



# A simple procedure to estimate reactivity with good noise filtering characteristics

Yoichiro Shimazu

Research Institute of Nuclear Engineering, University of Fukui, 1-2-4, Kanawa, Tsuruga, Fukui 914-0055, Japan



## ARTICLE INFO

### Article history:

Received 20 December 2013

Received in revised form 8 July 2014

Accepted 9 July 2014

### Keywords:

Reactivity estimation

Inverse Point Kinetic equation

Noise filtering

Subcriticality

Reactivity monitoring

## ABSTRACT

A new and simple on-line reactivity estimation method is proposed. The estimator has robust noise filtering characteristics without the use of complex filters. The noise filtering capability is equivalent to or better than that of a conventional estimator based on Inverse Point Kinetics (IPK). The new estimator can also eliminate the burden of selecting optimum filter time constants, such as would be required for the IPK-based estimator, or noise covariance matrices, which are needed if the extended Kalman filter (EKF) technique is used. In this paper, the new estimation method is introduced and its performance assessed without and with measurement noise.

© 2014 Elsevier Ltd. All rights reserved.

## 1. Introduction

One of the difficulties in the estimation of (sub-)criticality using a measured neutron signal is the filtering of noise in the signal. A commonly used method for on-line reactivity monitoring is a digital reactivity meter based on the Inverse Point Kinetic (IPK) equation. IPK-based reactivity meters have been used for the estimation of sub-criticality and also for PWR reactor physics tests (Shimazu et al., 1987, 2006). The IPK based reactivity meter is quite simple and has a sufficient capability to estimate reactivity accurately even under noisy conditions if an adequate filter is applied (Naing et al., 2005; Shimazu et al., 2003; Shimazu and Naing, 2005). In practice, the filter can be a simple first order delay filter. In that case, the parameter to be selected is the time constant for the filter.

Recently, a procedure based on the extended Kalman filtering (EKF) technique was presented (Bhatt et al., 2013). This procedure is claimed to have a better performance than the IPK-based method from the point of view of yielding higher accuracy, better noise suppression, and more robustness. However the EKF-based method requires the calculation and tuning of noise covariance matrices.

In order to improve these problems a very simple feedback mechanism is proposed for reactivity estimation. The calculation cost is negligible, and requires no information about covariance matrices or the time constant. The noise filtering capability is com-

parable to that of IPK. Comparison of noise filtering capability of the EKF-based method with that of the IPK-based method has been discussed in reference Shimazu and Rooijen (2014). The conclusion is as follows; when the noise level is not so high, the performance of the IPK-based method can be superior or comparable with the performance of the EKF-based method. Therefore, the present paper discusses only the comparison of our proposed new method with the IPK-based method. In the following sections, the principle and characteristics of the proposed procedure are described.

## 2. Principle of the procedure

The point kinetic equation is expressed as follows for a sub-critical system with external neutron source Q.

$$\frac{dn(t)}{dt} = \frac{\rho(t) - \beta}{\ell} n(t) + \sum_{i=1}^m \lambda_i C_i(t) + Q \quad (1)$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\ell} n(t) - \lambda_i C_i(t), i = 1, 2, \dots, m \quad (2)$$

$$\beta = \sum_i \beta_i, \quad (3)$$

where  $\rho(t)$  denotes the reactivity,  $n(t)$  denotes the neutron flux,  $C_i(t)$  denotes the concentration of the  $i$ -th group of delayed neutron precursors,  $m$  is the total number of delayed neutron precursor families,  $\lambda_i$  are the precursor decay constants,  $\ell$  denotes the prompt

E-mail address: [shimazu@u-fukui.ac.jp](mailto:shimazu@u-fukui.ac.jp)

neutron lifetime and  $Q$  denotes an independent neutron source. The effective strength of the neutron source has been assumed as constant and expressed (Shimazu et al., 2003) in terms of the initial stable sub-criticality as

$$Q = \frac{-\rho_0 n_0}{\ell}, \quad (4)$$

where  $n_0$  denotes the neutron flux at an arbitrary, sub-critical steady state and  $\rho_0$  denotes the corresponding reactivity obtained from some other means (e.g. reactor physics calculations or a measurement, e.g. by a reactor period measurement).

### 1) Inverse Point Kinetic (IPK) based method

The well-known IPK-based reactivity is estimated as follows. From the point kinetics model (1) and (2), the following equation can be derived:

$$\rho(t) = \frac{\ell}{n(t)} \left[ \frac{dn(t)}{dt} + \sum_{i=1}^m \frac{dC_i(t)}{dt} - Q \right]. \quad (5)$$

Suppose the samples of the neutron flux (measurement) are denoted as  $n_k$ , obtained at time instant  $t = kT_s$ ,  $k = 0, 1, 2, \dots$ , where  $T_s$  is the sampling interval. Then from Eq. (2)

$$C_{i,k} = e^{-\lambda_i T_s} C_{i,k-1} + \frac{1}{\lambda_i} (1 - e^{-\lambda_i T_s}) \frac{\beta_i}{\ell} n_k. \quad (6)$$

Further in (5) the derivative can be approximated as

$$\frac{dn}{dt}|_k = \frac{n_k - n_{k-1}}{T_s} \quad (7)$$

$$\frac{dC_i}{dt}|_k = \frac{C_{i,k} - C_{i,k-1}}{T_s}. \quad (8)$$

And thus

$$\rho_k = \frac{\ell}{n_k} \left[ \frac{n_k - n_{k-1}}{T_s} + \sum_{i=1}^m \frac{C_{i,k} - C_{i,k-1}}{T_s} - Q \right]. \quad (9)$$

Which along with Eq. (6), is suitable for implementation on a digital computer. The result will be an estimate of the reactivity at different time instants. If there are strong fluctuations in the neutron flux, for instance during a measurement at deep sub-criticality, the reactivity estimates obtained by such a method show significant fluctuations. This is because the IPK equations involve differentiation of the neutron flux with respect to time. In fact, Eqs. (7) and (8) are simple backward differences, which have only a "first order" accuracy. However such difficulty can be avoided as follows. Firstly, the differentiation term of the neutron flux can be omitted due to the fact that the magnitude of this term is negligible in comparison with the other terms; this simplification is widely adopted in digital reactivity meters in actual use (Shimazu et al., 1987). Secondly, the usual method of smoothing the neutron flux is to apply a low pass filter to the measured neutron flux before using the IPK estimator (Shimazu et al., 1987). The low pass filter is a digital first order delay filter which is given as follows.

$$n_k = n_{k-1} + \frac{dt}{dt + T_s} (\hat{n}_k - n_{k-1}), \quad (10)$$

where  $n_k$  and  $n_{k-1}$  are neutron flux and  $\hat{n}_k$  is the measured neutron flux. Since the time constant of the first order delay is selected as small as 0.2 s (Shimazu et al., 1987), accuracy is not lost.

### 2) Proposed new method

Application of our new method is limited to sub-critical systems, or slightly super-critical systems. Under such circumstances, the derivative of the neutron flux is small. Multiplying both sides of Eq. (1) by the prompt neutron lifetime, the following equation is obtained.

$$\ell \frac{dn(t)}{dt} = (\rho - \beta)n(t) + \ell \left( \sum_{i=1}^m \lambda_i C_i(t) + Q \right). \quad (11)$$

Considering that the neutron life time is small, the left side of Eq. (11) can be set to be zero as follows. This is the familiar prompt jump approximation (Bell and Glasstone, 1970).

$$0 = (\rho - \beta)n(t) + \ell \left( \sum_{i=1}^m \lambda_i C_i(t) + Q \right). \quad (12)$$

Thus, neutron flux can be expressed as

$$n(t) = \frac{\ell \left( \sum_{i=1}^m \lambda_i C_i(t) + Q \right)}{\beta - \rho(t)}. \quad (13)$$

The proposed new procedure estimates reactivity as follows. The following discussion assumes one family of delayed neutron precursors for the simplicity of explanation. (In the actual application, Eq. (13) is used, i.e. taking into account more than one precursor family.) At the moment, we assume no noise on the measurement signal.

Based on the above discussion, the expected neutron flux is calculated as

$$n_{ex,k+1} = \frac{\ell(\lambda C_{k+1} + Q)}{\beta - \rho_k}. \quad (14)$$

Note here that the reactivity in Eq. (14) is the reactivity determined in the previous time step. The precursor concentration,  $C_{k+1}$ , is calculated directly from the point kinetic equations, written in the prompt jump form. In any system of reactivity estimation, it is assumed that the change of neutron flux is due to the change of reactivity. Thus, if the reactivity is assumed to change slightly between the previous measurement and the present time by an amount,  $\delta\rho$ , the true (measured) neutron flux is

$$n_{t,k+1} = \frac{\ell(\lambda C_{k+1} + Q)}{\beta - (\rho_k + \delta\rho)}. \quad (15)$$

The difference of neutron flux,  $\delta n$ , between the measured value  $n_{t,k+1}$  and the expected value  $n_{ex,k+1}$  is calculated as

$$\begin{aligned} \delta n &= \ell(\lambda C_{k+1} + Q) \left\{ \frac{1}{\beta - (\rho_k + \delta\rho)} - \frac{1}{\beta - \rho_k} \right\} \\ &= \ell(\lambda C_{k+1} + Q) \left\{ \frac{\delta\rho}{(\beta - \rho_k)(\beta - (\rho_k + \delta\rho))} \right\} \end{aligned} \quad (16)$$

Here, Eq. (15) is used to get

$$\delta n = \frac{\delta\rho}{\beta - \rho_k} n_{t,k+1}. \quad (17)$$

This is a relation between the difference of the true neutron flux and the expected neutron flux, and the reactivity error. Thus, the correction for the reactivity becomes:

$$\delta\rho = \frac{\beta - \rho_k}{n_{t,k+1}} \delta n. \quad (18)$$

Finally, the estimated reactivity is corrected at the  $k+1$ -th step as follows:

$$\rho_{k+1} = \rho_k + \delta\rho. \quad (19)$$

From here on this method of reactivity estimation will be referred to as "Simplest Reactivity Estimator", SRE.

Note here that this estimation requires that one has knowledge of the initial reactivity of the reactor. It is assumed that this information is available, for instance from calculations, or from measurements, such as a reactor period measurement. In order to prove the validity of the new procedure, a simple simulation has

been done. A reactivity transient is given to a point reactor simulator to generate a neutron flux transient. Then the neutron flux is given to the reactivity estimator. This procedure was found to give the correct or true reactivity in accordance with the input neutron flux. However, in the actual (practical) application, the feedback component of the reactivity is directly calculated using the measured neutron flux as shown in Eq. (18). Thus, when neutron flux has noise, the corrected reactivity also has a direct influence from the noise. The estimated reactivity then fluctuates according to the neutron flux noise. In order to solve this problem, some theoretical considerations are given in the next section.

### 3. Theoretical discussion

In this section, the theoretical background of the proposed estimator is discussed and the elimination of noise component is explained, and justified on theoretical grounds. As expressed in Eq. (18), the right side of the equation can be interpreted as a difference of the neutron flux which is multiplied by a feedback gain. This concept is the same as in the EKF technique. In the Kalman filtering technique, the feedback gain is theoretically evaluated such that the estimated reactivity is optimized under the assumption that the noise components are mutually independent and have a Gaussian distribution. In order to evaluate the optimum feedback gain, many calculations are required. However, for the estimation of reactivity, it is not always necessary to use the optimum feedback gain. In many practical applications, the neutron flux change under normal reactivity monitoring regime is not so fast, for example, under sub-critical conditions. Thus it is expected that it is possible to estimate the reactivity with a reasonably good accuracy using a rather simplistic model for the feedback gain.

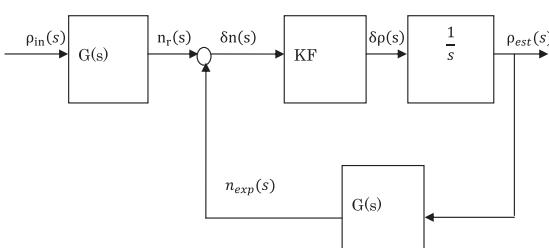
Using a linearized version of the classical point kinetic model with one group of delayed neutron precursors, a system diagram for the reactivity estimation can be expressed as shown in Fig. 1. Based on this diagram, the transfer function of the input reactivity to the estimated reactivity is expressed as

$$\rho_{\text{est}}(s) = \frac{KG(s)}{s + KG(s)} \rho_{\text{in}}(s), \quad (20)$$

where

$\rho_{\text{est}}(s)$  : estimated reactivity

$\rho_{\text{in}}(s)$  : input reactivity



where

- $\rho_{\text{in}}(s)$ : reactivity input to reactor
- $\rho_{\text{est}}(s)$ : estimated reactivity
- $n_r(s)$ : neutron flux of the reactor
- $n_{\text{exp}}(s)$ : expected neutron flux
- $\delta n(s)$ : neutron flux difference  $n_r - n_{\text{exp}}$
- $\delta\rho(s)$ : feedback reactivity
- $G(s)$ : Transfer function of the reactor
- $K$ : feedback gain
- $F$ : gain adjuster

Fig. 1. Block diagram of the estimation system.

$K$  : feedback gain

$G(s)$  : transfer function of the linearized point kinetic model

$$= \frac{s + \lambda}{s(sl + \lambda l + \beta)} n_0$$

$n_0$  : initial neutron flux

Note that the gain adjuster,  $F$ , expressed in Fig. 1 is assumed to be unity.

Here, we further simplify the model by assuming the reactivity is close to zero, and thus the feedback gain of Eq. (18) or Eq. (20) can be expressed as

$$K = \frac{\beta}{n_{t,k+1}}. \quad (21)$$

When the neutron flux is changing with a small amplitude around the initial value, then we can further simplify the feedback gain as

$$K = \frac{\beta}{n_0}. \quad (22)$$

Then, Eq. (20) can be expressed as

$$\rho_{\text{est}}(s) = \frac{\frac{\beta}{n_0} \frac{s + \lambda}{s(sl + \lambda l + \beta)} n_0}{s + \frac{\beta}{n_0} \frac{s + \lambda}{s(sl + \lambda l + \beta)} n_0} \rho_{\text{in}}(s). \quad (23)$$

Thus, the transfer function can be expressed as

$$\rho_{\text{est}}(s) = \frac{\beta(s + \lambda)}{s^2(sl + \lambda l + \beta) + \beta(s + \lambda)} \rho_{\text{in}}(s). \quad (24)$$

From the derivations, this transfer function is valid for the estimation of reactivity during a short time interval after a small reactivity addition to a reactor which is initially critical at steady state conditions. The estimated reactivity based on Eq. (24) following a step reactivity is shown in Fig. 2. The response is expressed as the relative value to the input reactivity. As can be seen the reactivity estimation takes a few seconds (See plot for  $F = 1.0$ ). Reactivity estimations with various gain adjusters are also shown in the figure. It is clear that the reactivity can be estimated correctly with any value of the feedback gain. However, the time required to converge to the correct reactivity depends on the feedback gain: the larger the feedback gain, the quicker the estimator responds. In the case of the smallest gain, the estimator also converges to the correct reactivity, but with an oscillation (overshoot). When the estimation is calculated in a discretized model with a finite sampling time interval, the estimation becomes oscillatory and finally diverges if the feedback gain is too large. This is due to the interplay between the sampling time step and the feedback gain. It can be said that a smaller feedback gain, such as less than unity, is safe to avoid divergence.

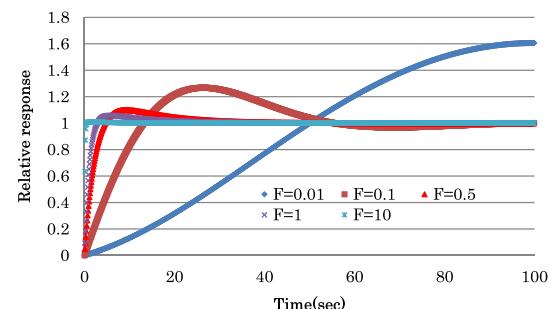


Fig. 2. Reactivity estimation following a step reactivity input.

**Table 1**

Example of modification of feedback gain  
( $F = \sqrt{n_{t,k+1}}$ ;  $n_{t,k+1} < 1.0$ ).

Normalized neutron flux	$F$
>1.0	0.1
0.5	0.707
0.1	0.316
0.05	0.2236
0.01	0.1

From this point of view, we adjusted the feedback gain using a gain adjuster  $F$  as follows.

$$\delta\rho = F \frac{\beta - \rho_k}{n_{t,k+1}} \delta n. \quad (25)$$

Then Eq. (24) is rewritten as

$$\rho_{\text{est}}(s) = \frac{F\beta(s + \lambda)}{s^2(sl + \lambda l + \beta) + F\beta(s + \lambda)} \rho_{\text{in}}(s). \quad (26)$$

When the normalized neutron flux value is smaller than 1.0 the gain adjuster is chosen as  $\sqrt{n_{t,k+1}}$ , which is smaller than unity. If the reactivity is negative, the neutron flux time rate of change is limited by the decay of the delayed neutron precursors, thus the feedback gain can safely be chosen to be closer to unity. When the amplitude of

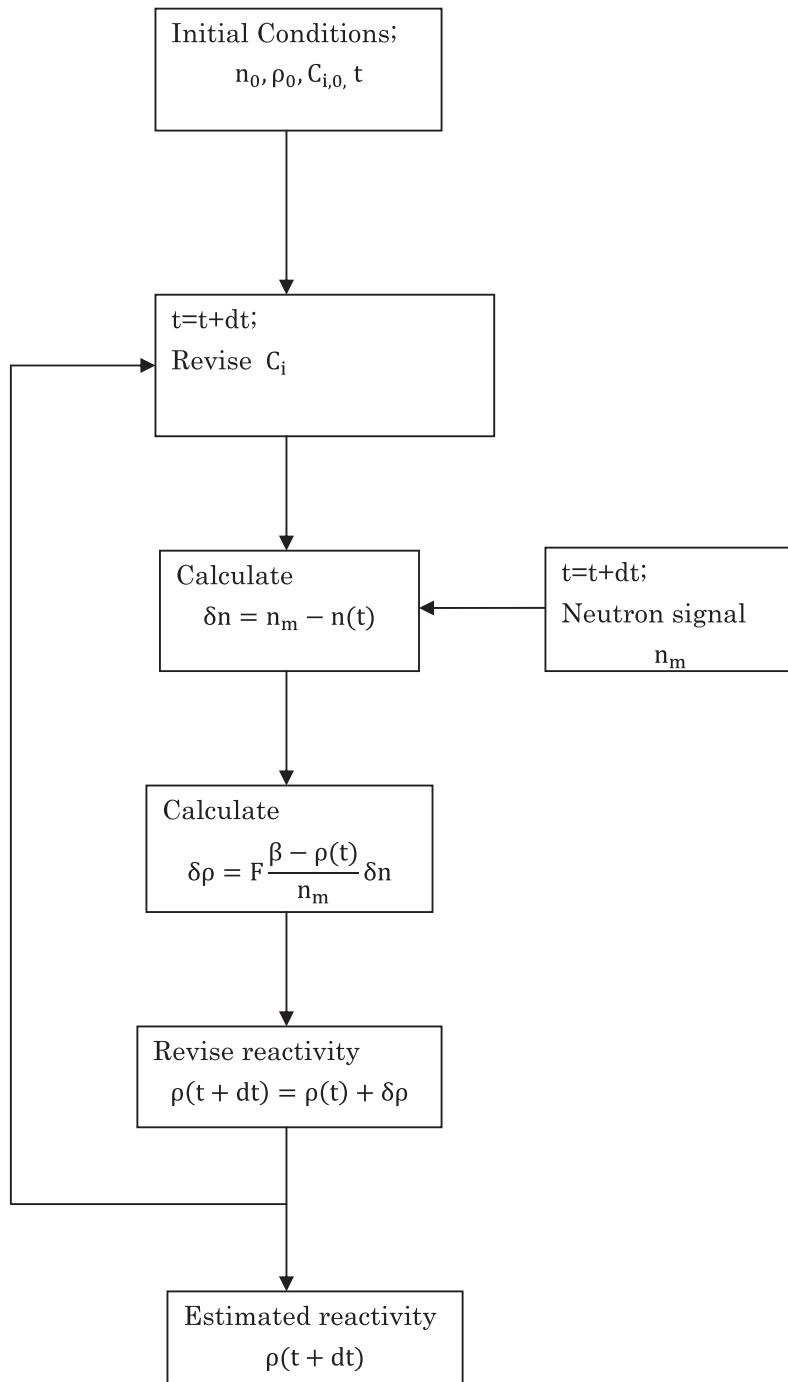


Fig. 3. Flow chart of the estimation.

the neutron flux decreases, the relative noise component increases (it is assumed that the noise in the measurement has a constant component). An example is shown in Table 1. As shown in Table 1, the feedback is quite small when the neutron flux decreases. As the gain becomes smaller, the feedback component of reactivity becomes smaller than the value obtained directly from Eq. (18). Thus the influence of noise component is reduced as the neutron flux decreases. As a result, the estimated reactivity is quite insensitive to noise. On the other hand, when the neutron flux becomes large, the influence of neutron noise component becomes small. Thus, when the normalized neutron flux value,  $n_{t,k+1} = n(t)/n_0$  is larger than 1.0, then the feedback gain adjuster  $F$  is set to 0.1. The validity of this procedure is shown in the next section.

There remains a freedom of selection of the gain adjuster, other than the square root. In order to obtain the gain adjuster automatically, the feedback component is calculated as follows instead of Eq. (18).

$$\delta\rho = \frac{\beta - \rho_k}{\sqrt{n_{t,k+1}}} \delta n. \quad (27)$$

Then, the gain adjuster  $F$  is calculated as  $(n_{t,k+1})^{\frac{r-1}{r}}$ . We have chosen  $r = 2$ , but we can select any other value of  $r$ . The general tendency of the reactivity estimation as function of the value of  $r$  is as follows. When increasing  $r$ , the noise filtering capability increases. But the error of the reactivity and the response time delay also increase. For example, with a ramp reactivity addition, of  $-2 \text{ pcm/s}$  for  $10,000 \text{ s}$ , the error at the final reactivity value becomes  $-0.14\%$ ,  $-0.73\%$ ,  $-2.77\%$  and  $-7.48\%$  corresponding to the value of  $r = 2, 3, 5$  and  $10$ , respectively. Also as the gain adjuster becomes smaller with increasing  $r$ , the response becomes too slow such that as shown in Fig. 2. When we reduce the number, i.e.  $r < 2$ , the noise filtering capability decreases. Based on some trial and error, the best value was found to be  $r = 2$ .

#### 4. Reactivity estimation and noise filtering capability in comparison with Inverse Point Kinetic method

In order to clarify the reactivity estimation process, a flowchart of the SRE is shown in Fig. 3. The reactivity estimation and noise filtering characteristics are compared for various reactor transients with noise. Here, we simply assume that a random noise component with a fixed amplitude. Reactivity estimations were compared with the conventional IPK method with a typical time constant ( $T = 0.5 \text{ s}$ ) for the first order delay filter.

The same reactivity transient is given to a point reactor simulator to generate a neutron flux signal. The reactor kinetic parameters are listed in Table 2 (Bhatt et al., 2013). The neutron life time is assumed to be 13 micro-seconds, as a typical value for an LWR. Then, a random noise, for example, random noise of 0.005% band of the initial neutron flux level, is added. The noise fraction looks very small. The reason is as follows. When the sub-criticality is deep or the sub-critical transient interval is long, the neutron flux becomes quite low. Thus, care is taken to ensure that the neutron flux does not become negative due to the addition of the noise. The above fraction is selected from such consideration. The extent

of noise fraction does not affect the comparison of noise filtering capability because reactivity fluctuation is proportional to the noise fraction. The reactivity is estimated both with the SRE-estimator and the IPK-based estimator. Examples of the reactivity comparison are shown for the typical reactivity transients as follows. All of the transients are calculated with a time interval of  $0.1 \text{ s}$ .

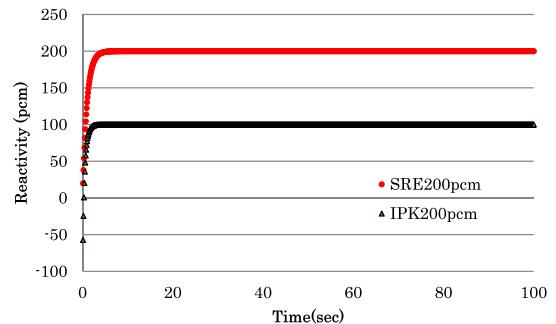


Fig. 4. Comparison of step reactivity addition.

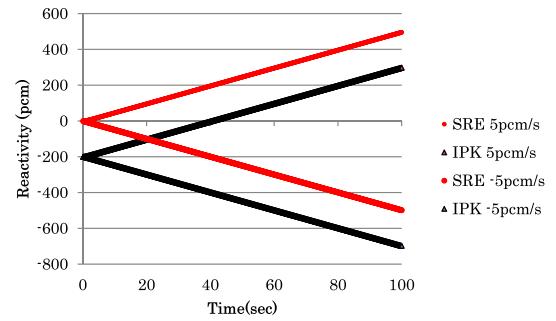


Fig. 5. Comparison of ramp reactivity addition.

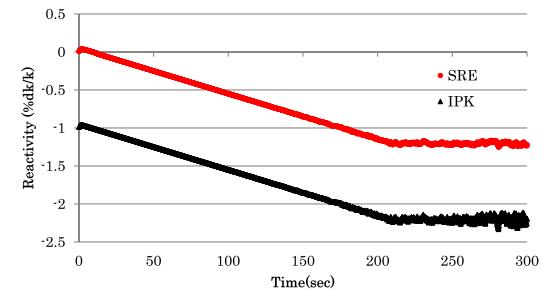


Fig. 6. Dynamic rod worth measurement simulation.

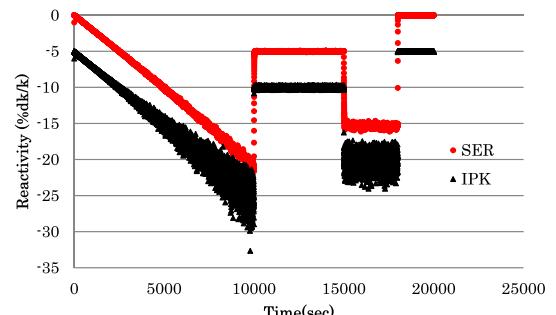


Fig. 7. Long subcritical transient with deep sub-criticality.

Table 2  
Reactor point kinetic parameters.

Precursor family	$\beta_i (\times 10^{-3})$	$\lambda_i (\text{s}^{-1})$
1	0.2487	0.0120
2	1.3800	0.0317
3	1.1990	0.1183
4	2.6270	0.3101
5	1.3790	0.9617
6	0.6799	2.8930

- (1) Step reactivity addition; [Fig. 4](#). Note that IPK data are shown with -100 pcm offset to avoid overlapping plots.
- (2) Ramp reactivity addition; [Fig. 5](#). Note that IPK data are shown with -200 pcm offset to avoid overlapping plots.
- (3) Simulation of dynamic rod worth measurement ([Chao et al., 2000](#)); [Fig. 6](#). Note that IPK data are shown with -1% dk/k offset to avoid overlapping plots.
- (4) Long subcritical transient with deep sub-criticality; [Fig. 7](#). Note that IPK data are shown with -5% dk/k offset to avoid overlapping plots.

It is shown that the proposed method gives a similar or better reactivity estimation and noise filtering capability than the conventional IPK-based estimator. It does not need a first order delay filter or time constant. It is sufficient to normalize the flux level to the expected flux level at the initial condition.

The IPK method has been shown to be a valid method in some practical applications. An example of a practical application is illustrated in reference ([Bhatt et al., 2013](#)), where the results of reactivity estimation for actual reactor transients by IPK and EKF are discussed. Since our new method performs at least equally well as the IPK method, one can have some confidence in the newly proposed method for practical applications.

## 5. Conclusions

A new and simple reactivity estimator is proposed. It has been shown that this estimator has robust noise filtering capability without any type of noise filters. As the noise filtering capability

can be better than IPK-based reactivity meters, this estimator might be suitable for sub-criticality monitoring.

## Acknowledgements

The author expresses his sincere gratitude to Dr. W.F.G. van Rooijen of the Research Institute of Nuclear Engineering, University of Fukui, Japan for his assistance in the preparation of this manuscript.

## References

- Bhatt, T.U. et al., 2013. Estimation of sub-criticality using extended Kalman filtering technique. *Ann. Nucl. Energy* 60, 98–015.
- Bell, G.I., Glasstone, S., 1970. Nuclear Reactor Theory, first ed. Van Nostrand Reinhold Company, New York.
- Chao, Y.A., Chapman, D.M., Hill, D.J., Grobmeyer, L.R., 2000. Dynamic rod worth measurement. *Nucl. Technol.* 132 (3), 403–412.
- Naing, W., Tsuji, M., Shimazu, Y., 2005. Subcriticality measurement of pressurized water reactor during criticality approach using a digital reactivity meter. *J. Nucl. Sci. Technol. (Tokyo, Japan)* 42, 145–152.
- Shimazu, Y. et al., 1987. Development of a digital reactivity meter and a reactor physics data processor. *Nucl. Technol.* 77 (6), 247–254.
- Shimazu, Y., Naing, W., 2005. Some technical issues on continuous subcriticality monitoring by a digital reactivity meter during criticality approach. *J. Nucl. Sci. Technol. (Tokyo, Japan)* 42, 515–524.
- Shimazu, Y., Rooijen, W.F.G., 2014. Qualitative performance comparison of reactivity estimation between the extended Kalman filter technique and the inverse point kinetic method. *Ann. Nucl. Energy* 66, 161–166.
- Shimazu, Y., Unesaki, H., Suzuki, N., 2003. Subcriticality monitoring with a digital reactivity meter. *J. Nucl. Sci. Technol. (Tokyo, Japan)* 40, 970–974.
- Shimazu, Y., Naing, W., Tsuji, M., 2006. Feasibility study of continuous subcriticality monitoring using a digital reactivity meter to overcome the criticality accident. *J. Nucl. Sci. Technol. (Tokyo, Japan)* 43, 1414–1421.