Problem Set 4

Assigned: Feb. 28
Due: Mar. 27

Problem 1

One can carry out prediction in the maze world of programming assignment 2 in Datalog; that is, you can write axioms that, given the starting state and the sequence of moves, allow you to infer the state at eath time. can infer the sequence of state.

Let Ω be the set of locations, objects, and instants of time. Let $\mathcal{L}$ be a Datalog language with the following predicates:

`A(l,t).` The player is at location `l` at time `t`.
`H(o,t).` The player has object `o` at time `t`.
`M(la,lb,t).` At time `t`, the player moves from location `la` to `lb`.
`X(ta,tb).` Time `tb` is the next time after time `ta`.
`S(l,o).` Location `l` supplies object `o`.

For the three-location maze in programming assignment 2, you could then assert the following axioms, using constants `START, A, B, GOLD, WAND, T0, T1, T2, T3` with the obvious meanings.

Domain constraint:
1. `A(l,t)` $\wedge$ `S(l,o)` $\Rightarrow$ `H(o,t).`

Causal axiom
2. `M(la,lb,ta)` $\wedge$ `X(ta,tb)` $\Rightarrow$ `A(lb,tb).`

Frame axiom:
3. `M(la,lb,ta)` $\wedge$ `H(o,ta)` $\wedge$ `X(ta,tb)` $\Rightarrow$ `H(o,tb).`

:**Starting state**
4. `A(START,T0).`
5. `S(A,GOLD).`
6. `S(B,WAND).`

Time sequence: 7. `X(T0,T1).`
8. `X(T1,T2).`
9. `X(T2,T3).`

Moves executed:
10. `M(START,A,T0).`
11. `M(A,START,T1).`
12. `M(START,B,T2).`

Show the result of doing forward chaining on these axioms, in the style of the handout on Datalog.

Problem 2

You have a box with 4 coins, all weighted.
1 coin comeis up heads with probability 0.1 (Category 1).
2 coins come up heads with probability 0.3 (Category 2).
1 coin comes up heads with probability 0.9 (Category 3).

A. You pick a coin out of the box at random and flip it. What is the probability that it will come up heads?

B. You pick a coin out of the box and flip it twice. What is the probability of two heads?

C. You pick two coins out of the box together (at the same time) and flip each of them once. What is the probability of two heads?

D. You pick a coin of the box at random, flip it, put it back, again pick a coin at random, and flip it. What is the probability of two heads?

E. You pick a coin out of the box at random and flip it. It comes up heads. What is the probability that it is Category 1? Category 2?

F. You pick a coin out of the box at random and flip it. It comes up tails. What is the probability that it is Category 1? Category 2?

G. You pick a coin out of the box at random and flip it. It comes up heads. You flip it again. What is the probability that the second flip will be heads?

H. You pick a coin out of the box at random and flip it twice. It comes up heads both times. What is the probability that it is Category 1? Category 2?

I. You pick a coin out of the box at random and flip it 10 times. What is the expected number of heads? (Hint: This is easy, once you've worked out part A. If you start calculating the probability distribution over the number of heads, you're on the wrong track.)

Problem 3

Suppose that X and Y are random variables with the following joint distribution:

	Y=1	**Y=2**	**Y=6**
X=0	0.2	0.14	0.12
X=4	0.08	0.02	0.04
X=5	0.24	0.10	0.06

A. Compute the following quantities: (a) P(X=4). (b) P(Y=2). (c) P(X=Y−1). (d) P(X $<$ Y). (e) P(Y=2 | X=4). (f) P(X=4 | Y=2). (g) Exp(X). (h) Exp(Y).

B. What is the probability distribution for the random variable X+Y? Compute Exp(X+Y) directly from that probability distribution.

Problem 4

You are at the racetrack, with \$100 to spend. You have good feelings about two horses in the third race, Silver and Dawn. Dawn is the favorite, and she is listed at even odds: that is, if you bet \$X and win, you will gain \$X; if she does not win you lose your \$X. Silver is a long shot currently listed at 10 to 1; that is, if you bet \$Y and he wins, you will gain 10·\$Y; if he loses, you lose \$Y. The other horses in the race are not worth considering.

What is a little worrisome is that there is a rumor going around that Dawn is somewhat fatigued from too many races.

The way you figure it is this: If the rumor is false and Dawn is well (assume this is a Boolean state), then she will certainly beat Silver. If the rumor is true and Dawn is fatigued, then it is a toss-up between her and Silver; each has a probability of 1/2 of winning.

In computing the probability that Dawn is actually fatigued, you make the following judgments: The prior probability (in the absence of the rumor) that she is fatigued is 1/10. If she is actually well, the probability that a (false) rumor that she is fatigued would have spread is 1/90. If she is actually fatigued, the probability that a (true) rumor that she is fatigued would have spread is 1/20.

A. Given that the rumor has spread, what is the probability that Dawn will win?

B. What independence assumption did you make in answering (A)?

C. If you bet \$X on Silver and \$Y on Dawn, what is your expected return? What is the optimal strategy: Bet the whole \$100 on Silver, bet it all on Dawn or split it, and if you split it, then how? What is your expected return under the optimal strategy?

D. While you are contemplating, you find that there is another option. You recognize one of the stablehands from the racetrack. You know him well, and you know that for \$40 he will tell you for sure whether Dawn is fatigued. However, that \$40 comes out of your budget for the day; if you bribe him, you have only \$60 left to bet. Ignoring ethical considerations, and the sharply negative utility that would ensue if the racetrack discovers that you were bribing their employees, what is the expected return if you bribe the stablehand and proceed to bet on Dawn if she is well and on Silver if Dawn is fatigued?

Note: I do not recommend betting on horse races or approve of dishonestly acquiring insider information. The situation here is purely hypothetical.