Proton energy and range

Andriy Zatserklyaniy, zatserkl@fnal.gov

May 8, 2015

1 Proton energy at depth x

Proton stopping power may be approximate by expression

$$\frac{dE}{dx} = -247.93E^{-0.7637} \qquad \frac{MeV}{g/cm^2}$$

Rewrite and solve this differential equation.

$$\frac{dE(x)}{dx} = s(E) = -aE^{-b}$$

In terms of $y(x) = E(x)$

$$\frac{dy(x)}{dx} = f(y)$$

where $y(x) = E(x)$ and $f(y) = s(E)$

Separate the variables

$$\frac{dy}{f(y)} = dx$$

Integrate the equation

$$x = -\int \frac{dy}{ay^{-b}} + C$$

$$x = -\frac{y^{b+1}}{a(b+1)} + C$$

Initial condition is that proton has energy E at depth x:

$$0 = -\frac{E_0^{b+1}}{a(b+1)} + C$$
$$C = \frac{E_0^{b+1}}{a(b+1)}$$

and

$$x = \frac{E_0^{b+1}}{a(b+1)} - \frac{y^{b+1}}{a(b+1)}$$
$$y = \left(E_0^{b+1} - a(b+1)x\right)^{\frac{1}{b+1}}$$

Finally,

$$E = \left(E_0^{b+1} - a(b+1)x\right)^{\frac{1}{b+1}} \tag{1}$$

or

$$E(x) = \left(E_0^{1.7637} - 247.93 \cdot 1.7637 \cdot x\right)^{\dfrac{1}{1.7637}}$$

See plot on the Fig.1.

Example: If we installed polystyrene degrader ($\rho = 1.05\ g/cm^3$) with thickness of 200 mm in front of the scintillator block, the proton will exit it with kinetic energy of about 80 MeV. This energy will be deposited in the scintillator block.

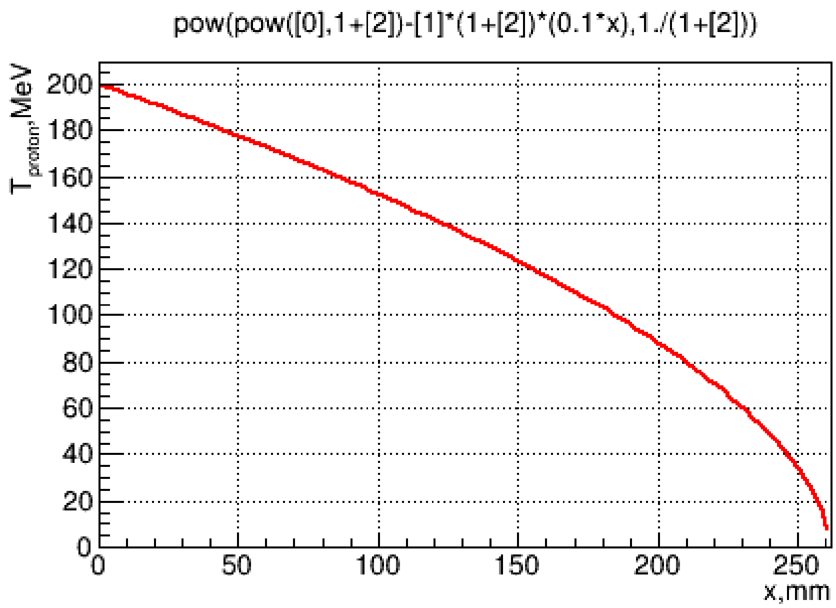

Figure 1: Kinetic energy of proton with initial energy 200 MeV vs depth

2 Proton Range

$$
\begin{aligned}
R(E) &= \int_0^E \frac{dE}{dE/dx} \\
&= \int_0^E \frac{dE}{aE^{-b}} \\
&= \frac{1}{a}\int_0^E E^b dE \\
&= \frac{E^{b+1}}{a(b+1)} \\
&= \frac{E^{1.7637}}{247.93 \cdot 1.7637}
\end{aligned}
$$

See result in the Fig.2.

Fig.3 shows standard dependence of the dE/dx vs E, while the Fig.3 plots dependence of the dE/dx on x, derivative of the expression 1.

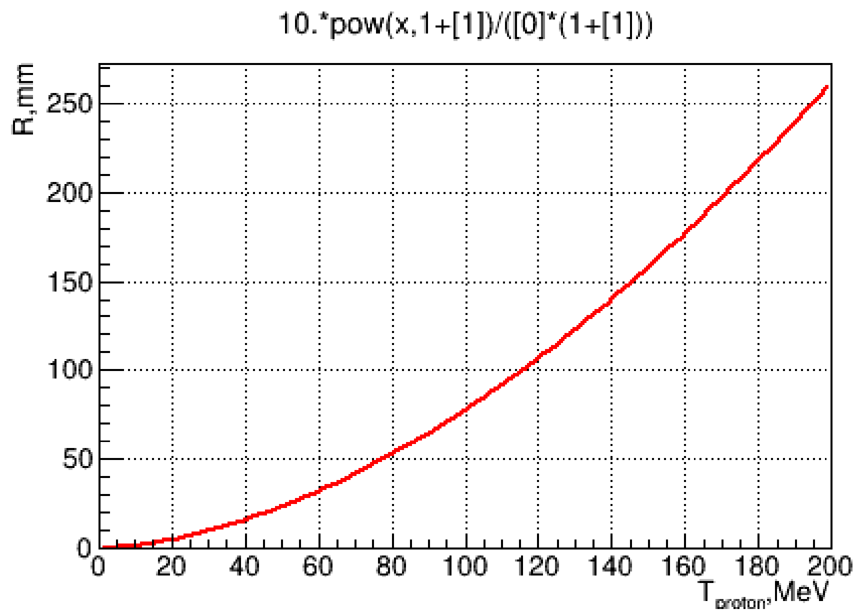

Figure 2: Proton range vs initial energy

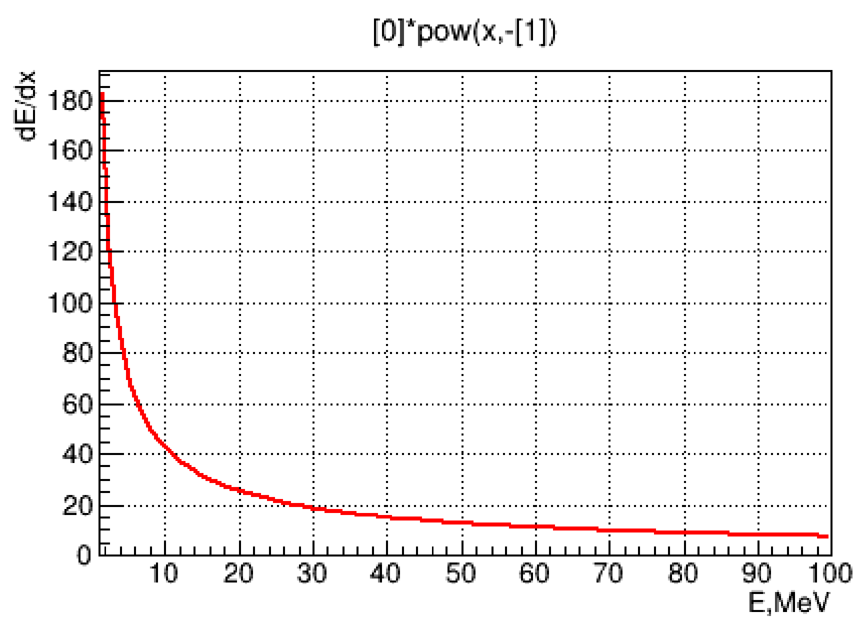

Figure 3: dE/dx vs E

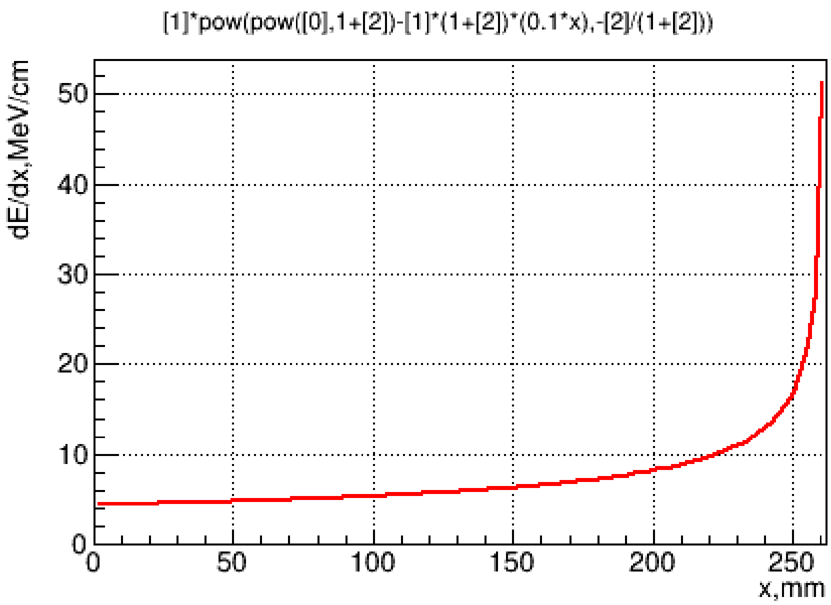

Figure 4: dE/dx vs x