Comparing Two Proportions

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IRD Global

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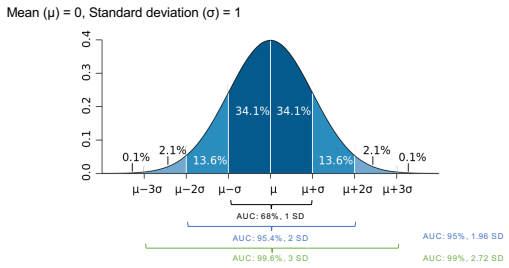
Normal distribution



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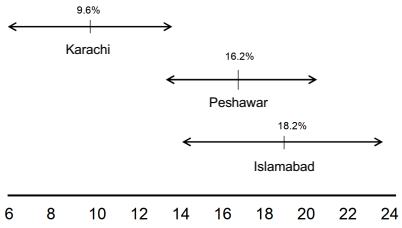

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Comparing two proportions

Statistical test of significance for the comparison of two proportions

- The Z-test for two proportions

95% confidence interval for the difference in two proportions

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Example

Clinical trial for advanced breast cancer

Patients randomly assigned to a treatment

Tumour response = Shrinkage of tumour surface area by ≥50% for at least two weeks

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		Treatment		
		CMF	L-PAM	Total
Tumour Response	Yes	49 (53%)	18 (20%)	67 (37%)
Outcome Response Dependent	No	44	73	117
	Total	93	91	184

It **looks** like CMF is more effective than L-Pam

How **strong is the evidence** that CMF is better than L-Pam?

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The Null Hypothesis – H_0

H_0: the treatments are equally effective

If H_0 were true then the true % response on CMF would be identical to the true % response on L-Pam

Notation

Group	Population	Sample
1	π_1	p_1
2	π_2	p_2

H_0: $\pi_1 = \pi_2 = \pi$

Basically, we try to **disprove** the null hypothesis

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Example

The question:

- If H_0 were true, what is the chance of getting as big (or bigger) a difference in the two proportions/percentages as that observed?

- If CMF and L-Pam were truly equally effective, what is the chance (or probability) of observing in our sample a treatment difference as large as (or larger than) 53% vs 20%?
- This probability is denoted by P and is known as the P-value and is calculated from a significance test

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Z-test for comparing two proportions

Example

H_0: $\pi_1 = \pi_2 = \pi$

$\pi_1 - \pi_2 = 0$

Observed difference in percentages

$p_1 - p_2 = 52.7 - 19.8 = 32.9\%$

We need the standard error of the difference in the two percentages to determine how far, on average, we might expect $p_1 - p_2$ to differ from zero due to sampling variability

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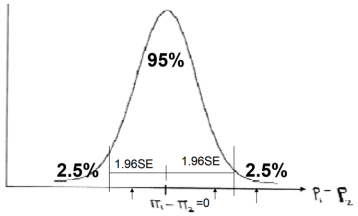
Sampling distribution of $p_1 - p_2$



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To calculate the SE ($p_1 - p_2$) - first need to make an estimate of the common response rate π

$$p = \frac{49 + 18}{93 + 91} = \frac{67}{184} = 36.4\%$$

$$SE(p_1 - p_2) = \sqrt{\{p \times (100 - p) \times (1/n_1 + 1/n_2)\}}$$

$$= \sqrt{\{36.4 \times (63.6) \times (1/93 + 1/91)\}}$$

$= 7.1\%$

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We determine how many standard errors our observed difference ($p_1 - p_2$) is from 0

We compute

$$Z = \frac{\text{observed difference in percentages}}{\text{Standard error of difference}}$$

$$= \frac{32.9}{7.1} = 4.63$$

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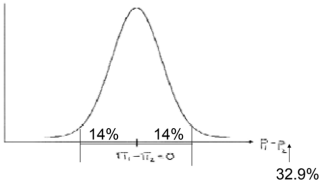
If the null hypothesis is true,

The larger value of Z, the smaller the probability P (the P-value)



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In our example

Z = 4.63 and thus P < 0.001

If H_0 were really true (ie if CMF and L-Pam were truly equally effective), **the chance of observing such a large difference in tumour response (32.9%) is less than 1 in 1000.**

The difference in percentages is **statistically significant** at 0.001 or 0.1% level.

There is very strong evidence that the CMF patients had a better response rate than the L-Pam patients.

Our chance of being wrong in drawing this conclusion is less than 0.1%.

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A second example

Clinical trial for MI patients

	Anturane	Placebo	Total
Dead	32 (4.1%)	44 (5.6%)	76
Alive	743	739	1482
Total	775	783	1558

Observed difference in percentages
5.62% - 4.13% = 1.49%.

SE (difference) = 1.09%

Hence Z = $\frac{1.49}{1.09}$ = 1.37

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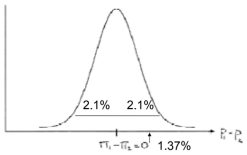
H_0 - death rates are the same



P = 0.17
If H_0 were true, the chance of getting such a difference in % dead are greater than 1 in 10

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Interpretation of P value

- The larger the value of Z, the smaller the probability P (the P-value)
- The smaller the P-value, the less likely it is that we would observe a difference in percentages as large as the one we have, if the null hypothesis were true.
- As the P-value gets smaller and smaller, our evidence gets stronger that there is a difference in the true percentages/ proportions.

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Interpretation of P value

- For small P-values, we tend to say that we reject the null hypothesis and assume that a true difference exists.
- For larger P-values, we say that we cannot reject the null hypothesis and that our observed difference probably arose by chance (due to sampling variability)

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How small is small?!

- Many people use P=0.05 as their cut-off point for rejecting or not the null hypothesis
- Caution – the "grey area"
- It is better, where possible, to present the actual P-value rather than simply say it is <0.05 or >0.05

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How small is small?!

- Actual P-values (or almost) can be obtained from statistical tables or computer programs
- We will use R in today's exercise.

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Exercise

Women aged 19-24

- 4893 vegetarian, of whom 1429 were anaemic $p_1 = 29.2\%$
- 11031 non-vegetarian, of whom 3011 were anaemic $p_2 = 27.3\%$
- Z-test to compare 2 proportions → P=0.01
- Conclusion?

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95% confidence interval for a difference in two percentages

Observed difference ± 1.96 x SE (difference)

Example 1 CMF vs L-Pam (P<0.001)

95%CI is 52.7% - 19.8% ± 1.96 x6.65%

= **19.9% to 45.9%**

Example 2 anturane trial (P=0.17)

95%CI is 5.6% - 4.1% ± 1.96x1.1%

= **-0.7% to 3.7%**

Note: close link between significance testing and confidence intervals

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Summary

- To compare results between two groups, must take account of sampling variability. Use a significance test.
- Formulate a null hypothesis, H_0 (no difference in the two true values) and try to disprove it.
- Today - Z-test for the comparison of two proportions (or %s).
- Compute Z-statistic which depends on the magnitude of the difference in the proportions and the sample sizes.

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Summary

- Calculate P-value from Normal distribution (in R or using statistical tables)
- P-value tells us how likely it is that we would observe a result like we have in our sample, if H_0 was true.
- Small P-values, reject H_0, true difference is likely.
- Large P-values, cannot reject H_0 and observed difference likely due to chance.
- Balance statistical significance with clinical/public health importance

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More exercises

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1. Peppermint Eases Pain?

The following report appeared in a popular magazine:

PEPPERMINT EASES PAIN

Peppermint oil has for a long time been known to help relieve indigestion. Now, doctors at two Danish hospitals have carried out a joint clinical trial using Obbekjaers Peppermint Oil Capsules on patients with irritable bowel syndrome - an uncomfortable, often chronic, condition where the bowel goes into spasm, which can cause a combination of pain, wind, constipation and/or diarrhoea. They found that out of 19 sufferers given peppermint oil, 13 (68%) found relief compared to only 6 (26%) from the other group of 23 patients who were not given the oil. Peppermint oil appears to have a relaxing effect on the bowel spasms which cause the symptoms. Packs of 90 capsules are priced at £4.45 (recommended dose between three and six a day) and are available from most health food stores including Holland and Barrett.

a) Assess the evidence for there being a difference in the proportion getting relief between those given peppermint oil and those not given the oil.

b) Discuss what design issues need to be considered in assessing the reliability of these data.

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2. Treatment of mild hypertension

The UK Medical Research Council Trial of Hypertension in Older Adults reported its findings in the British Medical Journal on 15 February 1992.

The trial recruited from general practices 4,396 patients aged 65-74 whose systolic blood pressure was between 160 and 209 mm Hg and whose diastolic blood pressure was less than 115 mm Hg. Patients were randomly assigned to initial therapy with a diuretic or a beta-blocker or a matched placebo, and then followed up for an average 5.8 years. The main objective was to see if a policy of anti-hypertensive treatment reduces the risk of stroke, coronary heart disease and death.

The main results are as follows:

	Active treatment		
	Diuretic	Beta-blocker	Placebo
No. of patients	1081	1102	2213
Strokes	45	56	134
Coronary events	48	80	159
Deaths	134	167	315

Assess the evidence for there being differences in the risk of :

a) death between those taking diuretic and those taking placebo

b) stroke between those taking beta-blocker and those taking placebo

Optional

c) Assess the evidence for there being differences in the risk of each of:
(i) stroke, (ii) coronary event (iii) death
between those taking active treatment (i.e. beta-blocker and diuretic groups combined) and those taking placebo

d) What issues in the design of this study need to be considered when interpreting these findings?

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