Distributed Pressure Drops: an Axisymmetric Approach

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A.Y. 2024/2025

Introduction

Distributed Pressure Drops are estimated using **Experimental Relationships** to define **Friction Factors** dependent on flow, fluid parameters and the specific application.

The aims of this work are:

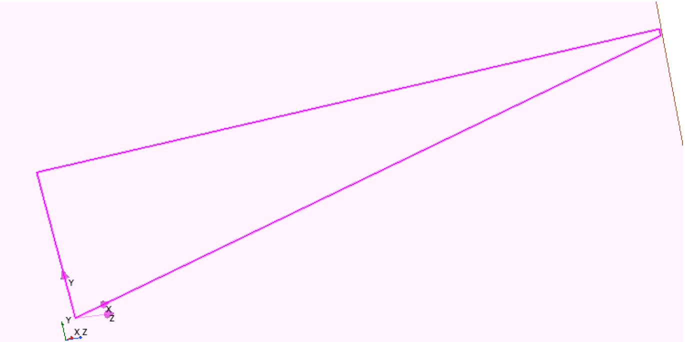
- Evaluate through a **CFD code** the friction factors in a pipe of circular cross section.
- Identify the factors affecting the CFD simulation result.
- Compare the results with the experimental relationships.

The simulations were performed using StarCCM+.

CFD computation: Laminar regime and Turbulent smooth pipe

Geometry

A circular pipe was modeled using a **2D axisymmetric approach**.
Length $L = 2.25\mathrm{m}$ and diameter $D = 3.75$ cm $\implies L/D = 60$.



Boundary Conditions

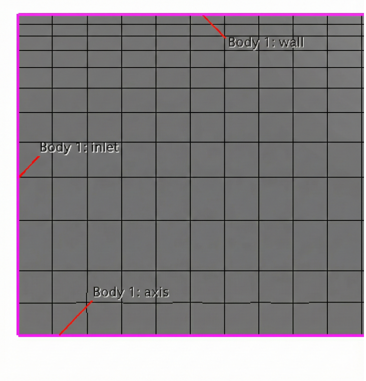
The fluid used is water with constant density $\rho = 997.561\,\mathrm{kg\,m^{-3}}$ and constant dynamic viscosity $\mu = 8.8871 \times 10^{-4}\,\mathrm{Pa\,s}$. The **Reynolds** numbers selected for the laminar and turbulent region were:

$$\begin{aligned} Re_{laminar} &= (1000, 1500, 2000) \\ Re_{turbulent} &= (4000, 16000, 20000, 45000, \\ &\quad 63000, 250000, 1000000) \end{aligned}$$

The **Inlet Type** was imposed as **Velocity Inlet**, while the **Outlet Type** was set as a **Pressure Outlet**.

Mesh

The mesh selected for the simulation is an **Automated Axisymmetric Two-Dimensional** mesh. In particular, a **Quadrilateral Mesher** is selected for the center of the pipe and a **Prism Layer Mesher** for the region near the wall.



Properties:

- **Base size**: $2\mathrm{mm}$;
- Prism layer **Total Thickness**: $15\mathrm{mm}$;
- **Number of prism layers**: 10;
- Prism layer **Stretching**: 1.2.

On the selection of the Mesh

The chosen properties depend on:

- **Reynolds number and velocity inlet**, higher velocities enhance the turbulence;
- **Wall Y+** values, it is common to have $Y^+ = 30$ in the center of the first cell on the wall;
- The **Physics Models** used in the simulation;
- **Wall roughness**, a thickness of the first prismatic cell comparable with the roughness height of the pipe leads to a strong variation in the results.

Wall Y+ estimates

At which distance y from the wall does $Y^+ = 30$ appear? An initial estimate of y comes from the relation:

$$f = 8\frac{\tau_w}{\rho w^2} \implies y = \frac{Y^+ D}{Re\sqrt{f/8}} \tag{1}$$

f was evaluated using the known relationships. Meanwhile, the first cell height h_1 is calculated as:

$$h_1 = H_{Tot}\frac{r-1}{r^n-1} \tag{2}$$

Where r is the prism layer stretching, n the number of prism layers and H_{tot} the total thickness. These two values are then compared. For the chosen properties $h_1 = 0.58$mm.

Wall Y+ estimates

Table: Computed distance at which $Y^+ = 30$ is expected and effective Y^+ value for $h_1 = 0.58$mm.

Reynolds	$y(Y^+ = 30)$ [mm]	Y^+ for $y = h_1/2$
4000	4	2.2
16 000	1.2	7.5
20 000	0.98	9
45 000	0.48	18
63 000	0.36	25
250 000	0.1	94
1 000 000	0.03	293

Wall Y+ estimates

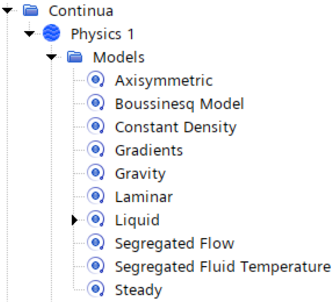
Which is the most suitable model for the **wall treatment**?
Different treatments could be adopted in StarCCM+:

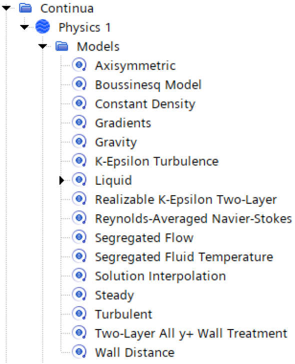
- **High Y+**, assumes $Y^+ > 30$, permits the use of a coarse mesh, less time consuming;
- **Low Y+**, assumes $Y^+ < 5$, a fine layered mesh should be used, computationally demanding;
- **All Y+**, this treatment could adapt the Wall function to the type of mesh adopted, for intermediate meshes and works fine for $5 < Y^+ < 30$;
- **Two Layer All Y+**, same as All Y+, the turbulent dissipation rate is imposed at the center of the first cell.

The **Two Layer All Y+** treatment was selected for all simulated cases.

The Physics Model

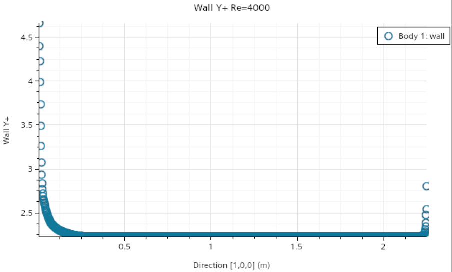
The **Two layer All Y+** approach overcomes the $Y^+ = 30$ problem. The adoption of this treatment imposes the use of Realizable $k - \epsilon$ model.

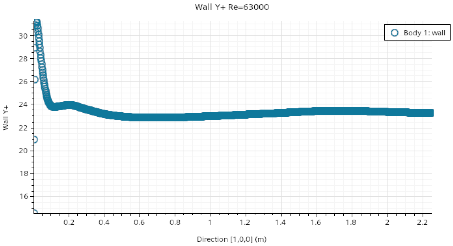




The Physics Model

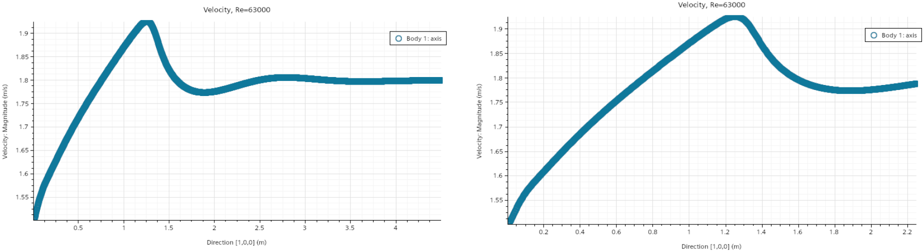
Wall Y^+ computed with the **Two Layer All Y+ approach**.



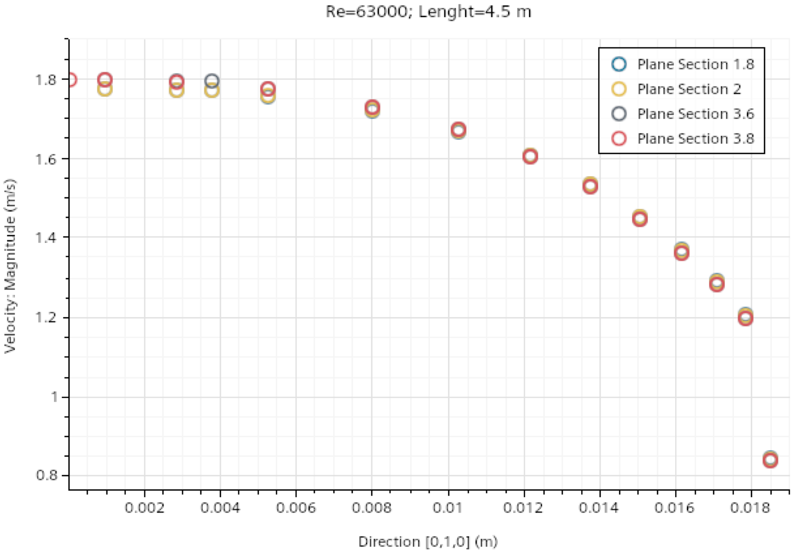


Derived parts and ΔP determination

The region of the pipe with a **fully developed flow** is the one selected for the estimation of the pressure drop ΔP. The velocity profile was observed. Was the pipe long enough?



Derived parts and ΔP determination

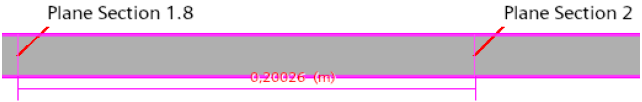


Derived parts and ΔP determination

Creating some **derived parts**, i.e. plane sections, it was possible to run a report and estimate the ΔP.

ΔP L/D=60	ΔP L/D=120
0.107kPa	0.108kPa

The local pressure has been evaluated at 1.8 m and 2 m. $\Delta L = 0.2$ m was used for all cases.



Friction factors evaluation

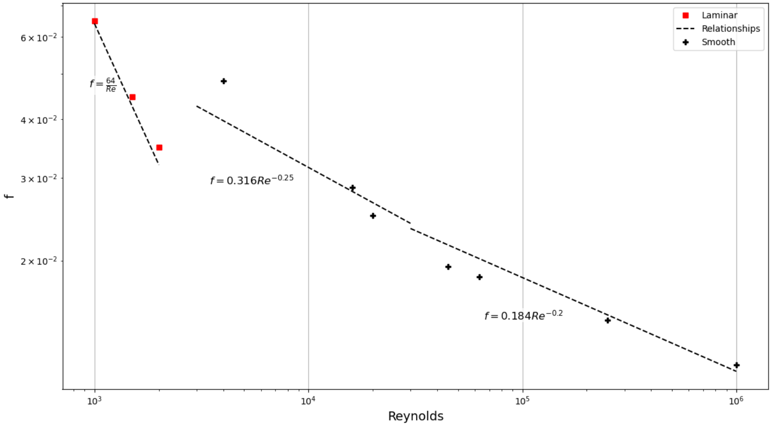
To compute the friction factors coming from the simulation, f_{CFD}, the **Darcy-Weisbach** relation was used:

$$f_{CFD} = 2\frac{\Delta P D}{\rho v^2 \Delta L}$$

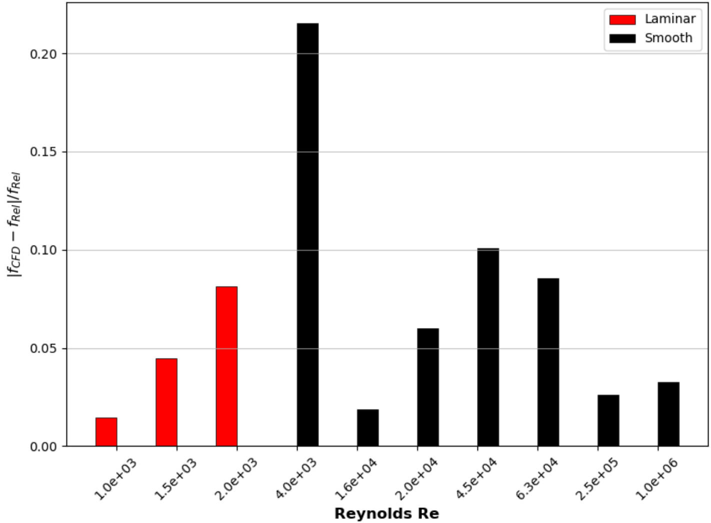
while, as a reference, the *Poiseuille, Blasius and McAdams* relationships were adopted.

$f_{\text{Poiseuille}} = 64/\text{Re}$	Laminar Regime
$f_{\text{Blasius}} = 0.316\text{Re}^{-0.25}$	Smooth, $\text{Re} < 30000$
$f_{\text{McAdams}} = 0.184\text{Re}^{-0.2}$	Smooth, $30000 < \text{Re} < 10^6$

Results


Laminar
Relationships
Smooth
$f = \frac{64}{Re}$
$f = 0.316Re^{-0.25}$
$f = 0.184Re^{-0.2}$
6×10^{-2}
4×10^{-2}
3×10^{-2}
2×10^{-2}
f
10^3
10^4
10^5
10^6
Reynolds

Results


Laminar
Smooth
0.20
0.15
0.10
0.05
0.00
$|f_{CFD} - f_{Rel}|/f_{Rel}$
1.0e+03
1.5e+03
2.0e+03
4.0e+03
1.6e+04
2.0e+04
4.5e+04
6.3e+04
2.5e+05
1.0e+06
Reynolds Re

Results

Some comments on the results:

- The discrepancy between the simulation values and the relationship predictions grows as the Reynolds number approaches the **Transition Region**.
- For $20000 < \mathrm{Re} < 250000$ the simulations underestimate the friction factors values.
- Note the high relative error for $\mathrm{Re} = 4000$.

CFD computation: Turbulent regime Rough pipe

Roughness and pre-processing

Selected Relative Roughness values:

$$\epsilon/D = (0.0001, 0.001, 0.002, 0.005, 0.01)$$
$$\epsilon = (3.75 \times 10^{-6}, 3.75 \times 10^{-5}, 7.5 \times 10^{-5}, 1.875 \times 10^{-4}, 3.75 \times 10^{-4})\ \mathrm{m}$$

It is evident that the last two **roughness height** values are comparable to h_1. How does the simulation behave with the same physics and mesh?

Reference Relationships

To evaluate f_{CFD} the Darcy-Weisbach relation is used. For the analytical comparison, the following relationships were adopted:

1. Churchill ($Re = 4000$):

$$f = 8\left[\left(\frac{8}{Re}\right)^{12} + \frac{1}{(A+B)^{1.5}}\right]^{\frac{1}{12}}$$

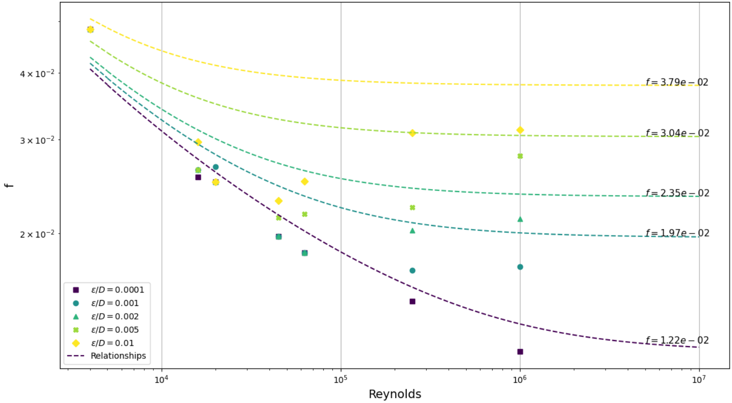
where: $$A = \left[2.457 \ln\left\{\frac{1}{(7/Re)^{0.9} + 0.27(\varepsilon/D)}\right\}\right]^{16}$$

$$B = \left[\frac{37530}{Re}\right]^{16}$$

2. Swamee-Jain ($Re > 4000$):

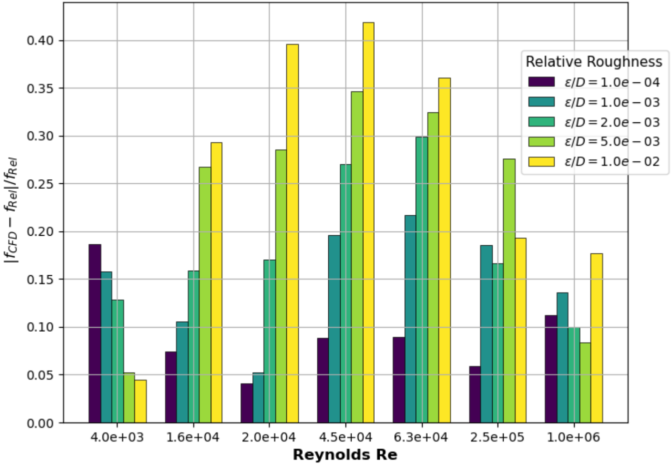
$$f = \frac{0.25}{\left[\log_{10}\left(\frac{1}{3.7(D/\varepsilon)} + \frac{5.74}{Re^{0.9}}\right)\right]^2} \quad \text{valid for} \begin{cases} 5000 < Re < 10^8 \\ 10^{-6} < \varepsilon/D < 10^{-2} \end{cases}$$

Results



4×10^{-2}
3×10^{-2}
2×10^{-2}
f
10^4
10^5
10^6
10^7
Reynolds
$f = 3.79e-02$
$f = 3.04e-02$
$f = 2.35e-02$
$f = 1.97e-02$
$f = 1.22e-02$
$\varepsilon/D = 0.0001$
$\varepsilon/D = 0.001$
$\varepsilon/D = 0.002$
$\varepsilon/D = 0.005$
$\varepsilon/D = 0.01$
Relationships

Results



0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
$|f_{CFD} - f_{Rel}|/f_{Rel}$
4.0e+03
1.6e+04
2.0e+04
4.5e+04
6.3e+04
2.5e+05
1.0e+06
Reynolds Re
Relative Roughness
$\varepsilon/D = 1.0e-04$
$\varepsilon/D = 1.0e-03$
$\varepsilon/D = 2.0e-03$
$\varepsilon/D = 5.0e-03$
$\varepsilon/D = 1.0e-02$

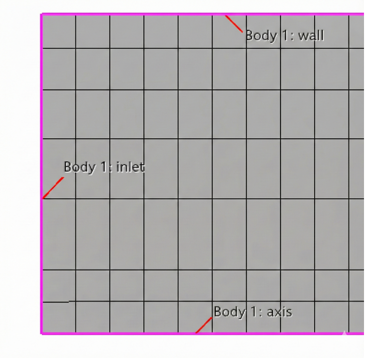
Results

Some comments on the results:

- In general, the simulations understimated the friction factors compared to the relationships.
- As anticipated, the **largest discrepancies** between references and simulations appear for the points with **highest relative roughness** values.
- The formation of a dip region for $16000 < \mathrm{Re} < 63000$ is evident.

New mesh settings

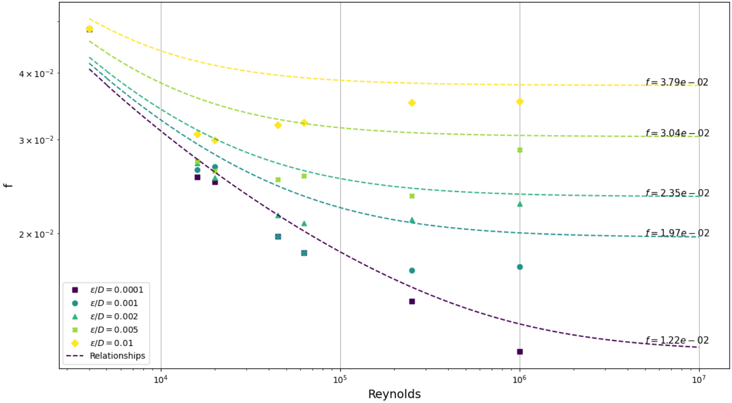
The similar values of roughness height and first cell height suggest that the **mesh is inadequate** to resolve the turbulence. Reducing the number of prism layers for the two higher roughness cases:



Properties:

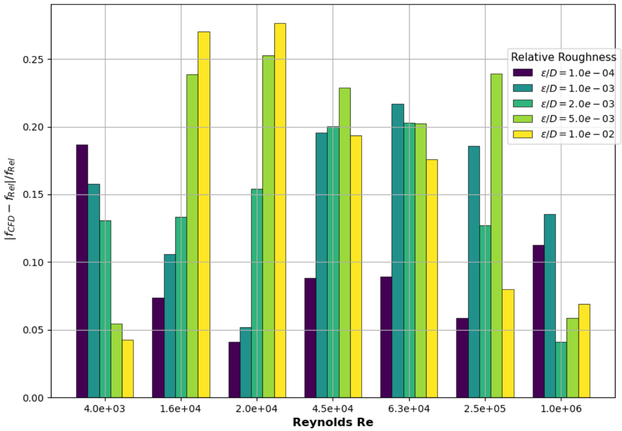
- **Base size**: 2mm;
- **Prism layer Total Thickness**: 15mm;
- **Number of prism layer**: 5;
- **prism layer Stretching**: 1.2;
- first cell height $h_1 \approx 2$mm.

New results



4×10^{-2}
3×10^{-2}
2×10^{-2}
f
10^4
10^5
10^6
10^7
Reynolds
$f = 3.79e-02$
$f = 3.04e-02$
$f = 2.35e-02$
$f = 1.97e-02$
$f = 1.22e-02$
$\varepsilon/D = 0.0001$
$\varepsilon/D = 0.001$
$\varepsilon/D = 0.002$
$\varepsilon/D = 0.005$
$\varepsilon/D = 0.01$
Relationships

New results

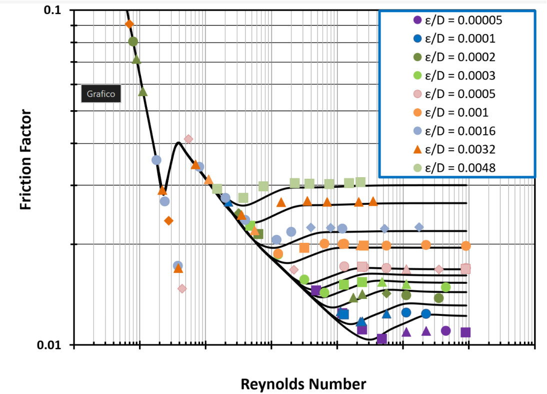

Relative Roughness
$\varepsilon/D = 1.0e-04$
$\varepsilon/D = 1.0e-03$
$\varepsilon/D = 2.0e-03$
$\varepsilon/D = 5.0e-03$
$\varepsilon/D = 1.0e-02$
$|f_{CFD} - f_{Rel}|/f_{Rel}$
0.00
0.05
0.10
0.15
0.20
0.25
4.0e+03
1.6e+04
2.0e+04
4.5e+04
6.3e+04
2.5e+05
1.0e+06
Reynolds Re

New results

Some comments on the results:

- With the new mesh the discrepancies are reduced.
- Again, the simulations underestimated the friction factor.
- The phenomenon in the region $16000 < \mathrm{Re} < 63000$ still remains. This phenomenon is related to how the code deals with the roughness. In particular, StarCCM+ reproduces a similar trend to Nikuradse's experiment.

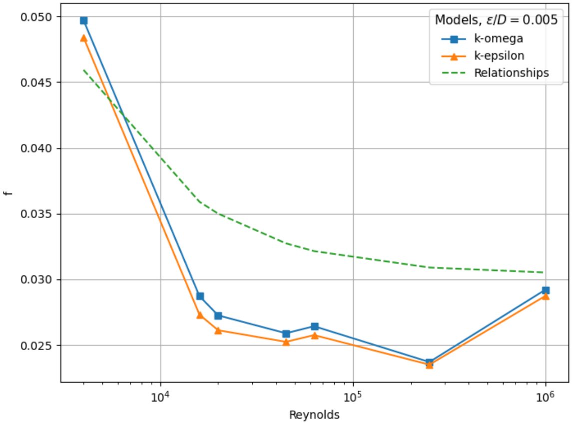
On Nikuradse



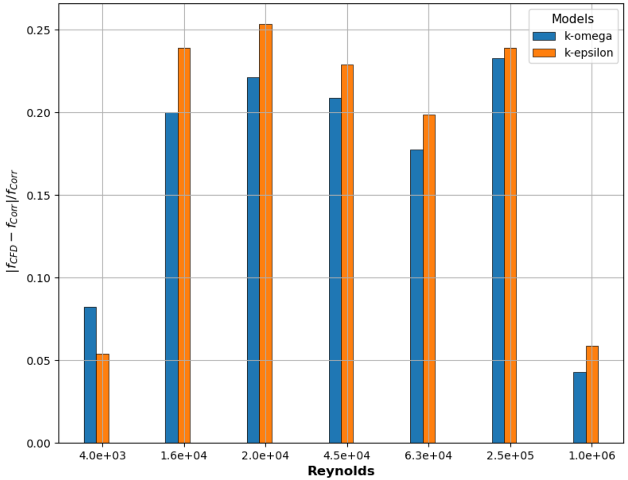
Nikuradse created artificially roughened pipes to control the geometry of the surface irregularities. He achieved this by gluing uniform sand grains of known sizes onto the inner walls of the pipes. We can recognise the same behaviour in the graph on the left.

What about changing the RANS model?

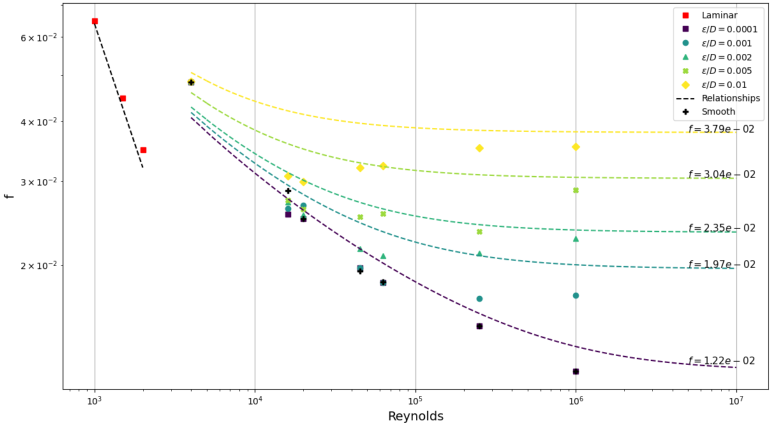
Moving from a $k - \epsilon$ to a $k - \omega$ model, an **All Y+** model was adopted. The mesh remains the same.



What about changing the RANS model?



Total result



Conclusion

This work explored the possibility of computing the Friction-Factor through an axisymmetric approach.

- **One of the factors affecting the results is the mesh choice**: the choice of the **Wall Y+ treatment** requires for **low Reynolds** numbers a **coarser mesh**; for **high Reynolds** numbers the **turbulent Boundary layer is thinner**, and a **finer mesh** is required; the **roughness** value requires a minimum height of the first cell.
- StarCCM+ reproduces the *sand-grain* trends of Nikuradse, a different approach respect the relationships taken as references.
- Besides all the different aspects that are related to the discrepancies, the **RANS code and relationships are associated with an intrinsic error**, due to the complexity of fluid dynamics problems.

Bibliography

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[2] M. Eckert. "Pipe flow: a gateway to turbulence.". In: *Arch. Hist. Exact Sci.* 75 (2021). DOI: https://doi.org/10.1007/s00407-020-00263-y.

[3] Siemens Digital Industries Software. *Simcenter STAR-CCM+ User Guide v. 2021.1*. Version 2021.1. 2021.