Problem Set 6

Assigned: April 5
Due: April 17

Problem 1

Suppose that a classifier computes a numerical score for an item based on its "confidence" that the item is a member of the target category. In using the classifier, you set a threshhold, accept the items whose score is higher than the thresshold, and reject items whose score is lower.

For instance, suppose you have the following labelled data set, with the true labels (T if the item is actually in the category; F if it is not) and scores.

Name	a	b	c	d	e	f	g	h	i	j
Label	T	T	T	F	T	F	T	T	F	T
Score	.95	.92	.91	.84	.81	.75	.68	.63	.62	.56

Name	k	l	m	n	o	p	q	r	s	t
Label	F	F	T	F	F	T	F	F	F	F
Score	.52	.51	.43	.42	.41	.32	.21	.15	.08	.01

If you then set the threshhold at 0.50, the classifier will accept items a-l and reject items m-t.

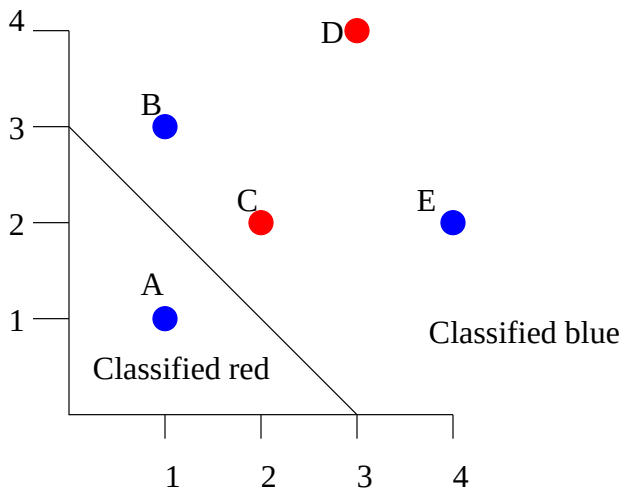
Compute precision, recall, and F-score for threshholds 0.9, 0.8, 0.6, and 0.4.

Problem 2

Suppose that you have the following collection T of data points in two dimensions:

x	1	1	2	3	4
y	1	3	2	4	2
	B	B	R	R	B

The picture below illustrates these points, together with the classifier
if $x + y - 3 > 0$ **then** BLUE **else** RED,
corresponding to the weight vector $\langle 1, 1, 3 \rangle$.



A. Compute the value of the error function for this classifier:

$$E_T(\vec{w}) = \sum_{\mathbf{p}\in T, \mathbf{p} \text{ misclassified}} |w_1\mathbf{p}_x + w_2\mathbf{p}_y - w_3|$$

B. Compute the gradient of the error function with respect to the weight vector $\vec{w}$. The gradient is a vector $\vec{\nabla}E|_{\vec{w}} = \langle g_1, g_2, g_3\rangle$ computed as follows: For a given weight vector $\vec{w}$ and data point $\mathbf{p} \in T$ let

$$s_{\vec{w}}(\mathbf{p}) = \begin{cases} 1 & \text{if } \mathbf{p} \text{ is labelled RED in } T \text{ but is classified BLUE by } \vec{w} \\ -1 & \text{if } \mathbf{p} \text{ is labelled BLUE in } T \text{ but is classified RED by } \vec{w} \\ 0 & \text{if } \mathbf{p} \text{ is correctly classified by } \vec{w} \end{cases}$$

Then

$$\vec{\nabla}E|_{\vec{w}} = \sum_{\mathbf{p}\in T} s_{\vec{w}}(\mathbf{p}) \cdot \langle \mathbf{p}_x, \mathbf{p}_y, -1\rangle$$

C. Compute the new weight vector after one step of gradient descent: $\vec{w}' = \vec{w} - \delta \cdot \vec{\nabla}E|_{\vec{w}}$ where $\delta = 0.1$.

D. How does the new weight vector $\vec{w}'$ classify the points?

E. What is the value of the error function at the new weight vector $\vec{w}'$?