

2019½ 2020

$$10 \qquad 1 \qquad \int_a^{+\infty} \frac{1}{x(\ln x)^2} dx \quad (a > 1)$$

$$10 \qquad 2 \qquad \int_0^{+\infty} \frac{x - \sin x}{x^3} dx$$

$$10 \qquad \sum_{n=1}^{\infty} \frac{\sin nx}{\sqrt{x}} \qquad \left[\frac{\pi}{2}, \pi \right]$$

$$10 \qquad \sum_{n=1}^{\infty} \left(1 - \frac{1}{n} \right)^{n^2} x^n$$

$$15 \qquad \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{n+1}}{n(n+1)}$$

$$15 \qquad 2\pi \qquad f(x) = \begin{cases} -\frac{\pi}{4} & -\pi < x < 0 \\ \frac{\pi}{4} & 0 \leq x < \pi \end{cases}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$$

$$10 \qquad I(\theta) = \int_0^{\frac{\pi}{2}} \ln(\sin^2 x + \theta^2 \cos^2 x) dx$$

$$10 \qquad \int_0^{+\infty} e^{-t} \frac{\sin \alpha t}{t} dt \quad 0 < \alpha < \infty$$

$$10 \qquad \{\varphi_n(x)\} \qquad [a, b]$$

$$\int_a^b \varphi_m(x) \varphi_n(x) dx = \begin{cases} 0 & m \neq n \\ 1 & m = n \end{cases} \quad f(x) \quad [a, b]$$

$$\alpha_n = \int_a^b f(x) \varphi_n(x) dx$$

$$1 \quad \sum_{k=1}^{\infty} \alpha_k^2$$

$$2 \quad \sum_{k=1}^{\infty} \alpha_k^2 \leq \int_a^b f^2(x) dx$$