0.1. Following the same line of argument that led to Eq. (5.28), show that the error on the integral evaluated using Simpson's rule is given, to leading order n, by Eq. (5.29).

Let us define two step sizes, $h_1 = (b-a)/N$ and $h_2 = (b-a)/2N = h_1/2$. Integrating using Simpson's rule gives us an approximation error of order h^4, so we may write

$$I = I_1 + ch_1^4$$

where I is the true value of the integral, I_1 is the numerical value obtained using Simpson's rule, and ch_1^4 is the error (c is an unknown constant).

We may do the same with the smaller step size, writing $I = I_2 + ch_2^4$. Equating these two gives us

$$I_1 + ch_1^4 = I_2 + ch_2^4.$$

Using $h_1 = 2h_2$, we may rearrange this to find

$$I_2 - I_1 = 15ch_2^4.$$

Identifying the error on our second evaluation as $\epsilon_2 = ch_2^4$, we see

$$\epsilon_2 = \frac{1}{15}(I_2 - I_1).$$