

Exercises 19– 23 relate to inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth and knaves always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the ways described. If you cannot determine what these two people are, can you draw any conclusions?

The way to work these problems is by building an assumption table, assigning a T/F value to each statement, and then deciding if it is possible based on the fact that a liar can never tell the truth and a truth-teller can never lie. Any assertion that is possible may be an acceptable answer.

Template for knight/knave

<i>if A is:</i>	<i>if B is;</i>	<i>Truth values of statements</i>	<i>Is assertion possible?</i>
<i>knight</i>	<i>knight</i>	<i>A(?), B(?)</i>	
<i>knight</i>	<i>knave</i>	<i>A(?), B(?)</i>	
<i>knave</i>	<i>knight</i>	<i>A(?), B(?)</i>	
<i>knave</i>	<i>knave</i>	<i>A(?), B(?)</i>	

19. A says “At least one of us is a knave” and B says nothing.

<i>if A is:</i>	<i>if B is;</i>	<i>Truth values of statements</i>	<i>Is assertion possible?</i>
<i>knight</i>	<i>knight</i>	<i>A(F)</i>	<i>No</i>
<i>knight</i>	<i>knave</i>	<i>A(T)</i>	<i>Yes</i>
<i>knave</i>	<i>knight</i>	<i>A(T)</i>	<i>No</i>
<i>knave</i>	<i>knave</i>	<i>A(T)</i>	<i>No</i>

A knave can not say he is a knave, so A is a knight and his statement is true. Therefore B is a knave.

20. A says “The two of us are both knights” and B says “A is a knave.”

Since the two statements can not both be correct, A is a knave. Therefore B's statement is correct and B is a knight

<i>if A is:</i>	<i>if B is;</i>	<i>Truth values of statements</i>	<i>Is assertion possible?</i>
<i>knight</i>	<i>knight</i>	<i>A(T), B(F)</i>	
<i>knight</i>	<i>knave</i>	<i>A(F), B(F)</i>	
<i>knave</i>	<i>knight</i>	<i>A(F), B(T)</i>	<i>Yes</i>
<i>knave</i>	<i>knave</i>	<i>A(F), B(T)</i>	

21. A says “I am a knave or B is a knight” and B says nothing.

<i>if A is:</i>	<i>if B is;</i>	<i>Truth values of statements</i>	<i>Is assertion possible?</i>
<i>knight</i>	<i>knight</i>	<i>A(T)</i>	
<i>knight</i>	<i>knave</i>	<i>A(F)</i>	
<i>knave</i>	<i>knight</i>	<i>A(F)</i>	<i>Yes</i>
<i>knave</i>	<i>knave</i>	<i>A(F)</i>	

If A is a knave, then he would be telling the truth which he can not do, so he is a knight and B must also be a knight for his statement to be true

22. Both A and B say “I am a knight.”

<i>if A is:</i>	<i>if B is;</i>	<i>Truth values of statements</i>	<i>Is assertion possible?</i>
<i>knight</i>	<i>knight</i>	<i>A(T), B(T)</i>	<i>Yes</i>
<i>knight</i>	<i>knave</i>	<i>A(T), B(F)</i>	<i>Yes</i>
<i>knave</i>	<i>knight</i>	<i>A(F), B(T)</i>	<i>Yes</i>
<i>knave</i>	<i>knave</i>	<i>A(F), B(F)</i>	<i>Yes</i>

It is possible for either A or B to be either a knight or a knave.

23. A says, “We are both knaves” and B says nothing.

<i>if A is:</i>	<i>if B is;</i>	<i>Truth values of statements</i>	<i>Is assertion possible?</i>
<i>knight</i>	<i>knight</i>	<i>A(F)</i>	
<i>knight</i>	<i>knave</i>	<i>A(F)</i>	
<i>knave</i>	<i>knight</i>	<i>A(F)</i>	<i>Yes</i>
<i>knave</i>	<i>knave</i>	<i>A(T)</i>	

Since a knave can only lie, his statement is false and B is a knight.

Exercises 24– 31 relate to inhabitants of an island on which there are three kinds of people: knights who always tell the truth, knaves who always lie, and spies (called normals by Smullyan who can either lie or tell the truth. You encounter three people, A, B, and C. You know **one of these people is a knight, one is a knave, and one is a spy**. Each of the three people knows the type of person each of other two is. For each of these situations, if possible, determine whether there is a unique solution and determine who the knave, knight, and spy are. When there is no unique solution, list all possible solutions or state that there are no solutions.

Template for knight/knave/spy

<i>A</i>	<i>B</i>	<i>C</i>	<i>Truth values of statements</i>	<i>Is it possible?</i>
<i>knight</i>	<i>knave</i>	<i>spy</i>	<i>A()</i> , <i>B()</i> , <i>C()</i>	
<i>knight</i>	<i>spy</i>	<i>knave</i>	<i>A()</i> , <i>B()</i> , <i>C()</i>	
<i>knave</i>	<i>knight</i>	<i>spy</i>	<i>A()</i> , <i>B()</i> , <i>C()</i>	
<i>knave</i>	<i>spy</i>	<i>knight</i>	<i>A()</i> , <i>B()</i> , <i>C()</i>	
<i>spy</i>	<i>knight</i>	<i>knave</i>	<i>A()</i> , <i>B()</i> , <i>C()</i>	
<i>spy</i>	<i>knave</i>	<i>knight</i>	<i>A()</i> , <i>B()</i> , <i>C()</i>	

24. A says "C is the knave," B says, "A is the knight," and C says "I am the spy."

<i>A</i>	<i>B</i>	<i>C</i>	<i>Truth values of statements</i>	<i>Is it possible?</i>
<i>knight</i>	<i>knave</i>	<i>spy</i>	<i>A(F)</i> , <i>B(T)</i> , <i>C(T)</i>	
<i>knight</i>	<i>spy</i>	<i>knave</i>	<i>A(T)</i> , <i>B(T)</i> , <i>C(F)</i>	<i>Yes</i>
<i>knave</i>	<i>knight</i>	<i>spy</i>	<i>A(F)</i> , <i>B(F)</i> , <i>C(T)</i>	
<i>knave</i>	<i>spy</i>	<i>knight</i>	<i>A(F)</i> , <i>B(F)</i> , <i>C(F)</i>	
<i>spy</i>	<i>knight</i>	<i>knave</i>	<i>A(T)</i> , <i>B(F)</i> , <i>C(F)</i>	
<i>spy</i>	<i>knave</i>	<i>knight</i>	<i>A(F)</i> , <i>B(F)</i> , <i>C(F)</i>	

Only option is C is the knave, A is the knight, and C must be the spy.

25. A says "I am the knight," B says "I am the knave," and C says "B is the knight."

<i>A</i>	<i>B</i>	<i>C</i>	<i>Truth values of statements</i>	<i>Is it possible?</i>
<i>knight</i>	<i>knave</i>	<i>spy</i>	<i>A(T)</i> , <i>B(T)</i> , <i>C(F)</i>	
<i>knight</i>	<i>spy</i>	<i>knave</i>	<i>A(T)</i> , <i>B(F)</i> , <i>C(F)</i>	<i>Yes</i>
<i>knave</i>	<i>knight</i>	<i>spy</i>	<i>A(F)</i> , <i>B(F)</i> , <i>C(T)</i>	
<i>knave</i>	<i>spy</i>	<i>knight</i>	<i>A(F)</i> , <i>B(F)</i> , <i>C(F)</i>	
<i>spy</i>	<i>knight</i>	<i>knave</i>	<i>A(F)</i> , <i>B(F)</i> , <i>C(T)</i>	
<i>spy</i>	<i>knave</i>	<i>knight</i>	<i>A(F)</i> , <i>B(T)</i> , <i>C(F)</i>	

Only option is A is the knight, is the spy, and C must be the knave.

26. A says, "I am the knave," B says "I am the knave," and C says "I am the knave."

<i>A</i>	<i>B</i>	<i>C</i>	<i>Truth values of statements</i>	<i>Is it possible?</i>
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<i>knight</i>	<i>knave</i>	<i>spy</i>	<i>A(F), B(T), C(F)</i>	
<i>knight</i>	<i>spy</i>	<i>knave</i>	<i>A(F), B(F), C(T)</i>	
<i>knave</i>	<i>knight</i>	<i>spy</i>	<i>A(T), B(F), C(F)</i>	
<i>knave</i>	<i>spy</i>	<i>knight</i>	<i>A(T), B(F), C(F)</i>	
<i>spy</i>	<i>knight</i>	<i>knave</i>	<i>A(F), B(F), C(T)</i>	
<i>spy</i>	<i>knave</i>	<i>knight</i>	<i>A(F), B(T), C(F)</i>	

None are possible.

27. A says "I am the knight," B says "A is telling the truth," and C says "I am the spy."

<i>A</i>	<i>B</i>	<i>C</i>	<i>Truth values of statements</i>	<i>Is it possible?</i>
<i>knight</i>	<i>knave</i>	<i>spy</i>	<i>A(T), B(T), C(T)</i>	
<i>knight</i>	<i>spy</i>	<i>knave</i>	<i>A(T), B(T), C(F)</i>	<i>Yes</i>
<i>knave</i>	<i>knight</i>	<i>spy</i>	<i>A(F), B(F), C(T)</i>	
<i>knave</i>	<i>spy</i>	<i>knight</i>	<i>A(F), B(F), C(F)</i>	
<i>spy</i>	<i>knight</i>	<i>knave</i>	<i>A(F), B(F), C(F)</i>	
<i>spy</i>	<i>knave</i>	<i>knight</i>	<i>A(F), B(F), C(F)</i>	

Only option is A is the knight, B is the spy, and C must be the knave.

28. A says "I am the knight," B says, "A is not the knave," and C says "B is not the knave."

<i>A</i>	<i>B</i>	<i>C</i>	<i>Truth values of statements</i>	<i>Is it possible?</i>
<i>knight</i>	<i>knave</i>	<i>spy</i>	<i>A(T), B(T), C(F)</i>	
<i>knight</i>	<i>spy</i>	<i>knave</i>	<i>A(T), B(T), C(T)</i>	
<i>knave</i>	<i>knight</i>	<i>spy</i>	<i>A(F), B(F), C(F)</i>	
<i>knave</i>	<i>spy</i>	<i>knight</i>	<i>A(F), B(F), C(T)</i>	<i>Yes</i>
<i>spy</i>	<i>knight</i>	<i>knave</i>	<i>A(F), B(T), C(T)</i>	
<i>spy</i>	<i>knave</i>	<i>knight</i>	<i>A(F), B(T), C(F)</i>	

Only option is C is the knight, B is the spy, and A must be the knave.

29. A says "I am the knight," B says "I am the knight," and C says "I am the knight."

<i>A</i>	<i>B</i>	<i>C</i>	<i>Truth values of statements</i>	<i>Is it possible?</i>
<i>knight</i>	<i>knave</i>	<i>spy</i>	<i>A(T), B(T), C(F)</i>	
<i>knight</i>	<i>spy</i>	<i>knave</i>	<i>A(T), B(T), C(T)</i>	
<i>knave</i>	<i>knight</i>	<i>spy</i>	<i>A(F), B(F), C(F)</i>	

<i>knave</i>	<i>spy</i>	<i>knight</i>	<i>A(F), B(F), C(T)</i>	<i>Yes</i>
<i>spy</i>	<i>knight</i>	<i>knave</i>	<i>A(F), B(T), C(T)</i>	
<i>spy</i>	<i>knave</i>	<i>knight</i>	<i>A(F), B(T), C(F)</i>	

Only option is C is the knight, B is the spy, and A must be the knave.

30. A says "I am not the spy," B says "I am not the spy," and C says "A is the spy."

<i>A</i>	<i>B</i>	<i>C</i>	<i>Truth values of statements</i>	<i>Is it possible?</i>
<i>knight</i>	<i>knave</i>	<i>spy</i>	<i>A(T), B(T), C(F)</i>	
<i>knight</i>	<i>spy</i>	<i>knave</i>	<i>A(T), B(F), C(F)</i>	
<i>knave</i>	<i>knight</i>	<i>spy</i>	<i>A(T), B(T), C(F)</i>	
<i>knave</i>	<i>spy</i>	<i>knight</i>	<i>A(T), B(F), C(F)</i>	
<i>spy</i>	<i>knight</i>	<i>knave</i>	<i>A(F), B(T), C(T)</i>	
<i>spy</i>	<i>knave</i>	<i>knight</i>	<i>A(F), B(T), C(T)</i>	

Only option is A is the knight, B is the spy, and C must be the knave.

31. A says "I am not the spy," B says "I am not the spy," and C says "I am not the spy."

<i>A</i>	<i>B</i>	<i>C</i>	<i>Truth values of statements</i>	<i>Is it possible?</i>
<i>knight</i>	<i>knave</i>	<i>spy</i>	<i>A(T), B(T), C(F)</i>	
<i>knight</i>	<i>spy</i>	<i>knave</i>	<i>A(T), B(F), C(T)</i>	
<i>knave</i>	<i>knight</i>	<i>spy</i>	<i>A(T), B(T), C(F)</i>	
<i>knave</i>	<i>spy</i>	<i>knight</i>	<i>A(T), B(F), C(T)</i>	
<i>spy</i>	<i>knight</i>	<i>knave</i>	<i>A(F), B(T), C(T)</i>	
<i>spy</i>	<i>knave</i>	<i>knight</i>	<i>A(F), B(T), C(T)</i>	

None are possible.