Equations

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1 Variables

Temperature (K)

$$T$$

Heat Capacity ($\frac{\mathrm{J}}{\mathrm{K}}$)

$$C$$

Heat Energy (J)

$$Q$$

Internal Energy (J)

$$E$$

Volume (m^3)

$$V$$

Mass (Kg)

$$m$$

Density ($\frac{\mathrm{Kg}}{\mathrm{m}^3}$)

$$\rho$$

Mass-specific heat capacity ($\frac{\mathrm{J}}{\mathrm{kg}\cdot\mathrm{K}}$)

$$c_m$$

Thickness (m)

$$L$$

Area (m^2)

$$A$$

Thermal Conductivity (K-Value $\frac{\mathrm{W}}{\mathrm{m}\cdot\mathrm{K}}$)

$$K$$

Thermal Transmittance (U-Value $\frac{\mathrm{W}}{\mathrm{m}^2\cdot\mathrm{K}}$)

$$U$$

3 Simulation

Rate of Energy Transmission

$$\dot{Q}_{a\to b} = T_a \cdot UA_{a\leftrightarrow b}$$
$$\dot{Q}_{b\to a} = T_b \cdot UA_{a\leftrightarrow b}$$

Energy Transmitted

$$Q_{a\to b} = \dot{Q}_{a\to b} \cdot \Delta t$$
$$Q_{b\to a} = \dot{Q}_{b\to a} \cdot \Delta t$$

New Total Energy

$$E_a = E_a + Q_{b\to a} - Q_{a\to b}$$
$$E_b = E_b + Q_{a\to b} - Q_{b\to a}$$

New Temperature

$$T_a = \frac{E_a}{C_a}$$
$$T_b = \frac{E_b}{C_b}$$

Matrix Equation

$$\begin{bmatrix} T_{a-new} \\ T_{b-new} \end{bmatrix} = \begin{bmatrix} 1 - UA_{a\leftrightarrow b} \cdot \Delta t \cdot \frac{1}{C_a} & UA_{a\leftrightarrow b} \cdot \Delta t \cdot \frac{1}{C_a} \\ UA_{a\leftrightarrow b} \cdot \Delta t \cdot \frac{1}{C_b} & 1 - UA_{a\leftrightarrow b} \cdot \Delta t \cdot \frac{1}{C_b} \end{bmatrix} \cdot \begin{bmatrix} T_a \\ T_b \end{bmatrix}$$

$$\vec{T}_{new} = M \cdot \vec{T}$$

5 Time Step Matrix (M)

$$M = C \cdot (M_{adjacency} + E_{lost}) \cdot \Delta t + I$$

6 Superfast simulation

Some ways of simulating the change in heat area over 1024 timesteps with a 10*10 matrix:

$\vec{T}_{new} = M \cdot (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot \vec{T})))))))$ This way takes 102,500 multiplications

$\vec{T}_{new} = (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot M))))))) \cdot \vec{T}$ This way takes 1,023,100 multiplications

Both these ways take very long and are inefficient so to make it super fast I calculate it like this:

$\vec{T}_{new} = (((M \cdot M) \cdot (M \cdot M)) \cdot ((M \cdot M) \cdot (M \cdot M))) \cdot \vec{T}$

simplified it looks like this:

$\vec{T}_{new} = M^{(2^{10})} \cdot \vec{T}$ this way takes 10,000 multiplications