Zayn Wang, written in LaTeX

Homework

Question 1.

$$When\ x \in (0,1), 1-x^4 < 1-x^8 < 1 < 1+x^4 \Rightarrow \sqrt{1-x^4} < \sqrt{1-x^8} < 1 < \sqrt{1+x^4}$$
$$\Rightarrow \int_0^1 \sqrt{1-x^4}dx < \int_0^1 \sqrt{1-x^8}dx < \int_0^1 1dx < \int_0^1 \sqrt{1+x^4}dx$$
$$\Rightarrow J < L < 1 < K$$

Question 2.

$$Main\ theorem : [\int_0^x f(t)dt]' = f(x), [\int_x^1 t^2 f(t)dt]' = -x^2 f(x)$$
$$\Rightarrow f(x) = -x^2 f(x) + \frac{x}{2} + \frac{x^3}{2} \Rightarrow f(x)(x^2+1) = \frac{x}{2}(x^2+1) \Rightarrow f(x) = \frac{x}{2}$$
$$\int_0^x f(t)dt = \frac{x^2}{4}, \int_x^1 t^2 f(t)dt = \frac{t^4}{8}|_x^1 = \frac{1}{8} - \frac{x^4}{8}$$
$$\Rightarrow \frac{x^2}{4} = \frac{1}{8} - \frac{x^4}{8} + \frac{x^2}{4} + \frac{x^4}{8} + c \Rightarrow c = -\frac{1}{8}$$

Question 3.

$$(1)\ u = sinx \in (0,1), du = cosxdx \Rightarrow \int_0^1 u^n du = \frac{u^{n+1}}{n+1} + C|_0^1 = \frac{1}{n+1}$$
$$(2)\ u = e^x + 1, du = e^x dx = (u-1)dx \Rightarrow \int \frac{1 = u-(u-1)}{u(u-1)} du = \int (\frac{1}{u-1} - \frac{1}{u})du$$
$$= ln(u-1) - lnu + c = x - ln(e^x+1) + C$$
$$(3)\ u = \sqrt{1+x}, u^2 - 1 = x, 2udu = dx \Rightarrow \int (u^2-1)^2 u \cdot 2udu = 2\int u^6 - 2u^4 + u^2 du$$
$$= \frac{2u^7}{7} - \frac{4u^5}{5} + \frac{2u^3}{3} + C = \frac{2(1+x)^{\frac{7}{2}}}{7} - \frac{4(1+x)^{\frac{5}{2}}}{5} + \frac{2(1+x)^{\frac{3}{2}}}{3} + C$$
$$(4)\ u = sinv^3, du = 3v^2 cosv^3 dv \Rightarrow \int \frac{1}{3} du = \frac{u}{3} + C = \frac{sinv^3}{3} + C$$
$$(5)\ \int_{-\pi/4}^{\pi/4} \frac{t^4 \tan t}{2+\cos} dt, \because odd, \therefore = 0$$
$$(6)\ u = cosx, du = -sinxdx \Rightarrow \int -\frac{lnu}{u} du$$
$$v = lnu, dv = \frac{1}{u} du \Rightarrow \int -vdv = -\frac{v^2}{2} - C = -\frac{ln^2 u}{2} - C = -\frac{ln^2 cosx}{2} + C$$

Question 4.

$$(1)\ e^{arctan1} - e^{arctan0}$$
$$(2)\ = \frac{d}{dx} C = 0$$
$$(3)\ = \frac{d}{dx}(F(x) - C) = e^{arctanx}$$

Question 5.

$$(1)\ f(x) = kx\sqrt{1-x} \Rightarrow \int_0^1 kx\sqrt{1-x}dx = 1$$

$$u = \sqrt{1-x} \in (1,0), u^2 = 1-x, 2udu = -dx \Rightarrow \int_1^0 k(u^2-1)u \cdot 2udu = 2k\int_1^0 u^4 - u^2 du = 2k[\frac{u^5}{5} - \frac{u^3}{3} + C]_1^0$$

$$2k \cdot \frac{2}{15} = 1 \Rightarrow k = \frac{15}{4}$$

$$(2)\ P = \int_{0.5}^1 f(x)dx, u \in (\frac{1}{\sqrt{2}}, 0) \Rightarrow P = \frac{15}{2}[\frac{u^5}{5} - \frac{u^3}{3} + C]_{\frac{1}{\sqrt{2}}}^0 = \frac{15}{2}(\frac{1}{6\sqrt{2}} - \frac{1}{20\sqrt{2}}) = \frac{7}{8\sqrt{2}}$$

$$(3)\ u \in (\sqrt{1-m}, 0) \Rightarrow \frac{1}{2} = \frac{15}{2}[\frac{u^5}{5} - \frac{u^3}{3} + C]_{\sqrt{1-m}}^0 \Rightarrow 1 = 5(1-m)^{\frac{3}{2}} - 3(1-m)^{\frac{5}{2}} \Rightarrow m = 0.586$$

$$(4)\ \int_0^1 \frac{15}{4}x^2\sqrt{1-x}dx$$

$$u = \sqrt{1-x} \in (1,0), u^2 = 1-x, 2udu = -dx \Rightarrow \int_1^0 k(u^2-1)^2 u \cdot 2udu = 2k\int_1^0 u^6 - 2u^4 + u^2 du$$

$$= 2k[\frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3} + C]_1^0 = \frac{4}{7}$$

Question 6.

$$n \to \infty \Rightarrow \int_0^1 x^9 dx \Rightarrow \frac{x^{10}}{10}|_0^1 = \frac{1}{10}$$

Question 7.

$$(1)\ F'(x) = \sqrt{1+x^4}$$

$$(2)\ y = \int_C^{3x+1} sint^4 dt - \int_C^{2x} sint^4 dt, u = 3x+1, v = 2x, u' = 3, v' = 2$$

$$\Rightarrow y = \int_C^u sint^4 dt - \int_C^v sint^4 dt \Rightarrow y' = sinu^4 \cdot u' - sinv^4 \cdot v'$$

$$\Rightarrow y' = 3sin(3x+1)^4 - 2sin(2x)^4$$

$$(3) y = \int_{\sqrt{x}}^x \frac{e^t}{t}dt, u = \sqrt{x}, u' = \frac{1}{2\sqrt{x}} \Rightarrow y' = \frac{e^x}{x} - \frac{e^{\sqrt{x}}}{2x}$$

Question 8.

$$(1) False : f(x) = g(x) = 1$$

$$(2) False : f(x) = x$$

$$(3) True : f(x)\ is\ above\ g(x), then\ the\ accumulation(intergral)\ is\ greater.$$

$$(4) True : The\ basic\ theorem$$

$$(5) False : f(x) = x - 0.5$$