simplest minimization

Assume angular velocity ω_i at time t_i, and uniform acceleration α during the time interval t_i to t_{i+1}.

The new angular velocity will be

$$\omega_{(i+1)} = \omega_i + \alpha\Delta t$$

$${\omega_{(i+1)}}^2 = (\omega_i + \alpha\Delta t)^2 = {\omega_i}^2 + \alpha^2(\Delta t)^2 + 2\omega_i\alpha\Delta t$$

and the radial accelerations are therefore

$$a_{r(i)} = r{\omega_i}^2$$

$$a_{r(i+1)} = r{\omega_{(i+1)}}^2 = r[{\omega_i}^2 + \alpha^2(\Delta t)^2 + 2\omega_i\alpha\Delta t]$$

giving

$$\dot{a_r} = \frac{1}{\Delta t}\left[a_{r(i+1)} - a_{r(i)}\right]$$

$$= \frac{r}{\Delta t}[\alpha^2(\Delta t)^2 + 2\omega_i\alpha\Delta t] = r[\alpha^2\Delta t + 2\omega_i\alpha]$$

$$= r\left[\left(\frac{a_{t(i)}}{r}\right)^2\Delta t + 2\frac{a_{t(i)}}{r}\sqrt{\frac{a_{r(i)}}{r}}\right]$$

$$= \frac{\left(a_{t(i)}\right)^2}{r}\Delta t + 2a_{t(i)}\sqrt{\frac{a_{r(i)}}{r}}$$

where we used

$\alpha = \frac{a_{t(i)}}{r}$ (1)

By ignoring[1] the term in Δt, this simplifies to

$$\dot{a_r} = 2a_{t(i)}\sqrt{\frac{a_{r(i)}}{r}}$$

which can easily be solved for r.

$$r = 4{a_{t(i)}}^2\frac{a_{r(i)}}{{\dot{a_r}}^2}$$

[1] The assumption is that $\alpha^2(\Delta t)^2 \ll 2\omega_i\alpha\Delta t$, which implies $\alpha^2\Delta t \ll 2\omega_i\alpha$; $\alpha\Delta t \ll 2\omega_i$; $\Delta\omega_i \ll \omega_i$
i.e. the change in omega is much smaller than its current value. This assumption would seem to fail near zero speed, with large accelerations. eg a “kickstart” scenario.

after which alpha is easily found.

From the full expression, we can use minimization methods to find the optimal value for r, i.e.

form a cost function

$$c = \dot{a}_r - \frac{(a_{t(i)})^2}{r}\Delta t - 2a_{t(i)}\sqrt{\frac{a_{r(i)}}{r}}$$

and minimize against r.

<u>Warmup method</u> for minimization problems: Use the known value of alpha, and just minimize to find r.

i.e. use cost function

$$c = a_{t(i)} - \alpha r$$

where alpha is a known fixed value.