

Equations

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1 Variables

| | |
|-----------------------------------------------------------------------------|--------|
| Temperature (K) | T |
| Heat Capacity ($\frac{\text{J}}{\text{K}}$) | C |
| Heat Energy (J) | Q |
| Internal Energy (J) | E |
| Volume (m^3) | V |
| Mass (Kg) | m |
| Density ($\frac{\text{Kg}}{\text{m}^3}$) | ρ |
| Mass-specific heat capacity ($\frac{\text{J}}{\text{kg}\cdot\text{K}}$) | c_m |
| Thickness (m) | L |
| Area (m^2) | A |
| Thermal Conductivity (K-Value $\frac{\text{W}}{\text{m}\cdot\text{K}}$) | K |
| Thermal Transmittance (U-Value $\frac{\text{W}}{\text{m}^2\cdot\text{K}}$) | U |
| | 1 |

Thermal Resistance (R-Value $\frac{\text{m}^2 \cdot \text{K}}{\text{W}}$)

$$R$$

Thermal Conductance ($\frac{\text{W}}{\text{K}}$)

$$UA$$

2 Formulas

Definition of Mass-specific Heat capacity

$$c_m = \frac{C}{m}$$

Definition of Heat capacity

$$C = \frac{Q}{\Delta T}$$

Definition of Density

$$\rho = \frac{m}{V}$$

Definition of Internal energy

$$E = c_m \cdot T$$

Definition of Thermal Transmittance

$$U = \frac{1}{R} = \frac{K}{L}$$

Definition of Thermal Conductance

$$UA = U \cdot A$$

3 Simulation

Rate of Energy Transmission

$$\dot{Q}_{a \rightarrow b} = T_a \cdot UA_{a \leftrightarrow b}$$

$$\dot{Q}_{b \rightarrow a} = T_b \cdot UA_{a \leftrightarrow b}$$

Energy Transmitted

$$Q_{a \rightarrow b} = \dot{Q}_{a \rightarrow b} \cdot \Delta t$$

$$Q_{b \rightarrow a} = \dot{Q}_{b \rightarrow a} \cdot \Delta t$$

New Total Energy

$$E_a = E_a + Q_{b \rightarrow a} - Q_{a \rightarrow b}$$

$$E_b = E_b + Q_{a \rightarrow b} - Q_{b \rightarrow a}$$

New Temperature

$$T_a = \frac{E_a}{C_a}$$

$$T_b = \frac{E_b}{C_b}$$

4 Matrix

Substitution into base variables

$$T_{a-new} = \frac{(C_a \cdot T_a - (UA_{a \leftrightarrow b} \cdot T_a \cdot \Delta t) + (UA_{a \leftrightarrow b} \cdot T_b \cdot \Delta t))}{C_a}$$

$$T_{b-new} = \frac{(C_b \cdot T_b + (UA_{a \leftrightarrow b} \cdot T_a \cdot \Delta t) - (UA_{a \leftrightarrow b} \cdot T_b \cdot \Delta t))}{C_b}$$

Turning into matrix format

$$T_{a-new} = T_a(1 - UA_{a \leftrightarrow b} \cdot \Delta t \cdot \frac{1}{C_a}) + T_b \cdot (UA_{a \leftrightarrow b} \cdot \Delta t \cdot \frac{1}{C_a})$$

$$T_{b-new} = T_a \cdot (UA_{a \leftrightarrow b} \cdot \Delta t \cdot \frac{1}{C_b}) + T_b(1 - (UA_{a \leftrightarrow b} \cdot \Delta t \cdot \frac{1}{C_b}))$$

Matrix Equation

$$\begin{bmatrix} T_{a-new} \\ T_{b-new} \end{bmatrix} = \begin{bmatrix} 1 - UA_{a \leftrightarrow b} \cdot \Delta t \cdot \frac{1}{C_a} & UA_{a \leftrightarrow b} \cdot \Delta t \cdot \frac{1}{C_a} \\ UA_{a \leftrightarrow b} \cdot \Delta t \cdot \frac{1}{C_b} & 1 - UA_{a \leftrightarrow b} \cdot \Delta t \cdot \frac{1}{C_b} \end{bmatrix} \cdot \begin{bmatrix} T_a \\ T_b \end{bmatrix}$$

$$\vec{T}_{new} = M \cdot \vec{T}$$

5 Time Step Matrix (M)

$$M = C \cdot (M_{adjacency} + E_{lost}) \cdot \Delta t + I$$

Identity Matrix (I)

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Weighted Adjacency Matrix/Rate of Energy Transmitted into the mass ($M_{adjacency}$)

$$\begin{bmatrix} 0 & UA_{a \leftrightarrow b} \\ UA_{a \leftrightarrow b} & 0 \end{bmatrix}$$

Loosing Energy Matrix/Rate of Energy Transmitted out of the mass (E_{lost})

$$\begin{bmatrix} -UA_{a \leftrightarrow b} & 0 \\ 0 & -UA_{a \leftrightarrow b} \end{bmatrix} = \left(M_{adjacency} \cdot \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right) \cdot diag$$

Thermal Capacity Matrix (C)

$$\begin{bmatrix} C_a & 0 \\ 0 & C_b \end{bmatrix} = \begin{bmatrix} C_a \\ C_b \end{bmatrix} \cdot diag$$

6 Superfast simulation

Some ways of simulating the change in heat area over 1024 timesteps with a 10*10 matrix:

$\vec{T}_{new} = M \cdot (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot \vec{T})))))$ This way takes 102,500 multiplications

$\vec{T}_{new} = (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot M))))) \cdot \vec{T}$ This way takes 1,023,100 multiplications

Both these ways take very long and are inefficient so to make it super fast I calculate it like this:

$$\vec{T}_{new} = (((M \cdot M) \cdot (M \cdot M)) \cdot ((M \cdot M) \cdot (M \cdot M))) \cdot \vec{T}$$

simplified it looks like this:

$$\vec{T}_{new} = M^{(2^{10})} \cdot \vec{T} \text{ this way takes 10,000 multiplications}$$