Error on ratio

Andriy Zatserklyaniy, zatserkl@fnal.gov

May 24, 2012

1 Error on ratio

Derive the error on the ratio $r = x_1/x_2$. The differential of r is

$$\begin{aligned} dr &= \frac{dx_1}{x_2} - \frac{x_1 dx_2}{x_2^2} \\ &= r\frac{dx_1}{x_1} - r\frac{dx_2}{x_2} \end{aligned}$$

so

$$\frac{dr}{r} = \frac{dx_1}{x_1} - \frac{dx_2}{x_2}$$

Compute standard error as a square root of variance. Variance of sum of independent variables is

$$V(ax_1 + bx_2) = a^2 V(x_1) + b^2 V(x_2)$$

In our case

$$\frac{\Delta r}{r} = \sqrt{\left(\frac{\Delta x_1}{x_1}\right)^2 + \left(\frac{\Delta x_2}{x_2}\right)^2}$$

2 Error on N_{pe}

Given we fitted the amplitude spectrum with a Gaussian with mean μ and width σ, the number of photoelectrons can be estimated as

$$N_{pe} \approx \frac{A}{(\sigma/\mu)^2}$$

here A is an normalizing coefficient which we assume to be equal to 1.

Find uncertainty on the square of the ratio.

$$
\begin{aligned}
&N_{pe} = (\mu/\sigma)^2 \\
&\frac{\partial N_{pe}}{\partial \mu}\Delta\mu = 2N_{pe}\frac{1}{\sigma}\Delta\mu = 2N_{pe}^2\frac{\Delta\mu}{\mu} \\
&\frac{\partial N_{pe}}{\partial \sigma}\Delta\sigma = -2N_{pe}\frac{\mu}{\sigma^2}\Delta\sigma = 2N_{pe}^2\frac{\Delta\sigma}{\sigma}
\end{aligned}
$$

For variance we have

$$V[N_{pe}] = (2N_{pe})^2\left[\left(\frac{\Delta\mu}{\mu}\right)^2 + \left(\frac{\Delta\sigma}{\sigma}\right)^2\right]$$

Finally, an error on N_{pe} is

$$\Delta N_{pe} = 2N_{pe}\sqrt{\left(\frac{\Delta\mu}{\mu}\right)^2 + \left(\frac{\Delta\sigma}{\sigma}\right)^2}$$