

STA4020 Problem Set 5

Fall 2024

General Instructions:

This assignment contains 3 questions. Please ensure that all work is clearly shown and that final answers are boxed or highlighted. This assignment is due by **23:59:59 PM on Wednesday, November 27, 2024**. Late submissions will incur a penalty unless prior arrangements have been made.

Ethical and Responsible Usage of AI

While you are encouraged to explore modern tools such as AI and machine learning platforms to enhance your learning, it is essential to use these resources responsibly. All solutions must be your original work. You may use AI to assist with understanding concepts or to guide you, but copying AI-generated answers directly without comprehension is not permitted. Always strive to understand the underlying methods and derivations behind your solutions.

Additional Reminders:

1. Independent Work: Each student must submit their own work. Direct copying from others is considered academic misconduct.
2. Formatting: Please submit your answers as a typed document (preferably in PDF format) or as clearly scanned handwritten work. Both electronic submissions and hard copies are acceptable.
3. References: If you use any external sources, including textbooks, articles, or online resources, make sure to cite them appropriately.

If you have any questions or need clarification on any of the questions, feel free to reach out before the due date.

Good luck!

Question 1 (30 Points)

In the Barra factor model, pure factor portfolios represent the risk premiums associated with specific factors.

- (15 Points) Explain what the returns of country, industry, and style factors represent. In your response, discuss the type of risk or premium each type of factor (i.e., country, industry and style) aims to capture.
- (15 Points) Explain the exposure of each type's pure factor portfolio (country, industry, style) to the other factors in the model.

Question 2 (20 Points)

You are provided with historical daily returns for AAPL stock in the file `aapl_returns.csv`. Use this data to calculate and analyze the tail risk of the stock.

- (5 Points) Compute the 99% Value at Risk (VaR) using empirical distribution based on the historical return data.
- (5 Points) Compute the 99% VaR using the normal distribution assumption, where μ and σ are estimated from the historical return data.
- (10 Points) Compare the two VaR values obtained from the empirical method and the normal distribution assumption. Discuss the reasons for any differences and the implications for risk management in practice.

Question 3 (50 Points)

In this exercise, you will use weekly return data for 50 US stocks. The data is provided to you in `returns.pkl`. The objective is to construct a **minimum variance portfolio** (MVP) using two different methods to estimate the covariance matrix and evaluate the performance of the resulting portfolios.

Instructions:

1. **Covariance Estimation:** Using a rolling window of 100 weeks (including the current week), estimate the covariance matrix of stock returns every week using:
 - The sample covariance matrix (unadjusted).
 - The Ledoit and Wolf shrinkage estimator.

To estimate the covariance matrix using LedoitWolf shrinkage, use the `LedoitWolf` module in Python's `sklearn.covariance`. Make sure to carefully read the documentation to understand its input requirements. The document can be found at:

<https://scikit-learn.org/1.5/modules/generated/sklearn.covariance.LedoitWolf.html>.

2. **Portfolio Construction:** For each week, construct a minimum variance portfolio (MVP) using both covariance estimation methods. The weights of the portfolio should minimize the variance of portfolio returns, subject to the constraint that the portfolio weights sum to 1. **Short positions are allowed, meaning the weights of some stocks in the portfolio can be negative.** Repeat this process for each week in the dataset to generate out-of-sample (OOS) returns for both MVPs.
3. **Performance Comparison:** Calculate the standard deviation of the OOS returns for both portfolios. Compare the two portfolios in terms of their OOS standard deviations. Discuss whether the portfolio constructed using the LedoitWolf covariance matrix has a lower OOS standard deviation compared to the portfolio constructed using the sample covariance matrix. In your discussion, provide an interpretation of the results in terms of the accuracy of covariance matrix estimation and its impact on portfolio risk.

Please submit your Python code and ensure that it is clear and well-commented.

Grading:

- Code correctness and clarity: **30 points.**
- Correct calculation of OOS standard deviations of the two portfolios: **10 points.**
- Clarity and reasoning of the discussion: **10 points.**

Question 1 (30 Points)

In the Barra factor model, pure factor portfolios represent the risk premiums associated with specific factors.

- (15 Points) Explain what the returns of country, industry, and style factors represent. In your response, discuss the type of risk or premium each type of factor (i.e., country, industry and style) aims to capture.

① Country Factor: [Pure long portfolio, +ve exposure to all industry factors, 0 exposure to all style factors]

- (From Lecture) This is the market portfolio weighted by market capitalization
- it captures the systematic risk associated with a given country's economic and political conditions
- => Able to describe risk / returns based on readily identifiable fundamental characteristics of a country

↳ Risk Premium:

- this is the additional premium investors may demand as compensation when dealing with higher risk in investing in a given country
- essentially reflects overall performance of the market in this country.
- compensation for risks related to geopolitical and macroeconomic factors s.a.:
 - GDP growth
 - Inflation Rate
 - Currency (Exchange Rate) fluctuations

=> This represents the excess return expected from investing in a given country's market juxtaposed to a risk-free rate.

② Industry Factors: [Dollar Neutral, 100% long on given industry, 100% short on country factor, 0 to all style]

- (From Lecture) it represents the excess returns of the industry relative to the market portfolio
- this represents the risk premiums associated with a given sector or sectors of the economy
- => Able to model / describe risk that is accompanied by certain industry behaviour

↳ Risk Premiums:

- this captures performance of stocks of a local sector/industry relative to the entire market.
- risk premium returns here will indicate performance of industry factor compared to country factor => reflects compensation for investors dealing with risk unique to a given industry.
- such risks include:
 - Changes to Law & Regulation
 - Industry Competition
 - Technological Advancements

③ Style Factor: [Dollar Neutral, 1-unit exposure to itself, 0-exposure to other industry/style]

- For a given style factor S_2 :

- Unit exposure to itself (S_2)

- Zero exposure to all other style factors

\Rightarrow style factors are designed to capture specific investment characteristics

- Characteristics include:

- Size

- Value

- Growth

- Momentum

- Volatility

b) Risk Premiums:

- style factors capture risk/returns associated with varying investment styles

\hookrightarrow e.g.:

- Value stocks may outperform growth stocks over time

- Style factor will indicate excess returns as a result of the different style and characteristics of investment.

- (15 Points) Explain the exposure of each type's pure factor portfolio (country, industry, style) to the other factors in the model.

① Country Factor Exposure:

- Exposure to Industry Factors

- positive exposure to all industry factors since it reflects overall market performance and this includes multiple industries. But exposures unique to a given market can vary based on market conditions.

- Exposure to Style Factors

- zero exposure to style factors since country factors represent a broader market behaviour rather than stock-focused characteristics.

② Industry Factor Exposure:

- Since industry factor is Dollar Neutral \Rightarrow implies a balance b/t long and short positions across country and all other industry's factors

- Exposure to Country Factor

- For a given industry I_p , it is 100% short on country factor portfolio (+ve exposure) since performance is based on overall market conditions

- Exposure to other Industries

- For a given industry portfolio I_p , it is 100% long on that industry. There can also be factor exposures within industries, reflecting how different sectors/industry might perform relative to one another based on economic conditions.

③ Style Factor Exposure

- Zero exposure to Country and Industry Factors since they represent characteristics of varying investment styles.
 - Not influenced by broader market or industry changes
 - One unit of exposure to itself.
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