Consider a system of a one-dimensional particle under the action of a double-well potential and linearly coupled to a set of 30 harmonic oscillators. The system Hamiltonian is: $H = H_S + H_B$. Here, $H_S = \frac{p_s^2}{2} + U_s(x)$ is the Hamiltonian of the one-dimensional particle, with the potential energy $U_s(x) = U_0(x^2 - 1)^2$, and x, p_s are the position and momentum of the one-dimensional particle; $H_B = \sum_{j=1}^{N}\left(\frac{p_j^2}{2} + \frac{1}{2}\omega_j^2 q_j^2 - \gamma_j q_j x\right)$ is the Hamiltonian that accounts for the harmonic oscillators and their coupling with the one-dimensional particle, where $N = 30$, q_j, p_j, ω_j are the position, momentum and frequency of the j-th harmonic oscillator, γ_j is the coupling constant between the j-th harmonic oscillator and the one-dimensional particle. We choose $\omega_j^2 = c \cdot j$, for all $1 \le j \le N$, and $c = \sum_{j=1}^{N}(\frac{1}{j}) / \sum_{j=1}^{10}(\frac{1}{j})$, $\gamma_j = 1, 1 \le j \le N$.

Do molecular simulation for two conditions: a) $U_0 = 2, k_B T = 1$, b) $U_0 = 2.5, k_B T = 1$, where k_B is the Boltzmann constant and T is the temperature. For each condition, the initial positions are: $x = -1, q_j = 0, 1 \le j \le N$, and the initial momenta should be drawn from the Maxell distribution: $f(p) = \sqrt{\frac{1}{2k_B T}}\, e^{\frac{-p^2}{2k_B T}}$. After you draw 31 random numbers, $(p_s', p_1', \cdots, p_{31}')$, from this distribution, normalize them with $a = \left(\frac{0.5 \cdot (N+1) k_B T}{K'}\right)^{1/2}$, where $K' = \frac{p_s'^2}{2} + \sum_{j=1}^{N}\frac{p_j'^2}{2}$. That is, use $p_s = a \cdot p_s',\ p_j = a \cdot p_j'$ as the initial momenta for the 1-D particle and the 30 harmonic oscillators.

For each simulation, 1) plot the potential, kinetic and total energy of the system as a function of the simulation time, 2) plot position of x_s as a function of time, which should stochastically transition between the two basins located at 1 and -1. 3) Run your MD simulation long enough so that you can compute a reliable distribution for the waiting time for the transitions. 4) Discuss what you learned from the distributions for the waiting time.