

CS162 Discussion Session 3: λ Calculus

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Review

$$FV(x) = x$$

$$FV(\lambda x \rightarrow e) = \text{vars}(e) - x$$

$$FV(e_1 \ e_2) = FV(e_1) + FV(e_2)$$

$$FV(x \ y) = \{x, y\}$$

$$FV(\lambda y \rightarrow x \ y) = \{x\}$$

$$FV((\lambda x \rightarrow \lambda y \rightarrow y) \ x) = \{x\}$$

Figure 1: Free variables

$$\lambda x \rightarrow e =_{\alpha} \lambda y \rightarrow e[x := y]$$

- Rename a formal parameter and replace all its occurrences in the body
- $\lambda x \rightarrow e$ **α -equivalent** to $\lambda y \rightarrow e[x := y]$

Figure 2: α renaming

Rules of α renaming: (from Wikipedia - Lambda Calculus)

- when α renaming an abstraction, the **only** variable occurrences that are renamed are those that are **bound** to the same abstraction.
- α renaming is **not** possible if it would result in a variable getting captured by a different abstraction.

$$\beta\text{-Reduction: } (\lambda x \rightarrow e_1) e_2 =_{\beta} e_1[x := e_2]$$

where $e_1[x := e_2]$ means “ e_1 with all **free occurrences** of x replaced with e_2 ”
In other words, If you see an *abstraction* applied to an *argument*, take the *body* of the abstraction and replace all free occurrences of the *formal* by that *argument*

Figure 3: β reduction

Quiz

Quiz - λ Calculus

① Find all the free variables in the following expressions:

- $(\lambda x \rightarrow x y) (\lambda x \rightarrow y x)$
- $(\lambda x \rightarrow x z) (\lambda w \rightarrow w y z x)$

② True or False:

- $\lambda r \rightarrow r x = \alpha \lambda x \rightarrow x x$
- $\lambda x \rightarrow x x = \alpha \lambda x \rightarrow x y$
- $\lambda x \rightarrow x y = \alpha \lambda x \rightarrow x z$
- $\lambda x \rightarrow y z = \alpha \lambda y \rightarrow y z$
- $(\lambda x \rightarrow (\lambda x \rightarrow x) x) = \alpha (\lambda x \rightarrow (\lambda y \rightarrow y) x)$
 $= \alpha (\lambda z \rightarrow (\lambda y \rightarrow y) x)$

③ Apply β -reduction (and α -renaming if needed) to the following expressions as much as possible:

- $(\lambda y \rightarrow y) (\lambda z \rightarrow z x)$
- $(\lambda x \rightarrow (x y)) (\lambda z \rightarrow z)$
- $(\lambda y \rightarrow y (\lambda x \rightarrow x)) (\lambda x \rightarrow x)$

① Find all the free variables in the following expressions:

- $(\lambda x \rightarrow x \text{ y}) (\lambda x \rightarrow \text{y x})$
- $(\lambda x \rightarrow x \text{ z}) (\lambda w \rightarrow w \text{ y z x})$

② The α -renaming of the following expressions is correct or not:

- $\lambda y \rightarrow y x = \alpha \lambda x \rightarrow x x$ ✗
- $\lambda x \rightarrow x x = \alpha \lambda x \rightarrow x y$ ✗
- $\lambda x \rightarrow x y = \alpha \lambda x \rightarrow x z$ ✗
- $\lambda x \rightarrow y z = \alpha \lambda y \rightarrow y z$ ✗
- $(\lambda x \rightarrow (\lambda x \rightarrow x) x) = \alpha (\lambda x \rightarrow (\lambda y \rightarrow y) x)$ ✓
 $= \alpha (\lambda z \rightarrow (\lambda y \rightarrow y) x)$ ✓

③ Apply β -reduction (and α -renaming if needed) to the following expressions as much as possible:

- $(\lambda y \rightarrow y) (\lambda z \rightarrow z x)$ ✗
- $(\lambda x \rightarrow (x y)) (\lambda z \rightarrow z)$ y
- $(\lambda y \rightarrow y (\lambda x \rightarrow x)) (\lambda x \rightarrow x)$ identity function

Q & A