Hypothesis Testing

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Today:

Hypothesis testing:

a. General Framework
b. Parametric tests:
    i. t-test
c. Permutation test (if time)

Hypothesis Testing Framework

Why?

- Collect a sample from a distribution; calculate statistics of the sample, but what do these sample statistics tell us about the underlying population?

Hypothesis Testing Framework

Example Questions:

- Does a job program increase average worker productivity?

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- Does optogenetic inhibition of neurons in mPFC impair behavior in a working memory task?

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H_0: optogenetic inhibition in mPFC has no effect on behavior in working memory task

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1. Formulate the null hypothesis, H_0, and the alternative hypothesis H_1

Does optogenetic inhibition of neurons in mPFC impair behavior in a working memory task?

H_0: optogenetic inhibition in mPFC has no effect on behavior in working memory task

H_1: optogenetic inhibition in mPFC has an effect on behavior in working memory task (performance improves or decreases)

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H_0: optogenetic inhibition in mPFC does not improve performance in working memory task

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1. Formulate the null hypothesis, H_0, and the alternative hypothesis H_1: **not just one set of {H_0, H_1}**

Does optogenetic inhibition of neurons in mPFC impair behavior in a working memory task?

H_0: optogenetic inhibition in mPFC does not improve performance in working memory task

H_1: optogenetic inhibition in mPFC improves performance in working memory task

Hypothesis Testing Framework

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Does optogenetic inhibition of neurons in mPFC impair behavior in a working memory task?

- Behavior performance metrics:
  - Accuracy on task
  - Reaction time on task
  - Slope/bias/lapse rates of psychometric curve

Aside: psychometric curves

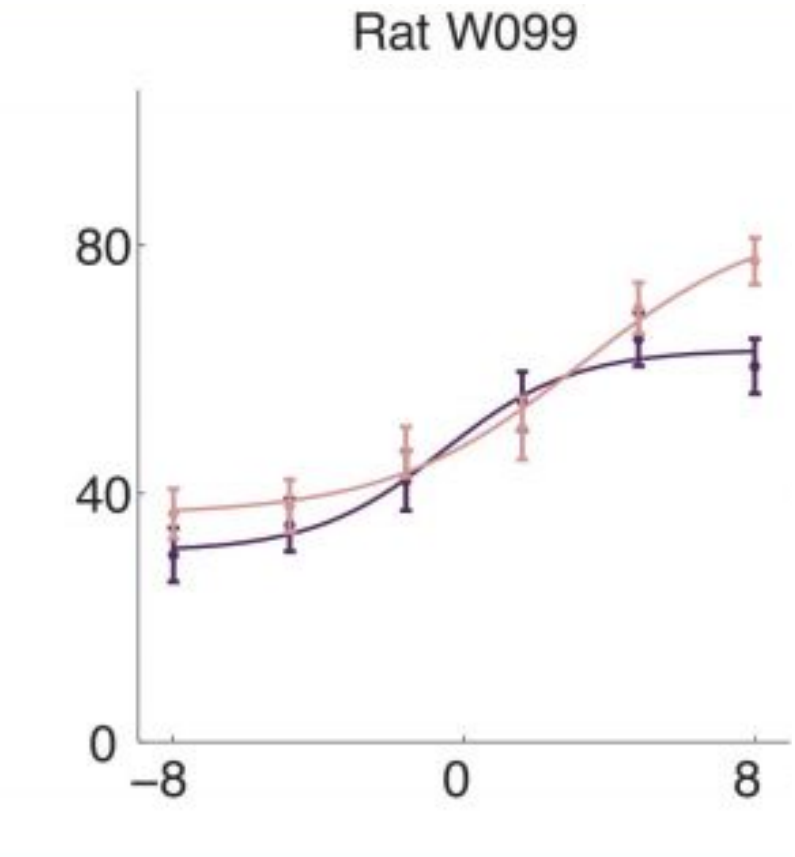

Figure: Akrami et al. (2018)

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- Hardest step. Either: make assumptions about distribution of test statistic under null hypothesis (parametric approach: t-tests, ANOVA, chi-squared test) or use permutation test (non-parametric approach).

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- This is what we will discuss for most of the rest of today (as well as step 2 since they are closely related)

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- Usually α is chosen to be 0.05. That is, under the null hypothesis, in 5% of cases, the null hypothesis will be rejected

Parametric Hypothesis Testing: t-test

1. Formulate the null hypothesis, H_0, and the alternative hypothesis H_1

Two types of t-test:

1. One-sample location test: Is population mean different from a particular value, μ_0?
2. Two-sample location test: Are the means of two populations different?

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H_0: $\mu=\mu_0$

H_1: $\mu\neq\mu_0$

Parametric Hypothesis Testing: t-test

1. Formulate the null hypothesis, H_0, and the alternative hypothesis H_1

Two types of t-test:

2. Two-sample location test: Are the means of two populations, μ_1 and μ_2, different?

H_0: $\mu_1=\mu_2$

H_1: $\mu_1\neq\mu_2$

Parametric Hypothesis Testing: t-test

2. Identify a test statistic that can be used to assess the truth of the null hypothesis

Two types of t-test:

1. One-sample location test: Is population mean different from a particular value, μ_0?

   Form t-statistic:

$$t = \frac{\sqrt{n}(\bar{y} - \mu_0)}{s} = \frac{\sqrt{n}(\bar{y} - \mu_0)}{se(\bar{y})}$$

Parametric Hypothesis Testing: t-test

2. Identify a test statistic that can be used to assess the truth of the null hypothesis

Two types of t-test:

2. Two-sample location test: Are the means of two populations, μ_1 and μ_2, different?

Form t-statistic:

$$t = \frac{(\bar{y}_1 - \bar{y}_2)}{\sqrt{(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2})}}$$

Parametric Hypothesis Testing: t-test

3. Compute the P-value: probability that a test statistic, as least as significant as the one observed, would be obtained assuming the null hypothesis were true. The smaller the P-value, the stronger the evidence against the null hypothesis

1. One-sample location test: Is population mean different from a particular value, μ_0?

Can show that, under the null hypothesis, the random variable $t = \frac{\sqrt{n}(\bar{y} - \mu_0}{s}$

has a t_{n-1} distribution.

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Can show that, under the null hypothesis, the random variable $t = \frac{\sqrt{n}(\bar{y} - \mu_0}{s}$

has a t_{n-1} distribution.

Calculate p-value by calculating probability mass that is to left or right of absolute value of test statistic (shown left is one-sided H_1)

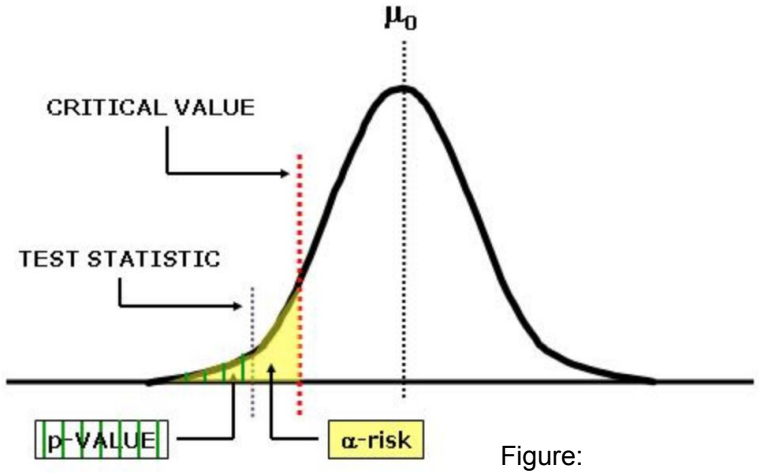

Figure: http://www.six-sigma-material.com/P-Value.html

Parametric Hypothesis Testing: t-test

4. Compare P-value to an acceptable significance level, α: if P less than or equal to α, then observed effect is statistically significant; null hypothesis is ruled out and alternative hypothesis is valid.

Reject the null hypothesis if P-value is smaller than 0.05

Aside:

Why does the random variable $t = \frac{\sqrt{n}(\bar{y} - \mu_0)}{s}$

have a t_{n-1} distribution under the null hypothesis?

Permutation Test

See separate slide deck