Cryptocurrency Investment DAOs As a New Venture Capital Platform: Power Law in Startup Returns

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ABSTRACT

Startup investors have considerably more misses than hits. We use the power law to model the performance of startup portfolios. With the power law model at hand, we identify the key factors that drive excess returns. We quantify the chance of an outlier in a portfolio of early-stage investments and its expected return multiple, we discuss the optimal portfolio size and the optimal holding period. We calibrate the tail index for cryptocurrency and NFT startups and compare it to that of traditional startup investments.

Furthermore, we introduce the concept of investment DAOs (decentralized autonomous organization) and evaluate startup portfolio construction and expected alpha in a comparative discussion of the traditional venture capital model. Unlike traditional venture capital, DAO investors have autonomy over investment decisions. We identify two characteristics innate to DAOs, the absence of the agency problem and the absence of a fixed investment period with the ensuing long-term perspective, that will provide DAOs a tailwind for investment performance.

KEYWORDS AND JEL CODES

- Startup returns follow the power law. The “tail-ness” of cryptocurrency and NFT returns is stronger than that of traditional startup returns.
- There were numerous launches of investment DAOs (decentralized autonomous organization) over the past year in the cryptocurrency sector. Proponents of DAOs highlight they have the potential to reshape the way we work, make group decisions, allocate resources, and distribute gains.
- We identify two characteristics typical to DAOs, the absence of the agency problem and the absence of a fixed investment period with the ensuing long-term perspective, that can help DAOs attain strong performance in startup investments.

TOPICS

Venture capital, cryptocurrency, portfolio theory, portfolio management

Inspection of startup return data reveals that only a small number of investments drives the entire portfolio performance. Startup investors realize early that they have considerably more misses than hits. In fact, one can make the proposition that venture investment is a business of extreme outliers where a very small percentage of the startup portfolio compensates for the losses from most investments, and on top of that generates the alpha – or excess returns – investors are aiming for. Beforehand, a startup investor even with the best diligence and differentiated support tailored to each startup portfolio company can't possibly know for certain which investment will turn out to be the outlier investment. In this article, we outline the "stochastic laws" behind this outlier phenomenon and then ask ourselves what this means for portfolio construction, optimal holding period and alpha generation.

We use the power law to describe and predict the performance of startup portfolios. With a power law model at hand, we are able to identify the key factors that will drive alpha.[1] Once equipped with this model, we narrow our focus on addressing the following four questions that are at the center of constructing and evaluating a venture portfolio:

1) What is the chance of an outlier in a portfolio of early-stage investments?
2) What return multiple can we expect for such an outlier?
3) What is the optimal portfolio size?
4) What is the optimal holding period?

In this discussion, we'll address the first question by using historical evidence that was gathered by established venture capital firms and investors in the past. The other three questions fall in the realm of the power law, and we address them coherently within that framework.

We apply the simplest specification of the power law. We assume that startup returns follow the Pareto distribution. More complex models were suggested in the literature, such as the Generalized Extreme Value Distribution or the Generalized Pareto Distribution (see, for example, Coles et al. 2001; Embrechts et al. 1997; McNeil et al. 2005). Although we acknowledge that the Pareto distribution could be restrictive, we believe that it has its advantages. More specifically, the Pareto distribution is parsimonious; in fact, we only need to estimate one parameter, the tail index which is commonly denoted as α in the academic literature. We will employ non-parametric estimators that were developed around the Hill's estimator which prove to be more robust in comparison to maximum likelihood, and the Hill's estimator itself.

A second aim of this article is to introduce the concept of investment DAOs (decentralized autonomous organization) and evaluate startup portfolio construction and expected alpha and investment performance in a comparative discussion of the traditional venture capital firm model. DAOs have had an incredible run over the past year in the cryptocurrency and NFT sectors, with numerous launches across a diverse range of themes. The reader can take a look at DeepDAO's website to get a sense of the existing and evolving landscape of active DAOs, their treasuries, number of token holders, members and their voting activities.[2] Supporters consider DAOs to have

[1] The power law distribution of startup returns was discussed by practitioners and academics: Korteweg and Sorensen 2011; Korver 2018; Malleby 2022; Neumann 2015, 2017; Thiel and Masters 2014.

[2] Their website https://deepdao.io/organizations contains a comprehensive dashboard of active DAOs and provides an interesting insight into the current DAO landscape in the cryptocurrency and NFT space.

the potential to reshape the way we work, make group decisions, allocate resources, and distribute the created wealth. DAOs are considered decentralized because the decision-making power ultimately rests on stakeholders instead of executives and corporate employees; instead of a top-down hierarchical structure, DAOs use Web 3 technologies coupled with incentive systems to distribute decision-making authority to its token holders and community members based on member participation, community contribution, but also individual possession of native tokens. Token holders have the right to submit and vote proposals, and at the same time participate in the upside of the DAO's market capitalization.

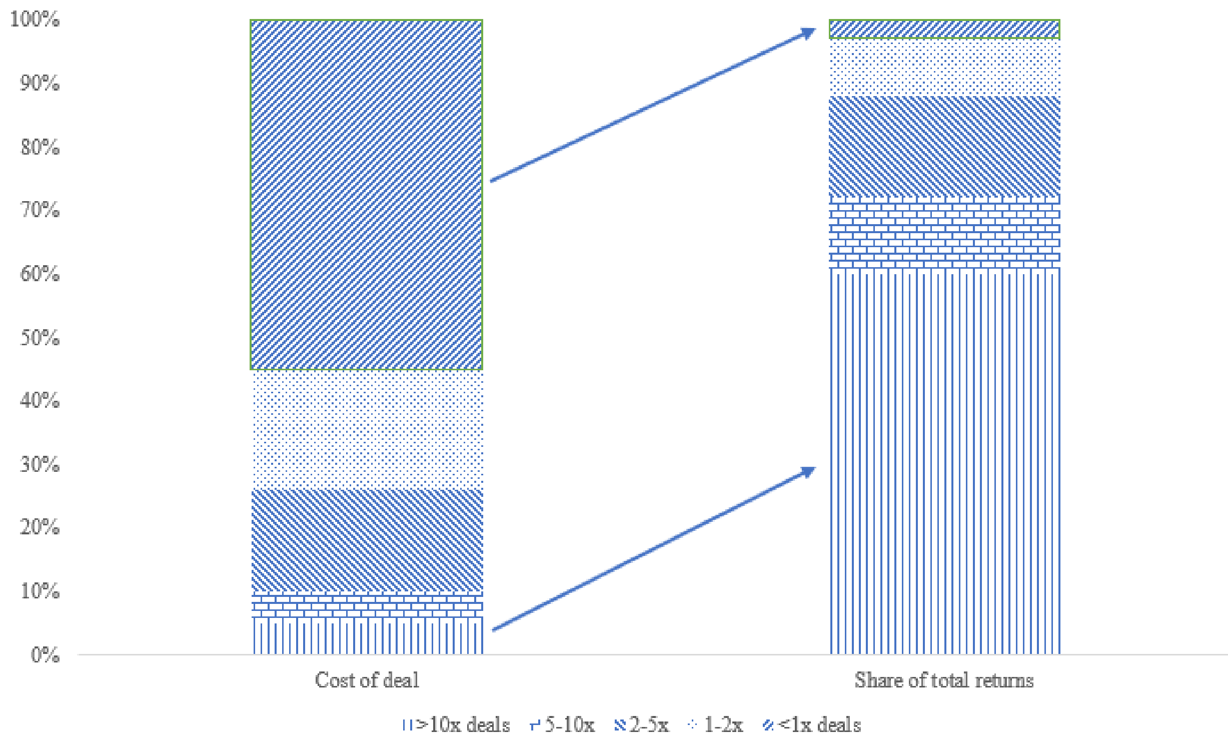
Unlike traditional venture capital or private equity funds in which limited partners such as institutional investors invest their money into a fund over which other people such as portfolio managers or general partners decide over how to invest the proceeds, investors in DAOs have autonomy over those investment decisions. Although investment DAOs today are characterized by a diverse set of governance and incentive models, they share some common features. In its essence, an investment DAO is a group of investors who pool their capital and make investments together. In some DAOs these pools are predetermined by an upfront commitment, in others, community members decide on a deal-by-deal basis whether they want to participate individually. Investment DAOs are innately social where community members are actively involved in the deals such as deal sourcing, founder calls, due diligence and ultimately funding although not every investment DAO member is expected to be involved in each one of these investment steps. Furthermore, investment DAOs aim to source high quality Web3 deals from its community members and tap its pool of community members with their diverse backgrounds and skill sets to support the portfolio startup companies in a tailored way in their growth trajectories. Proponents of investment DAOs anticipate that with the differentiated and community-driven support of the portfolio startups, the deal flow will improve both in quality and quantity leading to higher returns from which DAOs' treasuries will ultimately benefit. A stronger treasury will allow investment DAOs to provide services that will reinforce the investment community. Contributors are compensated with the native DAO token but are also rewarded a portion of the investment carry in the startups they help. In this article, we will make the case that the organizational form of investment DAOs is well suited to successfully implement the implications from the power law for early-stage investments.

WHAT IS THE PROBABILITY OF AN OUTLIER INVESTMENT IN A STARTUP FUND?

The odds to have an outlier in a startup fund are very low. This observation does not only apply to funds with a comparably poor track record. Tellingly, even the most stellar funds with their long-term impeccable track records face the same incredible uphill battle to find those outliers. We define here an investment outlier as a deal that generates a multiple on invested capital (MOIC) of 10 times or more. The summary chart in Exhibit 2 is based on data which was compiled by Horsely Bridge, a venture capital investor, and presented in Dixon (2015). The sample covers the entire US venture investment universe over 30 years from 1985-2014. The overall average probability of hitting a 10x MOIC is a mere 5%, and unsurprisingly, most deals (55% of all deals) return less than the invested capital.

EXHIBIT 1

U.S. Venture Investment Performance by Return Categories Measured in MOIC from 1985 – 2014 By Cost of All Deals and By Corresponding Share of Total Returns



Source: Blog post by Dixon (2015) using data compiled and summarized by Horsley Bridge, a venture capital limited partner and investor.

The information in Exhibit 1 provides us with almost all the necessary information to calculate expected returns of startup portfolios. In Exhibit 2, we use the information from the summary statistics from Exhibit 1 and present a schematic overview of how we will approach the question of constructing expected returns based on that data. Using the mid-points for the return multiple categories and their respective share of initial investments from Exhibit 1, we calculate a baseline return multiple of 1.3x on startup investments. This figure, however, does not include the return contributions from the outlier category of minimum 10x MOIC investments. The only information available at this stage of the analysis is that the outlier events generate a return of at least 10x and comprise 5% of the overall investments. Once we quantify expected returns conditional on a minimum threshold performance of 10x MOIC, we will use their corresponding 5% portfolio weight to populate total expected MOIC.

EXHIBIT 2

Schema of Breakdown of Startup Portfolio MOIC by Return Category

Return Multiple Category	**Numerical Proxy**	**Portfolio Weight**	**Return Multiple (Portfolio Weighted)**
>10x deals	*Use Pareto Distribution*	5%	*Expected MOIC driven by α*
5-10x	7.5	5%	0.4

2-5x	3.5	16%	0.6
1-2x	1.5	19%	0.3
<1x deals	0.2	55%	0.1
Total expected MOIC			**1.3 + weighted MOIC from >10x outliers**

Notes: α is the parameter for the tail index in the Pareto distribution. In the following sections, we will quantify α for various startup sectors including cryptocurrencies and NFTs. The discrepancy between the total baseline MOIC of 1.3x and the sum of the individual return multiple categories stem from rounding.
Source: Same data as displayed in Exhibit 1.

These expected returns depend on α, the tail index parameter of the Pareto distribution, our preferred choice of instrumentalizing the power law.

THE POWER LAW IN STARTUP RETURNS

The observation above that a single outlier company can make an entire investment fund suggests that the power law is at play in driving startup portfolio returns. The most important parameter in power law distributions is the tail index. The tail index captures the strength of the power law at play. We will use our findings to address the question of calculating conditional expected returns beyond pre-determined thresholds, in our case a 10x MOIC.

In a first step, we choose the Pareto distribution to specify the power law. There are a number of reasons for why we think that this is an appropriate specification. First, the Pareto distribution is the simplest model in extreme value theory. It is defined by only one parameter (up to a normalization parameter). Thanks to this parsimony of parameters, the estimation of the tail of the distribution will be more robust and less prone to specification errors and estimation challenges that typically come with multi-parameter maximum likelihood (see Embrechts et al. 1997 and McNeil et al. 2015 for a discussion of estimation techniques). Second, specifying the power law as a Pareto distribution will allow us to embed it into a simple and intuitive dynamic model of firm growth which in turn, together with the investment horizon, determines the size of the power law. Thanks to this model, we will be able to address in a coherent way the dynamic nature of optimal investment decisions in the early-stage sector, most importantly the determination of optimal holding periods. Lastly, many social phenomena, and more crucially, the fatness of tails in returns of various financial asset classes were estimated with the help of the Pareto distribution. Since this study is the first to our knowledge that attempts to quantify the size of the tail index in cryptocurrency projects and NFTs, cross-references to alternative asset classes will help us apply sanity checks to our estimates.

We will specify the random variable X as the MOIC as displayed in Exhibit 2, 1x, 2x, …, 10x etc. (we treat these multiples as a continuous variable). Based on historical data small return multiple outcomes are most likely, and large multiples are less likely. The probability distribution of the Pareto formula captures this feature as follows: the probability $p(x)$ that the random variable X is larger than some given number x is described as

$$p(x) = Cx^{-\alpha} \qquad (1)$$

where α is the tail index and defines the shape of the tail distribution and C is a normalization constant. One interpretation of C is the minimum value beyond which the power law applies in the tail. Equation (1) describes a survival function.

Equipped with this specification, we can calculate conditional expectations for return multiples X given that the multiple exceeds a particular extreme threshold. In our case, based on the summary statistics and schema from Exhibits 2 and 3, we choose this threshold to be a 10x return multiple. After applying some simplifying assumptions, the mean excess function for the Pareto distribution becomes

$$E[X-u|X>u]=\frac{u}{\alpha-1} \tag{2}$$

with u denoting the minimum threshold value. The derivation of (2) is outlined in the Appendix 1. We will set $u = 10$ later, representing the minimum threshold level of 10x MOIC.

Gabaix (2009), Newman (2005), and Reed and Hughes (2002) show that one mechanism that generates the power law is the exponential distribution. More specifically, we embed this suggested framework into a simple dynamic firm growth model (see also Neumann, 2015 and 2017). Assume that a company grows annually by a growth rate g. Then we can describe the value multiple $v(t)$ of the company with continuous compounding at horizon time t as:

$$v(t)=e^{gt}$$

If the growth rate g was a random variable, the company value multiple $v(t)$ would have an exponential distribution with horizon t a deterministic constant. The probability distribution of the company value multiple $p(v(t))$ is given by

$$p(v(t)=x)=\frac{1}{gt}x^{-\left(\frac{1}{gt}+1\right)}$$

which is a power law distribution with a Pareto specification as described in (1) with tail index

$$\alpha=\frac{1}{gt}+1 \tag{3}$$

We interpret t as exit time. This model describes that the tail index of the return multiple in the Pareto distribution is generated by a simple mechanism in which the tail index depends on the company growth and the time to exit, or the holding period. Equation (3) shows that the length of the holding period depends negatively on the tail index α. This implies that for a given firm growth rate g the exposure of the startup portfolio to the power law and hence the magnitude of the return multiples of outlier investments becomes larger the longer the investment horizon becomes. The more patient the startup investor is, the higher the expected return multiples are.

CRYPTOCURRENCIES AND NFTS ARE THE ASSET CLASS WITH THE FATTEST TAILS

In order to answer the questions outlined at the outset about expected outlier return multiples, optimal portfolio size and holding period, we need to estimate the tail index α of the Pareto distribution. For our comparative analysis, we cover tail index estimates for a wide range of financial asset classes. We rely on existing estimates of α from the literature for traditional startup sectors. For the cryptocurrency and NFT sectors, we estimate α ourselves by employing Hill's tail index estimator, a standard non-parametric estimation technique commonly used in the literature. This method is motivated by the Peak Over Threshold concept in extreme value theory in which the tail index is estimated based on observations that are greater than a given extremal threshold, which is set to include the top 15^{th} to 5^{th} percentile outliers.[3] In addition, we apply a modified version of the Hill's tail index estimator that is known to be more robust and less dependent on distribution specifications of the underlying data. The details are discussed in Appendix 2.

For cryptocurrencies, we use the market capitalization data from CoinMarketCap as of 4/4/2022. We constrained ourselves to the top 500 largest cryptocurrency projects in terms of their market capitalization based on the circulating supply and measured in USD. For NFTs, we use the data of the top 500 collectible projects from OpenSea in terms of cumulative transacted volumes in ETH as of 4/4/2022. We think that cumulative transacted volumes in ETH are a more reliable measure of an NFT collectible's overall valuation as they are directly indicative of the cashflows that they generate thanks to the perpetual royalties that flow to the NFT creators.

The non-parametric estimates for α for cryptocurrency and NFT projects are 0.85 and 1.0, respectively. We refer the reader to Appendix 3 for details. We are reluctant to use these estimates at face value without applying further sanity checks. First, an implication of a tail index value of less than or equal to one implies a non-finite mean in the Pareto distribution. This implication is clearly unrealistic as the market capitalization for any startup is bounded above by the size of the global economy in the most extreme case. We therefore require the tail index to be strictly larger than one.[4]

Second, as we have information on the size distribution of cryptocurrency projects and NFTs from the actual market capitalization data, we can calculate the empirical concentration ratios

[3] Hill's estimator is one of the most used estimators of the tail index of heavy-tailed distributions. We follow some of the good practices in employing the Hill's estimator as outlined in Aban et al. (2006), Cooke et al. (2011), Danielsson et al. (2019).

[4] There are estimators of the tail index for both the Pareto and the Generalized Pareto Distributions that explicitly allow to specify a lower and upper bound of the random variable. For instance, if the variable of interest is the company market capitalization, it may make sense to impose an upper bound to what the maximum firm value could become (see Embrechts et al., 2005; Falk et al. 2010; Haan and Ferreira, 2006; Taleb, 2020 for details).

and derive the implied α's associated with those concentration ratios. There is a clear mapping from the tail index α of the Pareto distribution to the level of market concentration. We refer the reader to Appendix 4 for the detailed derivation of that mapping. We use three market concentration ratios for our purposes to inspect the Pareto α's. These market concentration ratios are commonly used in the antitrust literature and empirical industrial organization: the C1 and C5 concentration ratios, which summarize the market shares of the top company and the top five companies, respectively, and the Herfindahl Index, which is a global measure of market concentration of an industry or sector as it makes use of market weights of every company in the sector.

Exhibit 3 presents the results. The concentration ratios for cryptocurrencies and NFTs are 39% and 18%, respectively for the C1 concentration ratio[5] and 67% and 42% for the C5 concentration ratio. The upper panel also presents the Herfindahl Index. The bottom panel of Exhibit 3 presents the corresponding α's that are associated with the three concentration ratios summarized in the upper panel. The power law tail indices range from 1.13 to 1.25 for cryptocurrency projects and from 1.27 to 1.55 for NFTs. We put more weight on the α implied by the Herfindahl Index as it represents a global concentration ratio using market weights of all projects. We acknowledge that our direct empirical estimations employing the Hill's and the moment estimator methods provided estimates of α that were too low.

EXHIBIT 3

Market Concentration Ratios in Cryptocurrency and NFT Sectors and Implied Power Law α's

	Empirical Market Concentration	
Concentration Ratio	**Cryptocurrency**	**NFT**
C1	39%	18%
C5	67%	42%
Herfindahl	0.20	0.05

	Implied Power Law α	
Concentration Ratio	**Cryptocurrency**	**NFT**
C1	1.25	1.55
C5	1.15	1.4
Herfindahl	1.13	1.27

Notes: C1 denotes the share of market capitalization of the largest project as a ratio to the top 500 cryptocurrency projects and NFT collectibles, respectively. C5 denotes the share of market capitalization of the five largest projects as a ratio to the top 500 cryptocurrency projects and NFT collectibles, respectively. The Herfindahl Index is defined as the sum of the squared market shares of the projects. The power law α's correspond to the empirical concentration ratios as reported in the top panel. They are derived from the empirical concentration ratios assuming that cryptocurrency and NFT market capitalizations have a Pareto distribution. The relationship between concentration ratios and the power law α are derived in detail in Appendix 4.
Source: Empirical market concentration ratios for cryptocurrency projects are calculated from market capitalization data from CoinMarketCap as of 4/4/2022 based on circulating supply. Empirical market concentration ratios for NFT projects are calculated from lifetime cumulative transaction volumes data in ETH from OpenSea as of 4/4/2022.

Nevertheless, we give some weight to our estimates of α. Combining these with the insights from Exhibit 3, we choose a tail index α of 1.05 for cryptocurrency projects and a tail index α of 1.15 for NFTs. In a next step in our "calibration" of cryptocurrency and NFT α's, we compare these estimates with tail indices of other "social phenomena". Exhibit 4 summarizes the findings.

[5] As of 4/4/2022, the top project for cryptocurrencies is Bitcoin and for NFT collectibles CryptoPunks.

Qualitatively, with the Pareto Distribution, the lower α is, the more extreme the outliers of that phenomenon are. In Exhibit 4 below, we present the "social phenomena" with their associated α's sorted in ascending order. The phenomena in the top rows of the table are characterized with most extreme outcomes whereas the bottom ones, yet still part of the power law family, have more benign outliers. In addition, we highlight in gray those phenomena that represent financial asset classes.

EXHIBIT 4

Approximate Power Law α Estimates for Various Social Phenomena and Financial Asset Classes

Phenomenon	**Power Law α (approximate)**	**Comments and Sources**
Intensity of wars	0.63	Cirillo and Taleb (2016, 2016)
Cryptocurrencies	1.05	Own estimation & calibration
Net worth in U.S.	1.1	Newman (2005)
NFTs	1.15	Own estimation & calibration
Frequency of words	1.2	Newman (2005)
Population of U.S. cities	1.3	Newman (2005)
Value of patents	1.3	Use as proxy for traditional early-stage investments; Scherer (1998); Scherer et al. (2000)
Number of hits on website	1.4	Newman (2005)
Company size	1.4	Axtell (2001), Luttmer (2007)
Number of books sold in the U.S.	1.5	Newman (2005)
Return multiples VC funds <$100m	1.6	Use as proxy for traditional VC growth stage; Neumann (2015)
Return multiples VC funds >$100m	1.8	Use as proxy for traditional VC growth stage; Neumann (2015)
S&P 500	3	Own estimation, Taleb (2020)

Notes: Financial asset classes are highlighted in gray. We state the power law α's in Exhibit 4 as approximate as we are cautious about the sensitivities of the alpha estimates around the assumptions made in their respective estimations. Estimated α's crucially depend on estimation methods, such as maximum likelihood and non-parametric estimators, and the choice of the minimum value beyond which the power law applies. Some of the studies display unrealistic estimates of one or close to one, such as Axtell's (2001) study on company size, which implies non-finite averages.
Source: The estimates for the respective phenomena are indicated in the right-most column.

The most extremal phenomenon are wars, typically measured in people falling victim of belligerent conflicts relative to the population size (for example, the Thirty Years' War had a death toll of 20% of the entire Central European population with some areas seeing a death toll of as much as 60% of the population by some estimates, e.g., Outram 2002). As highlighted earlier, a power law α of less than one implies that the mean is non-finite. Among financial assets, cryptocurrencies and NFTs have the lowest power law α. For startup investments, a lower power law α implies higher expected return multiples above given extremal thresholds. That massive upside however is often constrained by finding enough companies that can generate the required amount of high growth within the investment holding period.

PORTFOLIO CONSTRUCTION AND INVESTMENT HORIZON FOR STARTUPS

We will use the information about the tail index α and explore the questions around expected conditional returns, optimal portfolio size and holding periods.

Expected Return Multiples of Investment Outliers

With the power law α's from Exhibit 4 above at hand, we are equipped with the information we need to answer the question of what return multiple we can expect for investment outliers for every financial asset class.

For a better intuition, we first highlight two aspects of the power law – extrapolation and scalability/self-similarity – through which the tail index α operates in determining return multiples of investment outliers.

- **Extrapolation**: Future outliers and maxima are poorly predicted by past data. As an alternative, we will rely on extreme value theory as it establishes a coherent framework to make more reliable out-of-sample extrapolations. The naively obtained "empirical" distribution will reveal the biggest outlier to date, that is the current maximum return multiple (e.g., Ethereum with, say, an MOIC of 10,000x). But, if that investment is the current maximum outlier, it had to have exceeded what was the previous past outlier, and the empirical distribution would have missed it. For thicker tails, the difference between past outliers and future expected outliers is larger than for thinner tailed distributions. The tail exponent α captures that low-probability deviation not seen in the data and allows for more meaningful extrapolations.

- **Scalability / self-similarity**: This feature of the power law is best described with an example. Let's say we invest in a startup with a \$10m post-money market cap, and another one with a \$100m post-money market cap. Scalability is simply defined by the property that the relative probability (as measured as the ratio of the probabilities) of exceeding 10x for the first investment is the same as exceeding 10x for the second investment. This property is however only at play to some limited degree as at one point the upside to an investment is capped by the size of the global economy, say.

We saw earlier in Exhibit 1 that only 5% of all startup investments over the 30-year period from 1985-2014 generated a return of 10x or more. But what return multiple did those successful startup investments generate *on average*? And what return multiples can we expect for cryptocurrency and NFT projects? We answer these questions by using the tail index α presented in Exhibit 4 and then extrapolate the investment multiples for each sector.

Exhibit 5 summarizes the expected MOIC for investments contingent on an already realized 10x performance employing the relationship of conditional expectations and the tail index α for the Pareto distribution described in equation (2). Expectedly, cryptocurrencies with their lowest α have the highest return multiple: if an investor-owned tokens that were already up by 10x, that investor – on average – can expect this cryptocurrency project to generate a 200x performance. Similarly, NFTs would on average return 67x contingent on their current performance for having already reached an MOIC of 10x. The conditional return multiples for traditional early-stage startups and venture capital growth funds are accordingly lower as their respective power law tail indices have higher values. Expected return multiples for traditional early-stage startups are 33x, for venture capital growth funds of smaller and larger size 17x and 12x, respectively. Even though the tail index for NFTs is only marginally lower than that for traditional early-stage startups, this small difference has a material impact on expected outlier performance. To illustrate this, for NFTs in comparison to traditional early-stage startup investments, conservatively, even if all the other 95% of the investments returned zero, the overall fund performance will still be up on average by 3.4x for NFTs but "only" by 1.65x for traditional startup funds.

The flipside of this observation is that the very extremal nature of the return distributions is that the market capitalization of most projects will go to zero or disappoint otherwise. This has important ramifications for the optimal portfolio size of startup holdings and NFT projects.

EXHIBIT 5

Expected MOIC for Investments by Financial Asset Class Conditional on at Least 10x MOIC

Sector	Power Law α	Expected MOIC for Investments Already 10x
Cryptocurrencies	1.05	200x
NFTs	1.15	67x
Traditional startups	1.3	33x
VC growth funds <$100m	1.6	17x
VC growth funds >$100m	1.8	12x

Notes: Power law α's and their respective sources for each sector are from Exhibit 5. Expected MOIC conditional of a minimum 10x return are calculated according to equation (2) setting $u = 10$ and utilizing the corresponding alphas reported in this table.

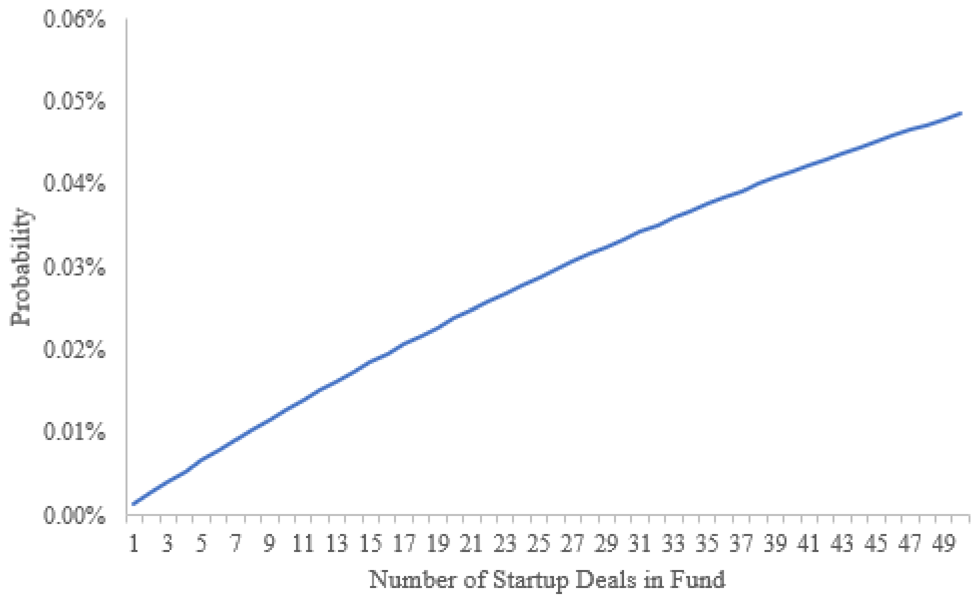
Optimal Portfolio Size

The power law reveals itself very slowly because we are dealing with outliers: outliers are rare events which take time to materialize. This observation has direct implications for the portfolio size of startup investments. We illustrate this by calculating the probability of attaining one outlier investment of a maximum of 50x MOIC as a function of the size of a hypothetical portfolio. The 50x MOIC benchmark is just an arbitrarily chosen multiple, and the takeaways from the following discussion applies to any other extremal MOIC value.

Exhibit 6 displays the results. The formulas and parametric values are presented in Appendix 5. We see that the probability of ending up with a homerun in form of one 50x deal in the portfolio increases monotonically with portfolio size. The chance of hitting a 50x MOIC outlier almost doubles from a portfolio of 20 to a portfolio of 50 deals. This illustrates that diversification benefits in portfolio construction are very tangible in the presence of the power law in the sense that the probability of realizing outlier investments which can carry the entire fund are more likely to materialize the larger the deal count in that fund is.

EXHIBIT 6

Probability That the Maximum Outlier Investment is 50x MOIC As a Function of Startup Deals in the Fund



Notes: The details to populate this graph are presented in Appendix 5. We assume here that $\alpha = 1.1$ and that the minimum value beyond which the power law applies is zero, using a value of C = 1e-20 as a proxy. In other words, we assume the power law applies to the entire domain of possible investment outcomes.

There are limits to grow a portfolio without bounds from a practical point of view. The right portfolio size is a judgment call based on the time horizon, investor base, opportunity set – and most crucially – the fund's capacity and its support model. Portfolio construction requires weighing the benefits of diversification against the fund's ability to source high quality deals and provide support to its portfolio companies. By actively helping companies, fund managers increase the chances of their portfolio companies to become commercially viable and successful. The portfolio size then becomes a matter of capacity of the fund management and its partners who need to strike the right balance of increasing the probability of hitting a homerun by steadily increasing the portfolio deal count against the number of companies the fund can support during the early stages of the life cycle of startup companies.

Optimal Holding Period

The estimates of the tail indices for various financial sectors displayed in Exhibit 4 are only a snapshot in time in the dynamics of company growth. As we have shown in equation (3), the tail index α can be endogenously determined by the company growth rate and the holding period of the investment.

If the model of exponential distribution of investment outlier occurrences holds, we can analyze how the tail index α varies with investment horizons and the average growth rates of startup companies. To illustrate the impact of the holding period on expected MOIC's, we assume that startup growth rates range from 30%-40%.[6] It is unrealistic to assume that companies can maintain

[6] Statista reports growth rates in the range from 45% to 75% for startup companies in the U.S. between 2000 to 2016: https://www.statista.com/statistics/693669/rate-of-start-up-growth-us/. The level of growth rates crucially depends on how the universe of startup companies is sampled, and how survival bias is addressed in the analysis. The takeaways

these high growth rates for an extended period, however. In a stylized representation, company growth rates follow an S-curve with an initial period of startups trying to find the right product market fit through multiple iterations. This period is characterized of comparably low growth rates. Startup companies that succeed in finding an initial product market fit enter a period in which they experience an explosion in growth as they gain momentum and quickly establish themselves in the market landscape. This phase is then followed by a period of a slowdown of growth rates as the companies reach a state of maturity (see Christensen, 1997 and 2003). We will investigate in our analysis below the interplay of investment horizon and power law α holding the company growth rate fixed. In addition, we truncate the investment horizon at 15 years. We think that this period encompasses the entire life cycle of startups before they reach the ultimate phase of maturity. In addition, 15 years is long enough to cover the investment cycle of most traditional venture capital firms which typically is around seven to ten years.

The results are summarized in Exhibit 7. The table presents the tail index α as a function of the investment horizon in years and the fixed 35% firm growth rate. We also report the expected MOIC contingent on the startup deal, reaching a 10x performance associated with the corresponding α. The model clearly predicts that as the expected time to exit becomes longer, the tail of the power law distribution gets fatter. The more patient the investor is, the more outsized the outliers can possibly become and hence the larger the upside of their investments is.

This is just another way to describe what's commonly known as the Lindy effect. It states that if something has been around for some time, the chances are it's going to be around for a longer time. In statistical terms, the hazard rate for a Pareto distribution is declining over time conditional on survival. Applied to startup investments, the longer a startup company has been around, the longer it's likely going to stay, eventually even grow to a 1,000x investment that early-stage investors are shooting for. This mechanism is at play independent of the usual compounding of the investment at the assumed growth rate.

EXHIBIT 7

Implied Power Law Alpha and Expected MOIC Conditional on >10x By Investment Horizon and Fixed Average Firm Growth Rate of 35%

Investment Horizon in Years	Power Law α (Implied by Investment Horizon)	Expected MOIC of an Investment Already 10x'ed
1	3.86	4
2	2.43	7
3	1.95	11
4	1.71	14
5	1.57	18
6	1.48	21
7	1.41	25

from this article don't depend on what growth rates we assume. We use the 30%-40% range for illustrative purposes only to highlight the importance of having a long-term investment horizon.

8	1.36	28
9	1.32	32
10	1.29	35
11	1.26	39
12	1.24	42
13	1.22	46
14	1.20	49
15	1.19	53

Notes: This table is using the relationship between power law alpha, investment horizon and firm growth rate as described in equation (3).

INVESTMENT DAOS VERSUS TRADITIONAL VENTURE FIRMS

Understanding the role of the power law behind the dynamics of early-stage investments and the implications that it has on the investment outcomes can help startup investors set themselves up for success. We will derive some key takeaways from the insights above when we address the four questions we posed at the outset of outlier spotting rates, expected return multiples of such outliers, optimal portfolio construction and ideal investment horizon. This will allow us to formulate a framework to identify the main drivers of alpha, that is, excess returns.

The four drivers of investment performance and PnL alpha that we identify with the help of the power law framework are:

- **In-depth research and diligence in bottom-up analysis**: This addresses the spotting rate. Simply continuing adding startups to a portfolio in the spirit of “spray and pray” is infeasible given the overwhelming number of new startups every year entering the market. Historically, the top venture capital firms differentiated themselves from their competitors by achieving higher success rates in holding outlier investments in their funds. Miss rates are high for everyone but getting only 1% “right” versus 5% for sitting on 100x investments means an additional 1x versus an additional 5x MOIC for the fund.
- **Differentiated support of portfolio companies**: Investors need to support portfolio companies in differentiated ways, leveraging their network for future funding rounds, hiring talent or advising them operationally. This engagement increases the probability of an outlier success.
- **Diversification**: The power law gets traction only after a certain portfolio size. If the hit rate of finding an outlier with a 100x MOIC is 5% and the portfolio contains 100 startups, on average the fund will have five such outliers. If the fund contains only ten portfolio companies, in contrast, hitting that homerun becomes more a matter of luck than skill.
- **Make time your friend**: Removing fixed investment cycle periods of seven to ten years can generate a considerable tailwind for early-stage investment performance. The statistical intuition behind this observation is a declining hazard rate – or the Lindy effect.

We think that the implications of the power law will provide strong tailwinds to the performance of startup funds of investment DAOs. With the backdrop of the insights from the preceding discussion, we identify two characteristics innate to investment DAOs that we anticipate being conducive to fund performance: the absence of the agency problem that traditional venture capital firms are exposed to and the absence of a fixed investment period with the ensuing long-term perspective. Regarding the former, the organizational form of investment DAOs ensures that the interests and incentives of the DAO and its community members as investors are very closely aligned. Community members qualify to participate in early-stage investment rounds via some stake ownership in the DAO. The health of a DAO's treasury and the affordability of investment services it can provide to its community and portfolio companies is directly dependent on the performance of an investment DAO's portfolio thanks to the carry compensation that flows to its treasury and, thus indirectly, to community members.

Furthermore, the autonomy of the community members of a DAO stemming from their direct involvement in the investment decision and own financing of the startup fundings tackles the agency problem at its core. Fully understanding the implications of the power law for the success of startup investments implies to "swing for the fences" in choosing portfolio companies and taking a long-term view. In contrast, the set investment cycles and a fee structure that is a combination of a fixed and incentive fee such as the common "2/20" compensation have the potential to shift the focus of general partners of traditional venture capital firms from the remuneration from the incentive fee to the recurring annual fixed fee from assets under management. A steady income flow from the fixed fee of a very large asset under management base with comparably decent performance incentive can have a limiting effect on the risk appetite of traditional venture firms to screen for those potential outlier projects in comparison to investment DAOs (in addition to increasing challenges to replicate very strong returns with larger funds). [7,8] The closed-end nature with investment cycles of seven to ten years is another potential challenge to performance of traditional venture funds in comparison to investment DAOs with DAO investors potentially being in a position to take a very long-term view of ten or even 15 years. As we illustrated in Exhibit 7, the tail index of the Pareto distribution decreases with the investment horizon with the associated MOIC accordingly getting larger.

We illustrated in Exhibit 6 that the chance of a startup outlier increases with the number of startup investments. In an unconstrained setting, a fund manager would accordingly continuously increase the number of startup investments in its portfolio to improve the outlook of hitting a homerun. As we highlighted earlier, this view is very simplistic as there are limits to a fund to grow the portfolio without bounds for practical reasons. Fund managers need to weigh the benefits of a large set of portfolio companies and the fund's support model for those portfolio companies: there is a tradeoff between the diversification benefits and the fund's ability to provide tailored support to its portfolio startup investments. We think that Investment DAOs can have an advantage over traditional venture capital funds in striking the right balance with these two conflicting goals. The formula boils down to available bandwidth. DAOs can leverage a vibrant community of cryptocurrency and NFT enthusiastic members with their depth and width of their skills whereas traditional venture firms face some limitations in form of the number of general partners and employees they have. It's against this backdrop that we anticipate that DAOs will be able to scale up their investments in numbers without compromising on the quality of their differentiated support of portfolio companies thus increasing their chances to hit the big homeruns for their funds.

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[7] For a discussion of incentive structures and contractual designs for delegated investment management and the role of fixed and incentive fees to align interests of money managers with those of their investors, see Ackermann et al. (1999), Li and Tiwari (2009). Shleifer and Vishny (1997) show how the agency problem imposes limits to arbitrage and, more crucially in our context, risk appetite of money managers.
[8] Chen et al. (2004) investigate the effect of fund size on fund performance in the active money management industry and find a negative relationship after taking into account fees, expenses and other control factors.

CONCLUSION

This article tries to address two questions related to startup investments in the cryptocurrency and NFT sectors. The first question centers around optimal portfolio construction with the backdrop of the power law governing investment returns. Empirical evidence of startup returns strongly suggests that venture investment is a business of extreme outliers. We use the Pareto distribution to instrumentalize the power law and estimate the tail index for cryptocurrency and NFT projects for that purpose. Based on our estimates and calibrations, we suggest tail index values of 1.05 and 1.15 for cryptocurrencies and NFTs, respectively. We find that these two sectors have the lowest power law α's among all financial asset classes, including traditional startup companies.

Expected returns of startup investments contingent on these returns to reach a value above a pre-specified extremal threshold depend on the tail index. Based on the estimates of the α's we calculate that expected returns for outlier ventures in the cryptocurrency and NFT sectors of at least a 10x MOIC are 200x MOIC and 67x MOIC, respectively. These outlier returns are considerably larger than the ones we can expect for traditional startup investments once they reach a 10x return multiple, which range from 17x to 33x, depending on whether these investments are early-stage or growth funds.

The likelihood of hitting a homerun in form of an investment that generates at least an MOIC of 10x is 5%. This figure was calculated in an empirical study based on all startup investments covering a 30-year period from 1985-2014 in the U.S. (see Dixon 2015). We assume that this statistic continues to hold for all venture sectors, also for cryptocurrencies and NFTs. Applying the 5% outlier hit ratio to the expected conditional outlier returns by sector and applying the corresponding estimates of the tail indices α, we calculate extra return multiples that would be added to a portfolio of corresponding startup investments: an additional 10x for cryptocurrencies, 3.4x for NFTs, 1.7x for early-stage traditional startup companies and 0.9x for traditional venture growth funds. Furthermore, we illustrate with the help of the Pareto model that the hit rate is not a given. We show how the hit rate increases with the number of holdings of startup companies in portfolio. A venture firm can increase the probability that it has an outlier investment in its fund by choosing a sufficiently large number of early-stage investments, ceteris paribus.

The second question we address in this article centers around the discussion of institutional design of successful early-stage investing. With the insights from the power law, we identify four criteria for successful early-stage investing: careful research and diligence in bottom-up investment analyses, differentiated support of portfolio companies, diversification by holding many portfolio startup companies, and long-term investment horizon. We introduce a new type of investment organization that has become popular in recent months in the cryptocurrency and NFT sectors, so-called investment DAOs. We highlighted two advantages that investment DAOs have over traditional venture capital firms in our outlook of their anticipated investment success. The first advantage stems from the lack of agency problems in investment DAOs which, in contrast, are present for traditional venture firms. Investment DAOs fund their investments directly through contributions of its members who take an active part in the investment decision making, and in addition participate in the upside of the investment DAOs' success via the carry that flows into their treasuries. The second advantage of investment DAOs over traditional venture firms stems from the fact that capacity constraints for tailored support of portfolio companies are less binding thanks to a broader community platform of specialized cryptocurrency enthusiastic members, and the investment horizon is not tied to seven to ten year closed-ended funds as is typical of venture firms.

It's still very early to safely predict a long-lasting success of cryptocurrencies and NFTs, let alone investment DAOs. Traditional venture capital firms by some estimate created nearly 76% of the total public-market capitalization of companies after 1995 even though venture capital-backed companies represent less than 0.5% of American companies created every year.[9] This investment model proved astoundingly successful. One hope of proponents of investment DAOs is to replicate this success but making it accessible to a broader range of investors with democratized access to the top Web 3 deals. In the end, only time will tell.

APPENDIX 1

Derivation of simplified the mean excess function for the Pareto distribution

We start with the mean excess function of the Generalized Pareto Distribution (GPD). The cumulative distribution function $G_{\mu,\sigma,\varepsilon}()$ of a random variable x with a GPD is

$$G_{\mu,\sigma,\varepsilon}(x) = 1 - \left(1 + \frac{\varepsilon(x-\mu)}{\sigma}\right)$$

with μ the location, σ the scale and $\varepsilon \neq 0$ the shape parameters, respectively. The mean excess function of the GPD is with minimal extremal threshold u is

$$E[x-u|x>u] = \frac{\sigma}{1-\varepsilon} + \frac{\varepsilon}{1-\varepsilon}u$$

In the special case of the Pareto distribution, the location parameter is set to $\mu = \sigma/\varepsilon$, and the shape parameter is $\varepsilon = 1/\alpha$. Although not relevant for deriving equation (2), the scale in the Pareto distribution $C = \sigma/\varepsilon$. When we set the scale parameter of the GPD to zero, $\sigma = 0$, we obtain the conditional expectation in equation (2). This assumption also implies that the power law applies to the entire domain of startup returns.

APPENDIX 2

Estimation methods of the Pareto tail index

Hill (1975) proposed a non-parametric estimator for the tail index parameter that is commonly used among practitioners and academics studying fat tails. For a sequence of x_1, x_2,

[9] See The Economist (2021). https://www.economist.com/finance-and-economics/2021/11/23/the-bright-new-age-of-venture-capital/21806438.

…, x_n independent and identically distributed random variables with order statistics $x_{1,n} \leq x_{2,n} \leq \ldots \leq x_{n,n}$, the Hill's estimator is defined as

$$H_{k,n} = \frac{1}{k}\sum_{j=1}^{k} log x_{n-j+1,n} - log x_{n-k,n} \tag{A1}$$

in a sample of *n* observations with positive integer $1 \leq k < n$. For the Pareto distribution, we use the inverse of the Hill's estimator to populate the estimate for its tail index parameter: $\hat{\alpha} = 1/H_{k,n}$.

In addition to the Hill's estimator, we employ an adaptation of the Hill's estimator that is more robust. Beirlant et al. (2005) introduced an adaptation of the Hill estimator defined as

$$\gamma_{k,n}^{M} = H_{k,n} + 1 - \frac{1}{2}\left(1 - \frac{H_{k,n}^{2}}{S_{k,n}}\right)^{-1} \tag{A2}$$

with

$$S_{k,n} = \frac{1}{k}\sum_{j=1}^{k}\left(log x_{n-j+1,n} - log x_{n-k,n}\right)^{2}$$

and *n* and *k* as defined above. They propose this estimator of the tail index to be a moment estimator of the shape parameter of the GPD. By taking the inverse, we obtain the estimate of the tail parameter of the Pareto distribution $\hat{\alpha} = 1/\gamma_{k,n}^{M}$.

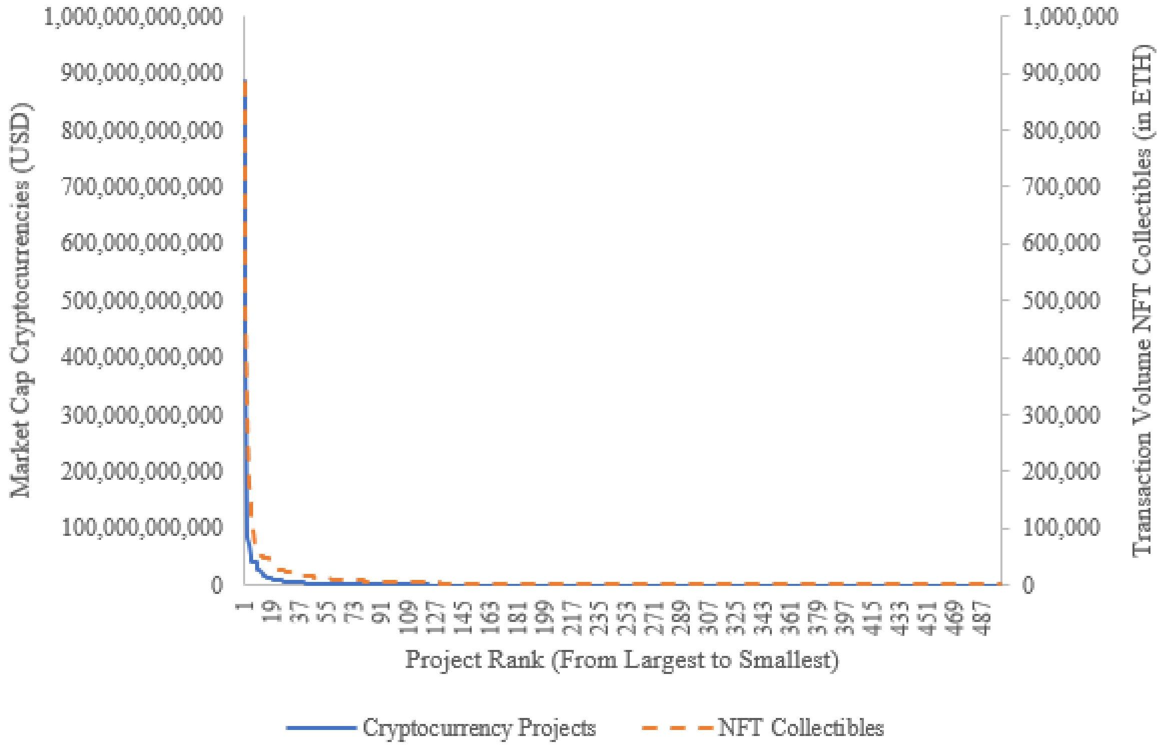
APPENDIX 3

Pareto tail index for cryptocurrencies and NFTs

We use the inverse of both the Hill's estimator described in equation (A1) and the moment estimator described in equation (A2) to obtain an empirical estimate of the tail index parameter α of the Pareto distribution for the cryptocurrency and NFT sectors. We identified the top 500 and top 1,000 cryptocurrency projects and NFT collectibles as of 4/4/2022. We used market capitalization based on circulating supply for cryptocurrencies from CoinMarketCap and transaction volumes of the top collectible projects from OpenSea. The estimated $\hat{\alpha}$ for both sectors do not depend on whether we use a sample of the top 500 or the top 1,000 projects. We present in this article the results based on the top 500 projects.

EXHIBIT 8

Market Capitalization for Cryptocurrencies / Transaction Volume for NFTs – Sorted from Largest to Smallest Top 500 Projects

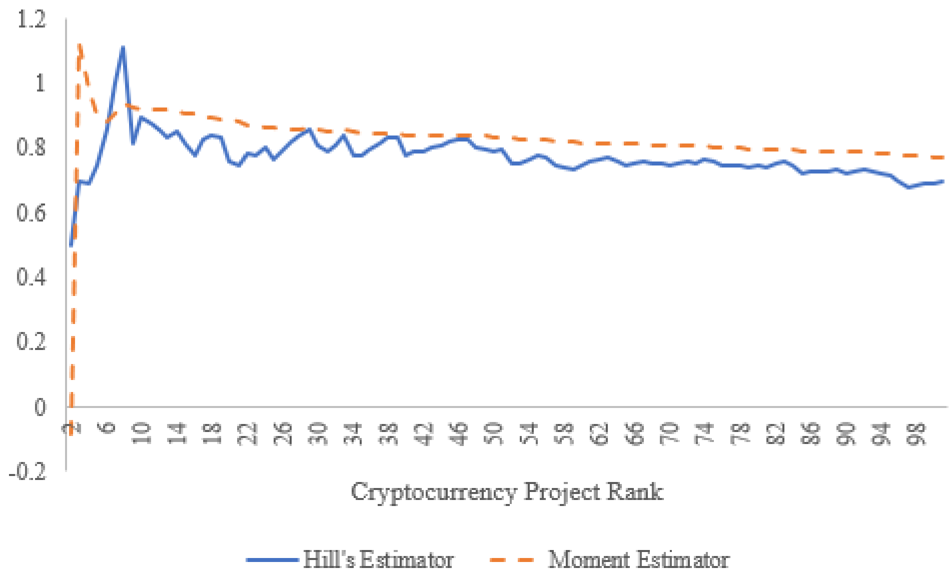


Source: CoinMarketCap for cryptocurrency projects as of 4/4/2022 and OpenSea for NFTs as of 4/4/2022. Cryptocurrency project market capitalization is based on circulating token supply and denominated in USD. NFT project capitalization is measured as cumulative transaction volumes since inception of the collectible and measured in ETH.

The graphs of the cryptocurrency and NFT projects in Exhibit 8 sorted by their market capitalization in descending order indicate a high concentration in the tail. Eyeballing the Hill's estimators and moment estimators for both sectors in Exhibits 10 and 11 indicates that the tail index estimators stabilize at around an $\hat{\alpha}$ of around 0.8 to 0.9 for cryptocurrencies and at around 1.0 for NFTs.

EXHIBIT 9

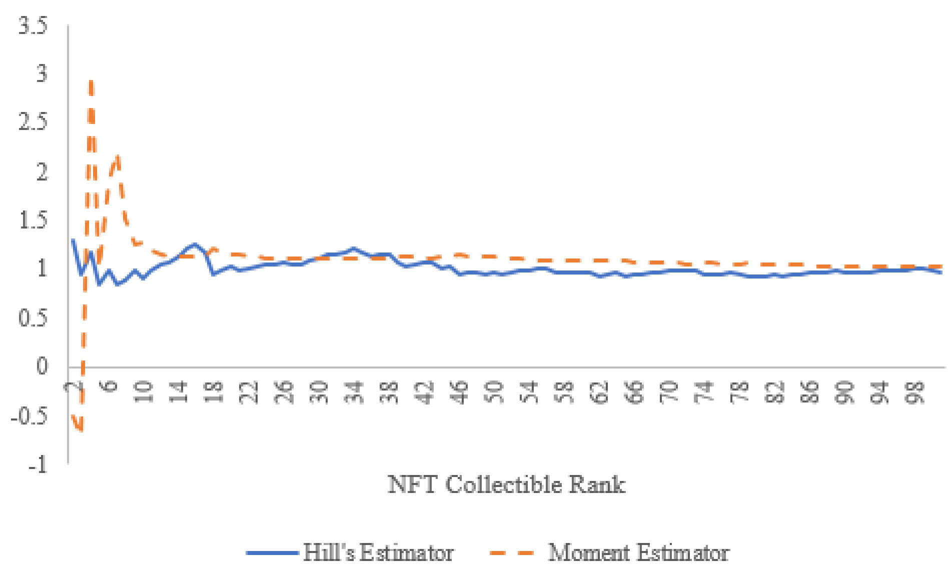
Tail Index Estimates for Cryptocurrency Projects



Source: CoinMarketCap for cryptocurrency projects as of 4/4/2022. Cryptocurrency project market capitalization is based on circulating token supply and denominated in USD.

EXHIBIT 10

Tail Index Estimates for NFT Collectibles



Source: OpenSea for NFTs as of 4/4/2022. NFT project capitalization is measured as cumulative transaction volumes since inception of the collectible and measured in ETH.

APPENDIX 4

Derivation of outlier probability as a function of fund size

The derivation of the formula to calculate the probability of the largest value to take a pre-specified value as a function of the number of trials n is derived in Newman (2005). We closely follow his exposition in this appendix. Similar accounts can be found in Gabaix (2009) and Reed and Hughes (2002).

We introduce x_{min} as the lowest value of a continuous real variable x at which the power law starts to apply. Applying the domain from x_{min} to ∞ to equation (1) and solving for the normalization constant C under the requirement that the probabilities integrate to one we obtain

$$p(x) = \frac{\alpha-1}{x_{min}}\left(\frac{x}{x_{min}}\right)^{-\alpha}$$

for the density function and

$$P(x) = \left(\frac{x}{x_{min}}\right)^{1-\alpha}$$

for the probability function. We let n be the number of draws from a power law distribution. We will interpret this variable as the number of startup companies in the portfolio. The probability that a particular sample we draw will have a maximum value of x and that all the other observations will be not greater than x is $p(x)dx[1-P(x)]^{n-1}$. As there are n ways to draw this value, the total probability $\pi(x)$ that the largest value takes value of x in a sample of n power law measurements is equal to

$$\pi(x) = np(x)[1-P(x)]^{n-1}$$

with the probability functions defined above. The graph in Exhibit 7 reflects this equation where we set $x = 50$, $x_{min} = 1\text{e-}20$ and $\alpha = 1.1$.

APPENDIX 5

Relationships between market concentration ratios and power law α

The relationship between the tail index α and the Herfindahl Index is derived in detail in Kondo et al. (2018) in their proposition 4. We merely replicate the relationship here.

For $\alpha \in (1,2)$, the Herfindahl Index $h_N^2(\alpha)$ with N denoting the number of firms is described by

$$h_N^2(\alpha) = \frac{1}{\left(N^{1-1/\alpha}\right)^2}$$

Next, regarding the concentration ratios C1 and C5, the top qth percentile concentration ratio is

$$\left(\frac{q}{100}\right)^{(\alpha-1)/\alpha} \tag{A3}$$

with $q = 1$ and $q = 5$, respectively. Let x_q be the value of the Pareto distributed random variable that is associated with the top qth percentile and x_{min} the minimum value of this random variable. Then the concentration ratio accruing to those companies above the qth percentile by using equation (1) can be written as:

$$\frac{\int_{x_q}^{\infty} \alpha C x^{-\alpha} dx}{\int_{x_{min}}^{\infty} \alpha C x^{-\alpha} dx} = \left(\frac{x_q}{x_{min}}\right)^{1-\alpha}$$

Applying the definitions of the qth percentile $\frac{q}{100} = C x_q^{-\alpha}$ and $x_{min} = C^{1/\alpha}$ utilizing the Pareto distribution specification, substituting these in above relationship, gives us the formula for the concentration ratio in (A3).

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