Unlucky inferences

The Unbearable Luck of Inference

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Test GR with a control parameter

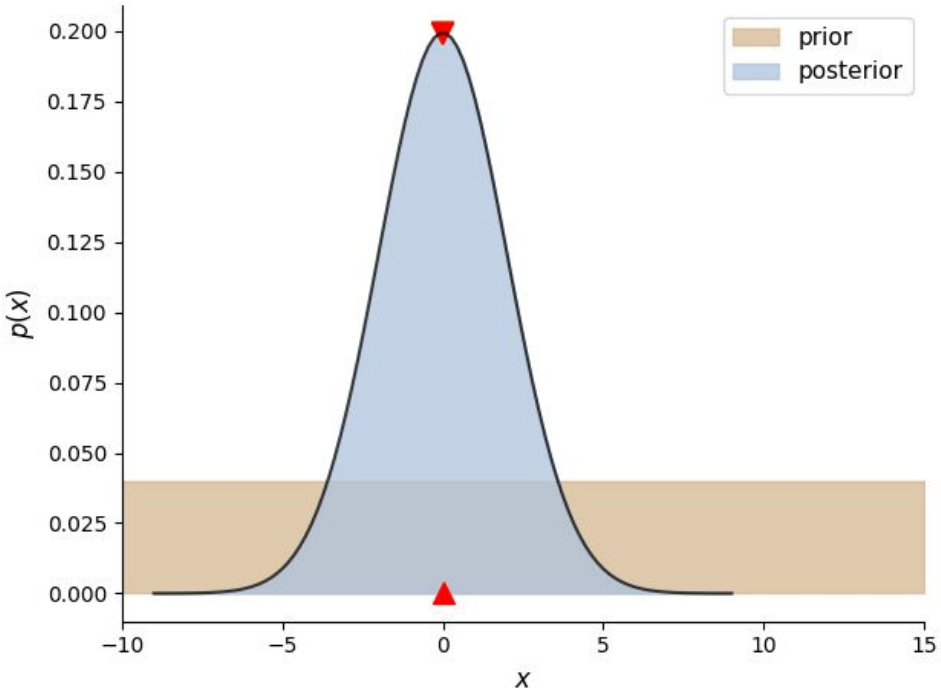
$$\mathcal{H}_0 = \{\text{GR is correct}\} \equiv \{x = 0\}$$
$$\mathcal{H}_1 = \{\text{GR is incorrect}\} \equiv \{x \neq 0\}$$

Bayesian Hypothesis Testing

Savage-Dickey Ratio

$$\mathcal{B}^0_1 = \frac{p(\mathcal{D}|\mathcal{H}_0)}{p(\mathcal{D}|\mathcal{H}_1)} = \frac{p(x=0|\mathcal{D})}{\pi(x=0)} > 0$$

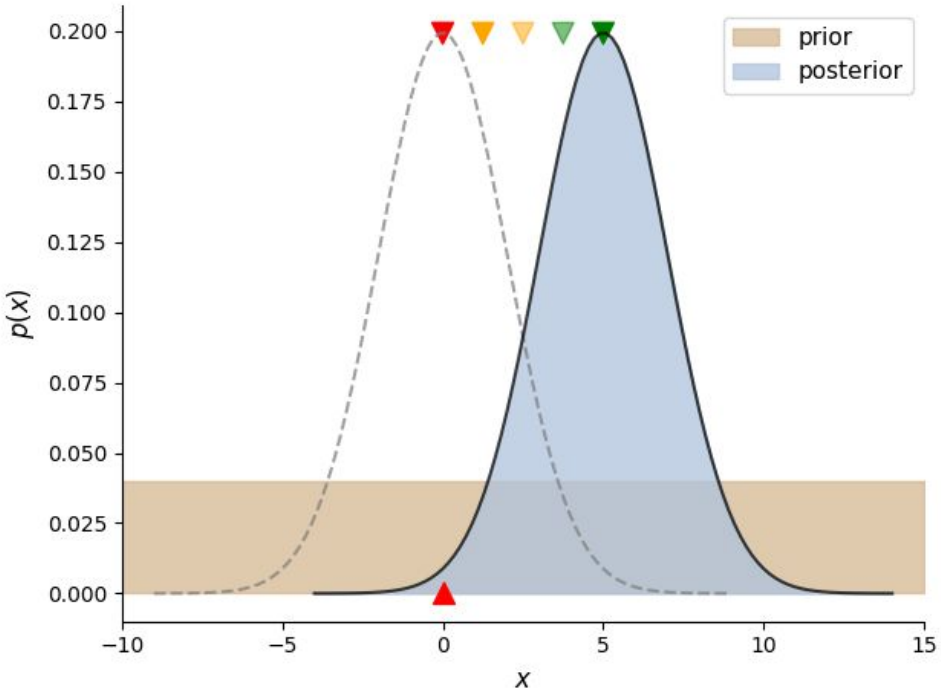
$$p(x|\mathcal{D}) = \mathcal{N}(0, \sigma_{\rm obs})$$



The noise of the instrument induces a spread in the measurement

$$\mathcal{B}_1^0 = \frac{p(x{=}0|\mathcal{D})}{\pi(x{=}0)} > 0$$

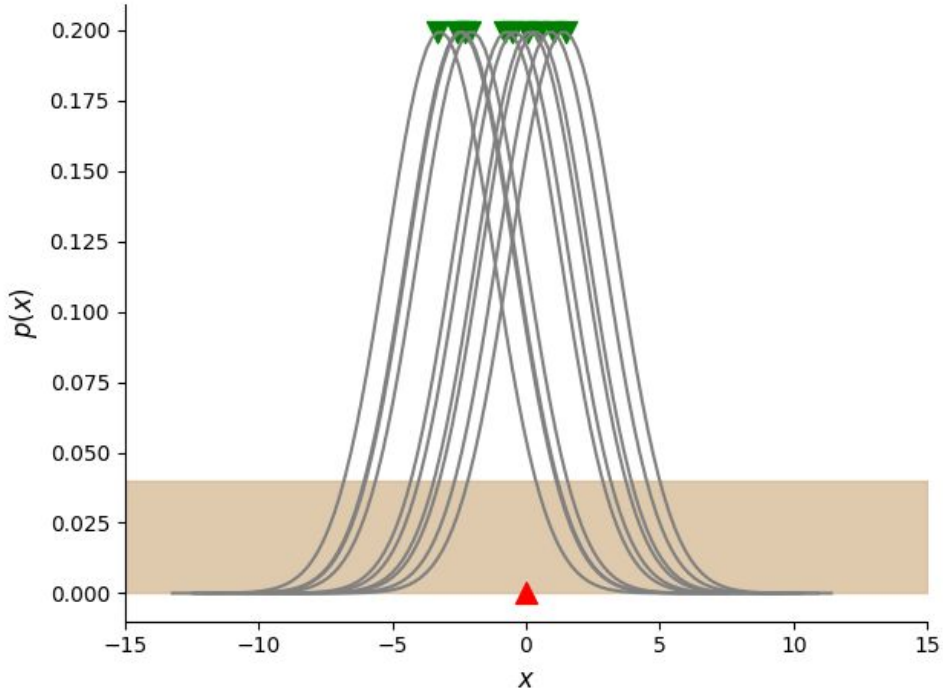
$$p(x|\mathcal{D}) = \mathcal{N}(x_\star, \sigma_{\rm obs})$$



The noise of the instrument induces a shift in the peak

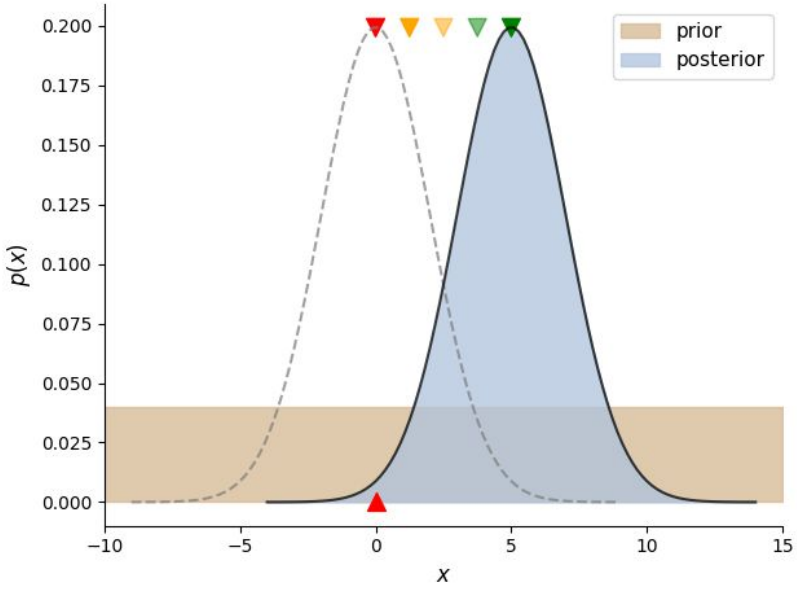
$$\mathcal{B}_1^0 = \frac{p(x{=}0|\mathcal{D})}{\pi(x{=}0)} < 0$$

$$p(x|\mathcal{D}) = \mathcal{N}(x_\star, \sigma_{\text{obs}})$$



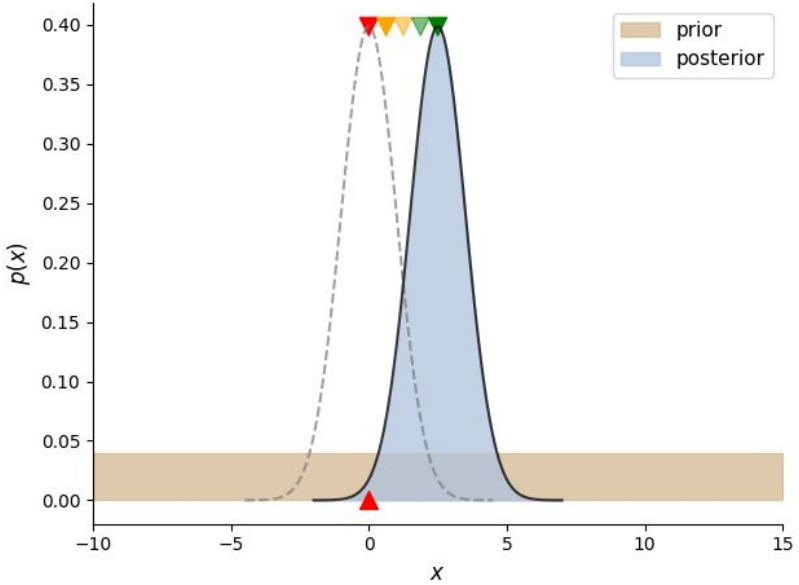
The posteriors and the distributions of the peaks have the same variance !

$$p(x_\star) = \mathcal{N}(0, \sigma_{\text{obs}})$$



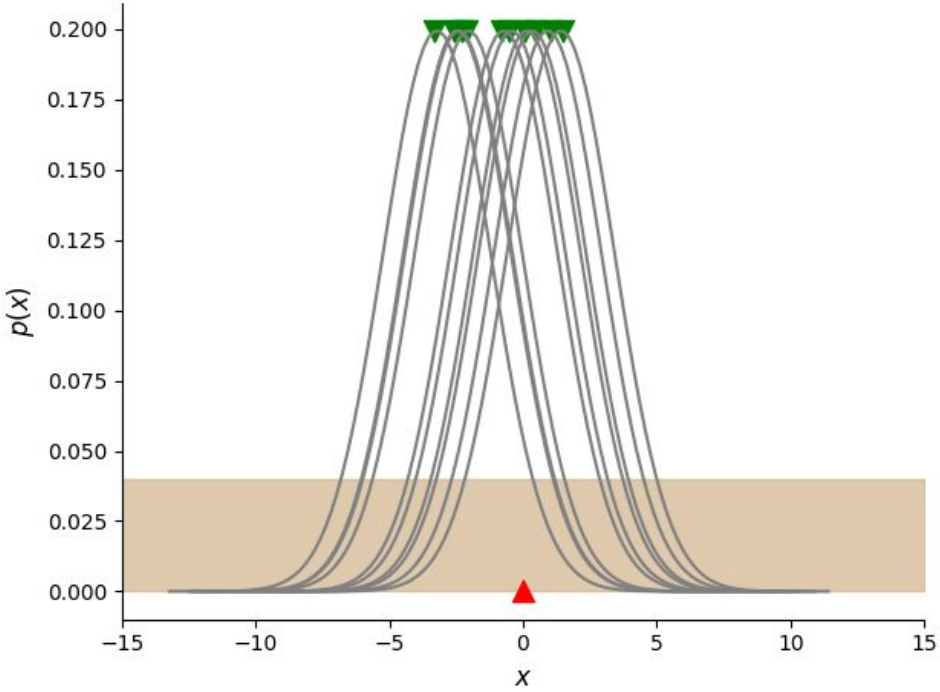
But you also reduce the spread by a corresponding amount !

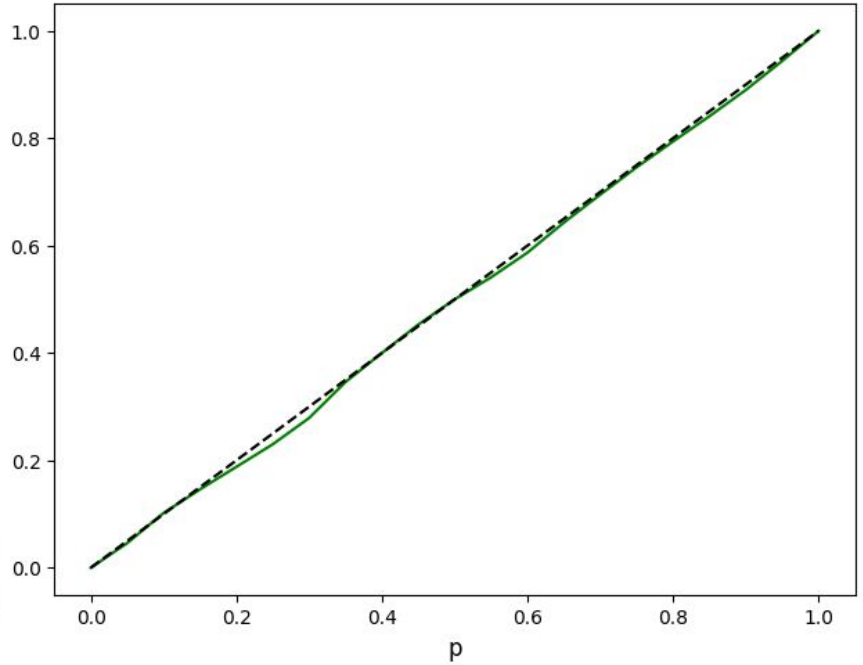
Can I reduce the bias by reducing noise? Yes

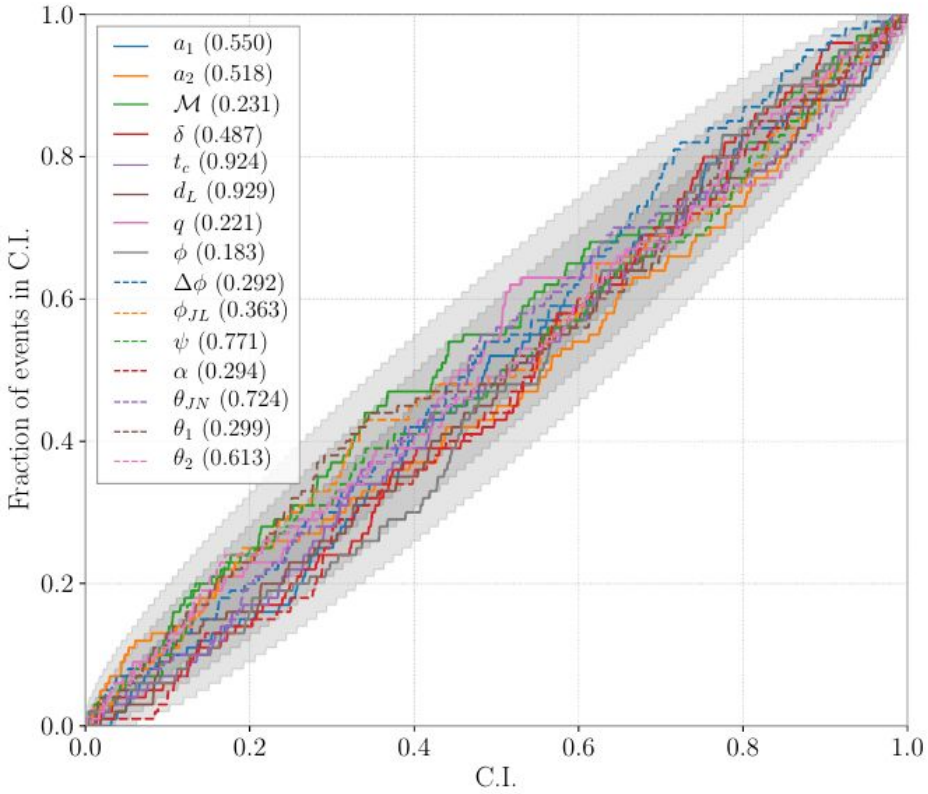


PP plot

Meaning: the true value is at the p % contour level p % of the time





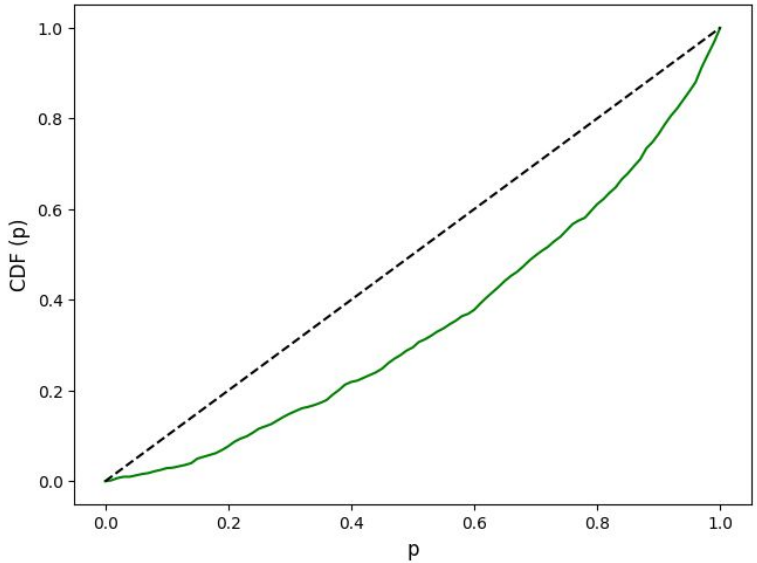


PP plot is a diagnostic that your sampler is working

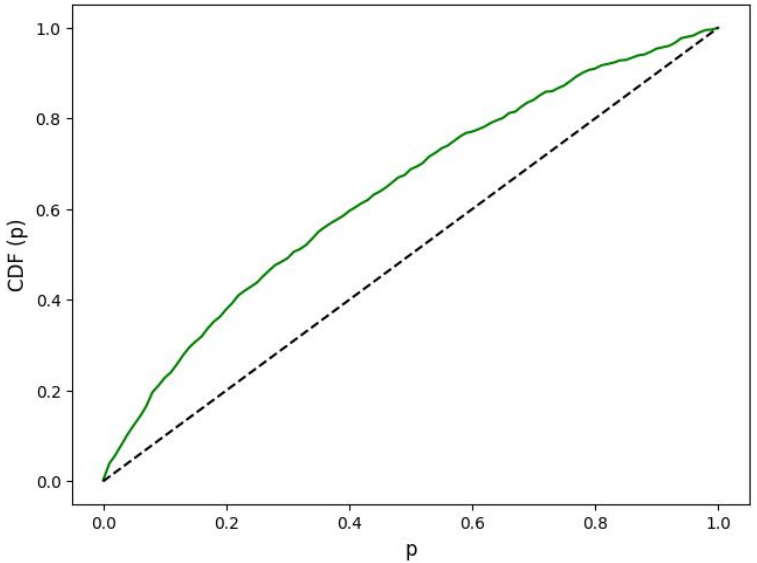
From the BILBY validation paper
https://arxiv.org/abs/2006.00714

Bias #1: shift in the peak of the posterior

Shifting to the left

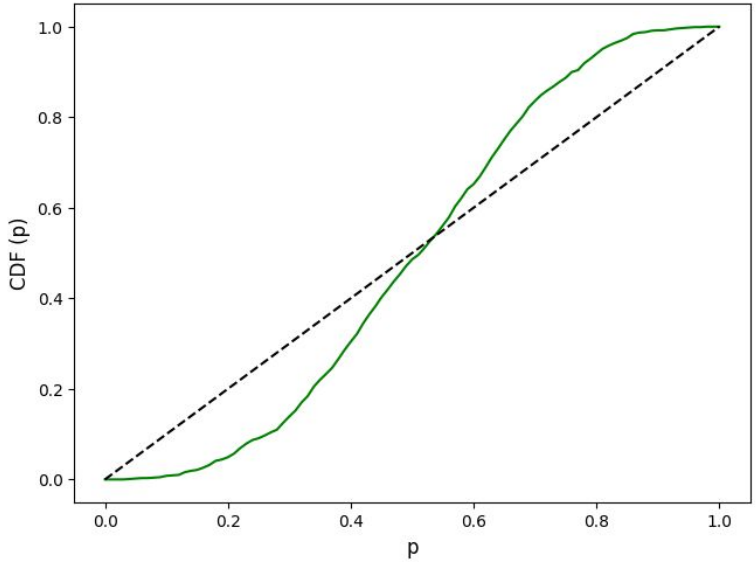


Shifting to the right

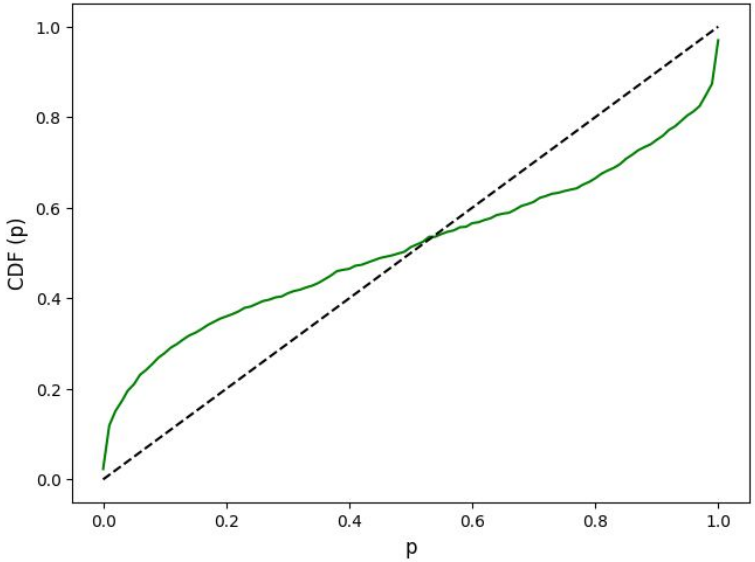


Bias #2: wrong spread of the posterior

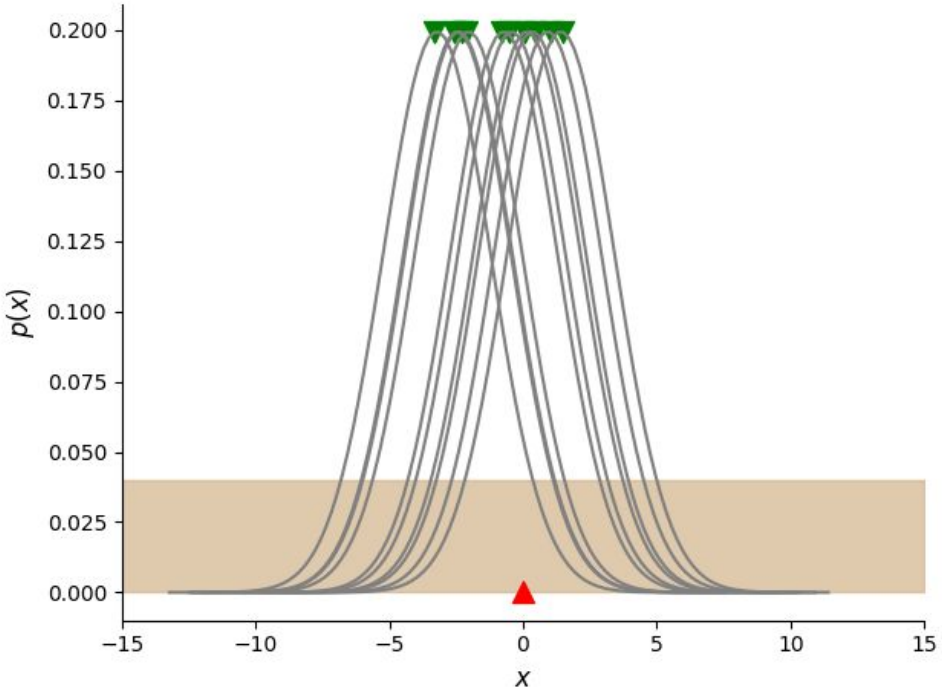
Under-constraining



Over-constraining



Stacking many events together



The real value is not known a priori

There can be “no” real value: it can be a distribution a.k.a. Population

This is addressed by hierarchical stacking techniques

Some useful references

- Introduction to Bayesian inference: https://arxiv.org/pdf/1809.02293.pdf

- Understanding pp plots: https://greg-ashton.physics.monash.edu/understanding-pp-plots.html