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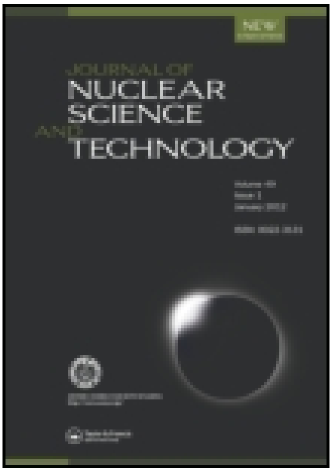

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Exact Solution of Point Reactor Kinetic Equation with Gaussian Reactivity Fluctuation Having Exponential-Type Correlation

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jet pump, even when both suction and drive flows are in the reverse direction. But in Model-1, this pressure change cannot be considered explicitly under the same flow situation.

(2) To take account of $\Delta P_{\text{mom, throat}}$ in Model-1, it has to be effectively included in the local pressure loss at the suction junction $\Delta P_{S,L}$ by using the equivalent reverse flow form loss coefficient K^{r}_{suction}.

[NOMENCLATURE]

A: Volume flow area
I: Inertia for junction
I': Half-volume inertia for junction
P: Thermodynamic pressure at volume center
$P_{,g}$: Elevation pressure
$\bar{V}$: Average fluid velocity in volume
W: Mass flow at junction
$\bar{W}$: Average mass flow in volume
ΔP_{mom}: Pressure change due to stream momentum change
$\Delta P_{,L}$: Local pressure loss
$\Delta P_{,W}$: Wall friction pressure loss

Subscripts
D: Drive junction
DC: Downcomer volume
JP: Jet pump volume
LP: Lower plenum volume
M: Mixing section
R: Recirculation pump discharge volume
S: Suction junction
T: Discharge junction

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SHORT NOTE

Exact Solution of Point Reactor Kinetic Equation with Gaussian Reactivity Fluctuation Having Exponential-Type Correlation

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KEYWORDS: point reactor, random reactivity noise, Gaussian processes, correlation of exponential type, mean neutron number, absence of delayed neutrons, reactor kinetic equations, exact solution, Kummer's confluent hypergeometric function

Given the point reactor kinetic equation in the case of absence of delayed neutrons

$$\dot{N}(t)=\frac{K(t)-1}{l}N(t)+S\,, \tag{1}$$

the solution becomes as

$$N(t)=\int_0^t d\xi\, S \exp\left[\int_\xi^t d\eta\, \frac{K(\eta)-1}{l}\right], \tag{2}$$

where $N(0)=0$ is postulated as the initial condition.

Suppose that the extraneous source S and the multiplication factor K are statistically independent. Their mean values are assumed to be independent of time and denoted by S_0 and K_0, respectively. Then, the mean of $N(t)$ can be written as

$$N_0(t)=\int_0^t d\xi\, S_0\, e^{-\alpha(t-\xi)}\left\langle \exp\int_\xi^t d\eta\, \frac{k(\eta)}{l}\right\rangle, \tag{3}$$

where $\alpha=(1-K_0)/l$

and $k(t)$ is the fluctuating part of the multiplication factor. The symbols $\langle f\rangle$ means to take the ensemble average for an arbitrary stochastic process f.

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We regard the $k(t)$ as the stationary Gaussian process with null mean. Then, Eq. (3) becomes as

$$N_0(t)=\int_0^t d\xi\, S_0\, e^{-\alpha(t-\xi)} \cdot \exp\left[\frac{1}{2l^2}\left\langle\left\{\int_\xi^t d\eta\, k(\eta)\right\}^2\right\rangle\right]. \quad (4)$$

Since the stationarity is assumed for $k(t)$, we have

$$\left\langle\left\{\int_\xi^t d\eta\, k(\eta)\right\}^2\right\rangle = 2(t-\xi)\int_0^{t-\xi} d\tau\left(1-\frac{\tau}{t-\xi}\right)\Gamma_k(\tau). \quad (5)$$

Suppose that the correlation function $\Gamma_k(\tau)$ of the fluctuation $k(t)$ is an exponential type and given by

$$\Gamma_k(\tau)=\sigma_k^2 e^{-\lambda_k|\tau|}. \quad (6)$$

Substitution of the above into Eq. (5) and a little algebra lead to the following result:

$$N_0(t)=\int_0^t d\xi\, S_0\, e^{-\alpha(t-\xi)} \cdot \exp\left[\frac{\sigma_k^2}{l^2\lambda_k}(t-\xi)\left\{1+\frac{e^{-\lambda_k(t-\xi)}-1}{\lambda_k(t-\xi)}\right\}\right]. \quad (7)$$

In the white noise limit, *viz.*, in the limit that

$$\lambda_k\to\infty,\ \sigma_k^2\to\infty \ \text{ and } \ 2\sigma_k^2/\lambda_k\to w \quad (8)$$

we have

$${}_wN_0(t)=\int_0^t d\xi\, S_0 \exp\left[-\left(\alpha-\frac{w}{2l^2}\right)(t-\xi)\right]. \quad (9)$$

The steady state value of the above becomes as

$${}_wN_{0s}={}_wN_0(\infty)=\frac{S_0}{\alpha-w/2l^2}. \quad (10)$$

This result is identical to that obtained by Akcasu & Karasulu[1], Quabili & Karasulu[2] and by Saito[3] among others[4]~[6] *via* various mathematical methods.

The case of the Gaussian white parametric excitation has constituted the unique case for which the exact result can be obtained and given by Eq. (10). The other cases where either Gaussianity or whiteness, or both of the properties might be missed are only approximatedly treated except for the dichotomic process[7], and the result (10) stands as the bench mark with which accuracy of each approximate method is consulted (*cf. e.g.*, Ref. (2)).

The present note points out that there is another case of the Gaussian excitations for which the exact result can be obtained. Namely, performing the Laplace transform of Eq. (7), we have the following exact result for the case where the Gaussian excitation has the correlation of an exponential type:

$$N_0[p]=\int_0^\infty dt\, e^{-pt}N(t) = \frac{S_0}{p}\cdot\frac{1}{R\lambda_k}\Phi(1,\ R+1\,;\,-\delta), \quad (11)$$

$$\text{where}\quad R=\frac{p+\alpha-\lambda_k\delta}{\lambda_k},\qquad \delta=\frac{\sigma_k^2}{l^2\lambda_k^2}.$$

The function Φ appearing in the result (11) denotes the Kummer's confluent hypergeometric function[8]. The steady value of Eq. (7) is obtained in the following form:

$$N_{0s}=\lim_{p\to 0} pN_0[p] = \frac{S_0}{r\lambda_k}\Phi(1,\ r+1\,;\,-\delta), \quad (12)$$

$$\text{where}\quad r=(\alpha-\lambda_k\delta)/\lambda_k.$$

In the limit given by Eq. (8), δ becomes null and $r\lambda_k$ tends to $\alpha-w/2l^2$. Since $\Phi(a,c\,;x)$ is unity when $x=0$, the result (12) naturally includes the case of the white noise limit given by Eq. (10).

The function $\Phi(1,r+1\,;-\delta)$ has the following forms of series expansion:

$$\Phi(1,\ r+1\,;\,-\delta) = 1-\frac{\delta}{1+r}+\frac{\delta^2}{(r+1)(r+2)} -\frac{\delta^3}{(r+1)(r+2)(r+3)}+\cdots, \quad (13a)$$

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$$=e^{-\delta}\left[1+\frac{r}{r+1}\delta+\frac{r}{2!(r+2)}\delta^2+\frac{r}{3!(r+3)}\delta^3+\cdots\right]. \tag{13b}$$

An approximate result obtained by Quabili *et al.*[2] *via* Bourret's integral equation method and another result by Saito[3] obtained with one ficton approximation are identical to the present exact result up to the order of δ.

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