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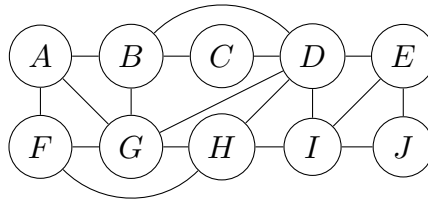
Problem Set 3

Assigned: Feb. 9

Due: Feb. 23

Problem 1

The simple INDEPENDENT SET problem is the special case of the BEST INDEPENDENT SET problem where all the vertices have value 1. In other words: You are given an undirected graph G and a number K . The task is to find a set S of vertices of size K such that no two vertices are connected by an edge. For instance, in the graph below, $\{A, C, H, J\}$ is an independent set of size 4. An INDEPENDENT SET problem can be translated into propositional calculus as follows: Use atoms of the form V_i meaning that V is the i th element in some listing of the set. For instance, for the above example, there would be 40 atoms $A1, A2, A3, A4, B1, B2, \dots, J4$. One valuation corresponding to the ordering $\{H, A, C, J\}$ would have $H1, A2, C3, J4$ assigned TRUE, and the other 36 atoms assigned FALSE.



Describe the categories of propositions needed to encode an INDEPENDENT SET problem as a problem in propositional calculus. Illustrate each category with a particular proposition for the above example.

Problem 2

Let D be a domain consisting of people and moments of time. Let L be the first-order language with the following primitives:

- $E(s, t)$ — Predicate. Time s precedes (is earlier than) time t .
- $A(p, t)$ — Predicate. Person p was alive at time t .
- $C(p, q)$ — Predicate. Person p is a child of person q .
- $O(p, q)$ — Predicate. Person p is older than person q .
- $F(p, q, t)$ — Predicate. Person p is a friend of person q at time t .

- Constants: D, E, F — Dora, Ed, Fiona. People.
- T — Constant. Some particular time.

Express the following statements in L :

1. Dora and Ed had a common friend at time T .
2. For all p and q , if p is a child of q , then q is older than p .
3. All the people who were friends of Ed at time T were older than Fiona.
4. For any two people p, q , p is older than q if and only if there was some time that p was alive that preceded any time when q was alive.
5. All of the people who were friends of Ed at time T had children.
6. None of Ed's children were ever friends of Fiona.
7. Dora had a friend at time T who is older than all of Ed's children.