and compilation

- **QEC:** Traditional design picks code distance $d$ from error rates. RTM adds a second axis: **cadence geometry**. Slight **desynchronization** (jitter/multi-phase) can raise $\alpha_{\mathrm{QEC}}$ at fixed $d$and decoder, often improving stability without extra overhead.
- **Compilation/runtime:** State-of-the-art routing minimizes depth/length. RTM asks also to minimize **time-variance** and **coincidence of long ops**, which can improve tails even if mean depth changes marginally.

---

### 10.3 Trade-offs and Pareto front

- **Throughput vs. layering:** Raising $\alpha$ by adding buffers/batching can reduce raw concurrency. We therefore optimize on a **Pareto front** (Sec. 7.1): increase $\alpha$ *subject to* throughput/fidelity floors.

- **Jitter vs. decoder timing:** Micro-jitter must stay within decoder validity; otherwise you trade higher $\alpha$ for logical failures.
- **Modular size:** Operating near $m^{\star}$ (Sec. 7.5) balances intra/inter costs, but drifting too far (bigger or smaller modules) can either flatten $\alpha$ (synchronization) or throttle bandwidth.

---

### 10.4 Failure modes (informative by design)

RTM's **collapse** gate turns failures into diagnostics:

- `NO_COLLAPSE`**:** curved log–log → missing mechanism (e.g., scale-dependent "clock" or nonlinear overhead).
- `REGIME_MIX`**:** kinks → hidden seams (firmware/scheduler swaps); rebin or split.
- **High $I^2$:** proxies disagree → do **not** fuse; inspect per-family controls.

Publishing these cases maps **scope boundaries** (where RTM does *not* apply), which is scientifically useful and prevents overreach.

---

### 10.5 Why a single fused indicator—and when not to use it

**Pros:** $\mathrm{ECI}_{\mathrm{QC}}(t)$ summarizes multiscale coherence, enabling **alerts** (Sec. 6.5) and trend tracking.
**Cons:** Fusion can hide heterogeneity. Hence the **gates** (at least two families, $I^2 < 50\%$, REML convergence). If they fail, publish **family-wise** $\hat{\alpha}_f$ only; the lack of fusion is itself a result ("the stack is speaking with different slopes").

---

### 10.6 Relation to time-changed diffusions and queueing

The PDE view (RTM as a **state-dependent clock**) explains why **tails** shrink when $\alpha$ rises: the **effective dynamic exponent** $z$increases, and exit/first-passage times scale more steeply with "radius" (Sec. 6 of the math paper). In queueing terms, scheduling that raises $\alpha$ **decorrelates** service bursts and dampens tail amplification.

---

### 10.7 External validity and portability

Because $\alpha$ is **gauge-invariant**, comparisons hold across labs and generations when bins are matched (environment keys). The same pipeline ports to **trapped ions**, **superconducting**,

**neutral atoms**, and **annealers** with layer-appropriate $(L, T)$. What changes is instrumentation; the **collapse logic** and **EIV estimation** remain.

---

### 10.8 Adoption path (practical)

1. **Shadow mode:** compute per-family $\hat{\alpha}_f$ and collapse panels without changing operations.
2. **Low-risk knobs:** enable **readout batching**, **staggered resets**, and tiny **cadence jitter** (≤3%).
3. **Close the loop:** bring $\mathrm{ECI}_{\mathrm{QC}}(t)$ into on-call dashboards with alert tiers and playbooks.
4. **Architectural planning:** measure $a, b, A, B$ (Sec. 7.5) to choose module sizes; iterate quarterly.

---

### 10.9 Open questions

- **Decoder co-design:** how to include $\alpha$ directly in decoders' scheduling/graph updates?
- **Learning controllers:** can RL tune $\alpha$ subject to KPI floors without violating collapse?
- **Holonomy tests:** practical statistics to distinguish curvature from topological obstructions (global collapse failure).
- **Cross-layer causality:** when do $\alpha$ changes at the physical layer *cause* changes at runtime vs. just correlate via utilization?

---

### 10.10 Takeaway

RTM-QC adds a **third axis—**the **geometry of tempo—**to fidelity and scale. With strict gates (collapse, heterogeneity) and modest controls (batching, jitter, routing variance), $\alpha$ becomes a reliable lever for **stability and throughput**, yielding early warnings and design guidance while respecting scientific falsifiability.

# 11. Limitations & Scope

**Bin dependence.** RTM is a **binwise** theory. If the environment (temperature, firmware, topology, decoder, utilization) drifts, the slope $\alpha$ is undefined until the bin is split. Results are only valid within clearly documented BIN keys.

**Proxy choice sensitivity.** $(L, T)$ proxies must reflect a **single dominant mechanism** per family. Mis-specified proxies (e.g., mixing readout and routing in the same $T$) induce curvature and validly fail collapse.

**Finite-window bias.** When $\alpha(u)$ drifts, any finite window of width $h$incurs $O(\varepsilon h)$ bias. Our adiabatic guidance mitigates but does not eliminate it; reported $\hat{\alpha}$ should be interpreted as **local**.

**EIV model assumptions.** ODR/TLS and SIMEX assume well-behaved errors (mean-zero, finite moments) and independence from $x$. Heavy-tailed or state-dependent errors require robustness checks (Theil–Sen, bootstrap, sensitivity bands).

**Fusion heterogeneity.** Random-effects fusion is appropriate only when families are **commensurate** and $I^2 < 50\%$. Otherwise the single-number indicator is withheld by design; RTM does not force agreement across mechanisms.

**Causality limits.** $\alpha$ is **structural but not causal** by default. Design sections propose interventions and A/B protocols, yet causal claims require the pre-registered controls and guardrails we specify.

**Scope boundaries.** Systems with **non-power** timing (persistent curvature), **scale-dependent clocks** (overheads that grow with $L$ inside a bin), or strong **holonomy** (global seams) lie **outside** RTM's applicability. In such domains, treat $\alpha$ as undefined and publish negative results.

# 12. Methods & Reproducibility

### 12.1 Data schema and BINs

- **BIN key:** {platform, temperature band, firmware hash (FPGA/DSP), topology ID, routing policy, syndrome cadence, utilization band}.
- **Tidy table (per bin):** [x=log L, y=log T, family, BIN tags, replicate_id, timestamp, weight].
- **Coverage gates:** ≥6 distinct $L$, span ≥0.6 in $\log L$.

### 12.2 Estimation pipeline (per family, per bin)

1. **Changepoint scan:** PELT/BIC on $(x, y)$ and on residuals if available; split if detected.

2. **Init:** Theil–Sen slope/intercept; flag catastrophics; build replicate weights.
3. **Primary fit:** ODR/TLS (orthogonal residuals) with replicate or bootstrap SEs.
4. **SIMEX (optional):** when $\sigma_\xi^2$ is estimable; extrapolate to $\lambda = -1$.
5. **Collapse test:** regress $\tilde{y} = y - \hat{\alpha}x - \hat{c}$on $x$; require $R^2_{\text{collapse}} < 0.05$, flat LOESS, clock placebo holds.
6. **Diagnostics:** leverage ≤25%; residual plots; window width $h$logged.
7. **Accept/Reject:** accept if all gates pass; else flag (`NO_COLLAPSE`, `REGIME_MIX`, `THIN_COVERAGE`, `LEVERAGE_RISK`, `EIV_FAIL`).

### 12.3 Fusion and heterogeneity (rolling)

- **Weights:** $w_f = 1/(\hat{\sigma}_f^2 + \hat{\tau}^2)$ with $\hat{\tau}^2$ via REML (DL as sensitivity).
- **Fused slope:** $\hat{\alpha}_{\text{QC}} = \sum w_f \hat{\alpha}_f / \sum w_f$; **variance:** $1/\sum w_f$.
- **Diagnostics:** fixed-effect baseline, **Cochran's** $Q$ and $I^2$.
- **Gates:** fuse only if $|\mathcal{F}| \geq 2$ and $I^2 < 50\%$. Otherwise publish family-wise.

### 12.4 Real-time operation and alerts

- **Rolling windows:** sliding horizon in $x$ (width $h$) and wall-clock (7–28 days).
- **Smoothing:** 3-point median; **Z-score** against 30-day EWMA.
- **Alert tiers:** Advisory/Watch/Warning thresholds (Sec. 6.5).
- **Playbooks:** throttle concurrency, stagger resets, cadence jitter, variance-aware routing; all interventions must re-pass **collapse**.

### 12.5 Robustness & sensitivity

- **Estimators:** publish ODR (primary), Theil–Sen, SIMEX bands.
- **Windows:** ±25% $h$ sensitivity; $\hat{\alpha}$ stability required.
- **Placebos & shuffles:** clock rescaling invariance; $L$-shuffles yield near-zero slopes.
- **Leave-one-family-out** fusion; **fixed-effect** vs **random-effects** comparison.

### 12.6 Provenance (methods YAML)

- BIN keys, estimator settings, bootstrap seeds, SIMEX $\Lambda$, window $h$, collapse thresholds, heterogeneity gates, versions of analysis code.

- All plots and numbers include hash of the methods YAML; re-runs with the same YAML reproduce numbers within bootstrap noise.

## 13. Conclusion & Outlook

We presented **RTM-aware quantum computing (RTM-QC)**: a **slope-first** framework that measures and **engineers** the geometry of time across scale. Inside stable bins, the characteristic time $T$ scales with a size proxy $L$ as $T \propto L^{\alpha}$; the **coherence exponent** $\alpha$ is invariant to clocks and thus comparable across devices, stacks, and labs. With **collapse** as a falsifiable gate and **errors-in-variables** estimation, $\alpha$becomes a reliable operational signal. Fusing clean, layer-wise slopes yields a real-time $\boldsymbol{ECI}_{\mathrm{QC}}$**(t)** that supports **early warnings** (decoherence events) and **design decisions** (scheduler, QEC cadence, modular sizing, I/O offsets).

**What this adds.** RTM-QC complements fidelity/QEC by introducing a third axis—**tempo geometry—**that explains and controls tails, queues, and synchronization cascades. Modest, reversible controls (batching, staggered resets, micro-jitter, low-variance routing) can **raise** $\alpha$ without degrading throughput or fidelity when used with guardrails.

**What it does not do.** RTM-QC does not replace physical improvements (fidelities, $T_1/T_2$), nor does it guarantee causality without the A/B protocols and guardrails we specify. Failures to collapse, high heterogeneity, or regime seams are **informative**, delineating scope boundaries rather than inviting post-hoc fixes.

**Near-term agenda.**

1. **Run the protocols** (Sec. 8) on superconducting and ion platforms; publish both successes and negatives with full collapse/fusion diagnostics.
2. **Close the loop**: deploy $\mathrm{ECI}_{\mathrm{QC}}(t)$ dashboards and alert playbooks in production; evaluate lead–lag vs. KPI spikes.
3. **Co-design with decoders** and compilers so cadence and routing optimize $\alpha$ subject to throughput/fidelity floors.
4. **Standardize reporting**: figures/tables in Sec. 9, methods YAML, and open robustness artifacts.

**Longer-term questions.** Incorporate $\alpha$ into **time-changed diffusion models** of queues; develop **holonomy tests** to distinguish curvature from seams; extend to **modular networks** and **neutral-atom** platforms; integrate learning-based controllers that respect collapse gates.

**Bottom line.** RTM-QC gives quantum teams a **unit-robust, falsifiable lever** over multiscale timing. Measure the slope, **validate by collapse**, fuse when families agree, and **engineer** $\alpha$—not as a slogan, but as a reproducible practice to deliver more stable and efficient quantum computation.

# Appendices

### Appendix A — Mathematical Background (RTM essentials for QC)

#### A.1 Semigroup → power law

Assume binwise scale semigroup $T(bL) = f(b)T(L)$, $f(1) = 1$, and measurability near $b = 1$. Then $f(b) = b^{\alpha}$ and

$$T(L) = \kappa L^{\alpha}, v(u) = \log\, T = \alpha u + \log\, \kappa, u = \log\, L.$$

$\alpha$ is **gauge-invariant**; $\kappa$ is a **clock**.

#### A.2 1-form & collapse

Define the RTM 1-form $\omega = dv - \alpha\, du$. **Collapse** (residual independence of $v - \alpha u$ from $u$) is equivalent to **exactness** of $\omega$ on a simply connected bin:

$$\omega = d\psi(x), d\omega = 0, \psi \text{ independent of } u.$$

If $\alpha = \alpha(x,u)$, then $d\omega = -d\alpha \wedge du$; nonzero curvature breaks collapse.

#### A.3 Variable exponents (finite-window bias)

For slowly varying $\alpha(u)$:

$$v(u) = \int_{u_0}^{u} \alpha(s)\, ds + \log\, \kappa(u), \hat{\alpha}(u;h) = \alpha(u) + O(\varepsilon h),$$

and $R^2_{\text{collapse}} = O((\varepsilon h)^2)$ for window width $h$.

---

### Appendix B — Estimators & Algorithms

#### B.1 Orthogonal Distance Regression (TLS/ODR)

Minimize orthogonal residuals:

$$\min_{\alpha,c} \sum_i \frac{(y_i - \alpha x_i - c)^2}{\sigma_{y,i}^2 + \alpha^2 \sigma_{x,i}^2}.$$

**Init:** Theil–Sen; **CIs:** bootstrap pairs/cluster; **checks:** condition number < $10^4$; max leverage < 25%.

### B.2 Theil–Sen

Median of pairwise slopes $\alpha_{ij} = (y_j - y_i)/(x_j - x_i)$; robust to outliers; mild EIV attenuation.

### B.3 SIMEX (optional)

If $\sigma_\xi^2 = \mathrm{Var}(\xi)$ is estimable, simulate $x^{(\lambda)} = x^{obs} + \sqrt{\lambda}\tilde{\xi}$ and extrapolate $\hat{\alpha}(\lambda)$ to $\lambda = -1$.

### B.4 Collapse gate

Regress residuals $\tilde{y} = y - \hat{\alpha}x - \hat{c}$ on $x$; require $R^2_{\text{collapse}} < 0.05$ and flat LOESS; pass clock placebo.

---

## Appendix C — Protocol Cards (copy–paste templates)

### C.1 Physical (staggered resets + readout waves)

- **L/T:** $L$ =active cluster size; $T$ = stable calibration interval.
- **Arms:** Control vs RTM-aware (waves + 2–4% reset offsets).
- **Duration:** 2–4 weeks, interleaved.
- **Success:** $\Delta\alpha_{\text{phys}} \geq 0.15$ (95% CI excludes 0), throughput loss ≤5%, collapse passes.

### C.2 QEC (micro-jitter / multi-phase)

- **L/T:** $L = d$; $T$ = cycles to logical failure.
- **Arms:** Fixed period vs $Pk = P(1 + \eta k)$, $|\ \eta k\ | \leq 0.02$ or 2–3 phase groups.
- **Success:** $\Delta\alpha_{\text{QEC}} \geq 0.15$, no logical-error regression (>5%) at fixed $d$.

### C.3 Runtime (batching + low-variance routing)

- **L/T:** $L$ = post-mapping width; $T$ = makespan.
- **Arms:** Baseline vs wavefront + variance-penalized routing.

- **Success:** $\Delta\alpha_{\text{runtime}} \geq 0.10$ and p95/p50 latency ↓ ≥10%.

### C.4 I/O (phase-offset windows)

- **L/T:** $L =$ multiplexing degree; $T =$ readout latency p95 (and p50).
- **Arms:** Synchronous vs phase-offset windows.
- **Success:** $\Delta\alpha_{\text{IO}} \geq 0.10$, p95/p50 ≤ 1.6 over majority of $L$.

---

## Appendix D — Methods YAML (skeleton)


```yaml
bin:
 platform: "SC"          # or "IONS", "NA"
 temperature_band: "10-15mK"
 firmware_hash: "fpga_1.4.2_dsp_0.9.8"
 topology_id: "mesh-v3"
 routing_policy: "baseline"  # or "rtm-aware"
 syndrome_cadence: "P=3.2us, jitter=0%"
 utilization_band: "30-60%"

estimation:
 min_L_points: 6
 min_logL_span: 0.6
 eiv: "odr"
 odr:
  init: "theil-sen"
  leverage_cap: 0.25
  bootstrap: {clusters: true, reps: 2000, seed: 123}
 simex:
  enabled: false
  lambda: [0.5,1.0,1.5,2.0]
collapse:
 r2_threshold: 0.05
 placebo_clock: true
 changepoint_scan: {method: "PELT", penalty: "BIC"}
```

```yaml
fusion:
  heterogeneity_gate_I2: 0.5
  tau2_method: "REML"
  min_families: 2

eci_rt:
  window_logL: 0.8
  horizon_days: 14
  smoothing: "median3"
  alert:
    z_advisory: -1.5
    z_watch: -2.0
    z_warning: -2.5
```

### Appendix E — Notation Glossary

- $L$: scale proxy (layer-specific); $u = \log L$.
- $T$: characteristic time; $v = \log T$.
- $\alpha$: **coherence exponent** (slope; clock-invariant).
- **Bin**: environment slice with fixed {platform, temperature band, firmware hash, topology ID, routing policy, syndrome cadence, utilization band}.
- **Collapse**: $R^2(\tilde{y} \sim x) < 0.05$ for $\tilde{y} = y - \hat{\alpha}x$; residuals show no trend vs $x$.
- $\mathbf{ECI}_{\mathrm{QC}}(t)$: fused slope via random-effects at time $t$.
- $Q, I^2, \tau^2$: heterogeneity statistics for fusion.
- ODR/TLS, Theil–Sen, SIMEX: slope estimators under EIV.
- **Adiabatic window**: width $h$ in $u$ where $|\partial_u \alpha| h \ll 1$.

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### Appendix F — Reproducible Figure Recipes (minimal)

- **Collapse panel**:
    - Fit ODR; compute residuals $\tilde{y}$.
    - Plot $y$vs $x$+ ODR band; residual vs $x$ with LOESS.
    - Annotate $R^2_{\text{collapse}}$, $\#L$, span, leverage.
- **Forest plot**:
    - For accepted families, display $\hat{\alpha}_f$ $\pm$CI; compute $w_f$, $Q$, $I^2$, $\hat{\tau}^2$.
    - Overlay fused $\hat{\alpha}_{\text{QC}}$.
- **ECI**$_{\text{QC}}(t)$:
    - Rolling fusion; show 50/95% bands; background colored by $I^2$ tiers; mark alert tiers.