1 Quantum field theory

1.1. Explain why this gives states with the correct symmetries for bosons and fermions, as described in §23.8.

1.2. Make sense of all this (and verify this commutation law for creation and annihilation of particles of a given type) by referring to the index notation of §23.8 or the diagrammatic notation of Fig. 12.17, or both, using expressions like $\bar{\psi}_\alpha \psi^{[\alpha} \phi^\beta \cdots \chi^{\kappa]}$. Sort out all the factorial factors which preserve normalization of the state, both in the fermion and the boson case.

1.3. Explain this Clifford-algebra structure, spelling out the role of the scalar product more explicitly. (Take the defining laws for a Clifford algebra in the form $\gamma_p \gamma_q + \gamma_q \gamma_p = -2g_{pq}\mathbf{I}$.) *Hint*: g_{pq} need not be diagonal.

1.4. Explain what addition, and multiplication by a scalar constant, mean in this space.

1.5. Show this. (Don't worry about subtleties like 'fall-off conditions'.)

1.6. Explain why we can remove a specific state in this way, despite my earlier qualifications about what an annihilation operator actually does. (*Hint*: See Exercise [26.2].)

1.7. By referring back to Exercise [26.2] and Fig. 12.18, exhibit this algebraic difference in the abstract-index or diagrammatic notation.

1.8. Give a 'physical interpretation' of the history of Fig. 26.3b, in terms of particle creation and annihilation.

1.9. Try to make these statements more precise by referring to first-order changes in the path, using 'O' symbols (as in §14.5), and relating this to the discussion given in §20.1, concerning the meaning of 'stationary action'. (Assume that S is large in units of $\hbar$.)

1.10. Why?

1.11. Explain how this singularity arises, by first rewriting $(\not{P} - M + i\varepsilon)^{-1}$ as a quotient for which the denominator is $P_\alpha P^\alpha - M^2 - \varepsilon^2$.

1.12. Why not? Explain how 4-momentum conservation at each vertex determines the 4-momentum of the virtual photon. *Hint*: All electrons have the same mass!

1.13. What is this freedom?

1.14. Can you see why this should be?

1.15. Can you see why?