**Analysis of Loop Fragments**

//*Fragment 1*

**sum = 0;**

**for (int i = 0; i < N; ++i)**

**++sum;**

|  |  |
| --- | --- |
| Expression | # of expressions evaluated |
| i = 0 | 1 |
| sum = 0 | 1 |
| i < N | N + 1 |
| ++i | N |
| ++sum | N |
| **Total:** | **T(N) = 3N + 3** |

So, for Fragment 1, **T(N) = 3N + 3 = Θ(N)**

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//*Fragment 2*

**sum = 0;**

**for (int i = 0; i < N; ++i)**

**for (int j = 0; j < N; ++i)**

**++sum;**

First, we must consider the innermost loop. Let this loop be called S1 and let the number of evaluated expressions as a function of N be T1(N).

S1 = 

This is similar to *Fragment 1* except the extra **sum = 0** is missing after the comma operator.

So,

**T1(N) = 3N + 2** for S1.

Now consider the entire *Fragment 2*. This is written as:

**sum = 0;**

**for (int i = 0; i < N; i++)**

**S1;**

|  |  |
| --- | --- |
| Expression | # of expressions evaluated |
| i = 0 | 1 |
| sum = 0 | 1 |
| i < N | N + 1 |
| ++i | N |
| S1 | N \* T1(N) = N(3N +2) = 3N2 + 2N |
| **Total:** | **T(N) = 3N2 + 4N + 3** |

So, for Fragment 2, **T(N) = 3N2 + 4N + 3 = Θ(N2)**

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//*Fragment 3*

**sum = 0;**

**for (int i = 0; i < N; ++i)**

**for (int j = 0; j < N \* N; ++j)**

**++sum;**

First, we must consider the innermost loop. Let this loop be called S1 and let the number of evaluated expressions as a function of N be T1(N). S1 is similar to *Fragment 1,* except the extra **sum = 0** is missing and **j < N \* N** is an addition.

|  |  |
| --- | --- |
| Expression | # of expressions evaluated |
| j = 0 | 1 |
| j < N \* N | N2 + 1 |
| ++j | N2 |
| ++sum | N2 |
| **Total:** | **T1(N) = 3N2 + 2** |

So,

**T1(N) = 3N2 + 2**

|  |  |
| --- | --- |
| Expression | # of expressions evaluated |
| i = 0 | 1 |
| sum = 0 | 1 |
| i < N | N + 1 |
| ++i | N |
| S1 | N \* T1(N) = N(3N2 + 2) = 3N3 + 2N |
| **Total:** | **T(N) = 3N3 + 4N + 3** |

So, for Fragment 3, **T(N) = 3N3 + 4N + 3 = Θ(N3)**

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//*Fragment 4*

**sum = 0;**

**for (int i = 0; i < N; ++i)**

**for (int j = 0; j < i; ++j)**

**++sum;**

First, we must consider the innermost loop. Let this loop be called S1 and let the number of evaluated expressions as a function of N be T1(N).

|  |  |
| --- | --- |
| Expression | # of expressions evaluated |
| i = 1 + 2 + 3 … + n | (N+1)/2 |
| j = 0 | 1 |
| j < i | i + 1 |
| ++j | i |
| ++sum | i |
| **Total:** | **T1(i) = 3i + 2** |

So,

**T1(i) = 3i + 2**

|  |  |
| --- | --- |
| Expression | # of expressions evaluated |
| i = 0 | 1 |
| sum = 0 | 1 |
| i < N | N + 1 |
| ++i | N |
| S1 | 3(((N – 1)(N))/2) + 2N = (3/2)N2 + (½)N |
| **Total:** | **T(N) = (3/2)N2 + (5/2)N + 3** |

So, for Fragment 4, **T(N) = T(N) = (3/2)N2 + (5/2)N + 3 = Θ(N2)**

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//*Fragment 5*

**sum = 0;**

**for (int i = 0; i < N; ++i)**

**for (int j = 0; j < i \* i; ++j)**

**for (int k = 0; k < j; ++k)**

**++sum;**

First, we must consider the innermost loop. Let this loop be called S1 and let the number of evaluated expressions as a function of N be T1(N).

|  |  |
| --- | --- |
| Expression | # of expressions evaluated |
| j = 1, 2, 3...i2 | (i2 + 1)/2 |
| k = 0 | 1 |
| k < j | j + 1 |
| ++k | j |
| ++sum | j |
| **Total:** | **T1(i) = 3((i2 + 1)/2) + 2** |

So,

**T1(N) = 3((i2 + 1)/2) + 2**

Next, we must consider the second inner loop. Let this loop be called S2 and let the number of evaluated expressions as a function of N be T2(N).

|  |  |
| --- | --- |
| Expression | # of expressions evaluated |
| i = 1, 2, 3...n | (N+1)/2 |
| j = 0 | 1 |
| j < i \* i | i2 + 1 |
| ++i | i2 |
| S1 | i2 \* T1(N) = i2(3((i2 + 1)/2) + 2) = 3N4/32 + 3N3/8 + 23N2/16 + 17N/8 + 31/32 |
| **Total:** | **T2(N) = 3N4/32 + 3N3/8 + 23N2/16 + 25N/8 + 127/32** |

So,

**T2(N) = 3N4/32 + 3N3/8 + 23N2/16 + 25N/8 + 127/32**

|  |  |
| --- | --- |
| Expression | # of expressions evaluated |
| i = 0 | 1 |
| sum = 0 | 1 |
| i < N | N + 1 |
| ++i | N |
| S2 | N \* T2(N) = N(3N4/32 + 3N3/8 + 23N2/16 + 25N/8 + 127/32) = 3N5/32 + 3N4/8 + 23N3/16 + 25N2/8 + 127N/32 |
| **Total:** | **T(N) = 3N5/32 + 3N4/8 + 23N3/16 + 25N2/8 + 191N/32 + 3** |

So, for Fragment 5, **T(N) = 3N5/32 + 3N4/8 + 23N3/16 + 25N2/8 + 191N/32 + 3 = Θ(N5)**

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//*Fragment 6*

**sum = 0;**

**for (int i = 1; i < N; ++i)**

**for (int j = 1; j < i \* i; ++j)**

**On if (j % i == 0)**

**for (int k = 0; k < j; ++k)**

**++sum;**

First, we must consider the innermost loop. Let this loop be called S1 and let the number of evaluated expressions as a function of N be T1(N).

|  |  |
| --- | --- |
| Expression | # of expressions evaluated |
| if (j % i == 0) | 1/(2N) + 1 |
| k = 0 | 1 |
| k < j | 1/(2N) \* (N(N+1)/2) + 1 = N/4 + 5/4 |
| ++k | 1/(2N) \* (N((N+1)/2)) = N/4 + 1/4 |
| ++sum | 1/(2N) \* (N((N+1)/2)) = N/4 + 1/4 |
| **Total:** | **T1(N) = 3N/4 + 1(2N) + 15/4** |

So,

**T1(N) = 3N/4 + 1(2N) + 15/4**

Next, we must consider the second inner loop. Let this loop be called S2 and let the number of evaluated expressions as a function of N be T2(N).

|  |  |
| --- | --- |
| Expression | # of expressions evaluated |
| j = 1 | 1 |
| j < i \* i | ((N+1)/2)2 + 1 |
| ++j | ((N+1)/2)2 |
| S1 | ((N+1)/2)2 \* T1(N) = ((N+1)/2)2 \* (3N/4 + 1(2N) + 15/4) = 11N3/16 + 37N2/16 + 41N/16 + 15/16 |
| **Total:** | **T2(N) = 11N3/16 + 45N2/16 + 57N/16 + 55/16** |

So,

**T2(N) = 11N3/16 + 45N2/16 + 57N/16 + 55/16**

|  |  |
| --- | --- |
| Expression | # of expressions evaluated |
| i = 1 | 1 |
| sum = 0 | 1 |
| i < N | N + 1 |
| ++i | N |
| S2 | N \* T2(N) = N(11N3/16 + 45N2/16 + 57N/16 + 55/16) =11N4/16 + 45N3/16 + 57N2/16 + 55N/16 |
| **Total:** | **T(N) =** 11N4/16 + 45N3/16 + 57N2/16 + 87N/16 + 3 |

So, for Fragment 1, **T(N) = 11N4/16 + 45N3/16 + 57N2/16 + 87N/16 + 3 = Θ(N4)**

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