Some Handy Properties of the Gaussian

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1 Area of the Gaussian

Let's consider histogram with bin width of 1/2 units (MeV on the figure), Fig.1.

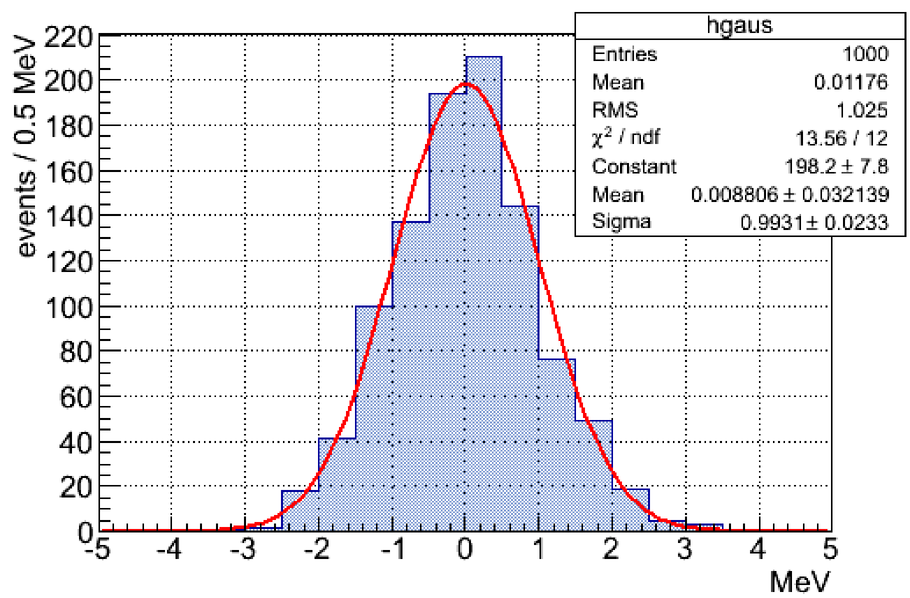

Figure 1: Histogram and Gaussian fit

Calculate the number of events in the histogram. The number of events is proportional to the histogram area

$$S = N \cdot s_1$$

where s_1 is area which corresponds to one event. The area of every histogram bin is a product of the bin width w and the number of events in the bin. Therefore, the area which corresponds to one event is a product of the bin width w and 1:

$$s_1 = w \cdot 1 = w$$

To calculate an area of the Gaussian-shape histogram in the Fig.1 let's approximate it by the area of the Gaussian

$$g(x) = Ae^{-\dfrac{(x-x_0)^2}{2\sigma^2}}$$

Fit results are:

$$A = 198.2$$
$$\sigma = 0.993$$

Calculate the area under the Gaussian

$$\begin{aligned} S &= \int_{-\infty}^{+\infty} Ae^{-\frac{(x-x_0)^2}{2\sigma^2}} \\ &= \sqrt{2\pi}\sigma A \underbrace{\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-x_0)^2}{2\sigma^2}}}_{=1} \\ &= \sqrt{2\pi}\sigma A \\ &\approx 2.5 \cdot \sigma \cdot A \end{aligned}$$

Express this area in terms of the number of events N and the area of the one event s_1:

$$\begin{aligned} S &= N \cdot s_1 \\ &= N \cdot w \end{aligned}$$

hence

$$N = S/w$$

or

$$N = \sqrt{2\pi} \cdot \sigma \cdot A/w$$

Because $\sqrt{2\pi} \approx 2.5066$

$$N \approx 2.5 \cdot \sigma \cdot A/w$$

The histogram on the Fig.1 was generated for 1000 events Gaussian-distributed with $\sigma = 1$. In our approximation

the area under the Gaussian

$$S \approx 2.5 \cdot 198.2 \cdot 0.993 = 492.0$$

the number of events for $w = 0.5$ MeV

$$N \approx 2.5 \cdot 198.2 \cdot 0.993/0.5 = 984.1$$