

CS 162 Programming languages

Lecture 7: Higher-order Functions

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max function

```
let max x y = if x < y then y else x;;

(* return max element of list l *)
let list_max l =
  let rec l_max l =
    match l with
    [] -> 0
    | h::t -> max h (l_max t)
  in
    l_max l;;
```

A better max function

```
let max x y = if x < y then y else x;;

(* return max element of list l *)
let list_max2 l =
  let rec helper cur l =
    match l with
    [] -> cur
    | h::t -> helper (max cur h) t
  in
    helper 0 l;;
```

Tail recursion

concat function

```
(* concatenate all strings in a list *)  
let concat l =  
    let rec helper cur l =  
        match l with  
        [] -> cur  
        | h::t -> helper (cur ^ h) t  
    in  
    helper "" l;;
```

What is the pattern?

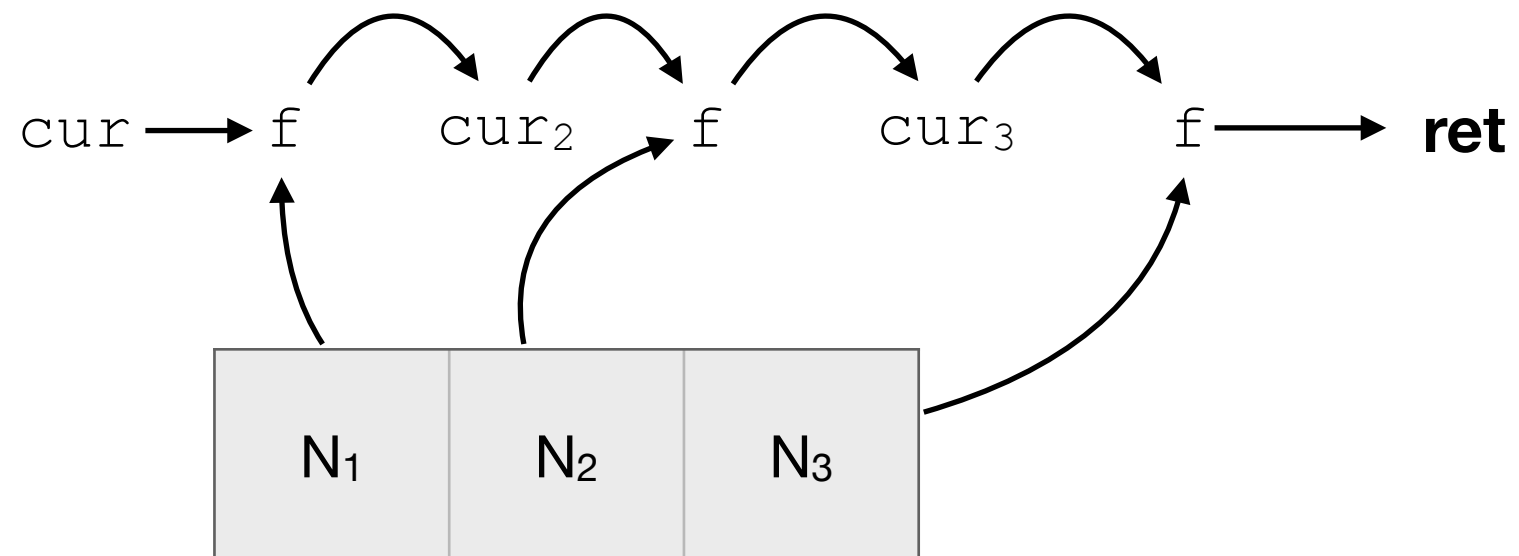
```
(* return max element of list l *)
let list_max2 l =
  let rec helper cur l =
    match l with
    [] -> cur
    | h::t -> helper (max cur h) t
  in
  helper 0 l;;
```

The two functions are sharing the same template!

```
(* concatenate all strings in a list *)
let concat l =
  let rec helper cur l =
    match l with
    [] -> cur
    | h::t -> helper (cur ^ h) t
  in
  helper "" l;;
```

fold

```
(* fold, the coolest function! *)  
let rec fold f cur l =  
  match l with  
  [] -> cur  
  | h::t -> fold f (f cur h) t;;
```



fold: examples

```
let list_max = fold max 0;;
```

```
let concat = fold (^) "";;
```

```
let multiplier = fold (*) 1;;
```

map

```
# (* return the list containing f(e)
   for each element e of l *)
let rec map f l =
  match l with
  [] -> []
  | h::t -> (f h) :: (map f t) ;;
```

```
let incr x = x+1;;

let map_incr = map incr;;

map_incr [1;2;3] ;;
```


Composing functions

$$(f \circ g)(x) = f(g(x))$$

```
# (* return a function that given an argument x
  applies f2 to x and then applies f1 to the result *)
let compose f1 f2 = fun x -> (f1 (f2 x));;

(* another way of writing it *)
let compose f1 f2 x = f1 (f2 x);;
```

Higher-order functions

```
let map_incr_2 = compose map_incr map_incr;;  
map_incr_2 [1;2;3];;  
  
let map_incr_3 = compose map_incr map_incr_2;;  
map_incr_3 [1;2;3];;  
  
let map_incr_3_pos = compose pos_filer map_incr_3;;
```

**Instead of manipulating lists, we are
manipulating the list manipulators!**

Benefits of higher-order functions

Identify common computation patterns

- Iterate a function over a set, list, tree ...
- Accumulate some value over a collection

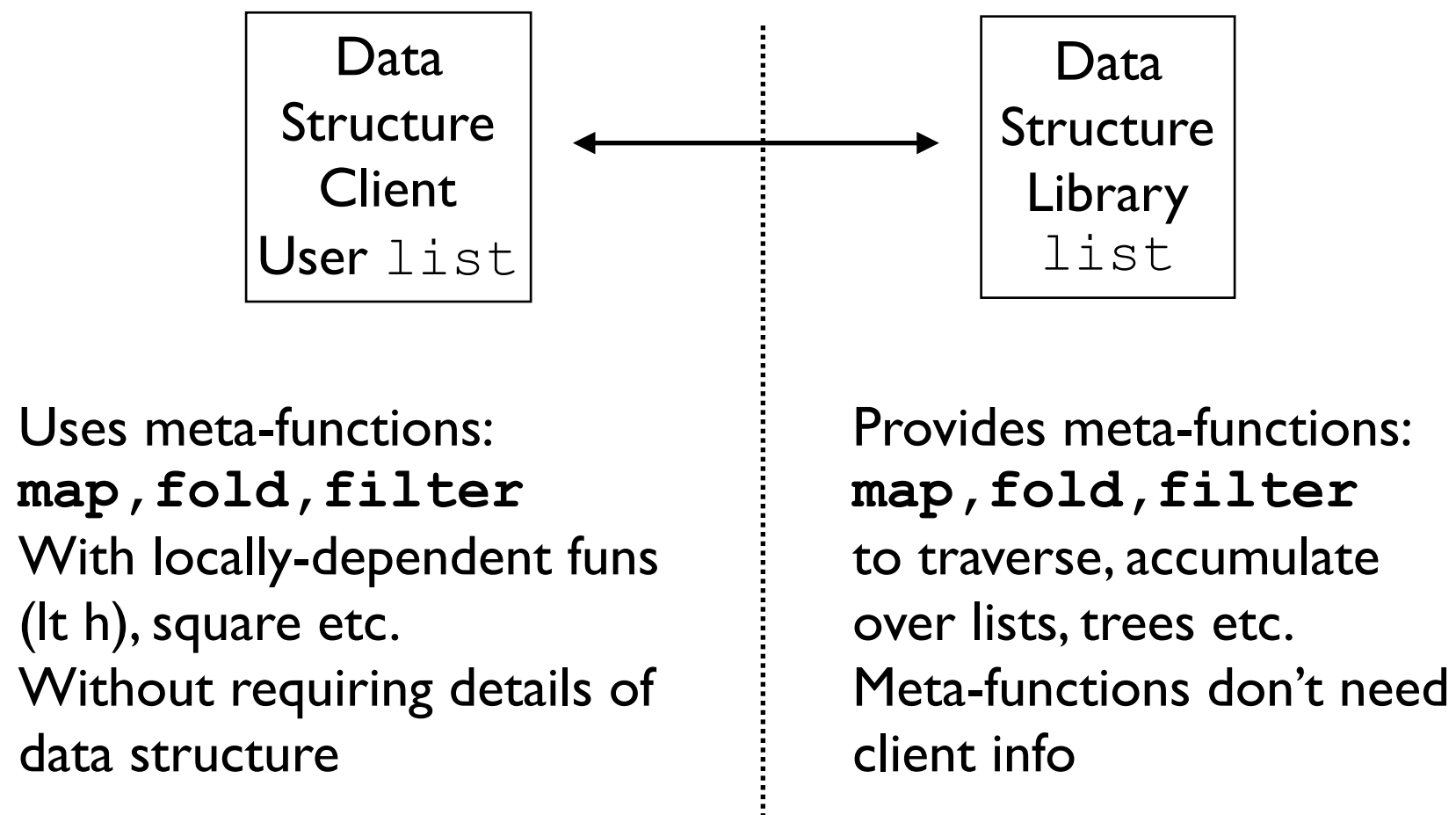
Pull out (factor) “common” code:

- Computation Patterns
- Re-use in many different situations

Functions as arguments/return

Higher-order functions enable modular code

- Each part only needs local information



Functions as arguments/return



“Free your mind”
– Morpheus

Different way of thinking about computation

- Manipulate the manipulators