B.P = 132 , PDF = 161 (1)

Q17) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\left[\frac{1}{2}(x+h)-\frac{1}{3}\right]-\left(\frac{1}{2}x-\frac{1}{3}\right)}{h}$

$\lim_{h\to 0} \frac{\cancel{\frac{1}{2}x}+\frac{1}{2}h \cancel{-\frac{1}{3}} \cancel{-\frac{1}{2}x} \cancel{+\frac{1}{3}}}{h} = \lim_{h\to 0} \frac{\frac{1}{2}\cancel{h}}{\cancel{h}}$

$\lim_{h\to 0} = \frac{1}{2} = \frac{1}{2}$ Ans.

Q18) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{[m(x+h)+b]-(mx+b)}{h}$

$= \lim_{h\to 0} \frac{\cancel{mx}+mh \cancel{+b} \cancel{-mx} \cancel{-b}}{h}$

$= \lim_{h\to 0} \frac{m\cancel{h}}{\cancel{h}} = m$ Ans.

Q19) $f(t) = 5t - 9t^2$

$= \lim_{h\to 0} \frac{f(t+h)-f(t)}{h} = \lim_{h\to 0} \frac{[5(t+h)-9(t+h)^2]-(5t-9t^2)}{h}$

$= \lim_{h\to 0} \frac{5t+5h-9(t^2+2th+h^2)-5t+9t^2}{h}$

$= \lim_{h\to 0} \frac{\cancel{5t}+5h \cancel{-9t^2}-18th-9h^2 \cancel{-5t} \cancel{+9t^2}}{h}$

$= \lim_{h\to 0} \frac{5h-18th-9h^2}{h} = \frac{\cancel{h}(5-18t-9h)}{\cancel{h}}$

$$= \lim_{h \to 0} (5 - 18t - 9h) = 5 - 18t \text{ A}$$

(20) — $f(x) = 1.5x^2 - x + 3.7$

$$f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[1.5(x+h)^2 - (x+h) + 3.7] - (1.5x^2 - x + 3.7)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{1.5x^2} + 3xh + 1.5h^2 - \cancel{x} - h + \cancel{3.7} - \cancel{1.5x^2} + \cancel{x} - \cancel{3.7}}{h}$$

$$= \lim_{h \to 0} \frac{3xh - 1.5h^2 - h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3x - 1.5h - 1)}{\cancel{h}}$$

$$= \lim_{h \to 0} 3x - 1.5h - 1 = 3x - 1 \text{ A}$$

(21) — $f(x) = x^3 - 3x + 5$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[(x+h)^3 - 3(x+h) + 5] - (x^3 - 3x + 5)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{3x} - 3h + \cancel{5} - \cancel{x^3} + \cancel{3x} - \cancel{5}}{h}$$

$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3 - 3h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2 - 3)}{\cancel{h}}$$

$$=\lim_{h\to 0}\left(3x^2+3xh+h^2-3\right)=3x^2+3x(0)+(0)^2-3$$ (3)

$$=3x^2-3$$

(22) $f(x)=x+\sqrt{x}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{x+h+\sqrt{x+h}-(x+\sqrt{x})}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}}+\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$=\lim_{h\to 0}\left[1+\frac{(x+h)-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}\right]$$

$$=\lim_{h\to 0}\left(1+\frac{\cancel{x}+h-\cancel{x}}{h\left(\sqrt{x+h}+\sqrt{x}\right)}\right)$$

$$=\lim_{h\to 0}1+\frac{\cancel{h}}{\cancel{h}\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$=\lim_{h\to 0}1+\frac{1}{\sqrt{x+h}+\sqrt{x}}$$

$$=1+\frac{1}{2\sqrt{x}}$$

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23) ___. $g(x) = \sqrt{1+2x}$ (4)

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{g}$$

$$= \lim_{h \to 0} \frac{\sqrt{1+2(x+h)} - \sqrt{1+2x}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{1+2(x+h)} - \sqrt{1+2x}}{h} \cdot \frac{\sqrt{1+2(x+h)} + \sqrt{1+2x}}{\sqrt{1+2(x+h)} + \sqrt{1+2x}}$$

$$\lim_{h \to 0} \frac{(1+2x+2h) - (1+2x)}{h\left(\sqrt{1+2x+2h} + \sqrt{1+2x}\right)}$$

$$= \lim_{h \to 0} \frac{2\cancel{h}}{\cancel{h}\sqrt{1+2x+2h} + \sqrt{1+2x}}$$

$$= \frac{2}{\sqrt{1+2x} + \sqrt{1+2x}}$$

$$= \frac{\cancel{2}}{\cancel{2}\sqrt{1+2x}} = \frac{1}{\sqrt{1+2x}}$$

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24) $f(x) = \dfrac{3+x}{1-3x}$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \frac{\dfrac{3+(x+h)}{1-3(x+h)} - \dfrac{3+x}{1-3x}}{h}$$

$$= \lim_{h \to 0} \frac{(3+x+h)(1-3x) - (3+x)(1-3x-3h)}{h(1-3x-3h)(1-3x)}$$

$$= \lim_{h \to 0} \frac{3-9x+x-3xh+h-3x^2-(3-9x-9h+x-3x^2-3hx)}{h(1-3x-3h)(1-3x)}$$

$$= \lim_{h \to 0} \frac{\cancel{3}-\cancel{9x}+\cancel{x}-3\cancel{xh}+h-\cancel{3x^2}-\cancel{3}+\cancel{9x}+9h-\cancel{x}+\cancel{3x^2}+3\cancel{hx}}{h(1-3x-3h)(1-3x)}$$

$$= \lim_{h \to 0} \frac{10\cancel{h}}{\cancel{h}(1-3x-3h)(1-3x)}$$

$$= \lim_{h \to 0} \frac{10}{(1-3x-3h)(1-3x)} = \frac{10}{(1-3x)^2}$$

A

25) $G(t) = \dfrac{4t}{t+1}$

$$G'(t) = \lim_{h \to 0} \frac{G(t+h) - G(t)}{h}$$

$$= \lim_{h \to 0} \frac{\dfrac{4(t+h)}{(t+h)+1} - \dfrac{4t}{t+1}}{h}$$

$$= \lim_{h \to 0} \frac{\dfrac{4(t+h)(t+1) - 4t\left((t+h)+1\right)}{\left[(t+h)+1\right](t+1)}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{4t^2} + \cancel{4ht} + \cancel{4t} + 4h - \cancel{4t^2} - \cancel{4ht} - \cancel{4t}}{h(t+h+1)(t+1)}$$

$$= \lim_{h \to 0} \frac{4\cancel{h}}{\cancel{h}(t+h+1)(t+1)}$$

$$= \lim_{h \to 0} \frac{4}{(t+h+1)(t+1)} = \frac{4}{(t+1)^2}$$

26) $g(t) = \dfrac{1}{\sqrt{t}}$

$$g'(t) = g(t+x) - g(t) \Big/ h$$

$$= \lim_{h \to 0} \frac{\frac{1}{\sqrt{t+h}} - \frac{1}{\sqrt{t}}}{h} = \lim_{h \to 0} \frac{\sqrt{t} - \sqrt{t+h}}{h(\sqrt{t+h})(\sqrt{t})}$$ (7)

$$= \lim_{h \to 0} \left(\frac{\sqrt{t} - \sqrt{t+h}}{h\sqrt{t+h}\sqrt{t}} \cdot \frac{\sqrt{t} + \sqrt{t+h}}{\sqrt{t} + \sqrt{t+h}} \right.$$

$$\lim_{h \to 0} \frac{\cancel{t} - \cancel{t} - \cancel{h}}{\cancel{h}(\sqrt{t+h})(\sqrt{t})(\sqrt{t} + \sqrt{t+h})}$$

$$\lim_{h \to 0} = \frac{-1}{(\sqrt{t+h})(\sqrt{t})(\sqrt{t} + \sqrt{t+h}}$$

$$= \frac{-1}{t(2\sqrt{t})} = -\frac{1}{2t^{3/2}}$$

27 — $f(x) = x^4$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h}$$

$$= \lim_{h \to 0} \frac{(x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4) - x^4}{h}$$

$$= \lim_{h \to 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(4x^3 + 6x^2h + 4xh^2 + h^4)}{\cancel{h}}$$

$$= 4x^3 \; A$$

B.P = 144 , PDF = 174

Q11:

$$Y(t) = 6t^{-9} \Rightarrow Y'(t) = 6(-9)t^{-10} = -54t^{-10} \; A$$

12 - $R(x) = \frac{\sqrt{10}}{x^7} = \sqrt{10}\, x^{-7} \Rightarrow R'(x) = -7\sqrt{10}\, x^{-8}$

$$= \frac{-7\sqrt{10}}{x^8} \; A$$

13 - $F(x) = \left(\frac{1}{2}x\right)^5 = \left(\frac{1}{2}\right)^5 x^5 = \frac{1}{32}x^5$

$$F'(x) = \frac{1}{32} 5x^4 = \frac{5}{32}x^4 \; A$$

14) $f(t) = \sqrt{t} - \frac{1}{\sqrt{t}} = t^{1/2} - t^{-1/2}$

$f'(t) = \frac{1}{2} t^{-1/2} + \frac{1}{2} t^{-3/2} = \frac{1}{2\sqrt{t}} + \frac{1}{2t\sqrt{t}}$

$\frac{t+1}{2t\sqrt{t}}$

15) $A(s) = -\frac{12}{s^5} = -12 s^{-5} = A'(s) \Rightarrow -12(-5 s^{-6})$

$= 60 s^{-6} = 60/s^6$

16) $B(y) = c y^{-6} \Rightarrow B'(y) \Rightarrow c(-6y^{-7})$

$= -6 c y^{-7}$

17) $y = 4\pi^2 \Rightarrow y' = 0$ Since $4\pi^2$ is a constant.

18) $g(u) = \sqrt{2}\, u + \sqrt{3u} = \sqrt{2}\, u + \sqrt{3}\sqrt{u}$

$g'(u) = \sqrt{2}(1) + \sqrt{3} \cdot \frac{1}{2} u^{-1/2}$

$= \sqrt{2} + \frac{\sqrt{3}}{2\sqrt{u}}$

19) $u = \sqrt[5]{t} + 4\sqrt{t^5} = t^{1/5} + 4t^{5/2}$

$u' = \frac{1}{5} t^{-4/5} + 4\left(\frac{5}{2} t^{3/2}\right)$

(10)

$$= \frac{1}{5}t^{-4/5} + 10t^{3/2} = \frac{1}{5\sqrt[5]{t^4}} + 10\sqrt{3}$$

20) $u = \left(\sqrt{x} + \frac{1}{\sqrt[3]{x}}\right)^2 = (\sqrt{x})^2 + 2\sqrt{x}\cdot\frac{1}{\sqrt[3]{x}} + \left(\frac{1}{\sqrt[3]{x}}\right)^2$

$$= x + 2x^{1/2 - 1/3} + \frac{1}{x^{2/3}}$$

$$= x + 2x^{1/6} + x^{-2/3}$$

$$u' = 1 + \cancel{2}\left(\frac{1}{\cancel{6}}x^{-5/6}\right) - \frac{2}{3}x^{-5/3}$$

$$= 1 + \frac{1}{3}x^{-5/6} - \frac{2}{3}x^{-5/3}$$

$$= 1 + \frac{1}{3\sqrt[6]{x^5}} - \frac{2}{3\sqrt[3]{x^5}}$$

37) $y = \frac{r^2}{1+\sqrt{r}}$

$$= \frac{(1+\sqrt{r})(2r) - r^2\left(\frac{1}{2}r^{-1/2}\right)}{(1+\sqrt{r})^2}$$

$$= \frac{2r + 2r^{3/2} - \frac{1}{2}r^{3/2}}{(1+\sqrt{r})^2}$$

(11)

$$= \frac{2r + \frac{3}{2}r^{3/2}}{(1+\sqrt{r})^2} = \frac{\cancel{4r +}}{\cancel{(1+\sqrt{r})^2}}$$

$$= \frac{\frac{4r + 3r^{3/2}}{2}}{(1+\sqrt{r})^2} = \frac{1}{2}\frac{r(4+3r^{1/2})}{(1+\sqrt{r})^2}$$

$$= \frac{r(4+3\sqrt{r})}{2(1+\sqrt{r})^2} \text{ Ans.}$$

38) $y = \frac{cx}{1+cx} \Rightarrow y' = \frac{(1+cx)(c)-(cx)(c)}{(1+cx)^2}$

$$= c + \cancel{c^2x} - \cancel{c^2x} / (1+cx)^2$$

$$= \frac{c}{(1+cx)^2} \text{ Ans.}$$

39) $y = \sqrt[3]{t}(t^2+t+t^{-1}) = t^{1/3}(t^2+t+t^{-1})$

$$= t^{7/3} + t^{4/3} + t^{-2/3}$$

$$y' = \frac{7}{3}t^{4/3} + \frac{4}{3}t^{1/3} - \frac{2}{3}t^{-5/3}$$

$$= \frac{1}{3}t^{-5/3}(7t^{9/3} + 4t^{6/3} - 2)$$

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$$= \frac{7t^3 + 4t^2 - 2}{3t^{5/3}}$$

40) $y = \frac{u^6 - 2u^3 + 5}{u^2} = \frac{\cancel{u^6}}{\cancel{u^2}} - \frac{2\cancel{u^3}}{\cancel{u^2}} + \frac{5}{u^2}$

$y = u^4 - 2u + 5u^{-2} \Rightarrow y' = 4u^3 - 2 - 10u^{-3}$

$y' = 2u^{-3}(2u^6 - u^3 - 5)$

$= \frac{2u^6 - u^3 - 5}{u^3}$

41) $f(x) = \frac{x}{x + c/x} \Rightarrow \frac{(x + c/x) - x(1 - c/x^2)}{\left(x + \frac{c}{x}\right)^2}$

$$= \frac{\cancel{x} + c/x - \cancel{x} + c/x}{\left(\frac{x^2 + c}{x}\right)^2}$$

$$= \frac{2c/x}{\left(\frac{x^2 + c^2}{x^2}\right)} = \frac{2cx}{(x^2 + c)^2}$$

42) $f(x) = \frac{ax+b}{cx+d} \Rightarrow \frac{(cx+d)(a)-(ax+b)(c)}{(cx+d)^2}$ (13)

$$= \frac{\cancel{acx}+ad-\cancel{acx}-bc}{(cx+d)^2}$$

$$= \frac{ad-bc}{(cx+d)^2}$$ Ans.

B.P = 154 , PDF = 183

Q-1) $f(x) = 3x^2 - 2\cos x$

$f'(x) = 6x - 2(-\sin x)$

$= 6x + 2\sin x$ Ans.

2) $f(x) = \sqrt{x}\sin x$

$= \sqrt{x}\cos x + \sin x\left(\frac{1}{2}x^{-1/2}\right)$

$= \sqrt{x}\cos x + \frac{\sin x}{2\sqrt{x}}$ Ans.

3) $f(x) = \sin x + \frac{1}{2}\cot x$

$$f'(x) = \cos x - \frac{1}{2}\csc^2 x$$

4) $y = 2\csc x + 5\cos x$

$$= 2\cos x \cot x - 5\sin x$$

5) $g(t) = t^3 \cos t$

$$= t^3(-\sin t) + \cos t \cdot 3t^2$$

$$= 3t^2 \cos t - t^3 \sin t$$

$$= t(3\cos t - t\sin t)$$

6) $g(t) = 4\sec t + \tan t$

$$g'(t) = 4\sec t \tan t + \sec^2 t$$

41)

$$= \lim_{t\to 0} \frac{\tan 6t}{\sin 2t} = 6t$$

$$= \lim_{t\to 0} \frac{\sin 6t}{t} \cdot \frac{1}{\cos 6t} \cdot \frac{t}{\sin 2t}$$

$$= \lim_{x\to 0} 6\sin t$$

41) $$= \lim_{t\to 0} \frac{\tan 6t}{\sin 2t}$$

$$= \lim_{t\to 0} \left(\frac{\sin 6t}{t} \cdot \frac{1}{\cos 6t} \cdot \frac{t}{\sin 2t} \right)$$

$$= \lim_{t\to 0} \frac{6\sin 6t}{6t} \cdot \lim_{t\to 0} \frac{1}{\cos 6t} \cdot \lim_{t\to 0} \frac{2t}{2\sin 2t}$$

$$= 6 \lim_{t\to 0} \frac{\sin 6t}{6t} \cdot \lim_{t\to 0} \frac{1}{\cos 6t} \cdot \frac{1}{2} \lim_{t\to 0} \frac{2t}{\sin 2t}$$

$$= 6(1) \cdot \frac{1}{1} \cdot \frac{1}{2}(1) = 3$$

42) $$= \lim_{\theta\to 0} \frac{\cos\theta - 1}{\sin\theta} = \lim_{\theta\to 0} \frac{\frac{\cos\theta - 1}{\theta}}{\sin\theta / \theta}$$

$$= \frac{\lim_{\theta\to 0} \frac{\cos\theta - 1}{\theta}}{\lim_{\theta\to 0} \frac{\sin\theta}{\theta}} = \frac{0}{1} = 0$$

43) $$= \lim_{\theta\to 0} \frac{\sin(\cos\theta)}{\sec\theta} = \frac{\sin\left(\lim_{\theta\to 0} \cos\theta\right)}{\lim_{\theta\to 0} \sec\theta}$$

$$= \frac{\sin 1}{1} = \sin 1$$

44)

$$= \lim_{t\to 0} \frac{\sin^2 3t}{t^2} = \lim_{t\to 0}\left(\frac{\sin 3t}{t}\cdot\frac{\sin 3t}{t}\right)$$

$$= \lim_{t\to 0}\frac{\sin 3t}{t}\cdot\lim_{t\to 0}\frac{\sin 3t}{t}$$

$$= \left(\lim_{t\to 0}\frac{\sin 3t}{t}\right)^2 = \left(3\lim_{t\to 0}\frac{\sin 3t}{3t}\right)^2$$

$$= (3\cdot 1)^2 = 9$$

45)

$$= \lim_{\theta\to 0}\frac{\sin\theta}{\theta+\tan\theta} = \lim_{\theta\to 0}\frac{\sin\theta/\theta}{1+\frac{\sin\theta}{\theta}\cdot\frac{1}{\cos\theta}}$$

$$= \frac{\lim_{\theta\to 0}\sin\theta/\theta}{1+\lim_{\theta\to 0}\frac{\sin\theta}{\theta}\cdot\lim_{\theta\to 0}\frac{1}{\cos\theta}}$$

$$= \frac{1}{1+1\cdot 1} = \frac{1}{2}$$

(17)

$$= \lim_{x \to 0} \frac{\sin(x^2)}{x} = \lim_{x \to 0} \left[x \cdot \frac{\sin(x^2)}{x \cdot x} \right]$$

$$= \lim_{x \to 0} x \cdot \lim_{x \to 0} \frac{\sin x^2}{x^2}$$

$$= 0 \cdot 1 = 0$$

47) —

$$= \lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{\sin x - \cos x} = \lim_{x \to \frac{\pi}{4}} \frac{\left(1 - \frac{\sin x}{\cos x}\right) \cdot \cos x}{(\sin x - \cos x) \cdot \cos x}$$

$$= \lim_{x \to \pi/4} \frac{\cos x - \sin x}{(\sin x - \cos x) \cos x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{-1}{\cos x} = \frac{-1}{1/\sqrt{2}}$$

$$= -\sqrt{2}$$

48) —

$$= \lim_{x \to 1} \frac{\sin(x-1)}{x^2 + x - 2} = \lim_{x \to 1} \frac{\sin \cancel{(x-1)}}{(x+2)\cancel{(x-1)}}$$

$$= \lim_{x \to 1} \frac{1}{x+2} \cdot \lim_{x \to 1} \frac{\sin(x-1)}{x-1}$$

$$= \frac{1}{3} \cdot 1 = \frac{1}{3}$$

(Book # 162)(PDF : 190)

37)

$$y = \cot^2(\sin\theta) = [\cot(\sin\theta)]^2$$

$$= 2[\cot(\sin\theta)] \cdot \frac{d}{d\theta}[\cot(\sin\theta)]$$

$$= 2\cot(\sin\theta) \cdot [-\cancel{\cos^2}\,\csc^2(\sin\theta) \cdot \cos\theta]$$

$$= -2\cos\theta \cot(\sin\theta)\operatorname{cosec}^2(\sin\theta)$$

38)

$$y = \left(ax + \sqrt{x^2+b^2}\right)^{-2}$$

$$y' = -2\left(ax + \sqrt{x^2+b^2}\right)^{-3} \cdot \left(a + \frac{1}{2}(x^2+b^2)^{-1/2}(2x)\right)$$

$$= \frac{-2\left(a + x/\sqrt{x^2+b^2}\right)}{\left(ax + \sqrt{x^2+b^2}\right)^3}$$

39) $y = \left[x^2 + (1-3x)^5\right]^3$

$y' = 3\left[x^2 + (1-3x)^5\right]^2 \left(2x + 5(1-3x)^4(-3)\right)$

$= 3\left[x^2 + (1-3x)^5\right]^2 \left(2x - 15(1-3x)^4\right)$

40) $y = \sin\left(\sin(\sin x)\right)$

$y' = \cos\left[\sin(\sin x)\right] \frac{d}{dx}\left(\sin(\sin x)\right)$

$= \cos\left[\sin(\sin x)\right] \cdot \cos(\sin x)\cos x$

41) $y = \sqrt{x + \sqrt{x}}$

$y' = \frac{1}{2}\left(x + \sqrt{x}\right)^{-1/2}\left(1 + \frac{1}{2}(x)^{-1/2}\right)$

$= \frac{1}{2\sqrt{x + \sqrt{x}}}\left(1 + \frac{1}{2\sqrt{x}}\right)$

42) $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

$y' = \frac{1}{2}\left(x + \sqrt{x + \sqrt{x}}\right)^{-1/2}\left(1 + \frac{1}{2}\left(x + \sqrt{x}\right)^{-1/2}\left(1 + \frac{1}{2}x^{-1/2}\right)\right)$

43) (20)

$g(x) = (2\gamma \sin \gamma x + n)^p$

$g'(x) = p(2\gamma \sin \gamma x + n)^{p-1}(2r \cos \gamma x \cdot r)$

$= p(2r \sin r x + n)^{p-1}(2\gamma^2 \cos \gamma x)$

44) $y = \cos^4(\sin^3 x)$

$y' = [\cos(\sin^3 x)]^4$

$= 4[\cos(\sin^3 x)]^3(-\sin(\sin^3 x))\, 3\sin^2 x \cos x$

$= -12 \sin^2 x \cos x \cos^3(\sin^3 x) \sin(\sin^3 x)$

46) $y = [x + (x + \sin^2 x)^3]^4$

$y' = 4[x + (x + \sin^2 x)^3]^3 \cdot [1 + 3(x + \sin^2 x)^2 \cdot (1 + 2\sin x \cos x)]$

47) $h(x) = \sqrt{x^2 + 1} = \frac{1}{2}(x^2+1)^{-1/2}(2x)$

$= \frac{x}{\sqrt{x^2+1}} \Rightarrow h''(x) \Rightarrow$

(21)

$$h''(x) = \frac{\sqrt{x^2+1}\cdot 1 - x\left[\frac{1}{2}(x^2+1)^{-1/2}\cdot(2x)\right]}{\left(\sqrt{x^2+1}\right)^2}$$

$$= \frac{(x^2+1)^{-1/2}\left[(x^2+1) - x^2\right]}{(x^2+1)}$$

$$= \frac{1}{(x^2+1)^{3/2}} \quad \#$$

48) —

$$y = \sin^2(\pi t) = \left[\sin(\pi t)\right]^2$$

$$y' = 2\left[\sin(\pi t)\right]\cos(\pi t)\cdot\pi$$

$$= \pi\sin(2\pi t)$$

$$y'' = \pi\cos(2\pi t)\cdot 2\pi = 2\pi^2\cos(2\pi t) \quad \#$$

49) —

$$H(t) = \tan 3t \Rightarrow H't \Rightarrow 3\sec^2 3t$$

$$= 2\cdot 3\sec 3t\,\frac{d}{dt}(\sec 3t)$$

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$$= 6\sec 3t\,(3\sec 3t\ \tan 3t) = 18\sec^2 3t\ \tan t$$

50)

$$y = \frac{4x}{\sqrt{x+1}}$$

$$y' = \frac{\sqrt{x+1}\,(4) - \cancel{4}x\left(\frac{1}{\cancel{2}}(x+1)^{-1/2}\right)}{\left(\sqrt{x+1}\right)^2}$$

$$= \frac{4\sqrt{x+1} - 2x/\sqrt{x+1}}{\left(\sqrt{x+1}\right)^2}$$

$$= \frac{4(x+1) - 2x}{\left(\sqrt{x+1}\right)^{3/2}}$$

$$= \frac{4x+4-2x}{\left(\sqrt{x+1}\right)^{3/2}}$$

$$= \frac{2x+4}{\left(\sqrt{x+1}\right)^{3/2}}$$

$$y'' = \frac{(x+1)^{3/2}\cdot\cancel{2}^{2} - (2x+4)\cdot\frac{3}{2\cancel{2}}(x+1)^{1/2}}{\left[(x+1)^{3/2}\right]^2}$$

$$= \frac{(x+1)^{1/2}\left[2(x+1) - 3(x+2)\right]}{(x+1)^3}$$

$$= \frac{2x+2-3x-6}{(x+1)^{5/2}}$$

$$= \frac{-x-4}{(x+1)^{5/2}}$$

S1)

$y = (1+2x)^{10} = y' = 10(1+2x)^9\cdot 2$

$= 20(1+2x)^9$

At $(0,1)$, $y' = 20(1+0)^9$

$= 20(1) = 20$

And equation of tangent line is:

$y-1 = 20(x-0)$

$y = 20x+1$

52) $y = \sin x + \sin^2 x$

$y' = \cos x + 2\sin x \cos x$

At $(0,0)$, $y' = \cancel{0}\ 1$

And the equation of tangent line is:

$y - y_1 = m(x - x_1)$

$y - 0 = m(x - 0)$

$y = 1(x)$

$y = x$

53) $y = \sin(\sin x)$

$y' = \cos(\sin x) \cdot \cos x$

At $(\pi, 0)$:

$y' = \cos(\sin \pi)\cos \pi$

$y' = \cos(0)(-1)$

$= (1)(-1) = -1$

Equation of Tangent line is:

$y - 0 = -1(x - \pi)$

$y = -x + \pi$

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54)

$y = \sqrt{5+x^2}$

$y' = \frac{1}{\cancel{2}}(5+x^2)^{-1/2}(\cancel{2}x)$

$= \frac{x}{\sqrt{5+x^2}}$

At $(2, 3)$:

$y' = \frac{2}{3}$, Equation of Tangent line is:

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{2}{3}(x - 2)$

$y - 3 = \frac{2}{3}x - \frac{4}{3}$

$y = \frac{2}{3}x - \frac{4}{3} + 3$

$y = \frac{2}{3}x - \frac{4+9}{3} = \frac{2}{3}x + \frac{5}{3}$

55) (a)

$y = f(x) = \tan\left(\frac{\pi}{4}x^2\right)$

$y = f'(x) = \sec^2\left(\frac{\pi}{4}x^2\right)\left(2 \cdot \frac{\pi}{\cancel{4}}x\right)$

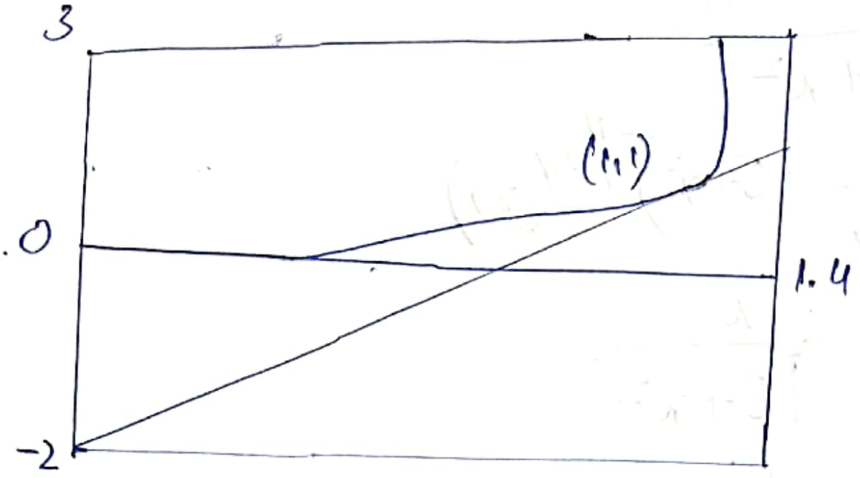
Slope of the Tangent at $(1, 1)$:

$f'(x) = \sec^2\frac{\pi}{4}\left(\frac{\pi}{2}\right) = \cancel{2} \cdot \frac{\pi}{\cancel{2}} = \pi$

Equation is:

$y - 1 = \pi(x - 1) \Rightarrow y = \pi x - \pi + 1$

(b)



8(a)

$f(x) = \sin 2x - 2\sin x$

$f'(x) = 2\cos 2x - 2\cos x$

$= 4\cos^2 x - 2\cos x - 2$

and, $4\cos^2 x - 2\cos x - 2 = 0$

$(\cos x - 1)(4\cos x + 2)$

$\cos x - 1 = 0$

$\cos x = 1$

or $\cos x = -\frac{1}{2}$

So, $x = 2n\pi$ or $(2n+1)\pi \pm 3$

n any integer.

B.P = 169 , PDF = 198

5) — $x^3+y^3=1$

$$\frac{d}{dx}(x^3+y^3)=\frac{d}{dx}(1)$$

$$3x^2+3y^2\cdot y'=0$$

$$3y^2\cdot y'=-3x^2$$

$$y'=\frac{-\cancel{3}x^2}{\cancel{3}y^2}=-\frac{x^2}{y^2}$$ A —

6) — $2\sqrt{x}+\sqrt{y}=3$

$$\frac{d}{dx}(2\sqrt{x}+\sqrt{y})=\frac{d}{dx}(3)$$

$$\cancel{2}\cdot\frac{1}{\cancel{2}}x^{-1/2}+\frac{1}{2}y^{-1/2}\cdot y'=0$$

$$\frac{1}{\sqrt{x}}+\frac{y'}{2\sqrt{y}}=0$$

$$\frac{y'}{2\sqrt{y}}=-\frac{1}{\sqrt{x}}$$

$$y'=-\frac{2\sqrt{y}}{\sqrt{x}}$$ A —

(7)

$$x^2 + xy - y^2 = 4$$

$$\frac{d}{dx}\left(x^2 + xy - y^2\right) = \frac{d}{dx}(4)$$

$$2x + x \cdot y' + y - 2yy' = 0$$

$$xy' - 2yy' = -2x - y$$

$$y'(x - 2y) = -2x - y$$

$$y' = \frac{-2x - y}{x - 2y} = \frac{2x + y}{2y - x}$$

(8)

$$2x^3 + x^2y - xy^3 = 2$$

$$\frac{d}{dx}\left(2x^3 + x^2y - xy^3\right) = \frac{d}{dx}(2)$$

$$6x^2 + x^2 \cdot y' + y(2x) - \left[x \cdot 3y^2 \cdot y' + y^3\right] = 0$$

$$x^2y' - 3xy^2y' = -6x^2 - 2yx + y^3$$

$$y'(x^2 - 3xy^2) = -6x^2 - 2xy + y^3$$

$$y' = \frac{-6x^2 - 2xy + y^3}{x^2 - 3xy^2}$$

9) $x^4(x+y) = y^2(3x-y)$

$$\frac{d}{dx}\left[x^4(x+y)\right] = \frac{d}{dx}\left[y^2(3x-y)\right]$$

$$x^4(1+y') + (x+y)4x^3 = y^2(3-y') + (3x-y)\cdot 2yy'$$

$$x^4 + x^4y' + 4x^4 + 4x^3y = 3y^2 - y^2y' + 6xyy' - 2y^2y'$$

$$5x^4 + 4x^3y + x^4y' = 3y^2 - y^2y' + 6xyy' - 2y^2y'$$

$$5x^4 + 4x^3y + x^4y' = 3y^2 + 6xyy' - 3y^2y'$$

$$3y^2y' + x^4y' - 6xyy' = -5x^4 - 4x^3y + 3y^2$$

$$y'(3y^2 + x^4 - 6xy) = -5x^4 + 4x^3y + 3y^2$$

$$y' = \frac{3y^2 - 5x^4 + 4x^3y}{x^4 + 3y^2 - 6xy}$$

10) $y^5 + x^2y^3 = 1 + x^4y$

$$\frac{d}{dx}\left(y^5 + x^2y^3\right) = \frac{d}{dx}\left(1 + x^4y\right)$$

$$5y^4\cdot y' + x^2\cdot 3y^2\cdot y' + y^3(2x) = x^4\cdot y' + y\cdot 4x^3$$

$$5y^4y' + 3x^2y^2y' - x^4y' = 4x^3y - 2xy^3$$

$$y'(5y^4 + 3x^2y^2 - x^4) = 4x^3y - 2xy^3$$

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$$y' = \frac{4x^3y - 2xy^3}{5y^4 + 3x^2y^2 - x^4}$$

11) $x^2y^2 + x\sin y = 4$

$$\frac{d}{dx}\left(x^2y^2 + x\sin y\right) = \frac{d}{dx}(4)$$

$$x^2 \cdot 2y \cdot y' + y^2 \cdot 2x + x\cos y\, y' + \sin y = 0$$

$$y'(2x^2y + x\cos y) = -2xy^2 - \sin y$$

$$y' = \frac{-2xy^2 - \sin y}{2x^2y + x\cos y}$$

12) $1 + x = x\sin(xy^2)$

$$\frac{d}{dx}(1 + x) = \frac{d}{dx}\left(x\sin(xy^2)\right)$$

$$1 = [\cos(xy^2)](x \cdot 2yy' + y^2)$$

$$1 = 2xy\cos(xy^2)y' + y^2\cos(xy^2)$$

$$1 - y^2\cos(xy^2) = 2xy\cos(xy^2)y'$$

$$y' = \frac{1 - y^2\cos(xy^2)}{2xy\cos(xy^2)}$$

13)

$$4\cos x \sin y = 1$$

$$\frac{d}{dx}(4\cos x \sin y) = \frac{d}{dx}(1)$$

$$4[\cos x \cdot \cos y \cdot y' + \sin y \cdot (-\sin x)] = 0$$

$$4\cos x \cos y\, y' = 4\sin x \sin y$$

$$y' = \frac{\cancel{4}\sin x \sin y}{\cancel{4}\cos x \cos y}$$

$$y' = \tan x \tan y$$

14)

$$y\sin(x^2) = x\sin(y^2)$$

$$\frac{d}{dx}(y\sin(x^2)) = \frac{d}{dx}[x\sin(y^2)]$$

$$y \cdot \cos x^2 \cdot 2x + \sin x^2 \cdot y' = x\cos y^2 \cdot 2yy' + \sin y^2$$

$$2xy\cos x^2 + y'\sin x^2 = 2xy\cos y^2 y' + \sin y^2$$

$$y'\sin x^2 - 2xy\cos y^2 y' = \sin y^2 - 2xy\cos x^2$$

$$y'(\sin x^2 - 2xy\cos y^2) = \sin y^2 - 2xy\cos x^2$$

$$y' = \frac{\sin y^2 - 2xy\cos x^2}{\sin x^2 - 2xy\cos y^2}$$

15)

$$\tan\left(\frac{x}{y}\right) = x + y$$

$$\frac{d}{dx}\left[\tan\left(\frac{x}{y}\right)\right] = \frac{d}{dx}(x+y)$$

$$\sec^2\left(\frac{x}{y}\right) \cdot \frac{y - xy'}{y^2} = 1 + y'$$

$$y\sec^2(x/y) - x\sec^2(x/y)\,y' = y^2 + y^2 y'$$

$$y\sec^2(x/y) - y^2 = y^2 y' + x\sec^2(x/y)\,y'$$

$$y\sec^2\left(\frac{x}{y}\right) - y^2 = y'\left(y^2 + x\sec^2\left(\frac{x}{y}\right)\right)$$

$$y' = \frac{y\sec^2(x/y) - y^2}{y^2 + x\sec^2\left(\frac{x}{y}\right)}$$

16)

$$\sqrt{x+y} = 1 + x^2y^2$$

$$\frac{d}{dx}\left(\sqrt{x+y}\right) = \frac{d}{dx}\left(1 + x^2y^2\right)$$

$$\frac{1}{2}(x+y)^{-1/2}(1+y') = x^2 \cdot 2yy' + y^2 \cdot 2x$$

$$\frac{1}{2\sqrt{x+y}} + \frac{y'}{2\sqrt{x+y}} = 2x^2yy' + 2xy^2$$

$$1+y' = 4x^2y\sqrt{x+y}\;y' + 4xy^2\sqrt{x+y}$$

$$y' - 4x^2y\sqrt{x+y}\;y' = 4xy^2\sqrt{x+y} - 1$$

$$y'\left(1 - 4x^2y\sqrt{x+y}\right) = 4xy^2\sqrt{x+y} - 1$$

$$y' = \frac{4xy^2\sqrt{x+y} - 1}{1 - 4x^2y\sqrt{x+y}}$$

17)

$$\cancel{\sqrt{xy} = 1+x^2y}$$

$$\cancel{\frac{d}{dx}\left(\sqrt{xy}\right) = \frac{d}{dx}\left(1+x^2y\right)}$$

$$\cancel{\frac{1}{2}(xy)^{-1/2}\cdot y' = x^2\cdot y' + y\cdot 2x}$$

$$\frac{1}{2\sqrt{xy}}\,y' = x^2y' + 2xy$$

$$\sqrt{xy} = 1+x^2y$$

$$\frac{d}{dx}\left(\sqrt{xy}\right) = \frac{d}{dx}\left(1+x^2y\right)$$

$$\frac{1}{2}(xy)^{-1/2}(xy'+y) = x^2y' + y\cdot 2x$$

$$\frac{x}{2\sqrt{xy}}y' + \frac{y}{2\sqrt{xy}} = x^2y' + 2xy$$

$$y'\left(\frac{x}{2\sqrt{xy}} - x^2\right) = 2xy - \frac{y}{2\sqrt{xy}}$$

$$y'\left(\frac{x - 2x^2\sqrt{xy}}{2\sqrt{xy}}\right) = \frac{4xy\sqrt{xy} - y}{2\sqrt{xy}}$$

$$y' = \frac{(4xy\sqrt{xy} - y)(\cancel{2\sqrt{xy}})}{(x - 2x^2\sqrt{xy})(\cancel{2\sqrt{xy}})}$$

$$y' = \frac{4xy\sqrt{xy} - y}{x - 2x^2\sqrt{xy}}$$

18)

$$\tan(x-y) = \frac{y}{1+x^2}$$

$$(1+x^2)\tan(x-y) = y$$

$$(1+x^2)\sec^2(x-y)\cdot(1-y') + \tan(x-y)2x = y'$$

$$(1+x^2)\sec^2(x-y) - (1+x^2)\sec^2(x-y)y' + 2x\tan(x-y) = y'$$

$$(1+x^2)\sec^2(x-y) + 2x\tan(x-y) = [1+(1+x^2)\sec^2(x-y)] \cdot y'$$

$$y' = \frac{(1+x^2)\sec^2(x-y) + 2x\tan(x-y)}{1+(1+x^2)\sec^2(x-y)}$$

19)

$$y\cos x = 1 + \sin(xy)$$

$$\frac{d}{dx}(y\cos x) = \frac{d}{dx}(1 + \sin(xy))$$

$$y(-\sin x) + \cos x \cdot y' = \cos(xy) \cdot (xy' + y)$$

$$\cos x y' - x\cos(xy)\, y' = y\sin x + y\cos(xy)$$

$$y'[\cos x - x\cos(xy)] = y\sin x + y\cos(xy)$$

$$y' = \frac{y\sin x + y\cos(xy)}{\cos x - x\cos(xy)}$$

20)

$$\sin x + \cos y = \sin x \cos y$$

$$\frac{d}{dx}(\sin x + \cos y) = \frac{d}{dx}(\sin x \cos y)$$

$$\cos x - \sin y\, y' = \sin x(-\sin y\, y') + \cos y \cos x$$

$$(\sin x \sin y - \sin y)y' = \cos x \cos y - \cos x$$

$$y' = \frac{\cos x(\cos y - 1)}{\sin y(\sin x - 1)}$$ Ans

B.P = 219 , PDF = 248

(1)

$$f(x) = 5 - 12x + 3x^2 \quad , \quad [1, 3]$$

Now: $f(1) = 5 - 12(1) + 3(1)^2$

$= 5 - 12 + 3$

$= -4$

Also: $f(3) = 5 - 12(3) + 3(3)^2$

$= 5 - 36 + 27$

$= -4$

$$\boxed{f(a) = f(b) = -4}$$

$$f(x) = 5 - 12x + 3x^2$$

$$f'(x) = \cancel{5} - 12 + 6x$$

Now.

$$f'(c) = 0$$

$$-12 + 6c = 0$$

$$6c = 12$$

$$\boxed{c = 2}$$

$\because \; c = 2 \in [1,3]$

2) —— $f(x) = x^3 - x^2 - 6x + 2 \;,\; [0,3]$

$$f(0) = 0 - 0 - 0 + 2$$

$$= 2 \text{ —— i}$$

Also;

$$f(3) = (3)^3 - (3)^2 - 6(3) + 2$$

$$= 27 - 9 - 18 + 2$$

$$= 2 \text{ —— ii}$$

$$\boxed{f(a) = f(b) = 2}$$

Now.

$$f'(x) = 3x^2 - 2x - 6$$

Let

$$f'(c) = 0$$

$$= 3c^2 - 2c - 6$$

Let $a=3$, $b=-2$, $c=-6$

$$= \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{4+72}}{6}$$

$$= \frac{2 \pm \sqrt{76}}{6}$$

$$= \frac{2 \pm 2\sqrt{19}}{6}$$

$$= \frac{\cancel{2}(1 \pm \sqrt{19})}{\cancel{6}}$$

$$= \frac{1 \pm \sqrt{19}}{3}$$

$$c = \frac{1 \pm \sqrt{19}}{3} \in [0, 3]$$

(3) —

$$f(x) = \sqrt{x} - \frac{1}{3}x, \quad [0,9]$$

$$f(0) = \sqrt{0} - \frac{1}{3}(0)$$

$$= 0 \text{ ———— i}$$

Also:

$$f(9) = \sqrt{9} - \frac{1}{\cancel{3}}(\cancel{9})^{3}$$

$$= 3 - 3$$

$$= 0 \text{ ———— ii}$$

$$f(a) = f(b) = 0$$

So,

$$f(x) = \sqrt{x} - \frac{1}{3}x$$

$$= \frac{1}{2}(x)^{-1/2} - \frac{1}{3}$$

$$= \frac{1}{2\sqrt{x}} - \frac{1}{3}$$

Let.

$$f'(c) = 0$$

$$\frac{1}{2\sqrt{c}} - \frac{1}{3} = 0$$

$$\frac{3-2\sqrt{c}}{6\sqrt{c}} = 0$$

$$3-2\sqrt{c} = 0$$

$$3 = 2\sqrt{c}$$

$$\sqrt{c} = \frac{3}{2}$$

$$\boxed{c = \frac{9}{4}}$$

$$c = \frac{9}{4} \in [0,9]$$ ——— •

(4) ——— • $f(x) = \cos 2x$, $\left[\frac{\pi}{8}, \frac{7\pi}{8}\right]$

5) —

$f(x) = 1 - x^{2/3}$

$f(-1) = 1 - (-1)^{2/3}$

$= 1 - 1 = 0$ ——— i

Also:

$f(1) = 1 - (1)^{2/3}$

$= 1 - 1 = 0$ ——— ii

$f'(x) = -\frac{2}{3} x^{-1/3}$

So,

$f'(c) = 0$ Has no solution

This Doesn't contradict Rolle's Theorem.

6) — $f(x) = \tan x$

$f(0) = \tan(0)$

$= 0$ ——— i

Also —

$f(\pi) = \tan x$

$= \tan \pi$ ——— ii

$f'(x) = \tan x$

$= \sec^2 x \geq 1$

So, $f'(c)=0$ Has No Solution.

11)

$f(x) = 3x^2+2x+5$, $[-1, 1]$

$f'(x) = 6x+2$

$f'(c) = 6c+2$

We Know that.

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$6c+2 = \frac{f(1)-f(-1)}{1-(-1)} \quad \text{———} \quad (i)$$

Let $f(1) = 3(1)^2+2(1)+5$

$= 3+2+5$

$f(1) = 10$

$f(-1) = 3(-1)^2+2(-1)+5$

$= 3-2+5$

$f(-1) = 6$

putting these values in eq (i)

$$6c+2=\frac{10-6}{2}$$

$$6c+2=\frac{\cancel{4}}{\cancel{2}}$$

$$6c+2=2$$

$$6c=0$$

$c=0$ which is in $(-1,1)$ #.

(12)

$$f(x)=x^3+x-1 \quad , [0,2]$$

$$f(0)=0+0-1$$
$$=-1 \quad \text{i}$$

$$f(2)=8+2-1$$
$$=10-1$$
$$=9 \quad \text{ii}$$

$$f'(x)=3x^2+1$$
$$f'(c)=3c^2+1$$

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

$$3c^2+1 = \frac{9-(-1)}{2}$$

$$3c^2 = 5-1$$

$$c^2 = \frac{4}{3} = \pm\frac{2}{\sqrt{3}}$$ but only

$\frac{2}{3}$ is in $(0,2)$

13)

$$f(x) = \sqrt[3]{x} \quad , [0,1]$$

$$f(0) = \sqrt[3]{0}$$

$$= 0 \text{ ——— i}$$

$$f(1) = \sqrt[3]{1}$$

$$= 1 \text{ ——— ii}$$

$$f'(x) = \frac{1}{3x^{2/3}}$$

$$f'(c) = \frac{1}{3c^{2/3}}$$

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$\frac{1}{3c^{2/3}} = \frac{1-0}{1}$$

$$\frac{1}{3c^{2/3}} = 1$$

$$c^{2/3} = \frac{1}{3}$$

$$c^2 = \left(\frac{1}{3}\right)^3$$

$$c^2 = \frac{1}{27} = \pm\sqrt{\frac{1}{27}} = \pm\frac{\sqrt{3}}{9}$$

but only $\frac{\sqrt{3}}{9}$ is in $(0,1)$ ~~f~~.

14)

$$f(x) = \frac{x}{x+2} \quad , \quad [1,4]$$

$$f(1) = \frac{1}{1+2} = \frac{1}{3} \text{ —— i}$$

$$f(4) = \frac{4}{4+2} = \frac{\cancel{4}}{\cancel{6}} = \frac{2}{3} \text{ —— ii}$$

$$f'(x) = \frac{\cancel{x}+2-\cancel{x}}{(x+2)^2} = \frac{2}{(x+2)^2}$$

$$f'(c) = \frac{2}{(c+2)^2}$$

(46)

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$\frac{2}{(c+2)^2} = \frac{\frac{2}{3}-\frac{1}{3}}{4-1}$$

$$\frac{2}{(c+2)^2} = \frac{\frac{2-1}{3}}{4-1}$$

$$\frac{2}{(c+2)^2} = \frac{\frac{1}{3}}{3}$$

$$\frac{2}{(c+2)^2} = \frac{1}{9}$$

$$18 = (c+2)^2$$

$$c+2 = \sqrt{18}$$

$$c = -2 \pm 3\sqrt{2}$$

$$= -2+3\sqrt{2} \simeq 2.24 \text{ is}$$

in $[1,4]$