Operating Systems Design
21. Cryptography: An Introduction

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Cryptography ≠ Security

Cryptography may be a component of a secure system

Adding cryptography may not make a system secure

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Terms

Plaintext (cleartext), message *M*

encryption, E(*M*)

produces ciphertext, *C*=E(*M*)

decryption: *M*=D(*C*)

Cryptographic algorithm, cipher

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Terms: types of ciphers

- Types
  - restricted cipher
  - symmetric algorithm
  - public key algorithm
- Stream vs. Block
  - Stream cipher
    - Encrypt a message a character at a time
  - Block cipher
    - Encrypt a message a chunk at a time

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Restricted cipher

Secret algorithm

- Vulnerable to:
  - Leaking
  - Reverse engineering
    - HD DVD (Dec 2006) and Blu-Ray (Jan 2007)
    - RC4
    - All digital cellular encryption algorithms
    - DVD and DIVX video compression
    - Firewire
    - Enigma cipher machine
    - Every NATO and Warsaw Pact algorithm during Cold War
- Not a viable approach!

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The key

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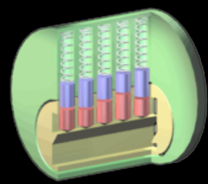
The key



Source: en.wikipedia.org/wiki/Pin_tumbler_lock

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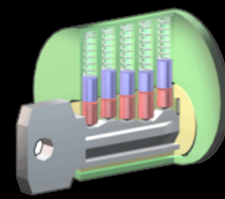
The key



Source: en.wikipedia.org/wiki/Pin_tumbler_lock

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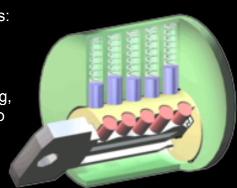
The key

- We understand how it works:
  - Strengths
  - Weaknesses
- Based on this understanding, we can assess how much to trust the key & lock.



Source: en.wikipedia.org/wiki/Pin_tumbler_lock

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Symmetric algorithm

Secret key

$$C = E_K(M)$$

$$M = D_K(C)$$

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Public key algorithm

Public and private keys

$$C_1 = E_{public}(M)$$
$$M = D_{private}(C_1)$$

also:

$$C_2 = E_{private}(M)$$
$$M = D_{public}(C_2)$$

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McCarthy's puzzle (1958)

The setting:

- Two countries are at war
- One country sends spies to the other country
- To return safely, spies must give the border guards a password

- Spies can be trusted
- Guards chat – information given to them may leak

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McCarthy's puzzle

Challenge

How can a guard authenticate a person without knowing the password?

Enemies cannot use the guard's knowledge to introduce their own spies

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Solution to McCarthy's puzzle

Michael Rabin, 1958

Use one-way function, $B = f(A)$
- Guards get *B*
  - Enemy cannot compute A if they know *A*
- Spies give *A*, guards compute *f(A)*
  - If the result is *B*, the password is correct.

Example function:

Middle squares
- Take a 100-digit number (A), and square it
- Let B = middle 100 digits of 200-digit result

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One-way functions

- Easy to compute in one direction
- Difficult to compute in the other

Examples:

Factoring:

$pq = N$	EASY
find p,q given N	DIFFICULT

Discrete Log:

$a^b \bmod c = N$	EASY
find b given a, c, N	DIFFICULT

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McCarthy's puzzle example

Example with an 18 digit number

A = 289407349786637777

A² = 837566141105253089484453338203501729

Middle square, B = 110525308948445338

Given A, it is easy to compute B

Given B, it is extremely hard to compute A

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Hash functions

- one-way function
  - Rabin, 1958: McCarthy's problem
  - middle squares, exponentiation, …
- [one-way] hash function
  - message digest, fingerprint, cryptographic checksum, integrity check
- encrypted hash
  - message authentication code
  - only possessor of key can validate message

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Popular hash functions

- SHA-2
  - Designed by the NSA; published by NIST
  - SHA-224, SHA-256, SHA-384, SHA-512
    - e.g., Linux passwords used MD5 and now SHA-512
- SHA-3
  - Under development
- MD5
  - 128 bits (not often used now since weaknesses were found)
- Derivations from ciphers:
  - Blowfish (used for password hashing in OpenBSD)
  - 3DES – used for old Linux password hashes

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Cryptography: what is it good for?

- Authentication
  - determine origin of message
- Integrity
  - verify that message has not been modified
- Nonrepudiation
  - sender should not be able to falsely deny that a message was sent
- Confidentiality
  - others cannot read contents of the message

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Cryptographic toolbox

- Symmetric encryption
- Public key encryption
- One-way hash functions
- Random number generators

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Popular symmetric algorithms

- AES (Advanced Encryption Standard)
  - FIPS standard since 2002
  - 128, 192, or 256-bit keys; operates on 128-bit blocks
- DES, 3DES
  - FIPS standard since 1976
  - 56-bit key; operates on 64-bit (8-byte) blocks
  - Triple DES recommended since 1999 (112 or 168 bits)
- Blowfish
  - Key length from 23-448 bits; 64-bit blocks
- IDEA
  - 128-bit keys; operates on 64-bit blocks
  - More secure than DES but faster algorithms are available

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Is DES secure?

56-bit key makes DES relatively weak
- 7.2×10^{16} keys
- Brute-force attack

Late 1990's:
- DES cracker machines built to crack DES keys in a few hours
- DES Deep Crack: 90 billion keys/second
- Distributed.net: test 250 billion keys/second

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The power of 2

Adding an extra bit to a key doubles the search space.

Suppose it takes 1 second to attack a 20-bit key:

- 21-bit key: 2 seconds
- 32-bit key: 1 hour
- 40-bit key: 12 days
- 56-bit key: 2,178 years
- 64-bit key: >557,000 years!

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AES

From NIST:

Assuming that one could build a machine that could recover a DES key in a second (i.e., try 2^{56} keys per second), then it would take that machine approximately 149 trillion years to crack a 128-bit AES key. To put that into perspective, the universe is believed to be less than 20 billion years old.

http://csrc.nist.gov/encryption/aes/

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Increasing The Key

Can double encryption work for DES?

- Useless if we could find a key K such that:

  $E_K(P) = E_{K2}(E_{K1}(P))$

- This does not hold for DES (luckily!)

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Double DES

Vulnerable to meet-in-the-middle attack

If we know some pair (P, C), then:

[1] Encrypt P for all 2^{56} values of K_1

[2] Decrypt C for all 2^{56} values of K_2

For each match where [1] = [2]

- test the two keys against another P, C pair
- if match, you are assured that you have the key

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Triple DES

Triple DES with two 56-bit keys:

$C = E_{K1}(D_{K2}(E_{K1}(P)))$

Triple DES with three 56-bit keys:

$C = E_{K3}(D_{K2}(E_{K1}(P)))$

Decryption used in middle step for compatibility with DES ($K_1=K_2=K_3$)

$C = E_K(D_K(E_K(P))) \equiv C = E_{K1}(P)$

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Secure Communication

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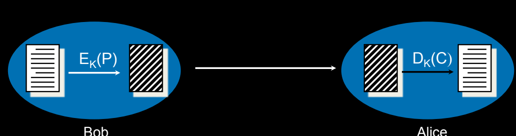
Communicating with symmetric cryptography

- Both parties must agree on a secret key, *K*
- Message is encrypted, sent, decrypted at other side



- Key distribution must be secret
  - otherwise messages can be decrypted
  - users can be impersonated

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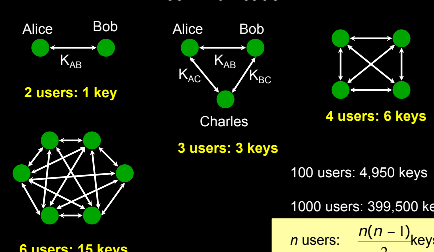
Key explosion

Each pair of users needs a separate key for secure communication



100 users: 4,950 keys

1000 users: 399,500 keys

n users: $\frac{n(n-1)}{2}$ keys

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Key distribution

Secure key distribution is the biggest problem with symmetric cryptography

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Key exchange

How can you communicate securely with someone you've never met?

Whit Diffie: idea for a *public key* algorithm

Challenge: can this be done securely?

Knowledge of public key should not allow derivation of private key

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Diffie-Hellman Key Exchange

Key distribution algorithm

- first algorithm to use public/private keys
- *not* public key encryption
- based on difficulty of computing discrete logarithms in a finite field compared with ease of calculating exponentiation

Allows two parties to compute a common key without fear from eavesdroppers. Then, they can securely transmit a session key.

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Diffie-Hellman Key Exchange

- All arithmetic performed in a field of integers modulo some large number
- Both parties agree on
  - a **large prime number *p***
  - and a number $\alpha < p$
- Each party generates a public/private key pair

private key for user *i*: X_i

public key for user *i*: $Y_i = \alpha^{X_i} \bmod p$

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Diffie-Hellman exponential key exchange

- Alice has secret key X_A
- Alice has public key Y_A
- Alice computes
- Bob has secret key X_B
- Bob has public key Y_B

$$K = Y_B^{X_A} \bmod p$$

K = (Bob's public key) $^{\textit{(Alice's private key)}}$ ***mod p***

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Diffie-Hellman exponential key exchange

- Alice has secret key X_A
- Alice has public key Y_A
- Alice computes
- Bob has secret key X_B
- Bob has public key Y_B
- Bob computes

$$K = Y_B^{X_A} \bmod p \qquad K' = Y_A^{X_B} \bmod p$$

K' = (Alice's public key) $^{\textit{(Bob's private key)}}$ ***mod p***

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Diffie-Hellman exponential key exchange

- Alice has secret key X_A
- Alice has public key Y_A
- Alice computes
  $K = Y_B^{X_A} \bmod p$
- expanding:

$$K = Y_B^{X_A} \bmod p = (\alpha^{X_B} \bmod p)^{X_A} \bmod p = \alpha^{X_B X_A} \bmod p$$

- Bob has secret key X_B
- Bob has public key Y_B
- Bob computes
  $K' = Y_A^{X_B} \bmod p$
- expanding:

$$K = Y_B^{X_A} \bmod p = (\alpha^{X_B} \bmod p)^{X_A} \bmod p = \alpha^{X_B X_A} \bmod p$$

$K = K'$

K is a common key, known *only* to Bob and Alice

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RSA: Public Key Cryptography

- Ron Rivest, Adi Shamir, Leonard Adleman created a true public key encryption algorithm in 1977
- Each user generates two keys:
  - private key (kept secret)
  - public key (can be shared with anyone)
- Difficulty of algorithm based on the difficulty of factoring large numbers
  - keys are functions of a pair of large (~200 digits) prime numbers

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RSA algorithm

Generate keys

- choose two random large prime numbers p, q
- Compute the product $n = pq$
- randomly choose the encryption key, e, such that:
  e and $(p - 1)(q - 1)$ are relatively prime
- use the extended Euclidean algorithm to compute the decryption key, d :
  $ed = 1 \bmod ((p - 1)(q - 1))$
  $d = e^{-1} \bmod ((p - 1)(q - 1))$
- discard p, q

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RSA Encryption

- Key pair: e, d
- Agreed-upon modulus: n
- Encrypt:
  - divide data into numerical blocks < n
  - encrypt each block:
    $c = m^e \bmod n$
- Decrypt:
  $m = c^d \bmod n$

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Communication with public key algorithms

Different keys for encrypting and decrypting

- no need to worry about key distribution

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Communication with public key algorithms

Alice | Bob

Alice's public key: K_A → ← Bob's public key: K_B

exchange public keys
(or look up in a directory/DB)

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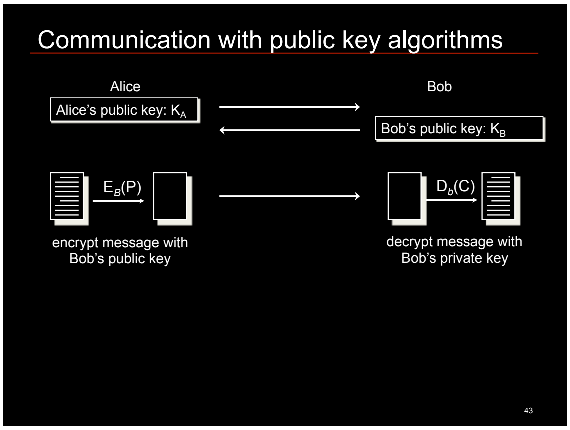
Communication with public key algorithms
Alice
Alice's public key: K_A
Bob
Bob's public key: K_B
$E_B(P)$
$D_b(C)$
encrypt message with Bob's public key
decrypt message with Bob's private key
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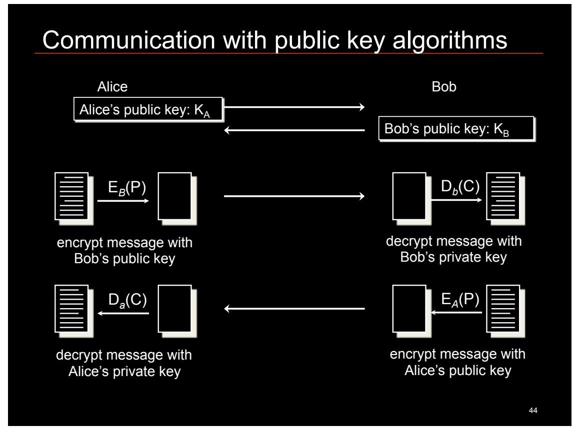
Communication with public key algorithms
Alice
Alice's public key: K_A
Bob
Bob's public key: K_B
$E_B(P)$
$D_b(C)$
encrypt message with Bob's public key
decrypt message with Bob's private key
$D_a(C)$
$E_A(P)$
decrypt message with Alice's private key
encrypt message with Alice's public key
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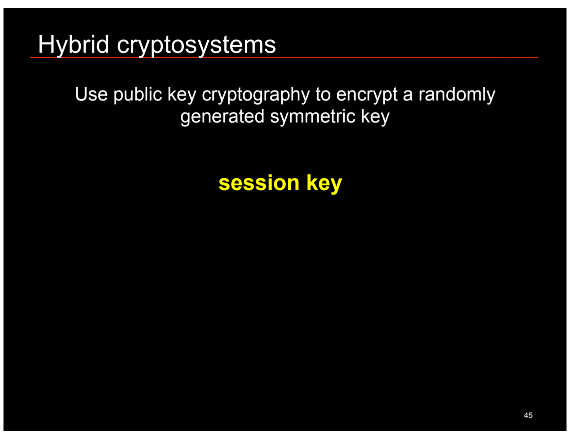
Hybrid cryptosystems
Use public key cryptography to encrypt a randomly generated symmetric key
session key
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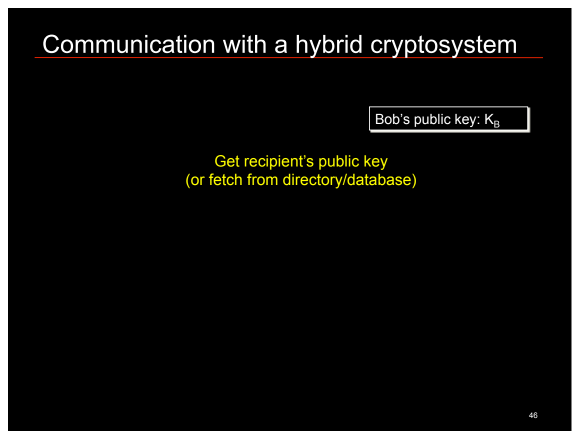
Communication with a hybrid cryptosystem
Bob's public key: K_B
Get recipient's public key
(or fetch from directory/database)
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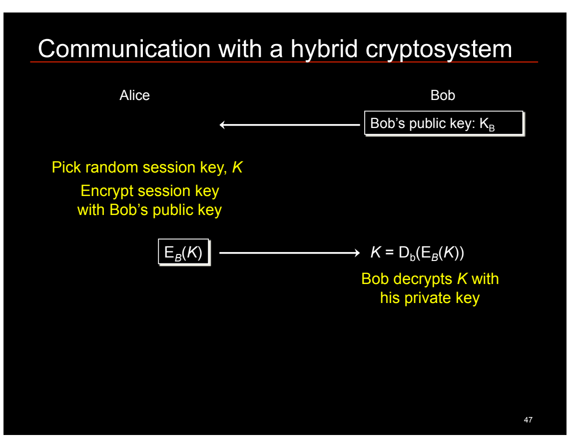
Communication with a hybrid cryptosystem
Alice
Bob
Bob's public key: K_B
Pick random session key, *K*
Encrypt session key with Bob's public key
$E_B(K)$
$K = D_b(E_B(K))$
Bob decrypts *K* with his private key
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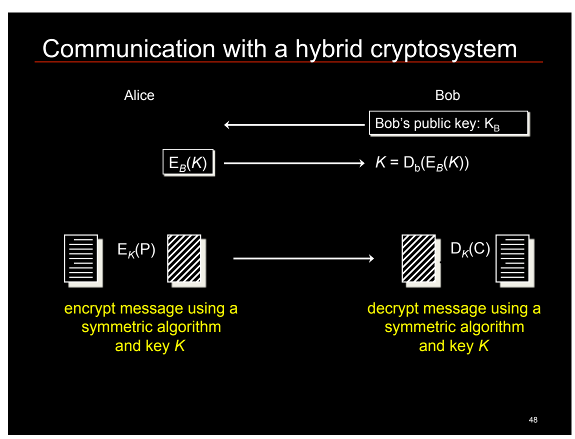
Communication with a hybrid cryptosystem
Alice
Bob
Bob's public key: K_B
$E_B(K)$
$K = D_b(E_B(K))$
$E_K(P)$
$D_K(C)$
encrypt message using a symmetric algorithm and key *K*
decrypt message using a symmetric algorithm and key *K*
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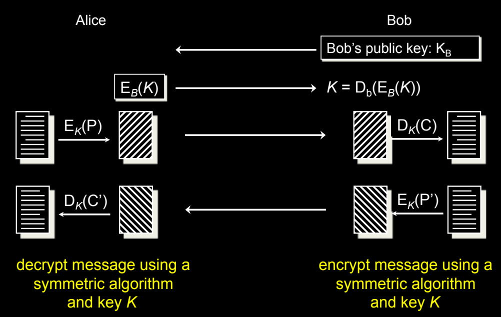
Communication with a hybrid cryptosystem



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Digital Signatures

- Validate the creator (signer) of the content
- Validate the the content has not been modified since it was signed
- The content does not have to be encrypted

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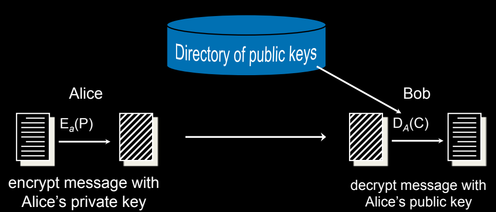
Digital signatures - public key cryptography

Encrypting a message with a private key is the same as signing it!



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But

- We don't want to permute/hide the content
  - If Alice was sending binary data to Bob, how would he deduce that it decrypted correctly?
- Public key encryption is considerably slower than symmetric encryption

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Signatures: Hashes to the rescue!

- Create a hash of the message
- Encrypt the hash with your public key and send it with the message
- Recipient validates the hash by decrypting it with your public key and comparing it with the hash of the received message
  - If the hashes don't match, that means either
    (a) the message was modified
    or (b) the encrypted hash was modified
    or (c) the hash was not encrypted by the correct party

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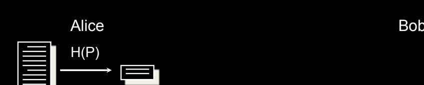
Digital signatures - public key cryptography



Alice generates a hash of the message

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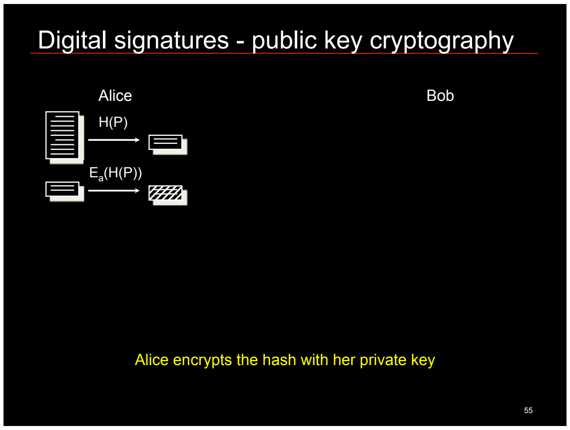
Digital signatures - public key cryptography
Alice
Bob
H(P)
E_a(H(P))
Alice encrypts the hash with her private key
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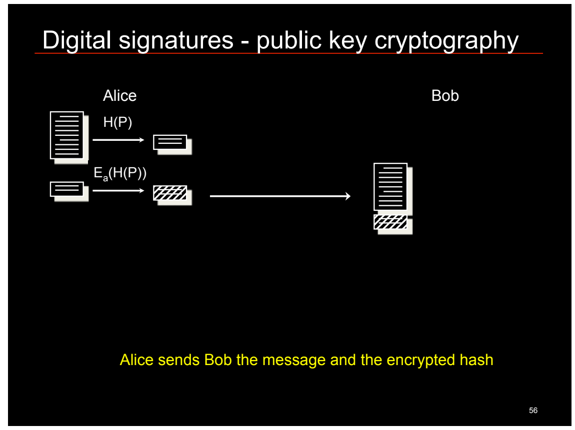
Digital signatures - public key cryptography
Alice
Bob
H(P)
E_a(H(P))
Alice sends Bob the message and the encrypted hash
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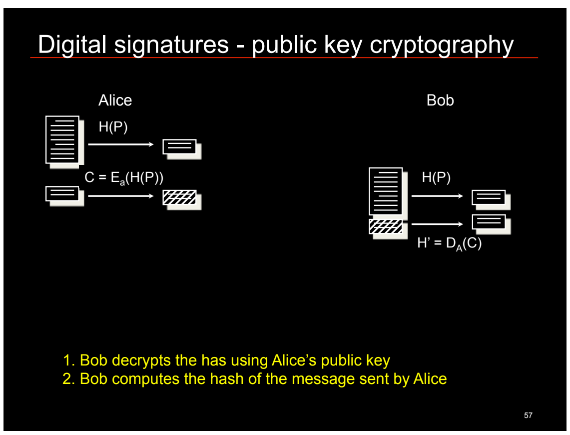
Digital signatures - public key cryptography
Alice
Bob
H(P)
C = E_a(H(P))
H(P)
H' = D_A(C)
1. Bob decrypts the has using Alice's public key
2. Bob computes the hash of the message sent by Alice
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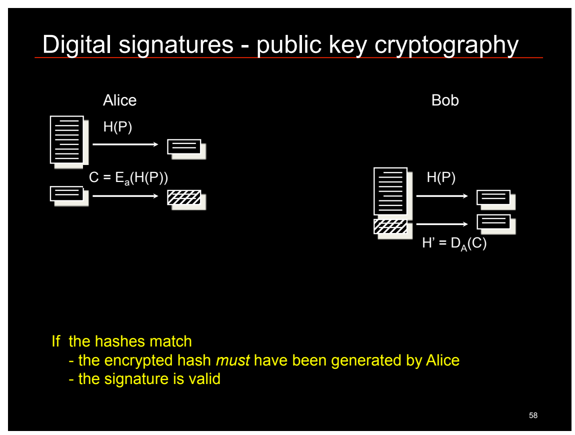
Digital signatures - public key cryptography
Alice
Bob
H(P)
C = E_a(H(P))
H(P)
H' = D_A(C)
If the hashes match
- the encrypted hash *must* have been generated by Alice
- the signature is valid
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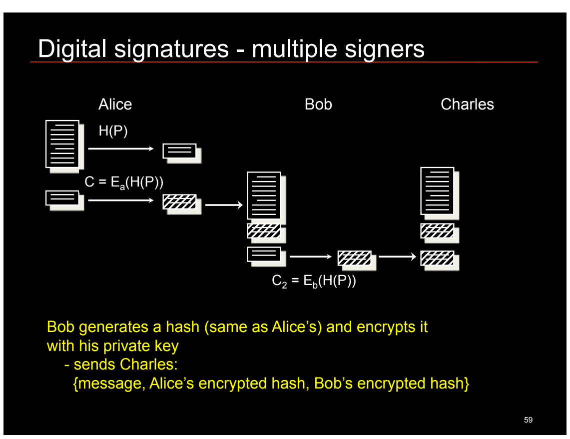
Digital signatures - multiple signers
Alice
Bob
Charles
H(P)
C = E_a(H(P))
C_2 = E_b(H(P))
Bob generates a hash (same as Alice's) and encrypts it
with his private key
- sends Charles:
{message, Alice's encrypted hash, Bob's encrypted hash}
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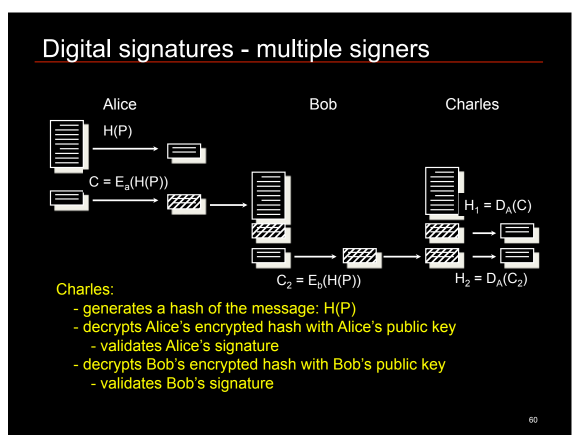
Digital signatures - multiple signers
Alice
Bob
Charles
H(P)
C = E_a(H(P))
C_2 = E_b(H(P))
H_1 = D_A(C)
H_2 = D_A(C_2)
Charles:
- generates a hash of the message: H(P)
- decrypts Alice's encrypted hash with Alice's public key
- validates Alice's signature
- decrypts Bob's encrypted hash with Bob's public key
- validates Bob's signature
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Covert AND authenticated messaging

If we want to keep the message secret

- combine encryption with a digital signature
- use a session key:
  pick a random key, *K*, to encrypt the message with a symmetric algorithm
- encrypt *K* with the public key of each recipient
- for signing, encrypt the hash of the message with sender's private key

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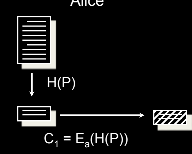
Secure and authenticated messaging



Alice generates a digital signature by encrypting the message digest with her private key.

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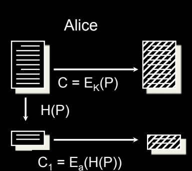
Secure and authenticated messaging



Alice picks a random key, *K*, and encrypts the message (P) with it using a symmetric algorithm.

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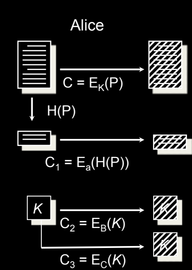
Secure and authenticated messaging



Alice encrypts the session key for each recipient of this message: Bob and Charles using their public keys.

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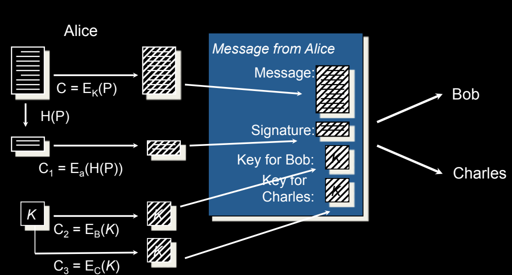
Secure and authenticated messaging



The aggregate message is sent to Bob and Charles

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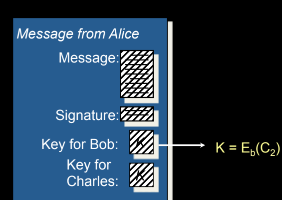
Secure and authenticated messaging



Bob receives the message:
- extracts key by decrypting it with his private key

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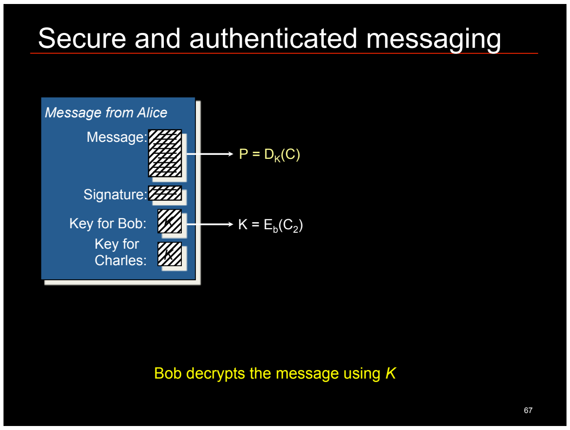
Secure and authenticated messaging
Message from Alice
Message:
Signature:
Key for Bob:
Key for Charles:
$P = D_K(C)$
$K = E_b(C_2)$
Bob decrypts the message using *K*
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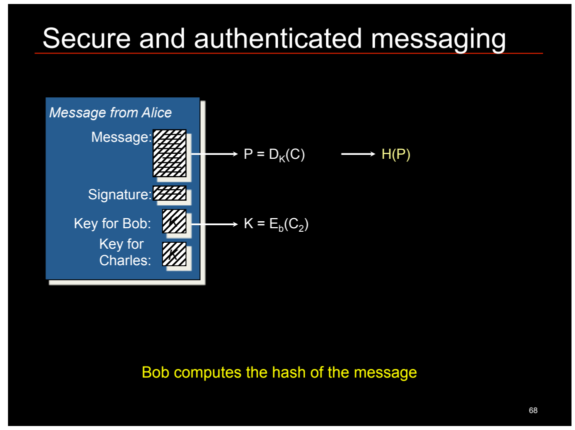
Secure and authenticated messaging
Message from Alice
Message:
Signature:
Key for Bob:
Key for Charles:
$P = D_K(C)$
$H(P)$
$K = E_b(C_2)$
Bob computes the hash of the message
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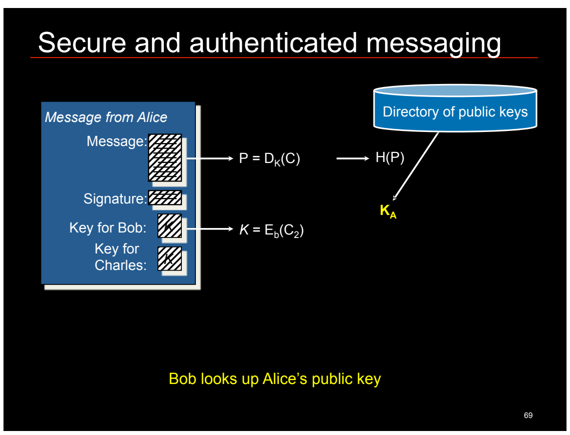
Secure and authenticated messaging
Directory of public keys
Message from Alice
Message:
Signature:
Key for Bob:
Key for Charles:
$P = D_K(C)$
$H(P)$
K_A
$K = E_b(C_2)$
Bob looks up Alice's public key
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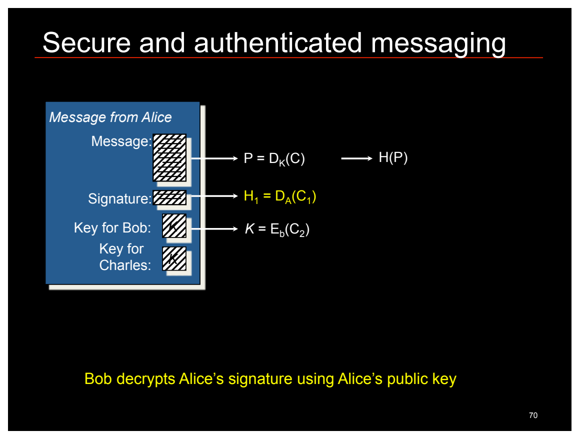
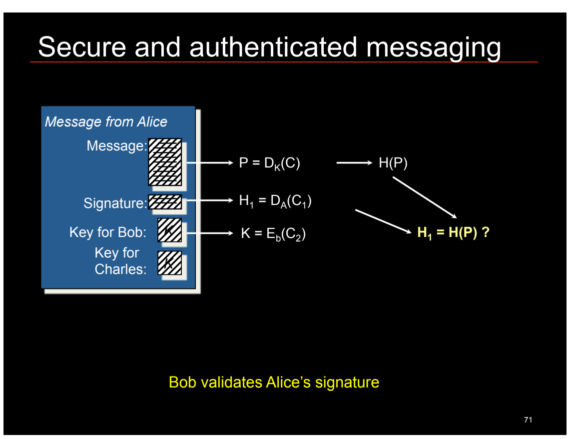

Secure and authenticated messaging
Message from Alice
Message:
Signature:
Key for Bob:
Key for Charles:
$P = D_K(C)$
$H(P)$
$H_1 = D_A(C_1)$
$K = E_b(C_2)$
Bob decrypts Alice's signature using Alice's public key
70


Secure and authenticated messaging
Message from Alice
Message:
Signature:
Key for Bob:
Key for Charles:
$P = D_K(C)$
$H(P)$
$H_1 = D_A(C_1)$
$K = E_b(C_2)$
$H_1 = H(P)$?
Bob validates Alice's signature
71


The End
72