Problem 4

Best-Practices Handbook: Addressing Multicollinearity in Regression Modeling

1. Definition

Multicollinearity occurs when two or more independent variables in a regression model are highly correlated, leading to unreliable estimates of the coefficients. Mathematically, if $X_1, X_2, \ldots, X_k$ are the independent variables, multicollinearity is present if:

$$\text{Corr}(X_i, X_j) \approx 1 \quad \text{or} \quad \text{Corr}(X_i, X_j) \approx -1 \quad \text{for } i \neq j$$

This results in inflated standard errors for the coefficients, making hypothesis tests unreliable.

2. Description

Multicollinearity complicates the interpretation of regression coefficients, as it becomes difficult to determine the individual effect of each predictor on the dependent variable. It can lead to unstable estimates and reduced statistical power.

In simpler terms: severe multicollinearity causes the model to become more sensitive to change in input data. **This is BAD for a quantitative model or strategy in practice**

3. Demonstration

Numerical Example

Consider a simulated dataset with three independent variables X_1, X_2, and X_3:

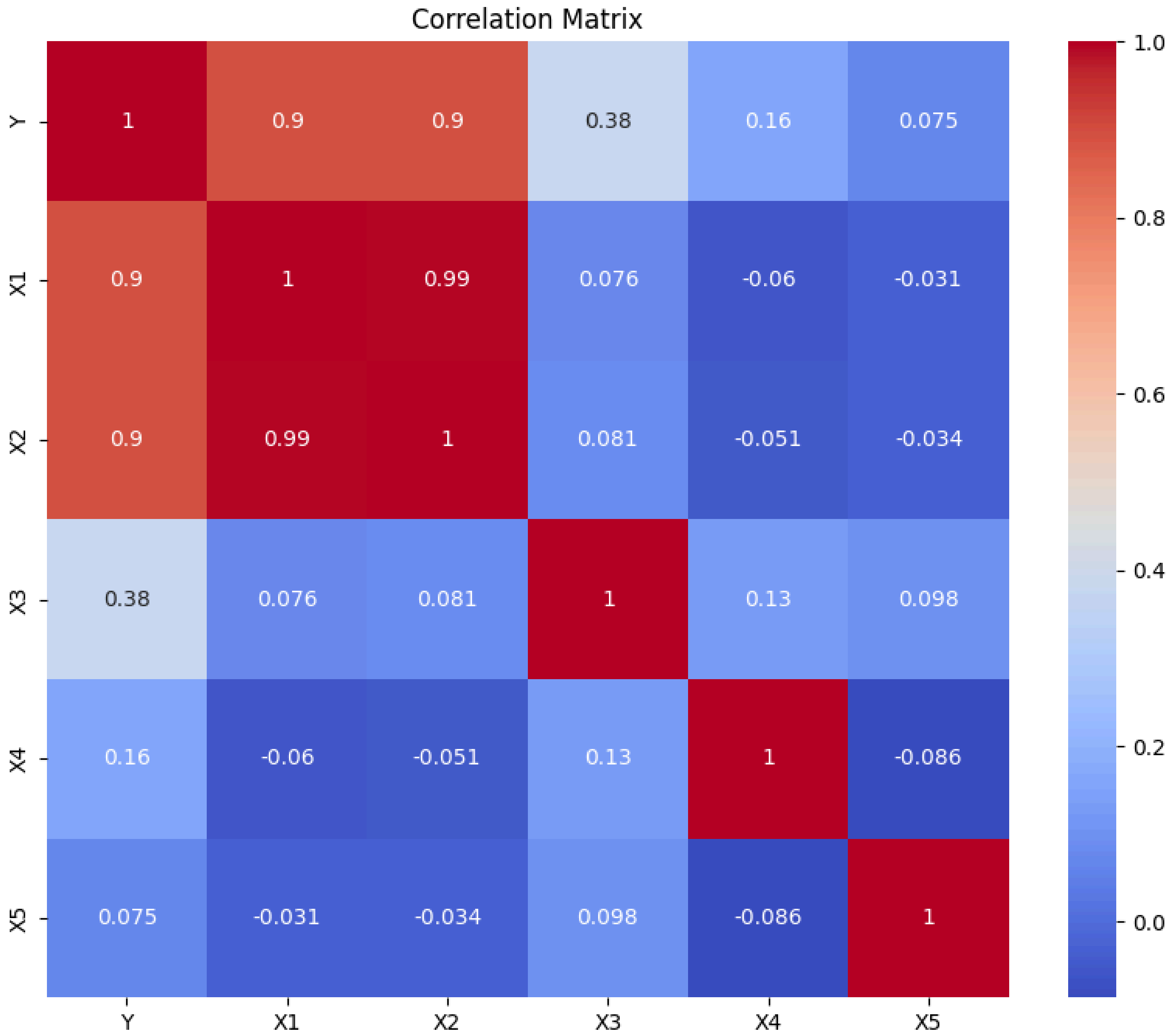
In [1]:
```
import numpy as np
import pandas as pd
import statsmodels.api as sm

# Simulating data
np.random.seed(0)
X1 = np.random.normal(0, 1, 100)
X2 = X1 + np.random.normal(0, 0.1, 100)  # Highly correlated with X1
X3 = np.random.normal(0, 1, 100)
X4 = np.random.normal(0, 1, 100)
X5 = np.random.normal(0, 1, 100)
Y = 3 + 2 * X1 + 1.5 * X2 + 1.0 * X3 + 0.5 * X4 + 0.25 * X5 + np.random.normal(0, 1, 100)

data = pd.DataFrame({'Y': Y, 'X1': X1, 'X2': X2, 'X3': X3, 'X4': X4, 'X5': X5})

# Fitting a regression model
X = sm.add_constant(data[['X1', 'X2', 'X3', 'X4', 'X5']])
model = sm.OLS(data['Y'], X).fit()
print(model.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      Y   R-squared:                       0.941
Model:                            OLS   Adj. R-squared:                  0.938
Method:                 Least Squares   F-statistic:                     298.7
Date:                Thu, 12 Sep 2024   Prob (F-statistic):           4.64e-56
Time:                        22:50:34   Log-Likelihood:                -137.88
No. Observations:                 100   AIC:                             287.8
Df Residuals:                      94   BIC:                             303.4
Df Model:                           5
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          2.8705      0.102     28.170      0.000       2.668       3.073
X1             2.9989      0.984      3.046      0.003       1.044       4.953
X2             0.4817      0.968      0.497      0.620      -1.441       2.404
X3             1.1596      0.106     10.931      0.000       0.949       1.370
X4             0.7706      0.109      7.094      0.000       0.555       0.986
X5             0.3446      0.097      3.566      0.001       0.153       0.536
==============================================================================
Omnibus:                        3.015   Durbin-Watson:                   2.083
Prob(Omnibus):                  0.221   Jarque-Bera (JB):                2.456
Skew:                          -0.367   Prob(JB):                        0.293
Kurtosis:                       3.222   Cond. No.                         20.1
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Visual Example

In [2]:
```python
import seaborn as sns
import matplotlib.pyplot as plt

# Visualizing the correlation matrix
plt.figure(figsize=(10, 8))
correlation_matrix = data.corr()
sns.heatmap(correlation_matrix, annot=True, cmap='coolwarm')
```

```
plt.title('Correlation Matrix')
plt.show()
```


Correlation Matrix
Y
X1
X2
X3
X4
X5
1 0.9 0.9 0.38 0.16 0.075
0.9 1 0.99 0.076 -0.06 -0.031
0.9 0.99 1 0.081 -0.051 -0.034
0.38 0.076 0.081 1 0.13 0.098
0.16 -0.06 -0.051 0.13 1 -0.086
0.075 -0.031 -0.034 0.098 -0.086 1
1.0
0.8
0.6
0.4
0.2
0.0

In [3]:
```python
# Show SE of the variables
print(model.bse)
```

```
const    0.101900
X1       0.984416
X2       0.968277
X3       0.106081
X4       0.108631
X5       0.096629
dtype: float64
```

Output Interpretation

The OLS regression summary shows us the inflated standard errors for X_1 and X_2 due to their multicollinearity.

Variable	Standard Error
const	0.101900
X1	*0.984416*
X2	*0.968277*
X3	0.106081
X4	0.108631
X5	0.096629

The correlation matrix also reflects strong correlation between X_1 and X_2 as can be seen from the bright orange and red squares.

4. Diagnosis

We use the following to measure the amount of multicollinearity:

- **Variance Inflation Factor (VIF)**: A VIF value greater than `5` is often taken as an indication of _*severe*_ multicollinearity. While a VIF value between `1` and `5` indicates no severe multicollinearity issue. This is defined for x_i as:

$$VIF_i = \frac{1}{1 - R_i^2}$$

Where R_i^2 is the R-squared value obtained by regressing the x_i against all other independent variables.

Obtaining VIF outputs

In [4]:
```
from statsmodels.stats.outliers_influence import variance_inflation_factor

vif = pd.DataFrame()
vif["Feature/Variable"] = X.columns
vif["VIF"] = [variance_inflation_factor(X.values, i) for i in range(X.shape[1])]
print(vif)
```

```
  Feature/Variable         VIF
0            const    1.057647
1               X1  100.269312
2               X2  100.256218
3               X3    1.038820
4               X4    1.038162
5               X5    1.023083
```

From the VIFs:

- Both predictor variables `X1` and `X2` have very large VIFs
- For e.g. the VIF of `X1` = `100.2693` > `5`. This number indicates severe multicollinearity among `X1` and `X2`
- For the rest of the variables, multicollinearity is not a big concern

5. Discussion

Consequences

The presence of multicollinearity can:

- Lead to large standard errors, making hypothesis tests unreliable.
- Cause coefficients to change dramatically with small changes in the model or data.

Techniques to Address Multicollinearity

1. **Remove Variables**: Consider removing one of the correlated variables.
2. **Combine Variables**: Create composite indices or use principal component analysis (PCA) to reduce dimensionality.
3. **Regularization Techniques**: Use Lasso or Ridge regression, which can handle multicollinearity by adding a penalty term to the loss function.

Practical Recommendations

- Always check for multicollinearity using VIF before finalizing your regression model.
- If multicollinearity is detected, consider the context of the variables and choose the most relevant ones for your analysis.