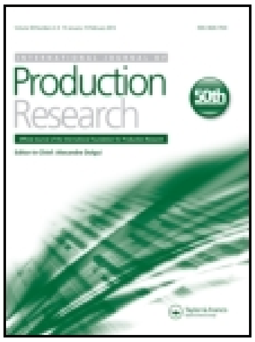

International Journal of Production Research

ISSN: 0020-7543 (Print) 1366-588X (Online) Journal homepage: https://www.tandfonline.com/loi/tprs20

Online versus bricks-and-mortar retailing: a comparison of price, assortment and delivery time

Zhaolin (Erick) Li, Qiang Lu & Masoud Talebian

International Journal of Production Research, 2015
Vol. 53, No. 13, 3823–3835, http://dx.doi.org/10.1080/00207543.2014.973074

Online versus bricks-and-mortar retailing: a comparison of price, assortment and delivery time

Zhaolin (Erick) Li[a*], Qiang Lu[a] and Masoud Talebian[b]

[a] *Discipline of Business Analytics, The University of Sydney Business School, Sydney, Australia;* [b] *School of Mathematical and Physical Sciences, The University of Newcastle, Newcastle, Australia*

(*Received 8 May 2013; accepted 22 August 2014*)

The popularity of online retailing has created great opportunities and challenges for retailers. A key challenge faced by many retailers is the choice between the clicks, the bricks-and-mortar, and the bricks-and-clicks strategies. In this paper, we develop a comprehensive framework for selecting the appropriate distribution channel given assortment, logistics and consumer characteristics. Under the traditional bricks-and-mortar retail model, the retailer displays and sells the assortment in a physical store, whereas under the online retail model, the retailer accepts orders online and delivers the products offline. The traditional retailer jointly determines the breadth, the depth and the price of the assortment, whereas the online retailer jointly determines the breadth and price of the assortment and the delivery time. After deriving the joint optimal solution for the traditional and the online retailers, we compare their performance. We analytically and numerically examine how inventory cost, delivery cost and consumer behaviours could affect the optimal distribution strategy and customer service.

Keywords: assortment; delivery time; E-business; multichannel; price-inventory joint optimisation; price-time joint optimisation

1. Introduction

Online retailing has become ubiquitous and traditional bricks-and-mortar retailers have been considering how to effectively incorporate the new distribution channel into their existing networks. In this research, we compare the three distinct retailing strategies, namely, the clicks, the bricks-and-mortar, and the bricks-and-clicks retailing models. Our goal is to develop a comprehensive framework for selecting the appropriate retailing strategy given the assortment, logistics and consumer characteristics.

These different distribution strategies can be found in a wide variety of firms. For instance, ALDI, an Australian supermarket chain operates only bricks-and-mortar stores. In contrast, online retailers such as Buygrocery in Australia, eBags, Bluelight.com, Netgrocer.com and onlinefoodgrocery.com in the USA exclusively use the online channel to sell goods to the end consumers directly. It is worth noting that some companies use the third strategy (the bricks-and-clicks strategy). For example, True Religion Brand Jeans (NASDAQ ticker TRLG), which is one of the higher performing shares in the apparel industry, sells garments of unique styles in its 83 retail stores and its virtual online store. Dell used the online strategy before 2007 and changed to the dual-channel strategy in June 2007 by working with retail giants such as Wal-Mart. In Australia, Coles and Woolworths, the two largest supermarket chains operate both online and offline channels. The co-existence of these three channel strategies suggests that we need to address the strategic choice on choosing channel strategies by considering relevant operational details.

The online channel works and functions very differently from the traditional channel. Consequently, a very different set of strategies is required to manage an online store compared to managing a traditional bricks-and-mortar store (see the discussions in Bhargava, Sun, and Xu 2006; Sun 2008; Sun, Ryan, and Shin 2008). One of the key differences between these two strategies is the shopping experience. If a consumer orders online, he/she must wait for the delivery of the product, whereas if he/she purchases at a traditional store, he/she can receive the product right away. This difference causes the online retailer and the traditional retailer to focus on two different operational factors: the delivery time and inventory. Specifically, the traditional retailer needs to keep the optimal amount of inventory in the store to avoid stock-out. However, the inventory costs often prevent the retailer from carrying excessive inventory. In contrast, an online retailer needs to consider how quickly it can deliver the goods to the consumer. From the consumer's perspective, faster delivery is better. However, from the retailer's perspective, faster delivery results in a higher delivery cost. Thus, the retailer has to optimise its delivery time given its logistics capability.

*Corresponding author. Email: erick.li@sydney.edu.au
The order of authorship is alphabetical. Each author made an equal contribution to the manuscript.

Furthermore, the online retailer and the traditional retailer may adopt very different assortment strategies because the online retailer enjoys an inventory cost advantage. Several online retailers (e.g. Dell) hold almost zero inventory, and they direct the customers' orders to the manufacturer, which delivers the product to customers directly. This distributional strategy is called the drop-shipping strategy, and it requires a substantial investment in supply chain reconfiguration and coordination. On the contrary, the traditional store has a cost disadvantage if it offers a larger assortment because of the shelf space and the restocking costs. Thus, online retailers usually offer a high level of variety, whereas the traditional retailers provide the convenient returnability and short response time. A short trip to the Victoria Secret Store after browsing the company website tells us that there are more choices of garments (with different colours, patterns, designs and fabrics) at the online store than in the retail store (Kumar 2005). However, consumers who order online must wait a few days for the delivery, whereas those who visit the store can walk away with the garments. The wide variety of distribution strategies and the contrast in shopping experiences give rise to the following question: 'Which distribution model can better serve the retailers and their customers?'

The answer to this question requires a careful examination of several important elements: assortment, price, delivery time, inventory and consumer characteristics. This research begins by presenting two alternative models: the online retail model, whereby the retailer accepts orders placed to the virtual store and delivers the products offline; and the traditional retail model, whereby the retailer sells the assortment in a traditional store. In the online retail model, the retailer determines the breadth and the price of the assortment as well as the delivery time (or lead time reduction). In the traditional retail model, the retailer determines the breadth of the assortment (the number of variants included in the assortment), the price of each item in the entire assortment and the inventory level for each item included in the assortment (the depth of the assortment). After deriving the optimal joint solutions for these two models, we compare their performances.

The trade-off for planning the distribution strategy involves choosing between the benefits of carrying a high level of variety, the savings in delivery cost and the competitiveness in price and delivery time. We find that high inventory cost and low demand volume are the main forces that keep the traditional retailer from offering a high level of product variety, whereas consumer characteristics and the cost of lead time reduction are the main forces that keep the online retailer from charging a premium for its goods. If the ability to offer a high level of variety offsets the long delivery time, then the online retail model prevails. Similarly, if the product can be distributed in a short time and at a low cost, then the online retailing model also prevails. These results are consistent with the observation that firms such as Amazon can distribute their products purely through online channel because they have established a low cost and efficient supply chain (Chaturvedi et al. 2013).

Furthermore, if the delivery cost is high and consumers are impatient, the traditional retail model prevails. This result is consistent with the observation that products such as soft drinks and bottled water are almost exclusively sold through the traditional retail channel.

The remainder of this paper is organised as follows. Section 2 reviews the literature. Section 3 analyses the assortment, price and delivery time joint optimisation of an online retailer. Section 4 analyses the assortment, price and inventory joint optimal decisions of a traditional retailer. Section 5 compares the performance of the two distribution models, and Section 6 concludes the research. All of the proofs are presented in the Appendix.

2. Literature review

Our work is positioned at the interface between marketing and operations (see Porteus and Whang 1991 for a review about the interface). The multichannel retailing literature in the marketing field has focused on the impact of the new online channel on the traditional channel and whether traditional retailers should embrace the new channel (Deleersnyder et al. 2002; Liu, Gupta, and Zhang 2006; Ofek, Katona, and Sarvary 2011; Yang, Essegaier, and Bell 2005; Zettelmeyer 2000; Zhang 2009). Our study extends this stream of research by simultaneously modelling the retailers' price, assortment and delivery time decisions and comparing the performance difference between the online and traditional retailers. Chiang and Monahan (2005) study online and traditional retailing by focusing on inventory costs where a one-for-one inventory control policy is applied, and show that the dual-channel strategy outperforms the online-only and retail-only strategies. Whereas, they consider a single product and assume that price is exogenously given; our research considers pricing and assortment as decision variables.

A few studies in marketing also investigate the impact of the Internet on prices (Lal and Sarvary 1999; Ofek, Katona, and Sarvary 2011; Pan, Ratchford, and Shankar 2004). For example, Lal and Sarvary (1999) challenge the conventional wisdom and show that the Internet can lead to higher prices and profits under some conditions. In our study, we go further to show that store traffic and assortment may also influence the online and offline retailers' pricing strategies.

In operations research, joint inventory and pricing optimisation have received increasing research attention since the 1990s. We refer to Petruzzi and Dada (1999) for a comprehensive review on the early literature and Chen and Simchi-Levi (2012) for the updated literature. This literature stream usually focuses on a single product.

The most relevant prior study in the area of price-inventory joint optimisation is Aydin and Porteus (2008). These authors study joint inventory management and pricing for multiple products. They adopt the multiplicative demand model in which the random components are identically and independently distributed non-negative random variables with an increasing failure rate. The consumer choice process is formulated as a general process that includes the multinomial logit model (MNL) as a special case. Aydin and Porteus (2008) show that the first-order condition can lead to the price-inventory joint optimal solution. Whereas, they do not study how to determine the optimal breadth of the assortment (i.e. the number of variants to offer) because they treat the assortment decision as exogenously given; we jointly optimise assortment, price and inventory level for the traditional retailing model.

Including the decision of assortment breadth in the model may post insurmountable challenges because the objective function becomes an ill-behaved multivariate function with integer and continuous variables. We refer to Kök, Fisher, and Vaidyanathan (2009) for a comprehensive review of the literature on assortment planning. One of the important early results on assortment planning was obtained by van Ryzin and Mahajan (1999). Under the assumption that each variant has an identical newsvendor ratio, these authors show that the optimal assortment consists of several of the most popular items. Subsequent studies extend the analysis to include such factors as consumer search (Cachon, Terwiesch, and Xu 2005) and endogenous pricing (Maddah and Bish 2007). Several other studies take a different route by relaxing the assumption of an identical newsvendor ratio. For example, Li (2007) and Alptekinoglu, Grasas, and Akcali (2009) show that the optimal assortment does not always include the most popular variants but instead includes the most profitable ones. The research performed by Maddah and Bish (2007) is closely related to the traditional retailing model analysed in our research. However, we extend the analysis by comparing the performance of two distinct retailing models.

In a more recent study, Maddah, Bish, and Tarhini (2014) investigate a newsvendor model under two settings: fixed assortment with endogenous price and inventory decisions, and fixed price with endogenous assortment and inventory decisions. They develop a Taylor-series approximation for the inventory cost and investigate its accuracy. In our traditional retailing model, we consider that a retailer determines assortment, price and inventory decisions jointly; in our online retailing model, we examine pricing decisions with time-sensitive consumers. These two aspects complement the work by Maddah, Bish, and Tarhini (2014).

There are several studies on delivery time strategy (e.g. Ho and Zheng 2004; Liu, Parlar, and Zhu 2007; Netessine and Rudi 2006; Shang and Liu 2011). These studies make two common assumptions. First, only a single product is involved. Second, the delivery time is exogenous and is not part of the joint optimisation. The main result of our research is that the choice of the retailing model is primarily determined by the logistics, assortment and consumer characteristics.

Our analysis is also related to the literature on choosing make-to-stock or make-to-order production systems (e.g. Kogan 1997; Kogan, Khmelnitsky, and Maimon 1998; and Kumar et al. 2006). In a make-to-order system, products are made after an order is received. In a make-to-stock system, products are made based on expected demand, therefore before an order is received. A bricks-and-mortar retailer usually adopts the make-to-stock strategy as consumers expect to obtain the product right away when they visit a store. However, an online retailer can use make-to-order strategy to avoid inventory costs. Our assumption of negligible inventory cost in the online channel somewhat overstates the benefit of choosing the online channel. As our model focuses on the relative inventory cost difference between offline and online retailing, for parsimony, we make this particular assumption to facilitate the comparison of three distinctive channel strategies.

3. Online retailing

The online retailer operates as follows. There are N identical variants that the retailer can offer in the assortment. The price of each item in the entire assortment is the same, which is common for many products such as clothes, cosmetics and frozen foods. We adopt the MNL framework developed by van Ryzin and Mahajan (1999) to model the consumer's choice process. Specifically, a typical consumer attains a utility of $U_i = u - p - rt + \epsilon_i$ by consuming one unit of variant i, where u is the maximum price that the consumer is willing to pay for the product, p is the retail price of the assortment and ϵ_i are independently and identically distributed Gumbel random variables with mean 0 and shape factor normalised to 1. Throughout this paper, we assume that p does not exceed u, otherwise, the store traffic will drop to zero. In our model, rt is the disutility associated with the response time t. In other words, r measures the consumer's patience. If consumers are more patient, r has a smaller value. This particular assumption on consumer's disutility is standard in marketing literature (e.g. Chen and Iyer 2002; Iyer 1998; Iyer and Soberman 2000; and Shaffer and Zettelmeyer 2004). As such, the probability that a consumer chooses a variant from the assortment that has $n \leq N$ items is given by

$$q_o = \frac{\exp(u - p - rt)}{n \exp(u - p - rt) + 1},$$

where we normalise the utility of the no-purchase option to be 0 (and, hence, $\exp(0) = 1$). The store traffic is assumed to be a Poisson random variable with mean λ. The per unit transportation cost is $g(t)$, which incurs when shipping a unit from the retailer to the consumer. We assume that the transportation costs are paid by the retailer and not by the consumer. Customers are paying the transportation costs indirectly through prices they are charged.

The exact form of the $g(t)$ function depends on the product's nature, but it is reasonable to assume that $g(t)$ is a continuous, weakly convex and weakly decreasing function. For clothes or cosmetics, which have a light weight, $g(t) > 0$ is convex decreasing in t. Also, there is an implicit assumption that the shipping costs are incurred by the retailer, and not by the consumer. Hereafter, we assume that the delivery cost is $g(t) = \frac{k}{t^2}$ in our numerical study. By varying the parameter k, we can examine how the delivery cost could affect the online store's profit.

A distinct benefit of adopting the online distribution strategy is that the retailer (for example, eBags and Bluelight.com) can provide a high level of service with a very low inventory cost. We assume that all customer demands are met. The online retailer's expected profit is

$$V_O = \max_{p,t,n} n(p - c - g(t))q_o\lambda, \tag{1}$$

where c is the per unit production cost. The following proposition determines the optimal values of the retailer's decisions, i.e. assortment, price and delivery time.

Proposition 1 *In the online retail model, the optimal assortment decision in the online retail model is to offer the full assortment (i.e. $n_O^* = N$). The optimal price-time joint solution is*

$$\begin{cases} g'(t^*) = -r \\ N\exp(u - p_o^* - rt^*) + 1 = (p_o^* - c - g(t^*)). \end{cases} \tag{2}$$

Proposition 1 indicates that the retailer offers all varieties. The result is quite intuitive: because the online retailer holds zero inventory, it is optimal for it to offer the full assortment. It also indicates that the optimal delivery time is independent of the price and is determined by the transportation cost and how patient the consumers are (i.e. the $g(t)$ function and parameter r). When $g(t)$ is differentiable, the optimal delivery time is set at a level where the marginal cost of reducing the delivery time is equal to the consumer's marginal utility. In the extreme case, where $g(t) = 0$ for any $t \geq 0$, then $t^* = 0$. This extreme case mimics the operations of Electronic Arts and Her Interactive. These two companies distribute PC games via online downloading. After we identify the optimal delivery time, the optimal price can be determined accordingly. In other words, the price-time joint optimisation can be solved sequentially: first, determine the delivery time and then the price. This finding supports the early research that considered exogenous delivery time (e.g. Netessine and Rudi 2006).

Using Equation (2), we obtain that

$$V_O^* = \frac{N\lambda(p_o^* - c - g(t^*))\exp(u - p_o^* - rt^*)}{N\exp(u - p_o^* - rt^*) + 1} = N\lambda\exp(u - p_o^* - rt^*). \tag{3}$$

We derive some monotone properties for the online retailer's pricing strategy and performance.

Corollary 1 (1) *The optimal price p_o^* is increasing along with N, is decreasing along with r and is unaffected by λ.* (2) *The online retailer's profit is increasing along with λ and N but is decreasing along with c and r.*

Corollary 1 offers several insights in relation to the online retailer's pricing strategy and performance. First, if the assortment is large, the consumers will have more choices if they purchase online because the online retailer offers the full assortment. Consumers are willing to pay more for more choices. It has been evidenced that higher prices are associated with higher product variety in ready-to-eat cereal (Benson 1990) and ski manufacturing industry (Corrocher and Guerzoni 2008). Furthermore, Zhang (2005) empirically shows that book prices in an online bookstore with large assortment (e.g. Amazon.com) tend to be higher than the ones in an online bookstore with small assortment (e.g. Buy.com). Second, high production costs and low store traffic can cause the retailer's profit to drop, which is quite straightforward. Third, high product variety can increase profit because it expands the market by meeting the needs of more customers. Finally, low customer patience can make online shopping less attractive; thus, the profit drops because the retailer must charge a lower price to attract consumers to purchase online.

4. Traditional retailing

We model the operations of the traditional retailer using a price-setting newsvendor with assortment decisions. Consistent with van Ryzin and Mahajan (1999), we assume that consumers do not make a second attempt if their first choice is out of stock. If a consumer makes a second attempt, it increases the profit of the traditional model. However, in the online model presented in Section 3, the consumer's demand for the first choice is always met. In other words, the second attempt does not exist in the online model. The absence of the second attempt will eliminate the difference in demand patterns between the online and the traditional models. As such, the profit difference between these two models is only affected by the nature of the distribution strategy. In the traditional retailing model, we use the same MNL framework as the one used in the online model to model consumer's choice process except that there is no disutility rt in the consumer utility here. The absence of the term rt in the consumer utility reflects that the consumers can pick up the product without further delay when they visit the retail store. Similarly, the store traffic is assumed to be a Poisson random variable with mean λ. Thus, the demand for variant i offered in the assortment D_i follows a Poisson distribution with mean λq_r, where

$$q_r = \frac{\exp(u-p)}{n\exp(u-p)+1}.$$

To facilitate the analysis, we approximate the Poisson random variable D_i as a normal random variable with mean λq_r and standard deviation $\sqrt{\lambda q_r}$. Denote the probability density function (pdf) and the cumulative distribution function (cdf) of the standard normal distribution with $\phi(\cdot)$ and $\Phi(\cdot)$, respectively. Define $(x)^+ = \max(0, x)$. The retailer's expected profit is

$$V_R = \max_{p,x_i,n}\left\{\sum_{i=1}^{n} pE\min(D_i, x_i) - cx_i - (h-s)(x_i - D_i)^+\right\},$$

where the first term inside the sigma operator is the expected sales revenue of variant i, the second term is the procurement cost of variant i and the third term is the expected holding cost of variant i net of salvage values. The retailer's decisions are of assortment, price and inventory levels. Without losing much of generality, we assume that the unsold inventory is salvaged at the procurement cost (i.e. $c = s$); thus, the newsvendor ratio equals $\frac{p-c}{p-c+h}$. Adopting the well-known newsvendor solution with normal demand distribution, the optimal inventory level for variant i is

$$x_i^* = x^* = \lambda q_r + z(p)\sqrt{\lambda q_r},$$

where $z(p) = \Phi^{-1}(\frac{p-c}{p-c+h})$ is the optimal safety stock factor.

The result clearly shows that the optimal level of inventory is influenced by the product demand. With any given n and p, the traditional retailer's attainable profit for any particular product variant in the assortment is

$$\begin{aligned} v(n,p) &= pE\min(D_i, x^*) - cx^* - (h-c)(x^*-D_i)^+ = (p-c)x^* - (p-c+h)E(x^*-D_i)^+ \\ &= (p-c)\left(\lambda q_r + z(p)\sqrt{\lambda q_r}\right) - (p-c+h)\sqrt{\lambda q_r}\left[\frac{(p-c)z(p)}{p-c+h} + \phi(z(p))\right]. \end{aligned} \tag{4}$$

When deriving the above equation, we use the identity that

$$E[(x-D)^+] = x - \lambda q_r + E[(D-x)^+] = x - \lambda q_r + \sqrt{\lambda q_r}L\left(\frac{x-\lambda q_r}{\sqrt{\lambda q_r}}\right),$$

where $L(z) = \int_z^\infty (t-z)f(z)dz = \phi(z) - z(1-\Phi(z))$ is the loss function of the standard normal distribution. Note that the pdf of the standard normal distribution is $\phi(z) = \frac{1}{\sqrt{2\pi}}\exp(-\frac{z^2}{2})$.

When n is fixed (which implies that q_r is fixed), the $v(n, p)$ function in Equation (4) is known to be quasi-concave in p. Formally,

Proposition 2 *For any given n, there exists a unique price $p^*(n)$ such that $v(n, p)$ is maximised, where*

$$p^*(n) = \arg_p\left\{\frac{\partial v(n,p)}{\partial p} = 0\right\}.$$

Proposition 2 establishes that the optimal price with any given assortment breadth can be found by solving the first-order condition. We are now ready to describe how to solve the price, assortment and inventory joint decisions in the traditional retail model. First of all, the optimal assortment includes n_r^* variants, where

$$n_r^* = \arg\max_n\left\{v(n, p^*(n))\right\}.$$

Next, the optimal price of the assortment is $p_r^* = p^*(n_r^*)$, and the optimal inventory level for each variant is $x^* = \lambda q_r^* + z(p_r^*)\sqrt{\lambda q_r^*}$, where

$$q_r^* = \frac{\exp(u-p_r^*)}{n_r^*\exp(u-p_r^*)+1}.$$

Essentially, we need to evaluate the N feasible assortments. For each assortment breadth n ($1 \le n \le N$), we solve the first-order condition associated with the $v(\cdot)$ function using Proposition 2. After that, we evaluate the resultant profit $v(n, p^*(n))$

Table 1. Retailer's optimal choice of assortment breadth and price $\lambda = 4$, $u = 5$, $h = 1.25$, $c = s = 1$, $k = 1$, and $r = 1$.

N	1	2	3	4	5
p_r^*	4.25	4.76	5.00	5.00	5.00
n_r^*	1	2	3	3	3
p_O^*	3.69	4.07	4.32	4.50	4.65
n_O^*	1	2	3	4	5

for each n. In the final step, we select the optimal assortment breadth n that provides the highest profit. We derive some monotone properties as follows.

COROLLARY 2 (1) *The traditional retailer's profit is increasing along with λ but is decreasing along with c and h.* (2) *The traditional retailer's profit asymptotically remains constant along with N and the optimal breadth of the assortment is weakly smaller than N.*

As discussed in van Ryzin and Mahajan (1999), when an extra variant is added to the assortment, the demand for each available variant becomes thinner. The coefficient of variation of D_i is $1/\sqrt{\lambda q_r}$, which is increasing in n and implies that the inventory cost of the entire assortment increases along with n. Hence, it is not always optimal to expand the assortment indefinitely. For any given cost parameters, there is a ceiling, denoted by $\bar{n}_r$, such that the traditional retailer's optimal profit V_R^* is increasing when the number of available variants N is less than $\bar{n}_r$ and remains the same when N exceeds $\bar{n}_r$. Table 1 illustrates how the optimal assortment breadth varies along with the number of available variants.

Because the online retailer offers a full assortment, the optimal assortment breadth in the online store is $n_O^* = N$ and the optimal price is increasing in N. On the other hand, the value of adding more variants has a limit. When there are more than 3 variants available, the traditional store only offers 3 variants and the optimal retail price is set at $p_r^* = u = 5$. We know that the profit function with a fixed price of $p = 5$ is quasi-concave in n because of the aforementioned 'thinning' effect. As such, the optimal number of variants carried by the traditional retailer is bounded and the optimal profit V_r^* is weakly increasing in N.

5. Channel comparison

5.1 *Profit comparison*

We first compare the profits of the traditional and the online channels. We are interested in studying how the logistics costs, consumer patience and holding costs could affect the performance of the two retailing models. These three factors only affect one of the channels but not both.

PROPOSITION 3 *The online retail model becomes more profitable than the traditional retail model when* (1) *customers become sufficiently patient (i.e. $r < r^b$);* (2) *the holding cost of the traditional model is sufficiently high (i.e. $h > h^b$); or* (3) *the delivery cost is sufficiently low (i.e. $k < k^b$). In addition, these thresholds exhibit pair-wise monotonic properties, which are summarised in the following Table 2.*

To illustrate the monotonic properties shown in Table 2, we construct Figure 1, in which the curves without any marker depict how the break-even threshold r^b changes along with another parameter such as h. We also consider another benchmark case where the online retail model incurs a small holding cost. In this case, the objective of the online retailer is modified to include inventory costs, similar to the traditional retailer. However, the inventory costs are lower in online stores due to lower costs of warehouses as opposed to store fronts.

Table 2. Monotonic properties of thresholds.

	r	h	g
r^b	N/A	Increasing	Decreasing
h^b	Increasing	N/A	Decreasing
g^b	Decreasing	Decreasing	N/A

We compute the corresponding thresholds and depict them using the curves with squared markers. Because the small holding cost reduces the profit of the online retail model, it shifts the curves of r^b downward. Similar observations are found with regard to the other two thresholds h^b and k^b. The other relevant curves are available through authors upon requests.

Next, we study how store traffic and assortment variety could affect the choice of the optimal distribution model. Note that these two factors affect both the channels.

Proposition 4 (1) *The online retail model becomes more profitable than the traditional retail model when the number of variants increases (i.e. N increases).* (2) *However, the increasing store traffic has an indefinite impact.*

We depict the profit functions V_O^* (represented by the curve without dots) and V_R^* (represented by the curve with dots) with respect to N and λ in Figure 1. The first chart shows that a break-even point N^b exists such that when $N > N^b$, the online retail model is more profitable than the traditional retail model. The second chart shows that the optimal transportation cost plays a key role in determining the optimal retailing strategy. When the optimal transportation cost is very low, the online retail model dominates. When the optimal delivery cost is very high, the traditional retail model dominates. When the optimal delivery cost is moderate, there exists a threshold λ^b such that the online retailing model is more profitable than the traditional retailing model if and only if the store traffic is $\lambda > \lambda^b$.

5.2 *Price comparison*

We compare the optimal price of the two retailing models.

Corollary 3 *When the number of available variants (N) increases, the optimal price of the online model (p_o^*) increases and that of the traditional model (p_r^*) increases initially and then levels off. Consequently, the price difference $p_o^* - p_r^*$ is weakly increasing along with N when N is sufficiently large.*

From Figure 2, we observe that when the store traffic λ increases, the optimal price of the online store remains constant with all else unchanged. This observation is consistent with Equation (2), which indicates that the optimal online price does

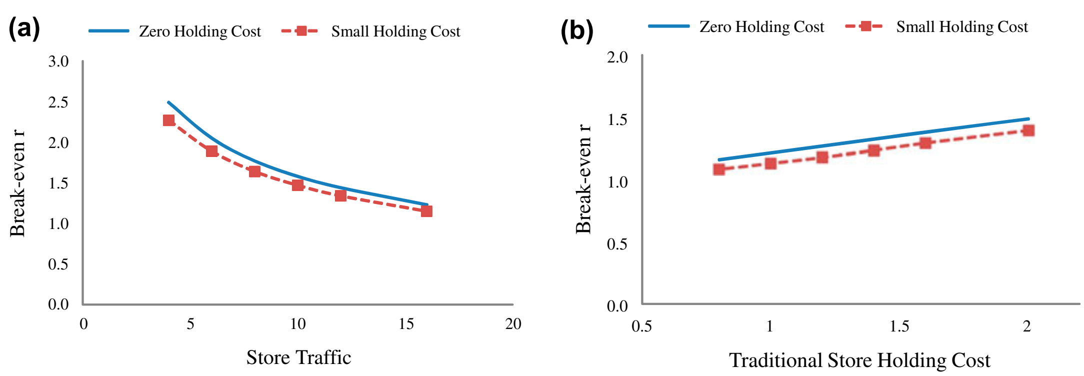

Figure 1. Sensitivity analysis on the thresholds. (a) When the consumers' patience (r) changes. (b) When the holding cost (h) for the traditional store changes.

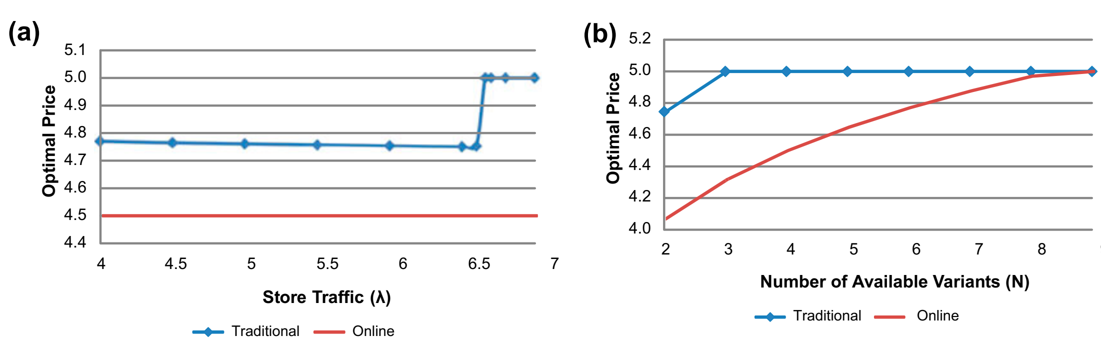

Figure 2. A comparison of optimal prices. (a) When store traffic (λ) changes. (b) When number of available variants (N) changes.

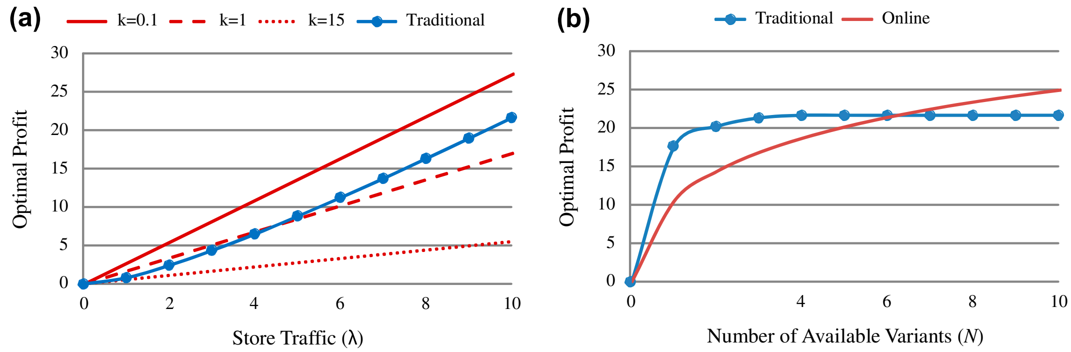

Figure 3. Sensitivity analysis on the number of available variants and store traffic. The curve with dots depicts the optimal profit of the traditional store and the curve without dots depicts the optimal profit of the online store. (a) When store traffic (λ) changes. (b) When number of available variants (N) changes.

not depend on the store traffic. The optimal price of the traditional store, however, exhibits some non-intuitive patterns because the objective function of the traditional store is ill-behaved. Generally, the optimal price of the traditional store is slowly decreasing in the store traffic and then jumps to a high level when the store traffic reaches a certain threshold. A theoretical proof of this result is intractable because of the difficulty in obtaining the closed-form solution of the optimal price and assortment breadth in the traditional model. Another observation that we can draw from Figure 2 is that the optimal prices of the online and traditional stores are increasing along with the number of variants available with all else unchanged.

6. Conclusion

In this study, we develop a framework to simultaneously investigate the retailers' price, assortment and delivery time decisions, and we make a holistic comparison between the online and traditional retailing strategies. The extant research has focused on the convenience of shopping and of the easy access to information (Zhang 2009), and some important and fundamental differences between the online and traditional channels are ignored. Our study fills this gap and provides insights about the fundamental differences between online and offline channels. First, online and traditional retailers have different assortment strategies. Our results show that the number of variants offered by an online retailer is weakly dominated by the number of variants offered by a traditional retailer. This finding is intuitive because the online retailers have a lower inventory cost. Second, online and traditional retailers have different pricing strategies. The offline price is influenced by store traffic, whereas the online price is not. The difference between online and offline prices is weakly increasing along with the number of available variants. Third, an online retailer must consider the speed to deliver the product to customers, whereas a traditional retailer must decide how much inventory to keep so that the costs of stockout and overstock are balanced. Going out of stock is a major concern for most traditional retailers because it causes losses (Che, Chen, and Chen 2012). Online purchasers are not going to receive the product right away and have to wait for the product to be delivered. Thus, online retailing is appealing only to patient consumers. This is a fundamental difference between online and offline retailing.

By comparing the results of the online and traditional retailing models, we find that if the product can be delivered quickly at a relatively low cost, the online channel is preferred, whereas if the delivery cost is high and consumers are impatient, the traditional channel is better. Our analysis adds insights to the current multichannel retailing literature (Ofek, Katona, and Sarvary 2011; Zhang 2009).

Our study also shows some other interesting results. First, inventory cost and demand volume are the two main factors that influence a traditional retailer's inventory decision. A traditional retailer needs to carefully balance the costs of holding surplus inventory and of having a stockout. Second, consumer patience and delivery cost are the two primary factors that influence an online retailer's pricing strategies. There is a negative relationship between delivery cost and time. The faster the delivery, the higher the logistics costs. Thus, if a consumer is very patient, it is not necessary to have a very quick delivery.

Our research has limitations. First, we only consider a monopoly model. We made this choice partially because of the technical difficulty: it is difficult to simultaneously optimise three decision variables in a duopoly model. Second, future empirical research that tests the results of our study might be able to provide more insights that are beyond the scope of this paper. Third, we assume a uniform delivery time for different items in the assortment. We consider standard assortments where items differ in some physical attributes that do not require high level of customisation. For these items, uniform delivery time is a common practice and by-law in countries like Australia. If we consider highly customised goods such as sail boats, it is then appropriate to consider feature-dependent delivery time. But this is not the scenario that we intend to study and we leave it for the future research.

References

Alptekinoglu, A., A. Grasas, and E. Akcali. 2009. "Is Assortment Selection a Popularity Contest? A Study of Assortment, Return Policy and Pricing Decisions of a Retailer." In *Consumer-Driven Demand and Operations Management Models*, edited by C. S. Tang and S. Netessine, 205–228. New York: Springer.

Aydin, G., and E. L. Porteus. 2008. "Joint Inventory and Pricing Decisions for an Assortment." *Operations Research* 56 (5): 1247–1255.

Benson, B. L. 1990. "Increasing Product Variety and Rising Prices." *Review of Industrial Organization* 5 (1): 31–51.

Bhargava, H. K., D. Sun, and S. H. Xu. 2006. "Stockout Compensation: Joint Inventory and Price Optimization in Electronic Retailing." *INFORMS Journal on Computing* 18 (2): 255–266.

Cachon, G. P., C. Terwiesch, and Y. Xu. 2005. "Retail Assortment Planning in the Presence of Consumer Search." *Manufacturing and Service Operations Management* 7 (4): 330–346.

Chaturvedi, N., M. Martich, B. Ruwadi, and N. Ulker. 2013. "The Future of Retail Grocery Online." McKinsey & Company Report. http://www.mckinsey.com/client_service/retail/latest_thinking

Che, H., X. Chen, and Y. Chen. 2012. "Investigating Effects of Out-of-stock on Consumer SKU Choice." *Journal of Marketing Research* 49 (4): 502–513.

Chen, Y., and G. Iyer. 2002. "Consumer Addressability and Customized Pricing." *Marketing Science* 21 (2): 197–208.

Chen, X., and D. Simchi-Levi. 2012. "Inventory Price Joint Optimization." In *The Oxford Handbook of Pricing Management*, edited by Ö. Özer and R. Phillips, 784–824. Oxford: Oxford Press.

Chiang, K., and G. E. Monahan. 2005. "Managing Inventories in a Two-echelon Dual-channel Supply Chain." *European Journal of Operational Research* 162: 325–341.

Corrocher, N., and M. Guerzoni. 2008. "Product Variety and Price Strategy in the Ski Manufacturing Industry." *Journal of Evolutionary Economics* 19 (4): 471–486.

Deleersnyder, B., I. Geyskens, K. Gielens, and M. G. Dekimpe. 2002. "How Cannibalistic is the Internet Channel? A Study of the Newspaper Industry in the United Kingdom and The Netherlands." *International Journal of Research in Marketing* 19 (2): 337–348.

Ho, T., and Y. Zheng. 2004. "Setting Customer Expectation in Service Delivery: An Integrated Marketing-operations Perspective." *Management Science* 50 (4): 479–488.

Iyer, G. 1998. "Coordinating Channels Under Price and Nonprice Competition." *Marketing Science* 17 (3): 338–355.

Iyer, G., and D. Soberman. 2000. "Markets for Product Modification Information." *Marketing Science* 19 (3): 203–225.

Kocabıyıkoğlu, A., and I. Popescu. 2011. "An Elasticity Approach to the Newsvendor with Price-sensitive Demand." *Operations Research* 59 (2): 301–312.

Kogan, K. 1997. "Discrete Event Control of Production Flows: Make-to-stock and Make-to-order Environments." *International Journal of Production Research* 35 (6): 1729–1742.

Kogan, K., E. Khmelnitsky, and O. Maimon. 1998. "Balancing Facilities in Aggregate Production Planning: Make-to-order and Make-to-stock Environments." *International Journal of Production Research* 36 (9): 2585–2596.

Kök, A. G., M. L. Fisher, and R. Vaidyanathan. 2009. "Assortment Planning: Review of literature and industry practice." In *Retail Supply Chain Management*, edited by N. Agrawal and S. Smiths, 99–153. New York: Springer.

Kumar, S. 2005. "Supply Chain Strategies in the Apparel Industry: The Case of Victoria's Secret." Thesis (M. Eng. in Logistics), Massachusetts Institute of Technology.

Kumar, V., M. Prakash, F. Tiwari, and T. S. Chan. 2006. "Stochastic Make-to-stock Inventory Deployment Problem: An Endosymbiotic Psychoclonal Algorithm Based Approach." *International Journal of Production Research* 44 (11): 2245–2263.

Lal, R., and M. Sarvary. 1999. "When and How is the Internet Likely to Decrease Price Competition." *Marketing Science* 18 (4): 485–503.

Li, Z. 2007. "A Single-period Assortment Optimization Model." *Production and Operations Management* 16 (3): 369–380.

Liu, Y., S. Gupta, and Z. Zhang. 2006. "Note on Self-restraint as an Online Entry-deterrence Strategy." *Management Science* 52 (11): 1799–1809.

Liu, L., M. Parlar, and S. X. Zhu. 2007. "Pricing and Lead Time Decisions in Decentralaized Supply Chains." *Management Science* 53 (5): 713–725.

Maddah, B., and E. K. Bish. 2007. "Joint Pricing, Assortment, and Inventory Decisions for a Retailer's Product Line." *Naval Research Logistics* 54 (3): 315–330.

Maddah, B., E. K. Bish, and H. Tarhini. 2014. "Newsvendor Pricing and Assortment under Poisson Decomposition." *IIE Transactions* 46 (6): 567–584.

Netessine, S., and N. Rudi. 2006. "Supply Chain Choice on the Internet." *Management Science* 52 (6): 844–864.

Ofek, E., K. Katona, and M. Sarvary. 2011. "'Bricks and Clicks': The Impact of Product Returns on the Strategies of Multi-channel Retailers." *Marketing Science* 30 (1): 42–60.

Pan, X., B. Ratchford, and V. Shankar. 2004. "Price Dispersion on the Internet: A Review and Directions for Future Research." *Journal of Interactive Marketing* 19 (4): 116–135.

Petruzzi, N. C., and M. Dada. 1999. "Pricing and the Newsvendor Problem: A Review with Extensions." *Operations Research* 47 (2): 183–194.

Porteus, E. L., and S. J. Whang. 1991. "On Manufacturing Marketing Incentives." *Management Science* 37 (9): 1166–1181.

Shaffer, G., and F. Zettelmeyer. 2004. "Advertising in a Distribution Channel." *Marketing Science* 23 (4): 619–628.

Shang, W., and L. Liu. 2011. "Promised Delivery Time and Capacity Games in Time-based Competition." *Management Science* 57 (3): 599–610.

Sun, D. 2008. "Dual Mechanism for an Online Retailer." *European Journal of Operational Research* 187 (3): 903–921.

Sun, D., J. Ryan, and H. Shin. 2008. "Why Do We Observe Stockless Operations on the Internet?" *Production and Operations Management* 17 (2): 139–149.

van Ryzin, G., and S. Mahajan. 1999. "On the Relationship Between Inventory Costs and Variety Benefits in Retail Assortment." *Management Science* 45 (11): 1496–1515.

Yang, Y., S. Essegaier, and D. R. Bell. 2005. "Free Shipping and Repeat Buying on the Internet: Theory and Evidence." *UC Davis Working Paper*.

Zettelmeyer, F. 2000. "Expanding to the Internet: Pricing and Communications Strategies When Firms Compete on Multiple Channels." *Journal of Marketing Research* 37 (3): 292–308.

Zhang, X. 2005. "Pricing in Online Book Retailing : The Role of Assortment and Consumer Perception of Product Availability." PhD thesis, The University of Toronto.

Zhang, X. 2009. "Retailers' Multichannel and Price Advertising Strategies." *Marketing Science* 29 (6): 1080–1094.

Appendix 1. Proofs

Proof of Proposition 1

We prove the first part of the proposition using the contradiction method. Suppose that $n^* < N$ and the optimal profit is V_O^*. We add one more variant into the assortment with the same price and delivery time. The profit of the new solution is denoted by V_O'. It can be shown that

$$\frac{V_O'}{V_O^*} = \frac{\frac{(n+1)\exp(u-p-\gamma t)}{(n+1)\exp(u-p-\gamma t)+1}}{\frac{n\exp(u-p-\gamma t)}{n\exp(u-p-\gamma t)+1}} = \frac{\exp(u-p-\gamma t)+1/n}{\exp(u-p-\gamma t)+1/(n+1)} > 1,$$

which contradicts the optimality of offering strictly less than N variants. Hence, $n^* = N$.

Using this result, we can rewrite V_O as follows:

$$V_O = \max_{p,t} \frac{N\lambda(p-c-g(t))\exp(u-p-rt)}{N\exp(u-p-rt)+1}.$$

The first-order conditions are:

$$\frac{\partial V_O}{\partial p} = 1 + N\exp(u-p-rt) - (p-c-g(t)) = 0,$$

and

$$\begin{aligned}
\frac{\partial V_O}{\partial t} &= [-g'(t)\exp(u-p-rt) - r(p-c-g)\exp(u-p-rt)](1+N\exp(u-p-rt)) \\
&\quad - [-rN\exp(u-p-rt)](p-c-g(t))\exp(u-p-rt) \\
&= \exp(u-p-rt)\left\{\begin{array}{l} -g'(t)(1+N\exp(u-p-rt)) - r(p-c-g(t)) \\ -rN(p-c-g(t))\exp(u-p-rt) \\ +rN(p-c-g(t))\exp(u-p-rt) \end{array}\right\} \\
&= \exp(u-p-rt)\left\{-g'(t)(1+N\exp(u-p-rt)) - r(p-c-g(t))\right\} = 0.
\end{aligned}$$

Because $\exp(u-p-rt) > 0$, we observe that the second equation yields

$$-g'(t)(1+N\exp(u-p-rt)) - r(p-c-g(t)) = 0.$$

Solving these two equations provides the stated results.

Proof of Corollary 1

Part (1) Equation (2) indicates that the optimal price and delivery time are determined by

$$(p_o^* - c - g(t^*)) - N\exp(u-p_o^*-rt^*) - 1 = 0.$$

Define $F(p,t) = (p-c-g(t)) - N\exp(u-p-rt) - 1$. The implicit function theorem implies that

$$\frac{\partial p_o^*}{\partial N} = -\frac{\partial F/\partial N}{\partial F/\partial p_o^*} = \frac{\exp(u-p_o^*-rt^*)}{1+N\exp(u-p_o^*-rt^*)} > 0.$$

In other words, the optimal price p^* is increasing along with N. Similarly, we find

$$\frac{\partial p^*}{\partial r} = -\frac{\partial F/\partial r}{\partial F/\partial p_o^*} = -\frac{Nt\exp(u-p_o^*-rt^*)}{1+N\exp(u-p_o^*-rt^*)} < 0.$$

From (2), we observe that the optimal price is not a function of λ, thus, $\frac{\partial p_o^*}{\partial r} = 0$.

Part (2) According to the envelope theorem, it holds that

$$\frac{dV_O^*}{dc} = \frac{dV_O}{dc}|_{p_o^*, t^*}.$$

The V_O function, (before it is optimised), is decreasing along with c. Hence, $\frac{dV_O^*}{dc} < 0$. Similarly, we find that $\frac{dV_O^*}{dr} < 0$. Using Equation (3), we find that

$$\frac{\partial V_O^*}{\partial \lambda} = N \exp(u - p_o^* - rt^*) > 0.$$

Because p_o^* is a function of N, we obtain

$$\begin{aligned}\frac{\partial V_O^*}{\partial N} &= \lambda \exp(u - p_o^* - rt^*) - N\lambda \exp(u - p_o^* - rt^*) \frac{\partial p_o^*}{\partial N} \\ &= \lambda \exp(u - p_o^* - rt^*) - N\lambda \exp(u - p_o^* - rt^*) \frac{\exp(u - p_o^* - rt^*)}{1 + N \exp(u - p_o^* - rt^*)} \\ &= \frac{\lambda \exp(u - p_o^* - rt^*)}{1 + N \exp(u - p_o^* - rt^*)} > 0.\end{aligned}$$

Proof of Proposition 2

We prove Proposition 2 using Corollary 2 of Kocabıyıkoğlu and Popescu (2011). In the OM/OR literature, the fundamental question 'under what conditions is the inventory-price newsvendor function quasi-concave or concave?' has been puzzling researchers since the 1990s. Recently, Kocabıyıkoğlu and Popescu (2011) provided an answer to this question. These authors proposed the Lost Sales Rate (LSR) elasticity, which is given by

$$\varepsilon(p, x) = \frac{pF_p(p, x)}{1 - F(p, x)},$$

where $F(p, x) = \Pr(D(p) \leq x)$ is the CDF of the price-dependent demand and $F_p(p, x)$ is the partial derivative with respect to price p. Kocabıyıkoğlu and Popescu (2011) proved that if the LSR elasticity exceeds 0.5 (globally, respectively, path-wise), then the

$$v(p, x) = pE \min(D(p), x) - cx - h(x - D(p))^+ \tag{A1}$$

function is jointly concave, and if the LSR elasticity is increasing along with inventory or price, then $v(n, p)$ is jointly quasi-concave.

In our model, the price-dependent demand is

$$D(p) = \frac{\lambda \exp(u - p)}{n \exp(u - p) + 1} + Z\sqrt{\frac{\lambda \exp(u - p)}{n \exp(u - p) + 1}} = \lambda q_j(n, p) + Z\sqrt{\lambda q_j(n, p)},$$

where the first term is the deterministic (price-dependent) component of the demand and the second term is the noise, where Z follows the standard normal distribution, which has an increasing failure rate function.

We apply Corollary 2 of Kocabıyıkoğlu and Popescu (2011) to prove that the newsvendor profit function $v(p, x)$ defined in Equation (A1) is jointly quasi-concave for any given $n \geq 1$. Towards this end, we need to verify that both $\frac{p\partial \lambda q_j(n,p)}{\partial p}$ and $\frac{p\partial\sqrt{\lambda q_j(n,p)}}{\partial p}$ are decreasing along with p. Recall that $\lambda > 0$ is a constant. If the term $\frac{p\partial q_j(n,p)}{\partial p}$ is decreasing along with p, then the term $\frac{p\partial\sqrt{q_j(n,p)}}{\partial p}$ is also decreasing along with p. With some algebra, we find that

$$p\frac{\partial}{\partial p}\left(\frac{\lambda \exp(u - p)}{n \exp(u - p) + 1}\right) = \frac{-\lambda p e^{u-p}}{2ne^{u-p} + n^2 e^{2u-2p} + 1}$$

and

$$\frac{\partial}{\partial p}\left(\frac{-\lambda p e^{u-p}}{2ne^{u-p} + n^2 e^{2u-2p} + 1}\right) = \frac{-\lambda e^{u-p}\left[1 + 2ne^{u-p} + n^2 e^{2u-2p} + p\left(n^2 e^{2u-2p} - 1\right)\right]}{4ne^{u-p} + 6n^2 e^{2u-2p} + n^4 e^{4u-4p} + 4n^3 e^{u-p} e^{2u-2p} + 1}.$$

Because $n^2 e^{2u-2p} \geq 1$ for any $p \leq u$ and $n \geq 1$, we observe that the denominator is positive and the numerator is negative. In other words, $\frac{p\partial q_j(n,p)}{\partial p}$ is decreasing in p for any given $n \geq 1$. Recall that the random noise D has an increasing failure rate. Our demand model satisfies all of the conditions stated in Corollary 2 of Kocabıyıkoğlu and Popescu (2011). Hence, we conclude that $\pi_i(p, x)$ is jointly quasi-concave.

The final step is to verify whether $pd(p)$ is concave in p, which is a prerequisite of Kocabıyıkoğlu and Popescu (2011). We thank the reviewer for pointing out this point. Note that $pd(p) = \frac{p\lambda \exp(u-p)}{1+n\exp(u-p)} + Zp\sqrt{\frac{\lambda \exp(u-p)}{1+n\exp(u-p)}}$. The second derivative with respect to p is

$$\frac{d^2}{dp^2}\left(\frac{p\lambda \exp(u - p)}{1 + n \exp(u - p)}\right) = \frac{\lambda\left(pe^{u-p} - 2e^{u-p} - 2ne^{2u-2p} - npe^{2u-2p}\right)}{3ne^{u-p} + 3n^2 e^{2u-2p} + n^3 e^{3u-3p} + 1}.$$

Because $n \geq 1$ and $u - p > 0$, the numerator is negative (i.e. $pe^{u-p} - npe^{2u-2p} < 0$ and $-2e^{u-p} - 2ne^{2u-2p} < 0$).

The second term also has a negative second derivative.

$$\frac{d^2}{dp^2}\left(Zp\sqrt{\frac{\lambda \exp(u-p)}{1+n\exp(u-p)}}\right)$$

$$= \frac{Z}{4\lambda e^{u-p} + 8n\lambda e^{2u-2p} + 4n^2\lambda e^{3u-3p}} \begin{pmatrix} 2p\left(\lambda \frac{e^{u-p}}{ne^{u-p}+1}\right)^{\frac{3}{2}} - 4\left(\lambda \frac{e^{u-p}}{ne^{u-p}+1}\right)^{\frac{3}{2}} \\ -8ne^{u-p}\left(\lambda \frac{e^{u-p}}{ne^{u-p}+1}\right)^{\frac{3}{2}} - p\lambda e^{u-p}\sqrt{\lambda \frac{e^{u-p}}{ne^{u-p}+1}} \\ -4n^2e^{2u-2p}\left(\lambda \frac{e^{u-p}}{ne^{u-p}+1}\right)^{\frac{3}{2}} - 2n^2pe^{2u-2p}\left(\lambda \frac{e^{u-p}}{ne^{u-p}+1}\right)^{\frac{3}{2}} \end{pmatrix}.$$

Because $2p\left(\lambda \frac{e^{u-p}}{ne^{u-p}+1}\right)^{\frac{3}{2}} - 2n^2pe^{2u-2p}\left(\lambda \frac{e^{u-p}}{ne^{u-p}+1}\right)^{\frac{3}{2}}$ is negative for any $p \leq u$, the second term is also concave.

Proof of Corollary 2

Part (1) According to the envelope theorem, it holds that

$$\frac{dV_R^*}{dc} = \frac{dV_R}{dc}|_{p_r^*}.$$

The traditional retailer's profit V_R (before it is optimised) is decreasing along with c. Hence, $\frac{dV_R^*}{dc} < 0$. Similarly, we find that $\frac{dV_R^*}{d\lambda} < 0$ and $\frac{dV_R^*}{dh} < 0$.

Part (2) Suppose that the price p and the assortment breadth n are given. van Ryzin and Mahajan (1999) proved that V_R is quasi-concave in n. This finding implies that a ceiling exists, denoted by $\bar{n}_r$, such that the traditional retailer's profit V_R is increasing along with n and decreasing along with n when it exceeds $\bar{n}_r$.

Note that this ceiling can be found by setting p equal to consumer's willingness to pay (u) because the optimal width of assortment is non-decreasing in selling price p. The traditional retailer's optimal assortment breadth is the smallest of N and $\bar{n}_r$. Hence, the traditional retailer's profit asymptotically remains constant along with N and the optimal breadth of assortment is weakly smaller than N.

Proof of Proposition 3

The first part of Proposition 3 is a direct consequence of Corollaries 1 and 2. These results are consistent with many empirical observations that firms sell highly customised handbags, shoes and T-shirts through a virtual store. The second part of the Proposition 3 is related to the monotonic properties of various thresholds. To save the space, we only provide the proof of one representative case because all the other cases can be proven using the implicit function theorem. By definition, when $r = r^b$, $V_O^* = V_r^*$ with all parameters being fixed. The threshold r^b is implicitly determined by $G = V_O^* - V_r^* = 0$. Suppose that we change the value of h (we change one parameter at the time). The threshold r^b changes. Using The implicit function theorem, we find that

$$\frac{\partial r^b}{\partial h} = -\frac{\frac{\partial G}{\partial h}}{\frac{\partial G}{\partial r}} = \frac{\frac{\partial V_r^*}{\partial h}}{\frac{\partial V_O^*}{\partial r}} > 0.$$

Proof of Proposition 4

First, define $\Delta = V_O^* - V_R^*$ as the difference in the optimal profit between the online store and the traditional store. It holds that

$$\frac{\partial \Delta}{\partial N} = \frac{\partial V_O^*}{\partial N} - \frac{\partial V_R^*}{\partial N}.$$

The proof of Corollary 1 indicates that $\frac{\partial V_O^*}{\partial N} > 0$, whereas Corollary 2 indicates that the optimal assortment breadth of the traditional store is weakly less than N. This finding implies that $\frac{\partial V_R^*}{\partial N} = 0$ when N is sufficiently large. We conclude that $\frac{\partial \Delta}{\partial N} > 0$ when N is sufficiently large. In other words, the optimal profit of the online store is asymptotically larger than that of the traditional store.

Next, we investigate the sign of

$$\frac{\partial \Delta}{\partial \lambda} = \frac{\partial V_O^*}{\partial \lambda} - \frac{\partial V_R^*}{\partial \lambda}.$$

The proof of Corollary 1 indicates that $\frac{\partial V_O^*}{\partial \lambda} = N\exp(u - p^* - rt^*) > 0$. In other words, V_O^* is linearly increasing in λ. Using the envelope theorem, we find that

$$\frac{\partial V_R^*}{\partial \lambda} = n_r^*\left[\left(p^* - c\right)q^* - \frac{p^* + h}{2}\sqrt{\frac{q^*}{\lambda}}\phi(z)\right],$$

where the second term goes to zero when $\lambda \to \infty$. This indicates that the optimal profit of the traditional store is increasing linearly and asymptotically along with λ. We cannot determine the sign of $\frac{\partial \Delta}{\partial \lambda}$. Figure 3 shows that there could be three cases. When the optimal transportation cost is very low, the online retailing model dominates. When the optimal transportation cost is very high, the traditional retailing model dominates. When the optimal transportation cost is moderate, there exists a threshold λ^b such that the online retailing model is more profitable than the traditional retailing model if and only if the store traffic is $\lambda > \lambda^b$.

Proof of Corollary 3

Corollary 1 states that $\frac{\partial p_o^*}{\partial N} > 0$ and Corollary 2 states that $\frac{\partial p_r^*}{\partial N} > 0$ for $N \leq \bar{n}_r$ and then $\frac{\partial p_r^*}{\partial N} = 0$ for $N > \bar{n}_r$ (because the traditional store does not add more variants and the optimal price p_r^* remains unchanged). We observe that for a sufficiently large N, it holds that $\frac{\partial p_o^*}{\partial N} - \frac{\partial p_r^*}{\partial N} \geq 0$. Therefore, the price difference $p_o^* - p_r^*$ is asymptotically and weakly increasing along with N. However, for $N \leq \bar{n}_r$, the sign of $p_o^* - p_r^*$ is indefinite.