Equations

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1 Variables

Temperature (K)

$$T$$

Heat Capacity ($\frac{\mathrm{J}}{\mathrm{K}}$)

$$C$$

Heat Energy (J)

$$Q$$

Internal Energy (J)

$$E$$

Volume (m^3)

$$V$$

Mass (Kg)

$$m$$

Density ($\frac{\mathrm{Kg}}{\mathrm{m}^3}$)

$$\rho$$

Mass-specific heat capacity ($\frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}}$)

$$c_m$$

Thickness (m)

$$L$$

Area (m^2)

$$A$$

Thermal Conductivity (K-Value $\frac{\mathrm{W}}{\mathrm{m} \cdot \mathrm{K}}$)

$$K$$

Thermal Transmittance (U-Value $\frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}}$)

$$U$$

Thermal Resistance (R-Value $\frac{\mathrm{m}^2 \cdot \mathrm{K}}{\mathrm{W}}$)

$$R$$

Thermal Conductance ($\frac{\mathrm{W}}{\mathrm{K}}$)

$$UA$$

2 Formulas

Definition of Mass-specific Heat capacity

$$c_m = \frac{C}{m}$$

Definition of Heat capacity

$$C = \frac{Q}{\Delta T}$$

Definition of Density

$$\rho = \frac{m}{V}$$

Definition of Internal energy

$$E = c_m \cdot T$$

Definition of Thermal Transmittance

$$U = \frac{1}{R} = \frac{K}{L}$$

Definition of Thermal Conductance

$$UA = U \cdot A$$

3 Simulation

Rate of Energy Transmission

$$\dot{Q}_{a \to b} = T_a \cdot UA_{a \leftrightarrow b}$$
$$\dot{Q}_{b \to a} = T_b \cdot UA_{a \leftrightarrow b}$$

Energy Transmitted

$$Q_{a \to b} = \dot{Q}_{a \to b} \cdot \Delta t$$
$$Q_{b \to a} = \dot{Q}_{b \to a} \cdot \Delta t$$

New Total Energy

$$E_a = E_a + Q_{b \to a} - Q_{a \to b}$$
$$E_b = E_b + Q_{a \to b} - Q_{b \to a}$$

New Temperature

$$T_a = \frac{E_a}{C_a}$$
$$T_b = \frac{E_b}{C_b}$$

4 Matrix

Substitution into base variables

$$T_{a-new} = \frac{(C_a \cdot T_a - (UA_{a\leftrightarrow b} \cdot T_a \cdot \Delta t) + (UA_{a\leftrightarrow b} \cdot T_b \cdot \Delta t))}{C_a}$$

$$T_{b-new} = \frac{(C_b \cdot T_b + (UA_{a\leftrightarrow b} \cdot T_a \cdot \Delta t) - (UA_{a\leftrightarrow b} \cdot T_b \cdot \Delta t))}{C_b}$$

Turning into matrix format

$$T_{a-new} = T_a(1 - UA_{a\leftrightarrow b} \cdot \Delta t \cdot \frac{1}{C_a}) + T_b \cdot (UA_{a\leftrightarrow b} \cdot \Delta t \cdot \frac{1}{C_a})$$

$$T_{b-new} = T_a \cdot (UA_{a\leftrightarrow b} \cdot \Delta t \cdot \frac{1}{C_b}) + T_b(1 - (UA_{a\leftrightarrow b} \cdot \Delta t \cdot \frac{1}{C_b})$$

Matrix Equation

$$\begin{bmatrix} T_{a-new} \\ T_{b-new} \end{bmatrix} = \begin{bmatrix} 1 - UA_{a\leftrightarrow b} \cdot \Delta t \cdot \frac{1}{C_a} & UA_{a\leftrightarrow b} \cdot \Delta t \cdot \frac{1}{C_a} \\ UA_{a\leftrightarrow b} \cdot \Delta t \cdot \frac{1}{C_b} & 1 - UA_{a\leftrightarrow b} \cdot \Delta t \cdot \frac{1}{C_b} \end{bmatrix} \cdot \begin{bmatrix} T_a \\ T_b \end{bmatrix}$$

$$\vec{T}_{new} = M \cdot \vec{T}$$

5 Time Step Matrix (M)

$$M = C \cdot (M_{adjacency} + E_{lost}) \cdot \Delta t + I$$

Identity Matrix (I)

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Weighted Adjacency Matrix/Rate of Energy Transmitted into the mass ($M_{adjacency}$)

$$\begin{bmatrix} 0 & UA_{a\leftrightarrow b} \\ UA_{a\leftrightarrow b} & 0 \end{bmatrix}$$

Loosing Energy Matrix/Rate of Energy Transmitted out of the mass (E_{lost})

$$\begin{bmatrix} -UA_{a\leftrightarrow b} & 0 \\ 0 & -UA_{a\leftrightarrow b} \end{bmatrix} = \left(M_{adjacency} \cdot \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right) .diag$$

Thermal Capacity Matrix (C)

$$\begin{bmatrix} C_a & 0 \\ 0 & C_b \end{bmatrix} = \begin{bmatrix} C_a \\ C_b \end{bmatrix} .diag$$

6 Superfast simulation

Some ways of simulating the change in heat area over 1024 timesteps with a 10*10 matrix:

$\vec{T}_{new} = M \cdot (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot \vec{T})))))))$ This way takes 102,500 multiplications

$\vec{T}_{new} = (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot M))))))) \cdot \vec{T}$ This way takes 1,023,100 multiplications

Both these ways take very long and are inefficient so to make it super fast I calculate it like this:

$\vec{T}_{new} = (((M \cdot M) \cdot (M \cdot M)) \cdot ((M \cdot M) \cdot (M \cdot M))) \cdot \vec{T}$

simplified it looks like this:

$\vec{T}_{new} = M^{(2^3)} \cdot \vec{T}$ this way takes 10,000 multiplications

$\vec{T}_{new} = M \cdot (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot \vec{T})))))))$ This way takes 102,500 multiplications

$\vec{T}_{new} = (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot (M \cdot M))))))) \cdot \vec{T} = M^{(2^3)} \cdot \vec{T}$

$\dot{Q}(energy\ flow) = \frac{\Delta T}{R(thermal\ resistance)}$ is analogous to $I = \frac{V}{R}$