Digital Signal Processing (DSP) Course

IIR filter design

by Zavareh Bozorgasl

Outline

Subject Page number

1. Brief problem statement

This homework is about designing IIR and FIR digital filters. In the report, we use LPF and BPF for lowpass filter and bandpass filter, respectively. In the first part, we are supposed to design a lowpass and 2 bandpass filters with the given specifications in the problem statement. Then, by comparing the results in the excel file, and looking at the different figures of a filter (magnitude, phase, group delay, impulse response), we can have some observations.

For the second BPF, I used fstop1 = 9 kHz, and fstop2 = 21 kHz, all other parameters are similar to the Bandpass filter #1.

We investigate some of IIR and FIR filter design methods. We don't go into some details, for example, for designing IIR filters, it is easy to convert an analog filter into a digital IIR filter (Invariance impulse response, bilinear transformation). Here, we mostly see the final result in terms of their parameters and the responses.

2. Some notes on the computations

Here I give some comments on the computations of cost, minimum pole distance to the unit circle, and the maximum group delay.

We can see the number of additions, number of states, and number of multiplications by the **cost** function in Matlab. About maximum radius, instead of doing by hand calculations, we can compute the poles by **zpk** function; then, by computing the absolute value of each pole (which is indeed the radius of each pole), we can compute the maximum of the vector, which is the maximum radius. By subtracting this number from 1, we can obtain the minimum distance to the point of instability (i.e., the unit circle). Moreover, we can compute the maximum in passband group delay by the **grpdelay** function is in a below piece of code.

```
[b,a] = sos2tf(IIR_filt.Coefficients); %SOS to numerator and denominator
 IIR = dsp.IIRFilter('Numerator',b,...
  'Denominator',a,'Structure','Direct form II');%Specify the configuration (Direct form I or II)
 cost(IIR) % Cost of the filter
 %info(IIR_filt)
 [z,p,k] = zpk(IIR_filt);%Zero poles of IIR filter
 Max_radius = max(abs(p))% The maximum radius of the poles
 Dis_circle = 1- Max_radius       %Distance to the unit circle for the pole with max radius
 grp_samples = max(grpdelay(IIR_filt))% Computing the number of samples of group delay
 grp_time = (max(grpdelay(IIR_filt))/Fs)*10^6 % Converting the samples to microSeconds
```

In addition, if we want to transfer second order sections (SOS) of IIR filter to a numerator and a denominator, we can use **sos2tf** function which gets the object made by **designfilt** function.

```
%Convert the second order section coefficients to tranfer function format
%Here b is the numerator and a is the denominator
[b,a] = sos2tf(filterObj.Coefficients);
```

I considered the cost of multipliers (which includes additions) as 2.5. By changing it, all the costs will be updated.

3. A comparison of FIR and IIR filters

In summary, in Table 1, we can see some differences among different filter types.

Table 1. Difference between FIR and IIR filters (a nutshell) [1]

Basis for Comparison	FIR Filter	IIR Filter
Stands for	Finite Impluse Response	Infinite Impulse Response
Nature	Non-recursive	Recursive
Computational Efficiency	Less	Comparatively more
Usage	Difficult	Quite easy
Feedback	Absent	Present
Stability	More	Less
Requirement to generate current output	Present and past samples of input.	Present and past samples of input along with past output.
Delay offered	High	Comparatively lower
Transfer function	Only zeros are present.	Both poles and zeros are present.
Memory requirement	More	Less
Sensitivity	Less	Comparatively more
Resolution offered at low frequencies	Less	More
Controllability	Easy	Quite Difficult

4. Investigation of plots and the excel file data

Figure 1 shows the magnitude of different IIR BPFs (for the BPF with the given specification). Figure 2 shows the passband of the magnitude of different IIR BPFs. Figure 3 presents the phase of these filters.

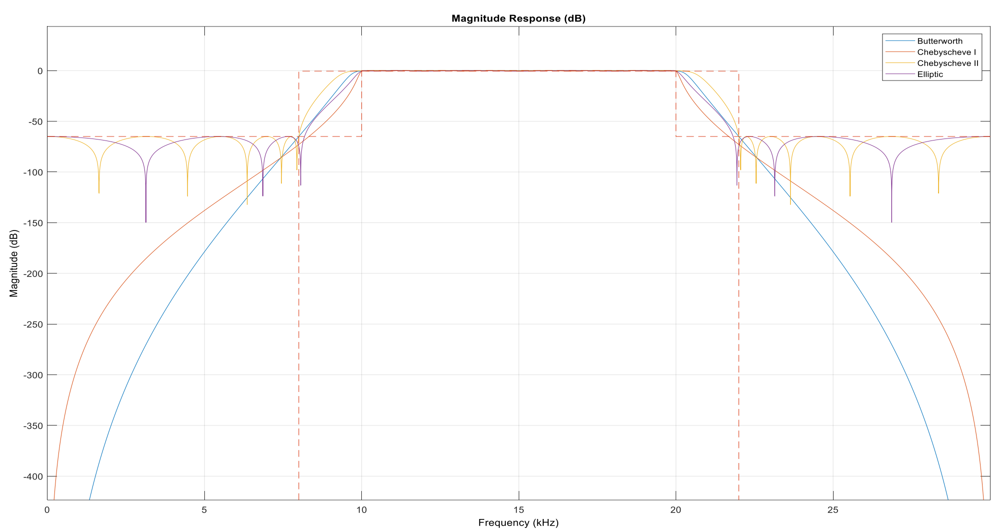

Fig. 1 The magnitude of different IIR BPFs.

The frequency response of the Butterworth filter has no ripples in the passband and the stopband. Therefore, it is called a maximally flat filter. The advantage of Butterworth filters is the smooth, monotonically decreasing frequency response in the transition region. A disadvantage is that this increases the cost of filter (as we can see from the excel file).

The frequency response of the Chebyshev I filter has a norrower transition range than the frequency response of the Butterworth filter which results in a passband with more ripples. The frequency response characteristics of Chebyshev I filters have an equiripple magnitude response in the passband, monotonically decreasing magnitude response in the stopband, and a sharper rolloff in the transition region as compared to Butterworth filters.

We see from this figure that Chebyshev I has more roll off in the transient band, then, are Elliptic, Butterworth, and Chebyshev II. Elliptic has ripples in both passband and stopband, which leads to using the maximum degree of freedom. To remind, we had the same discussion for the FIR filter design by the Kaiser window method and the Parks-McCellan algorithm.

Indeed, the Kaiser window method is not equiripple in both the passband and the stopband. In other words, the Kaiser window method cannot efficiently use all the degree of freedom for the design. We see that the filter which is designed by the Parks-McCellan algorithm is equiripple in both the passband and the stopband. Hence, the filter which is designed by Parks-McCellan has less cost and also the group delay.

The selection of the window function for a FIR filter is similar to the choice between Chebyshev and Butterworth IIR filters where you have to choose between side lobes near the cutoff frequencies and the width of the transition region.

With this is mind, we can have some intuition about the value of costs and also the phase (group) delay of each filter in the excel sheet. Indeed, these matches with the numbers that we attained in the excel file. Elliptic has the least value of cost (because it uses all the available freedom in the passband and stopband), Chebyshev I and II are similar to some extent, but Chebyshev II has more group delay than Chebyshev I, which mostly might be because of its sharper roll off in the transient region.

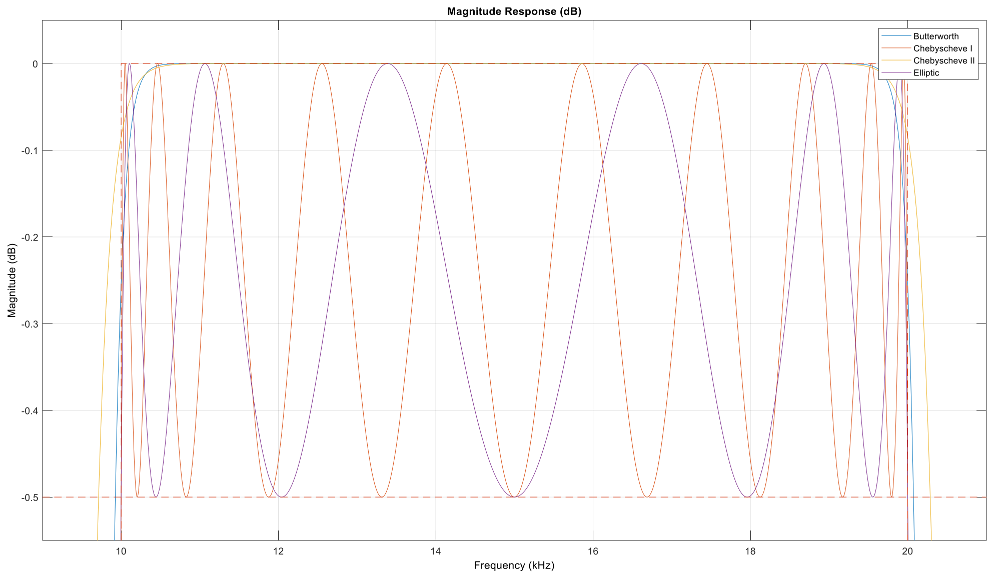

Fig. 2 The passband of the magnitude of different IIR BPFs.

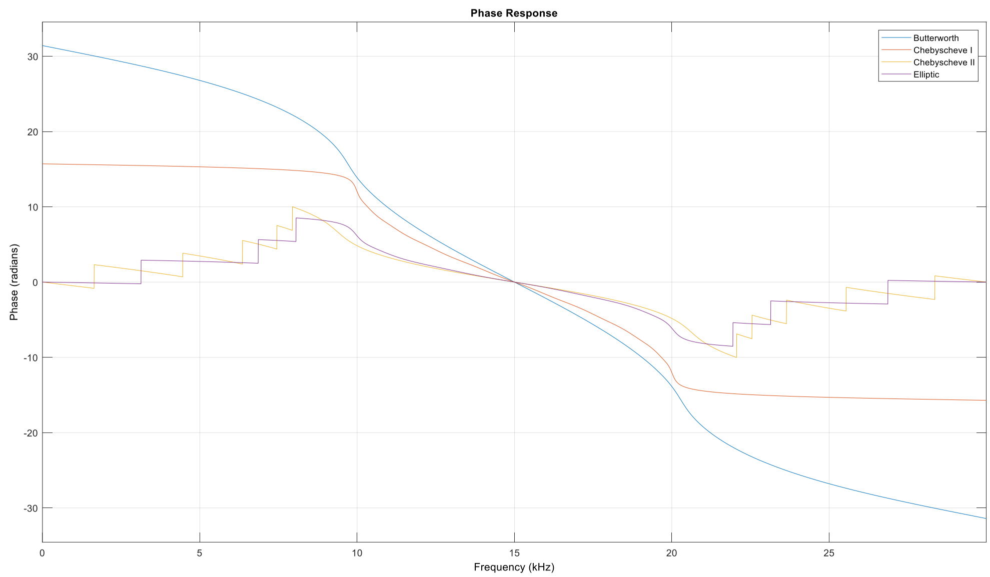

Fig. 3 The phase of different IIR BPFs.

A disadvantage of IIR filters is the nonlinear phase response. IIR filters are well suited for applications that require no phase information, for example, for monitoring the signal amplitudes (in applications like speech processing). Moreover, the phase of IIR filters is linear in some frequencies, which may fit our application. In general, FIR filters are better suited for applications that require a linear phase response [2].

Figures 4 and 5 show the magnitude and the passband of LPF IIR filters, respectively. Figure 6 presents their corresponding phases.

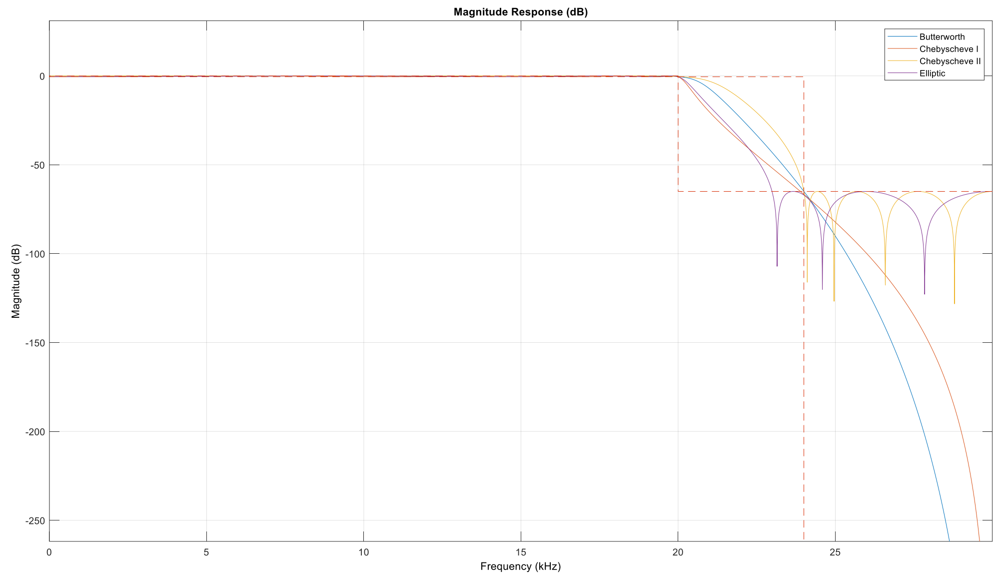

Fig. 4 The magnitude and the passband of LPF IIR filters.

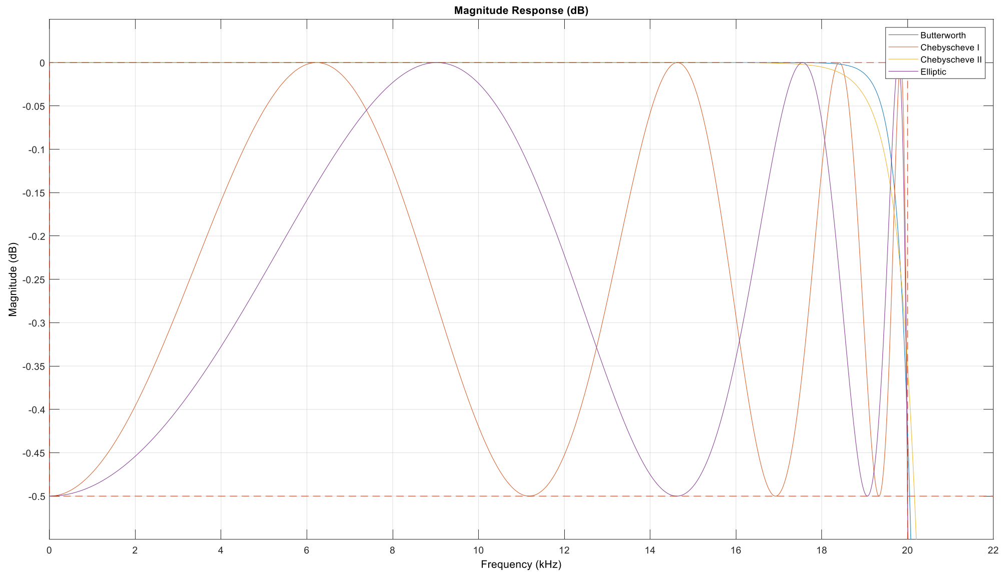

Fig. 5 The passband of LPF IIR filters

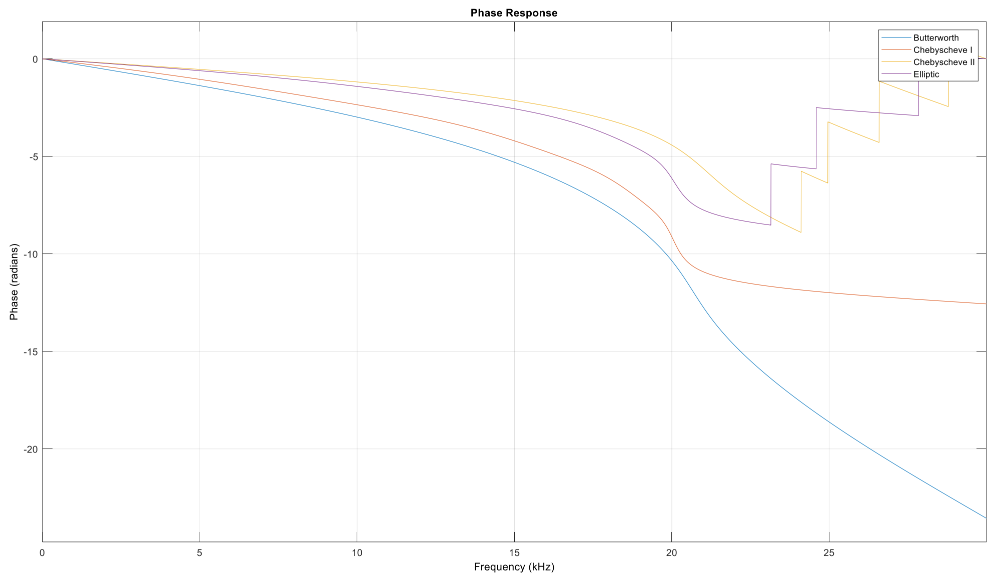

Fig. 6 The phase of LPF IIR filters

Figure 7, which shows all FIR and IIR filters for the bandpass filter 2, has some good points.

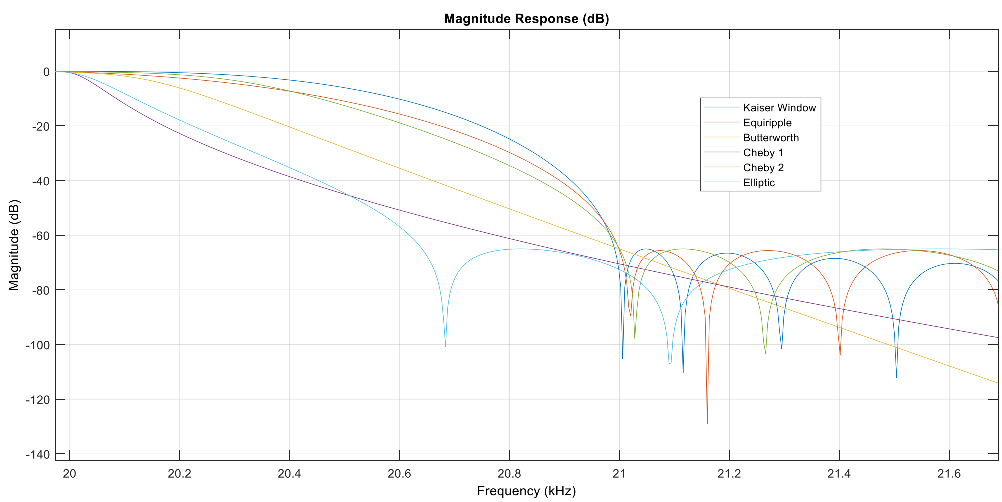

Fig. 7 BPF FIR and IIR filters

By using IIR filters, we'll be able to create frequency responses that we cannot create with FIR filters, e.g., shaper cutoffs and peaks. As a side note, the impulse response h[n] of an IIR filter is not as important for IIR filters as for FIR filters since it is not ordinarily used to compute the output, i.e., we don't ordinarily implement a convolution of h[n] with the input. Rather we implement the difference equation.

The advantage of IIR filters over FIR filters is that IIR filters usually require fewer coefficients to execute similar filtering operations, that IIR filters work faster, and require less memory space. But, IIR filter has a feedback loop so they will accumulate rounding and noise error. We can see the numbers in the excel sheet which certify this (the number of filter taps, the cost, the group delay).

We must notice that IIR filters don't offer the computational advantages of the FIR filter for the multi-rate application system.

IIR filters are more susceptible to the problem of line finite length arithmetic. Hence, when the poles are so close to the circle (as mentioned in the problem statement), we need greater arithmetic precision in our multipliers to be sure of the stability.

5. A summary

In general, it is very important to know the application that we want to design a filter for that. As we do not have a specific application here, I have just given some general tips for choosing a filter in the current report.

As a general note, IIR filters can heavily be sensitive to the finite length computations, and also quantization can lead to their instability. Also, they have a phenomena called limit cycle (instability) which has some useful application, such as in making oscillators.

Moreover, changing the cost of multipliers in the excel file, does not affect the results.

Direct form I or direct form II:

Criteria of choosing FIR or IIR is obvious as we discussed above. For both LPF and BPF IIR filters, direct form II is better than direct form I in terms of cost. This configuration needs lesser number of delays (half of the direct form I).

LPF:

For FIR, Equiripple is better than Kaiser window in terms of both the cost and also the group delay (we attained the same result in Homework 4).

For IIR filters, Elliptic filter has the lowest cost, but it has a rather larger group delay. If instability because of not having appropriate hardware be a concern, a better choice might be Chebyshev 2 which has a close cost to Elliptic but the cost is very lower than Butterworth (I choose this because the nearest pole to the unit circle has greater distances than other choices).

BPF 1 and BPF 2:

For FIR, the Equiripple filter is better than Kaiser window in terms of both the cost and also the group delay (we attained the same result in Homework 4).

For IIR filters, Elliptic filter has the lowest cost, but it has a rather larger group delay. If instability because of not having appropriate hardware be a concern, a better choice might be Chebyshev 2 which has a close cost to Elliptic but the cost is very lower than Butterworth (I choose this because the nearest pole to the unit circle has greater distances than other choices). Chebyshev 2 has cost more than Chebyshev 1 (not much difference), but it has about 1/3 group delay; that's why it might be better for more applications.

It is obvious that BPF 2, as we decreased the transient regions, the cost and also the maximum in passband group delay have increased (whether for IIR or FIR ones). Moreover, these filters have closer poles to the unit circle (with respect to the BPF 1), and needs more care about their stability.

6. References

[1] https://circuitglobe.com/difference-between-fir-filter-and-iir-filter.html

[2] https://www.ni.com/docs/en-US/bundle/diadem/page/genmaths/genmaths/calc_filterfir_iir.htm