

Part I

Implementation and Technical Data

1. Modeling of fuel cell

1.1. Governing equations

For this model the current is split into an ionic and electronic part. Protons flow through the membrane and form an ionic current. The electrons only flow through the solid matrix of electrodes, resulting in an electronic current. Therefore, it satisfies the following proton potential equation [1]

$$\nabla \cdot (-\sigma_s \nabla \phi_s) = S_s \quad (1)$$

$$\nabla \cdot (-\sigma_m \nabla \phi_m) = S_m \quad (2)$$

where ϕ_s and ϕ_m are the protonic and electronic potential. The electronic conductivity is listed in Table 1, the protonic conductivity in electrolyte σ_s is given by [1]

$$\sigma_s = \frac{1}{2} (0.5139\lambda - 0.326) \exp \left[1268 \left(\frac{1}{303} - \frac{1}{T} \right) \right] \quad (3)$$

and in porous electrode the protonic conductivity is

$$\sigma_{PE} = \varepsilon_{cm}^{1.5} \sigma_s. \quad (4)$$

S is the current source term and defined as

anode catalyst layer:

$$S_m = j_a \quad S_s = -j_a \quad (5)$$

cathode catalyst layer:

$$S_m = j_c \quad S_s = -j_c \quad (6)$$

The source terms in both species and charge equations are related to the transfer current density j_a and j_c , which was calculated by using a simplified Butler–Volmer equation given as

$$j_a = i_{0a} r_a \left(\frac{C_{H_2}}{C_{H_2,ref}} \right)^{0.5} \left(\frac{\alpha_a + \alpha_c}{RT} F \eta_a \right) \quad (7)$$

$$j_c = i_{0c} r_c \left(\frac{C_{O_2}}{C_{O_2,ref}} \right) \exp \left(-\frac{\alpha_c}{RT} F \eta_c \right) \quad (8)$$

where η represents the potential difference between solid matrix and electrolyte and is defined as

anode side:

$$\eta_a = \phi_s - \phi_m \quad (9)$$

cathode side:

$$\eta_c = \phi_s - \phi_m - V_{OC} \quad (10)$$

1.2. Fuel cell setup

PEMFC consists of gas diffusion layers, porous electrode layers at anode and cathode side and the membrane in between [2].

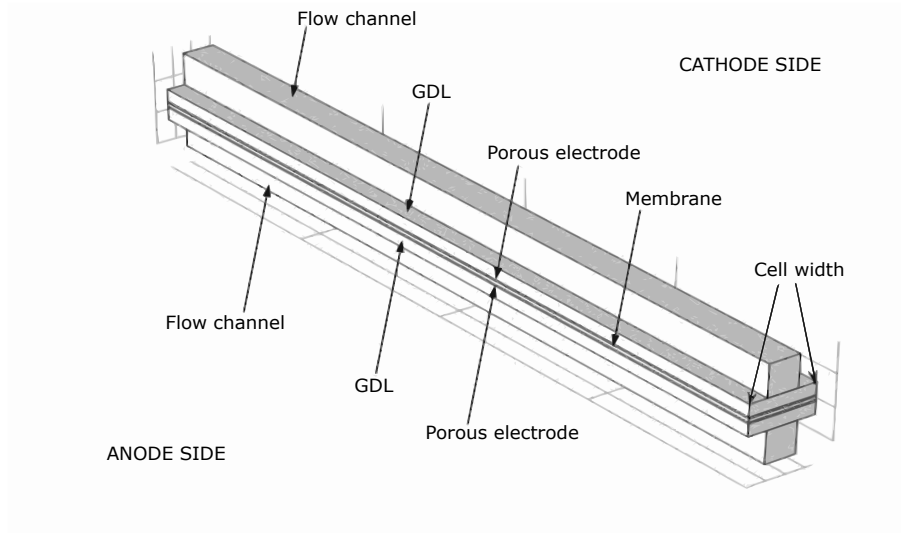


Figure 1: Geometry of the fuel cell model [2]

The coefficients are described in Table 1.

Table 1: Model parameters and values

Parameter	Symbol	Value	Unit
Cell length	L	10^{-2}	m
Gas diffusion layers (GDL) thickness	H_{gdl}	38×10^{-5}	m
Reference hydrogen molar concentration	$C_{H_2,ref}$	40.88	mol/m ³
Reference oxygen molar concentration	$C_{O_2,ref}$	40.88	mol/m ³
Electronic conductivity	σ_m	222	S/m
Exchange current density, anode	i_{0a}	1	A/m ²
Exchange current density, cathode	i_{0c}	1	A/m ²
Ratio of reaction surface, anode	r_a	10^9	1/m
Ratio of reaction surface, cathode	r_c	10^4	1/m
Hydrogen concentrations	C_{H_2}	60	mol/m ³
Oxygen concentrations	C_{O_2}	10	mol/m ³
Transfer coefficient, anode	α_a/α_c	1/1	-
Transfer coefficient, cathode	α_c	1	-
Open circuit voltage	V_{OC}	1.229	V
Universal gas constant	R	8.31	J/mol/K
Faraday constant	F	96485	C/mol
Cell width	W	2×10^{-3}	m
Membrane thickness	H_{mem}	100×10^{-6}	m
Porous electrode thickness	H_{el}	50×10^{-6}	m
Electrolyte phase volume fraction	ε_{cm}	1	-
Water content	λ	5	-
Cell Temperature	T	453.15	K

1.3. Variation of operation Fuel cell parameters

The average cell power density is one of the most significant magnitudes of fuel cells whose increase is of paramount importance. The Fig. 2 shows the plot of average cell power density depending on the required voltage for different geometry parameters like cell width $[W]$, membrane thickness $[H_{mem}]$ and porous electrode thickness $[H_{el}]$.

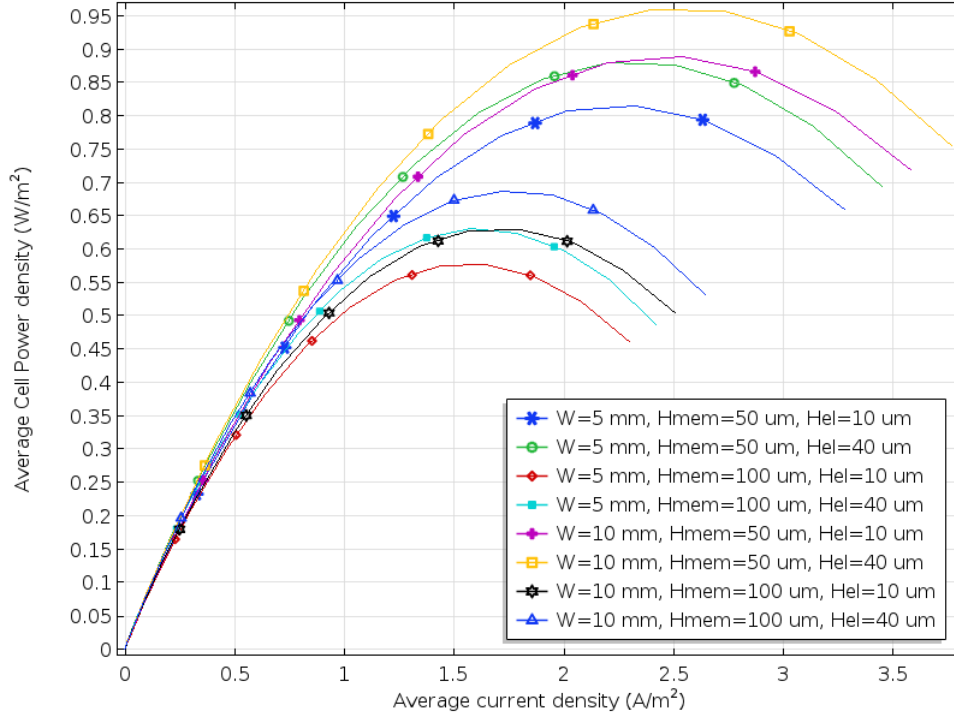


Figure 2: Variation of geometry parameters

The average cell power density is not only depended on cell geometry but also on other parameters listed in Table 1. In Fig 3 the average cell power density is plotted for other varying properties like hydrogen concentrations $[C_{H_2}]$, oxygen concentrations $[C_{O_2}]$ and the volume fraction of membrane in porous electrode $[\varepsilon_{cm}]$.

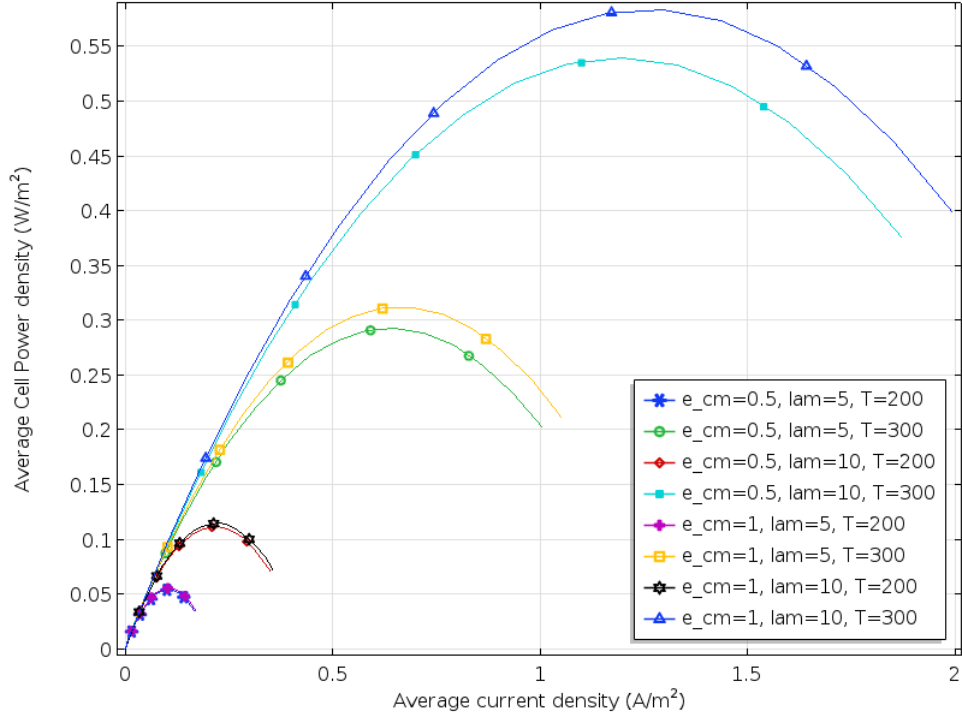


Figure 3: Variation of some other properties

The case is treated where a desired mean cell power density is given and the task is to find the parameters that achieve this power by variation. This is an optimization problem, and the genetic algorithm is used to solve the problem.

1.4. Parameter identification through optimization

In order to use a genetic algorithm, a solution space must first be specified. In our case, the following parameters are taken from Table 2 with a range and the step size is the resolution of the solution space.

Table 2: Solution space for genetic algorithms

Symbol	Range	Step size	Unit
W	$[1 \ 10]$	0.1	mm
H_{mem}	$[10 \ 110]$	10	μm
H_{el}	$[10 \ 110]$	1	μm
ε_{cm}	$[0.1 \ 1]$	0.1	-
λ	$[1.5 \ 15]$	1.5	-
T	$[200 \ 400]$	20	K

From the solution space an initial population is randomly generated which is a set of individuals or chromosomes. The position or locations in a chromosome are referred to as genes to which individual parameters are assigned.

Since the fitness function is a positive real number, it is obtained by scaling from the objective function, which corresponds to the optimization goal of the optimization problem. The objective function that is used is the average cell power density

$$F_p = \sqrt{\sum_{j=1}^N |p_j(W, H_{\text{mem}}, \dots, T) - \hat{p}_j|^2} \stackrel{!}{=} \min \quad (11)$$

where p_j is the simulated and \hat{p}_j the desired average cell power density. Index j represents various points on the average cell power density curve where the simulated and desired average cell power density are compared and evaluated. The number of points is chosen as $N = 21$ that means that the output power is evaluated for 21 different required output voltages. .

2. Modeling of reverberation chamber

2.1. Governing equations

A real life ERC differs from a cavity resonator because it normally contains one or more stirrers [3]. However, due to the simplicity of the model, it is still preferable to start by analyzing a cavity resonator and later supplement the model in order to achieve an ERC. Thus, the Helmolz equation is initially applied to the electric and magnetic fields of the cavity resonator through

$$\Delta \vec{E}(\vec{r}) + k^2 \cdot \vec{E}(\vec{r}) = 0 \quad (12)$$

$$\Delta \vec{H}(\vec{r}) + k^2 \cdot \vec{H}(\vec{r}) = 0 \quad (13)$$

$$k = \frac{\omega}{\sqrt{\varepsilon_0 \mu_0}} \quad (14)$$

where ε_0 the permittivity and μ_0 the permeability of the medium (air) is in which the wave is propagating. Due to their almost equal qualities, the permittivity and the permeability for the vacuum will be used instead of those for air. In addition, it is assumed that the tangential components of the electric field and the normal components of the magnetic field on the walls disappear as seen in

$$\vec{n} \times \vec{E} = 0 \quad (15)$$

$$\vec{n} \cdot \vec{H} = 0 \quad (16)$$

where \vec{n} is the surface normal vector and the convention is also used that the z-axis is chosen as the reference direction. This leads to the discrete solution in the direction of the z-axis for the electric and magnetic fields with the resonance frequency

$$f_{res} = \frac{1}{2} \frac{1}{\sqrt{\varepsilon_0 \mu_0}} k_{xyz} = \frac{1}{2} \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \sqrt{\left(\frac{l}{a}\right)^2 + \left(\frac{m}{b}\right)^2 + \left(\frac{n}{c}\right)^2} \quad (17)$$

for the number triple l, m and n applies, $l, m = 0, 1, 2, 3, \dots$ and $n = 1, 2, 3, \dots$

2.2. Reverberation chamber Setup

An overview of the setup of the aluminium resonance chamber was modeled and is displayed in Fig.4. A mode stirrer and a Vivaldi antenna - as a transmitting antenna exciting the field inside the ERC - were placed inside the chamber. The details describing the geometry and position for each item are listed in Table 3.

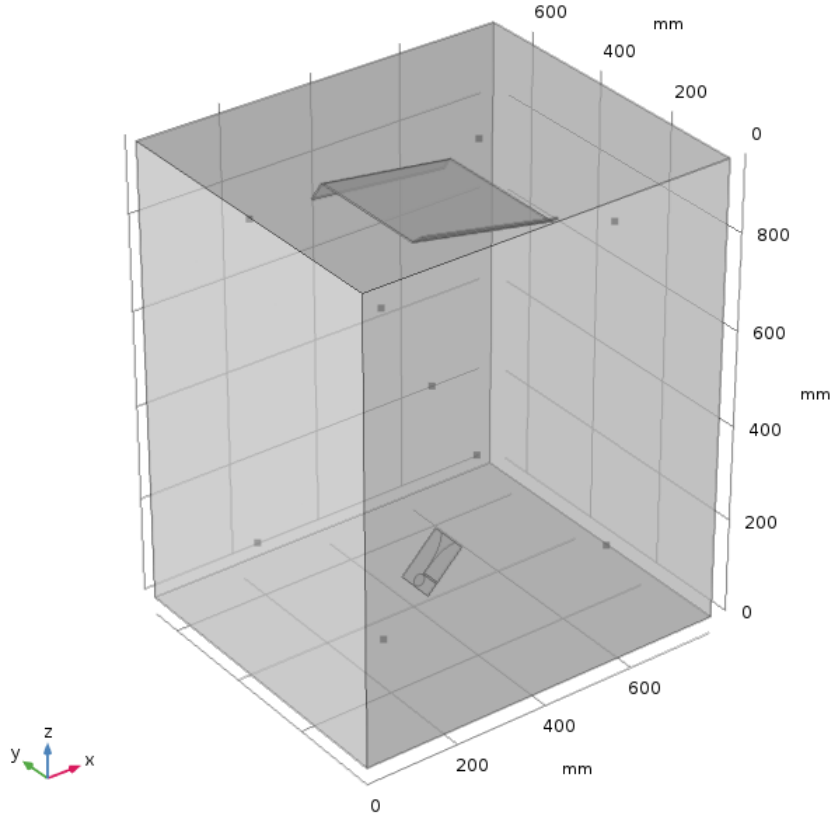


Figure 4: Geometry of the resonance chamber, see Table 3 for details

A Vivaldi antenna was chosen as the source for the simulation due to the wideband characteristic. As shown in Fig.5, the antenna was positioned in the center of the xy-plane of the chamber floor with an established angle and height from the chambers floor.

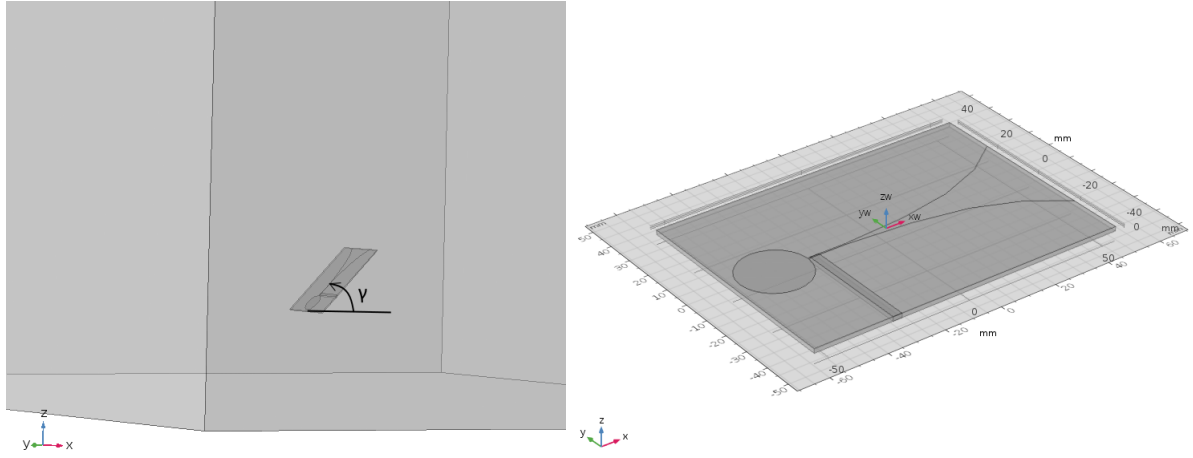


Figure 5: Vivaldi antenna geometry and position, see Table 3

Fig.6 shows a close up view of the stirrer and specifies the angle of the stirrer wings, the distances of the wings, and the length and width of the stirrer. Significant parameters this work will be concerned with are distances tr_1y_1 , tr_1y_2 , tr_2y_1 , and tr_2y_2 which correspond to the distance of the wing edges from the boundary of the stirrer length St_L .

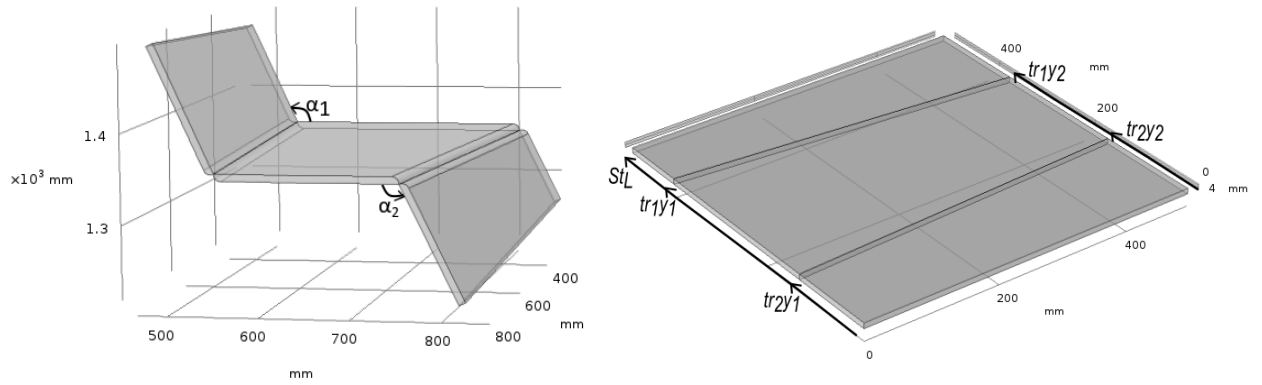


Figure 6: Stirrer geometry and angle of wings

All the parameters and the designated values that are represented in the figures shown above have been described in Table 3.

Table 3: Model parameters and values

Equipment	Parameter	Symbol	Value	Unit
Aluminum Chamber	Height	Ch_H	960	mm
	Width	Ch_W	790	mm
	Length	Ch_L	673	mm
Vivaldi Antenna	Height	An_H	80	mm
	Length	An_L	110	mm
	Thickness	An_{Th}	1.5	mm
	Angle in chamber	γ	45	degree
	Position in x-direction	$An_{x,pos}$	0	mm
	Position in y-direction n	$An_{y,pos}$	0	mm
	Position in z-direction	$An_{z,pos}$	100	mm
	Frequency	f	2.1603	GHz
	Characteristic impedance	Z_{ref}	50	Ω
	Voltage	V_0	7	V
	Substrate thickness	H	1.6	mm
Stirrer	Length	St_L	340	mm
	Width	St_W	300	mm
	Thickness	St_{Th}	3	mm
	Position in x direction	$St_{x,pos}$	0	mm
	Position in y direction	$St_{y,pos}$	0	mm
	Position in z direction	$St_{z,pos}$	860	mm
	Angle of stirrer wings	α_1	60	degree
	Angle of stirrer wings	α_2	40	degree
	Angle in relation to ceiling	β	10	degree
	Distance 1	tr_1y_1	240	mm
	Distance 2	tr_1y_2	100	mm
	Distance 3	tr_2y_1	240	mm
	Distance 4	tr_2y_2	100	mm

2.3. Variation of the stirrer shape and quantity in an ERC

2.3.1. Simulation Performance

As mentioned in the previous chapter, both the shape and the number of stirrers are crucial to achieve a statistically uniform and isotropic field. First, the shape of a stirrer is considered and changed. To do this, all parameters in the table 3 are left at the same value except for the four distance. These values change the shape of the wings. The model was simulated for 40 stirrer positions and 9 degree steps and measurements were recorded from 9 different points spaced out within the chamber. From these points a value of the electric

field in both imaginary and real parts in each direction was measured and averaged over 9 points. The following distribution of these fields is displayed for distance in table 3.

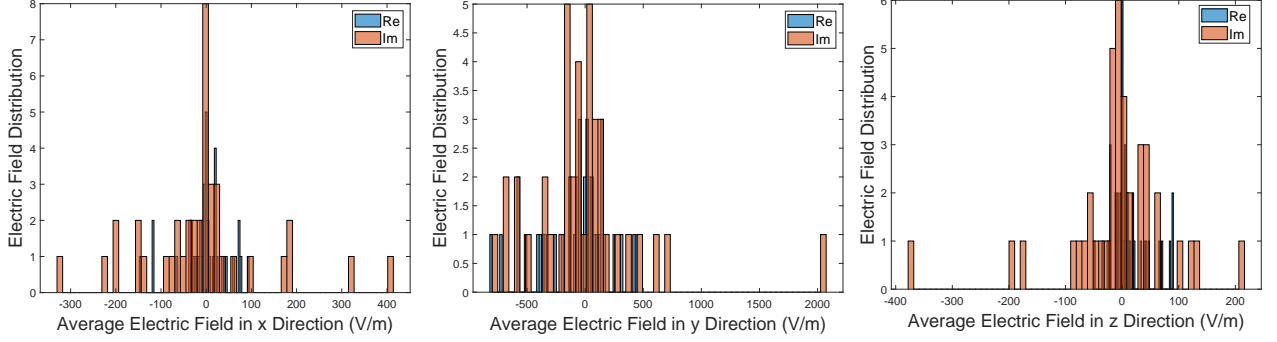


Figure 7: Distribution of the average electric field

Resulting from these electric field distributions, the following P -values were determined in Matlab.

Table 4: Resulting P -values of the simulation

Average Electric Field Distribution	Direction	P -value	σ in V/m
Real Part	x	0.0078	48, 75
	y	0.0505	296, 54
	z	0.0036	34, 25
Imaginary Part	x	0.0030	133, 67
	y	2.3123×10^{-5}	464, 76
	z	1.0217×10^{-4}	92, 64

The P -values from the simulation with the initial stirrer geometry do not reach the requirements discussed in the previous chapter. Consequently, these results lead to the construction of a new stirrer whose geometry allows for higher P -values. Thus, this issue will be considered as an optimization problem, that will be solved through the use of genetic algorithms.

2.3.2. Performance of the optimized stirrer geometry

The following parameters listed in Table 5 are the solution spaces for the algorithms, where the range and step size build the resolution of this space.

Table 5: Solution space for genetic algorithms

Symbol	Range	Step size	Unit	Description
tr_1y_1	[240 330]	9	mm	distance
tr_1y_2	[240 330]	9	mm	distance
tr_2y_1	[10 100]	9	mm	distance
tr_2y_2	[10 100]	9	mm	distance

The objective function that we used is the Shapiro-Wilk-Test value P for the real and imaginary average electric field in each of the three directions in terms of the different parameters or distance.

$$F_P = \sqrt{\sum_{i=1}^6 |P_i(tr_1y_1, tr_1y_2, tr_2y_1, tr_2y_2) - 1|^2} \rightarrow \min. \quad (18)$$

2.3.3. Performance of the optimized shape for three stirrers in chamber

In Fig.8, we observe the reverberation chamber with three stirrer whose geometries have will later be optimized.

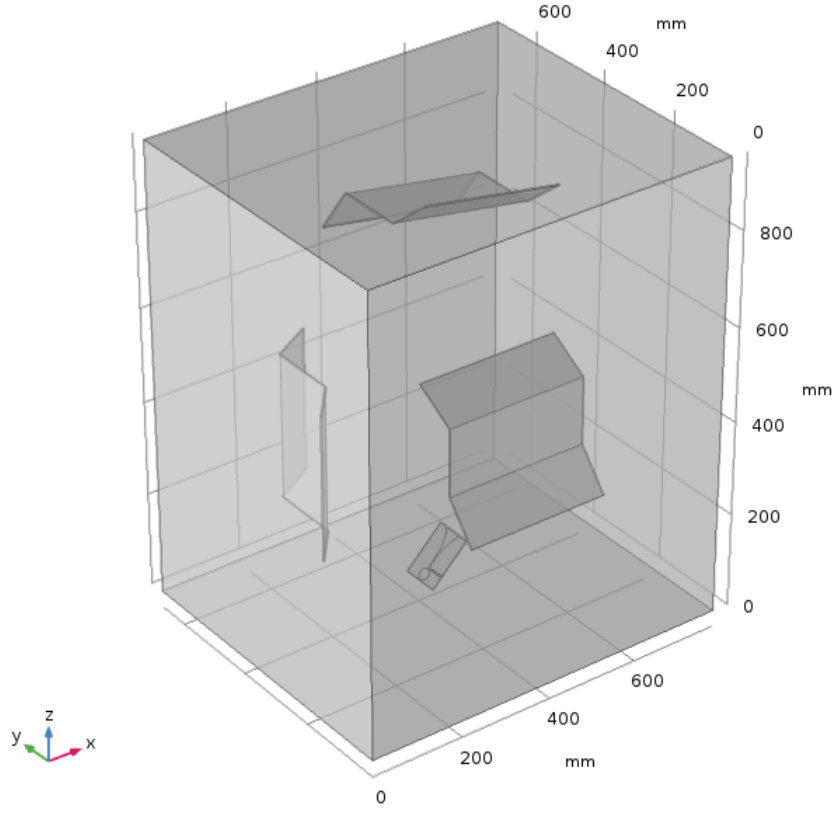


Figure 8: Setup of the resonance chamber with three stirrers, see Table 6 for their postions

The positions of all three stirrers are listed in Table 6.

Table 6: Model parameters and values

Equipment	Parameter	Symbol	Value	Unit
Stirrer 1	Position in x direction	$St_{x,1}$	0	mm
	Position in y direction	$St_{y,1}$	0	mm
	Position in z direction	$St_{z,1}$	860	mm
Stirrer 2	Position in x direction	$St_{x,2}$	0	mm
	Position in y direction	$St_{y,2}$	100	mm
	Position in z direction	$St_{z,2}$	0	mm
Stirrer 3	Position in x direction	$St_{x,3}$	100	mm
	Position in y direction	$St_{y,3}$	0	mm
	Position in z direction	$St_{z,3}$	0	mm

The parameters that have been chosen to be optimized are the shape of the wings for all

three stirrers inside the chamber. The newly formulated fitness function for these selected parameters is

$$F_P = \sqrt{\sum_{i=1}^6 |P_i(tr_{11}y_1, \dots, tr_{23}y_2) - 1|^2} \rightarrow \min. \quad (19)$$

A new solution space for the three stirrers is specified in Table 7.

Table 7: Solution space for genetic algorithms

Equipment	Symbol	Range	Step size	Unit	Description
Stirrer 1	$tr_{11}y_1$	[240 330]	9	mm	stirrer wing distance 1
	$tr_{11}y_2$	[240 330]	9	mm	stirrer wing distance 2
	$tr_{21}y_1$	[10 100]	9	mm	stirrer wing distance 3
	$tr_{21}y_2$	[10 100]	9	mm	stirrer wing distance 4
Stirrer 2	$tr_{12}y_1$	[240 330]	9	mm	stirrer wing distance 1
	$tr_{12}y_2$	[240 330]	9	mm	stirrer wing distance 2
	$tr_{22}y_1$	[10 100]	9	mm	stirrer wing distance 3
	$tr_{22}y_2$	[10 100]	9	mm	stirrer wing distance 4
Stirrer 3	$tr_{13}y_1$	[240 330]	9	mm	stirrer wing distance 1
	$tr_{13}y_2$	[240 330]	9	mm	stirrer wing distance 2
	$tr_{23}y_1$	[10 100]	9	mm	stirrer wing distance 3
	$tr_{23}y_2$	[10 100]	9	mm	stirrer wing distance 4

2.4. Variation of geometrie and amount of built-in objects in an ERC

2.4.1. Simulation of using different geometric objects in ERC

Arrays of the chosen two geometric objects, half spheres and cones, were inserted inside the chamber cavity 9 with parameters specified in Table 8. These equidistant objects are of the same radius ($R=120$ mm) and are modeled as such in order to ultimately compare the values of each simulation and, thus, posses the representation with the better results.

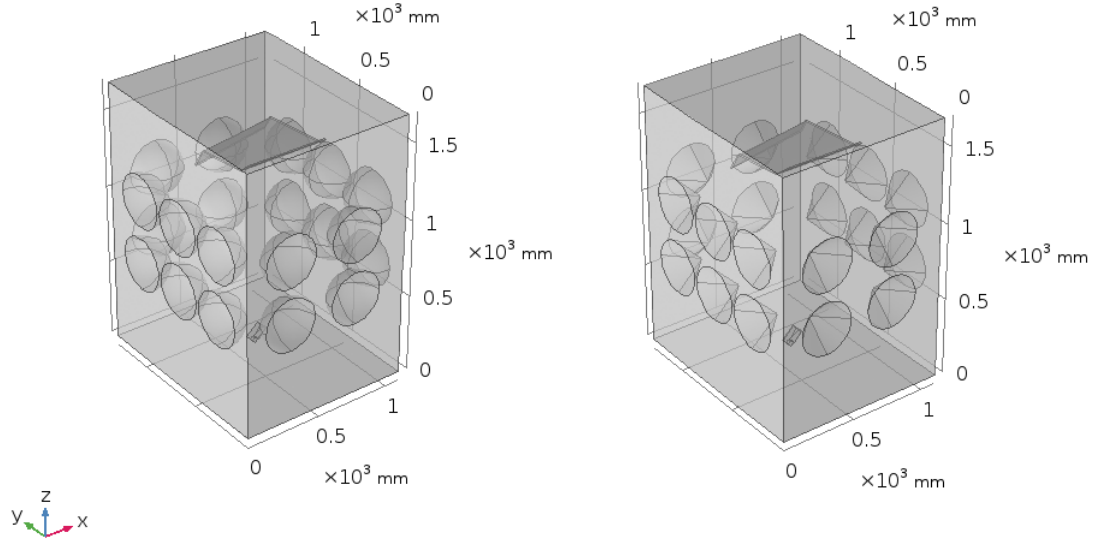


Figure 9: Electromagnetic resonance chamber with half spheres and cones placed inside with the same distance between each other

The dimensions of the chosen objects, as seen in Table 8, were similarly selected in order to definitively say which geometry produces more satisfactory results.

Table 8: Dimensions of used geometric objects

Symbol	Optimized Values	Unit	Description
Rad_{Sph}	120	mm	radius of the sphere
Rad_{Con}	120	mm	radius of the cone
Con_H	120	mm	cone height

References

- [1] HE, Mingyan ; HUANG, Ziping ; SUN, Pengtao ; WANG, Cheng: Modeling and Numerical Studies for a 3D Two-Phase Mixed-Domain Model of PEM Fuel Cell. In: *Journal of The Electrochemical Society* 160 (2013), Nr. 4. <http://dx.doi.org/10.1149/2.036304jes>. – DOI 10.1149/2.036304jes
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