LA-UR-02-6889

The Structure of Light Nuclei and Its Effect on Precise Atomic Measurements

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Summary. This review consists of three parts: (a) what every atomic physicist needs to know about the physics of light nuclei; (b) what nuclear physicists can do for atomic physics; (c) what atomic physicists can do for nuclear physics. A brief qualitative overview of the nuclear force and calculational techniques for light nuclei will be presented, with an emphasis on debunking myths and on recent progress in the field. Nuclear quantities that affect precise atomic measurements will be discussed, together with their current theoretical and experimental status. The final topic will be a discussion of those atomic measurements that would be useful to nuclear physics, and nuclear calculations that would improve our understanding of existing atomic data.

1 Introduction

....numerical precision is the very soul of science.....

This quote[1] from Sir D'Arcy Wentworth Thompson, considered by many to be the first biomathematician, could well serve as the motto of the field of precise atomic measurements, since precision is the *raison d'être* of this discipline. I have always been in awe of the number of digits of accuracy achievable by atomic physics in the analysis of simple atomic systems[2]. Nuclear physics, which is my primary field and interest, must usually struggle to achieve three digits of numerical significance, a level that atomic physics would consider a poor initial effort, much less a decent final result.

The reason for the differing levels of accuracy is well known: the theory of atoms is QED, which allows one to calculate properties of few-electron systems to many significant figures[3]. On the other hand, no aspect of nuclear physics is known to that precision. For example, a significant part of the "fundamental" nuclear force between two nucleons must be determined phenomenologically by utilizing experimental information from nucleon-nucleon scattering[4], very little of which is known to better than 1%. In contrast to that level of precision, energy-level spacings in few-electron atoms can be measured so precisely that nuclear properties influence significant digits in those energies[5]. Thus these experiments can be interpreted as either a measurement of those nuclear properties, or corrections must be applied to eliminate the nuclear effects so that the resulting measurement tests or measures non-nuclear properties. That is the purview of this review.

The single most difficult aspect of a calculation for any theorist is assigning uncertainties to the results. This is not always necessary, but in calculating nuclear corrections to atomic properties it is essential to make an effort. That is just another way to answer the question,"What confidence do we have in our results?" Because it is important for atomic physicists to be able to judge nuclear results to some degree, this discussion has been slanted towards answers to two questions that should be asked by every atomic physicist. The first is: "What confidence should I have in the values of nuclear quantities that are required to analyze precise atomic experiments?" The second question is: "What confidence should I have that the nuclear output of my atomic experiment will be put to good use by nuclear physicists?"

2 Myths of Nuclear Physics

Every field has a collection of myths, most of them being at least partially true at one time. Myths propagate in time and distort the reality of the present. A number of these are collected below, some of which the author once believed. The resolution of these "beliefs" also serves as a counterpoint to the very substantial progress made in light-nuclear physics in the past 15 years, which continues unabated.

My myth collection includes:

• The strong interactions (and consequently the nuclear force) aren't well understood, and nuclear calculations are therefore unreliable.

• Large strong-interaction coupling constants mean that perturbation theory doesn't converge, implying that there are no controlled expansions in nuclear physics.

• The nuclear force has no fundamental basis, implying that calculations are not trustworthy.

• You cannot solve the Schrödinger equation accurately because of the complexity of the nuclear force.

• Nuclear physics requires a relativistic treatment, rendering a difficult problem nearly intractable.

All of these myths had some (even considerable) truth in the past, but today they are significant distortions of our current level of knowledge.

3 The Nuclear Force

Most of the recent progress in understanding the nuclear force is based on a symmetry of QCD, which is believed to be the underlying theory of the strong interactions (or an excellent approximation to it). It is generally the case that our understanding of any branch of physics is based on a framework of symmetry principles. QCD has "natural" degrees of freedom (quarks and gluons) in terms of which the theory has a simple representation. The (strong) chiral symmetry of QCD results when the quark masses vanish, and is a more complicated analogue of the chiral symmetry that results in QED when the electron mass vanishes. The latter symmetry explains, for example, why (massless or high-energy) electron scattering from a spherical (i.e., spinless) nucleus vanishes in the backward direction.

The problem with this attractive picture is that it does not involve the degrees of freedom most relevant to experiments in nuclear physics: nucleons and pions. It is nevertheless possible to "map" QCD (expressed in terms of quarks and gluons) into an "equivalent" or surrogate theory expressed in terms of nucleon and pion degrees of freedom. This surrogate works effectively only at low energy. The small-quark-mass symmetry limit becomes a small-pion-mass symmetry limit. In general this (slightly) broken-symmetry theory has $m_\pi c^2 \ll \Lambda$, where the pion mass is $m_\pi c^2 \cong 140$ MeV and $\Lambda \sim 1$ GeV is the mass scale of QCD bound states (heavy mesons, nucleon resonances, etc.). The seminal work on this surrogate theory, now called chiral perturbation theory (or χPT), was performed by Steve Weinberg[6], and many applications to nuclear physics were pioneered by his student, Bira van Kolck[7]. From my perspective they demonstrated two things that made an immediate impact on my understanding of nuclear physics[8]: (1) There is an alternative to perturbation theory in coupling constants, called "power counting," that converges geometrically like $(Q/\Lambda)^N$, where $Q \sim m_\pi c^2$ is a relevant nuclear energy scale, and the exponent N is constrained to have $N \geq 0$; (2) nuclear physics mechanisms are severely constrained by the chiral symmetry. These results provide nuclear physics with a well-founded rationale for calculation.

This scheme divides the nuclear-force regime in a natural way into a long-range part (which implies a low energy, Q, for the nucleons) and a short-range part (corresponding to high energy, Q, between nucleons). This is indicated in Fig. (1), which is a cartoon of the potential between two nucleons meant only to indicate significant regions and mechanisms. Since χPT is effective only at low energies, we expect that only the long-range part of the nuclear force can be treated successfully by utilizing only the pion degrees of freedom. This would be the region with $r > b$. We need to resort to phenomenology (i.e., fitting to nucleon-nucleon scattering data) to treat systematically the short-range part of the interaction ($r < b$).

The long-range nuclear force is calculated in much the same way that atomic physics calculates the interactions in an atom using QED. Both are illustrated in Fig. (2). The dominant interaction between two nucleons is the

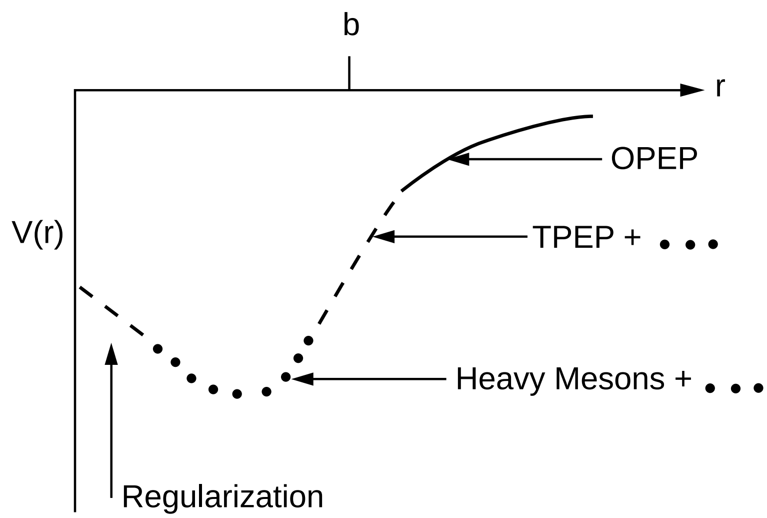

Fig. 1. Cartoon of the nuclear potential, $V(r)$, showing regions of importance

exchange of a single pion illustrated in Fig. (2b) (One-Pion-Exchange Potential or "OPEP") and denoted V_π. Its atomic analogue is one-photon exchange in Fig. (2a) (containing the dominant Coulomb force). Because it is such an important part of the nuclear potential, it is fair to call V_π the "Coulomb force" of nuclear physics. Smaller contributions arise from the two-pion-exchange potential in Fig. (2e) (called "TPEP"), which is the analogue of two-photon exchange between charged particles shown in Fig. (2d). There is even an analogue of the atomic polarization force in Fig. (2g), where two electrons simultaneously polarize their nucleus using their electric fields. The nuclear analogue involving three nucleons simultaneously is illustrated in Figs. (2h) and (2i), and is called a three-nucleon force[9]. Although relatively weak compared to V_π (a few percent), three-nucleon forces play an important role in fine-tuning nuclear energy levels. The final ingredient is an important short-range interaction (which must be determined by phenomenology) shown in Fig. (2c) that has no direct analogue in the physics of light atoms. Just as one can exchange three photons, three-pion-exchange is possible and is depicted in Fig. (2f).

It is worth recalling that the uncertainty principle tells us that exchanging light particles produces longer-range forces and exchanging heavier particles produces shorter-range forces. Thus OPEP has a longer range than TPEP, as illustrated in Fig. (1). Many mechanisms have been proposed for the short-range part of the nuclear force, such as heavy-meson exchange, for example. Any meson-exchange mechanism produces singular forces, which are regularized to make them finite. However one chooses to do this, the part of the nuclear force inside b must be adjusted to fit the nucleon-nucleon scattering data, and no individual parameterization of the short-range force is intrinsically superior (i.e., it doesn't matter how you do it).

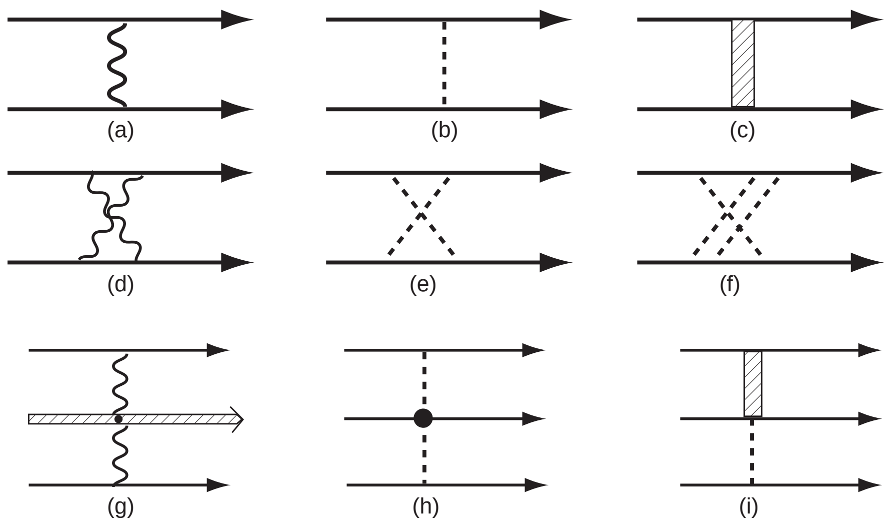

Fig. 2. First- and second-order (in α, the fine structure constant) atomic interactions resulting from photon exchange are shown in the left-most column, where solid lines are electrons, wiggly lines are photons, and the shaded line is a nucleus. The analogous nuclear interactions resulting from pion exchange are shown in the middle column, where solid lines are nucleons and dashed lines are pions. Nuclear processes involving short-range interactions (shaded vertical areas) are shown in the right-most column, together with a three-pion-exchange interaction

How all of this works in practice is indicated in Fig. (3). Imagine that you throw away *all* of the nuclear potential inside $r = b$ (with b chosen to be 1.4 fm) in Fig. (1), keeping only the tail of the force between two nucleons. Now compute a phase shift (the $^3\mathrm{P}_0$, for example). This very modest physics input predicts the basic shape of the phase shift (dashed line) as a function of energy. This variation with energy is a consequence of the small pion mass (compared to the energy scale in the figure). What is missing in this curve is a smooth (negative) short-range contribution that grows roughly in proportion to the energy. We can fill in the missing short-range interaction inside $r = b$ by adding a potential term specified by one short-range parameter. This produces the dotted line, which is a rather good fit, and adding two more terms (solid line) produces a nearly perfect fit to the experimental results. Fixing the short-range part of the potential looks very much like making an effective-range expansion. All useful physics is specified by a few parameters, and the details are completely unimportant.

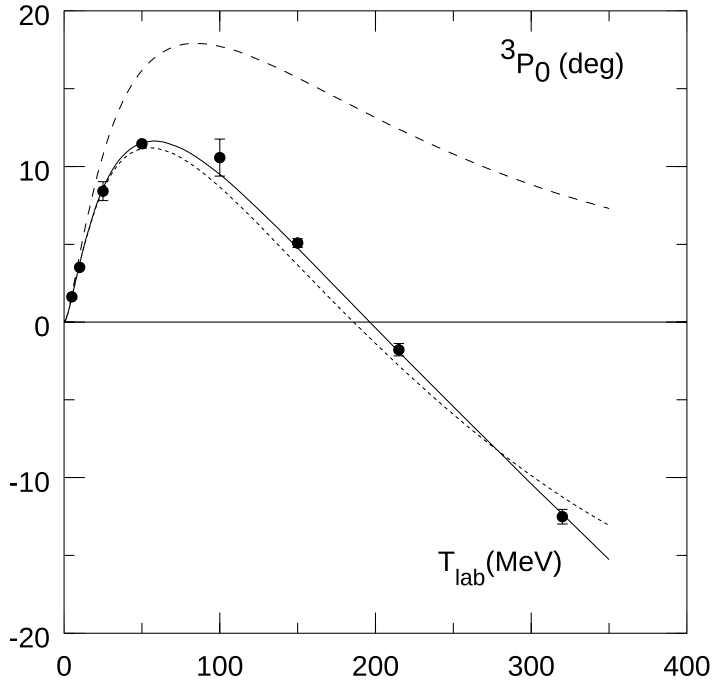

Fig. 3. $^3P_0$ phase shift (in degrees) calculated only with the OPEP tail for $r > b$ (dashed line), and with one (dotted) and three (solid) short-range-interaction terms added. The experimental results are indicated by separate points with error bars[10]

What are the consequences of exchanging a pion rather than a photon? The pseudoscalar nature of the pion mandates its spin-dependent coupling to a nucleon, and this leads to a dominant tensor force between two nucleons. Except for its radial dependence, the form of V_π mimics the interaction between two magnetic dipoles, as seen in the Breit interaction, for example. Thus we have in nuclear physics a situation that is the converse of the atomic case: a dominant tensor force and a smaller central force. In order to grasp the difficulties that nuclear physicists face, imagine that you are an atomic physicist in a universe where magnetic (not electric) forces are dominant, and where QED can be solved only for long-range forces and you must resort to phenomenology to generate the short-range part of the force between electrons and nuclei.

Although this may sound hopeless, it is merely difficult. The key to handling complexities is adequate computing power, and that became routinely available only in the late 1980s or early 1990s. Since then there has been explosive development in our understanding of light nuclei. Underlying all of these developments is an improved understanding of the nuclear force. It is convenient to divide nuclear forces and their history into three distinct time periods.

First-generation nuclear forces were developed prior to 1993. They all contained the one-pion-exchange force, but everything else was relatively crude. The fits to the nucleon-nucleon scattering data (needed to parameterize the short-range part of that force) were indifferent.

Second-generation forces were developed beginning in 1993[4]. They were more sophisticated and generally very well fit to the scattering data. As an example of how well the fitting worked, the Nijmegen group (which pioneered this sophisticated procedure) allowed the pion mass to vary in the Yukawa function defining V_π, and then fit that mass. They also allowed different masses for the neutral and charged pions that were being exchanged and found[11]

$$m_{\pi^\pm} = 139.4(10)\,\mathrm{MeV}\,, \tag{1}$$

$$m_{\pi^0} = 135.6(13)\,\mathrm{MeV}\,, \tag{2}$$

both results agreeing with free pion masses ($m_{\pi^\pm} = 139.57018(35)$ MeV and $m_{\pi^0} = 134.9766(6)$ MeV [12]). It is both heartening and a bit amazing that the masses of the pions can be determined to better than 1% using data taken in reactions that have no free pions! This result is the best quantitative proof of the importance of pion degrees of freedom in nuclear physics.

Third-generation nuclear forces are currently under development. These forces are quite sophisticated and incorporate two-pion exchange, as well as V_π. All of the pion-exchange forces (including three-nucleon forces) are being generated in accordance with the rules of chiral perturbation theory. One expects even better fits to the scattering data. This is clearly work in progress, but preliminary calculations and versions have already appeared[13].

4 Calculations of Light Nuclei

Having a nuclear force is not very useful unless one can calculate nuclear properties with it. Such calculations are quite difficult. Until the middle 1980s only the two-nucleon problem had been solved with numerical errors smaller than 1%. At that time the three-nucleon systems ^{3}H and ^{3}He were accurately calculated using a variety of first-generation nuclear-force models[14]. Soon thereafter the α-particle (^{4}He) was calculated by Joe Carlson, who pioneered a technique that has revolutionized our understanding of light nuclei: Green's Function Monte Carlo (GFMC)[15].

The difficulty in solving the Schrödinger equation for nuclei is easily understood, although it was not initially obvious. Nuclei are best described in terms of nucleon degrees of freedom. Nucleons come in two types, protons and neutrons, which have nearly the same masses and can be considered as the up and down components of an "isospin" degree of freedom. If one also includes its spin, a single nucleon thus has four internal degrees of freedom. Two nucleons consequently have 16 internal degrees of freedom, which is roughly the number of components in the nucleon-nucleon force (coupling spin, isospin

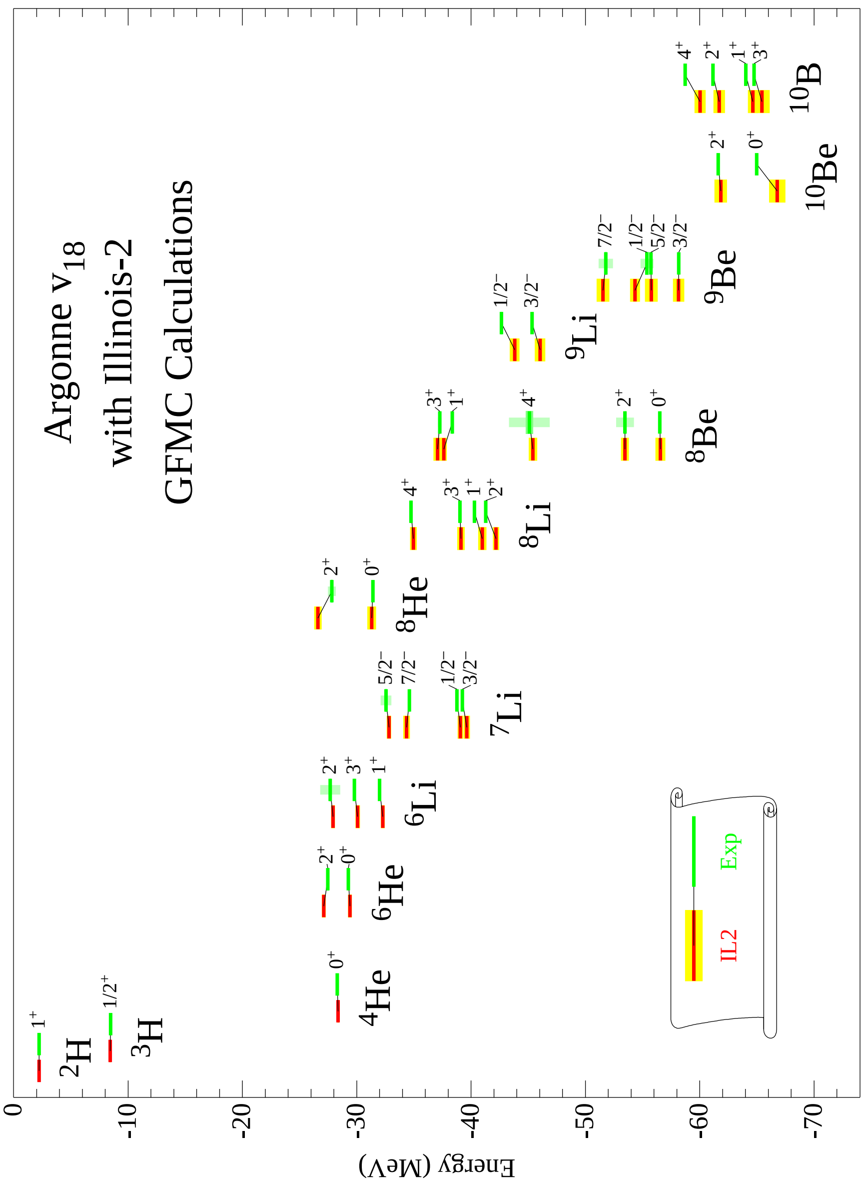

Fig. 4. GFMC calculations of the binding energies of the levels (labelled by their spins and parities) of light nuclei with as many as ten nucleons. These calculations use a common Hamiltonian and have a numerical uncertainty on the order of 1%. Heavy shaded lines to the left are calculated energies (with errors), while light shaded lines to the right are experimental energies. The label “IL2” refers to the Illinois-2 model of the three-nucleon force that is used in all of the calculations together with the Argonne V_{18} two-nucleon force

and orbital motion in a very complicated way). To handle this complexity one again requires fast computers, and that is a fairly recent development.

The GFMC technique has been used to solve for all of the bound (and some unbound) states of nuclei with up to 10 nucleons. One member of this collaboration (Steve Pieper[18]) calculated that the ten-nucleon Schrödinger equation requires the solution of more than 200,000 coupled second-order partial-differential equations in 27 continuous variables, and this can be accomplished with numerical errors on the order of 1%! A subset of the results of this impressive calculation are shown in Fig. (4)[19].

Although the nucleon-nucleon scattering data alone can predict the binding energy of the deuteron (^{2}H) to within about 1/2%, the experimental binding energy is used as input data in fitting the nucleon-nucleon potential. The nuclei ^{3}H and ^{3}He (not shown) are slightly underbound without a three-nucleon force, and that force can be adjusted to remedy the underbinding. This highlights both the dominant nature of the nucleon-nucleon force and the relative smallness of three-nucleon forces, which is nevertheless appropriate in size to account for the small discrepancies that result from using only nucleon-nucleon forces in calculations of nuclei with more than two nucleons.

Once the ^{3}H binding energy is fixed, the binding energy of ^{4}He is then accurately predicted to within about 1%. The five-nucleon systems (not shown) are unbound, but their properties are rather well reproduced. The six-nucleon systems are also well predicted. There are small problems with more neutron-rich nuclei (compare ^{9}Li with ^{7}Li or ^{8}He with ^{6}He or ^{4}He), but only 3 adjustable parameters in the three-nucleon force allow several dozen energy levels to be quite well reproduced[19]. Because nuclei are weakly bound systems, there are large cancellations between the (large) potential and (large) kinetic energies, leaving small binding energies. The results shown in Fig. (4) are quite remarkable, especially given that small (fractional) discrepancies in the energy components lead to large effects on the binding energies.

We note finally that power counting can be used to show that light nuclei are basically non-relativistic, and relativistic corrections are on the order of a few percent. Power counting is a powerful qualitative technique for determining the relative importance of various mechanisms in nuclear physics.

5 What Nuclear Physics Can Do for Atomic Physics

With our recently implemented computational skills we in nuclear physics can calculate many properties of light nuclei with fairly good accuracy. This is especially true for the deuteron, which is almost unbound and is computationally simple. Although nuclear experiments don't have the intrinsic accuracy of atomic experiments, many nuclear quantities that are relevant to precise atomic experiments can also be measured using nuclear techniques, and usually with fairly good accuracy.

What quantities are we talking about? The nuclear length scale is set by $R \sim 1$ fm $= 10^{-5}$ Å. The much larger atomic length scale of $a_0 \sim 1$ Å means that an expansion in powers of R/a_0 makes great sense, and a typical wavelength for an atomic electron is so large compared to the nuclear size that only moments of the nuclear observables come into play. This also corresponds to an expansion in α, the fine-structure constant, and $m_e R$, where m_e is the electron mass. This is a rapidly converging series.

For processes that have nuclear states inside loops (such as polarizabilities) the excitation energies of those states play a significant role. Although states of any energy can be excited in principle, in practice the effective energy of (virtual) excitation for light nuclei (call it $\bar{\omega}_N$) is within a factor of two of 10 MeV, except for the more tightly bound α-particle, which also has a smaller radius as a consequence. This number follows from the uncertainty principle and the fact that nucleons in a light nucleus have a radius of about 2 fm. The deuteron's weak binding generates the lowest values, which is about 6 MeV for the deuteron's electric polarizability ($\alpha_{\rm E} \sim 1/\bar{\omega}_N$). Using the value of $\bar{\omega}_N = 10$ MeV, we find $\bar{\omega}_N R \sim 1/10$, which is a reasonably small expansion parameter.

Table 1. Orders in α where various contributions to the Lamb shift for S-states have been calculated. The label "f.s." denotes a contribution from nuclear finite size or nuclear structure. Once nuclear physics enters a process at a given order, higher orders will also have nuclear corrections. A "–" indicates that although a complete calculation of nuclear contributions has not been made, such contributions are expected. Names refer to the person who first calculated the leading-order term of that type. References and the meanings of other labels are given in the text

Process	α^2	α^4	α^5	α^6
NR Coulomb	Bohr	f.s.	f.s.	f.s.
Rel. Coulomb		Dirac		f.s.
Recoil		Darwin	–	–
Nucl. Structure			f.s.	–
Vacuum Pol.			Uehling	f.s.
Radiative			Bethe	f.s.

At what levels do various nuclear mechanisms affect the Lamb shift? The (lowest) orders in α that receive nuclear contributions (for S-states) are sketched in Table (1). The various mechanisms are divided into static Coulomb (both non-relativistic and relativistic), recoil (inverse powers of the nuclear mass, M), nuclear structure, vacuum polarization, and radiative processes. The nuclear effects are conveniently divided into two categories: those that directly involve only the properties of the nuclear ground state, and those that involve virtual excited states and are traditionally called "nuclear structure." A radius is a good example of the former, while a polarizability is the prototype of the latter. Calculational techniques are quite different for these two categories.

It is beyond the scope of this review to list detailed formulae and extensive references to past work. I strongly recommend the recent review of [3], which is extremely well organized. An entire section is devoted to nuclear contributions, and these are listed in their Table (10) with references and numerical values for the hydrogen atom. A sketch of how these contributions scale is given below together with some of the more recent references.

The leading-order non-relativistic energy is simply the Bohr energy of order α^2. Nuclear finite-size contributions of non-relativistic type (i.e., generated by the Schrödinger equation) begin for S-states in order $(Z\alpha)^4$[20] and are proportional to R^2; they have also been calculated in order $(Z\alpha)^5$ and $(Z\alpha)^6$[21, 22]. The Dirac energy has a leading-order $(Z\alpha)^4$ term, while the nuclear finite-size contributions of relativistic type begin in order $(Z\alpha)^6$ and are proportional to R^2. A recent calculation exists for deuterium[23]. The non-relativistic finite-size corrections of order $(Z\alpha)^5$ and $(Z\alpha)^6$ are tiny for electronic atoms (they contain higher powers of $m_{\rm e}R$, which is very small), but are not necessarily small for muonic atoms ($m_\mu R$ is about 1 for most light nuclei), which was the original motivation for developing them. P-state finite-size effects begin in order $(Z\alpha)^6$ and are of both relativistic ($\sim R^2$) and non-relativistic ($\sim R^4$) types.

The most important nuclear-structure mechanism is the electric polarizability (which has a long history and will be discussed in more detail later), and this generates a leading contribution of order $\alpha^2(Z\alpha)^3$. Coulomb corrections of order $\alpha^2(Z\alpha)^4$ were developed in the context of a greatly simplified model of the polarizability in muonic atoms[24] (which would not be applicable to electronic atoms).

The Uehling mechanism for vacuum polarization is of order $\alpha(Z\alpha)^4$, while the first nuclear corrections are of order $\alpha(Z\alpha)^5$[26, 27, 28, 29]. The leading-order radiative process is also of order $\alpha(Z\alpha)^4$, while the nuclear finite-size corrections begin in order $\alpha(Z\alpha)^5$[28, 29]. Both of these nuclear corrections are proportional to R^2. Recoil corrections have a long and interesting history that predates the Schrödinger equation (C. G. Darwin derived the leading term of order $(Z\alpha)^4$ using Bohr-Sommerfeld quantization; see the references in [30]). To the best of my knowledge no published calculation exists for the nuclear-finite-size recoil corrections, which begin in order $(Z\alpha)^5$, although the techniques of [31] lead to a result proportional to R^2/M, which should be very small. We note finally the hadronic vacuum polarization, which (although not nuclear in origin) is generated by the strong interactions[32].

One quantity through which nuclear size manifests itself is the nuclear charge form factor (the Fourier transform of the nuclear ground-state charge density, ϱ), which is given by

$$F(\mathbf{q}) = \int d^3r\, \varrho(\mathbf{r})\, \exp(i\mathbf{q}\cdot\mathbf{r}) \cong Z(1 - \frac{\mathbf{q}^2}{6}\langle r^2\rangle_{\rm ch} + \cdots) - \tfrac{1}{2}\,\mathbf{q}^\alpha \mathbf{q}^\beta\, Q^{\alpha\beta} + \cdots\,, \quad (3)$$

where $\mathbf{q}$ is the momentum transferred from an electron to the nucleus, $Q^{\alpha\beta}$ is the nuclear quadrupole-moment tensor, Z is the total nuclear charge, and $\langle r^2\rangle_{\rm ch}$ is the mean-square radius of the nuclear charge density. These moments should dominate the nuclear corrections to atomic energy levels because $|\mathbf{q}|$ in an atom is set by the (very small) atomic scales. Using F to construct the electron-nucleus Coulomb interaction, one obtains

$$V_{\rm C}(\mathbf{r}) \cong -\frac{Z\alpha}{r} + \frac{2\pi Z\alpha}{3}\langle r^2\rangle_{\rm ch}\,\delta^3(\mathbf{r}) - \frac{Q\alpha}{2r^3}\frac{(3\,(\mathbf{S}\cdot\hat{\mathbf{r}})^2 - \mathbf{S}^2)}{S(2S-1)} + \cdots\,, \quad (4)$$

where $\mathbf{S}$ is the nuclear spin operator and Q is the nuclear quadrupole moment (which vanishes unless the nucleus has spin $S \geq 1$). The Fourier transform of the nuclear ground-state current density has a similar expansion

$$\mathbf{J}(\mathbf{q}) = \int d^3r\; \mathbf{J}(\mathbf{r})\, \exp(i\mathbf{q}\cdot\mathbf{r}) \cong -i\mathbf{q}\times\boldsymbol{\mu}\,(1 - \frac{\mathbf{q}^2}{6}\langle r^2\rangle_{\rm M} + \cdots) + \cdots\,, \quad (5)$$

where $\boldsymbol{\mu}$ is the nuclear magnetic moment and $\langle r^2\rangle_{\rm M}$ is the mean-square radius of the magnetization density. The first term generates the usual atomic hyperfine interaction.

Table 2. Values of the root-mean-square charge and magnetic radii and the quadrupole moment (if nonvanishing) of the nucleons and various light nuclei obtained by nuclear experiments, together with a selected reference. If two values are given, the second value is that obtained by an atomic or molecular measurement

Nucleus	$\langle r^2\rangle_{\rm ch}^{1/2}$ (fm)	ref.	$\langle r^2\rangle_{\rm M}^{1/2}$ (fm)	ref.	Q (fm^2)	ref.
H	0.880 (15)	[33]	0.836 (9)	[34]	–	
	0.883 (14)	[35]			–	
^{2}H	2.130 (10)	[36]	2.072 (18)	[37]	0.282 (19)	[38]
					0.2860 (15)	[39, 40]
^{3}H	1.755 (87)	[37]	1.84 (18)	[37]	–	
^{3}He	1.959 (34)	[37]	1.97 (15)	[37]	–	
	1.954 (8)	[41]			–	
^{4}He	1.676 (8)	[42]	–		–	
Nucleon	$\langle r^2\rangle_{\rm ch}$ (fm^2)	ref.	$\langle r^2\rangle_{\rm M}^{1/2}$ (fm)	ref.		
n	−0.1140 (26)	[43]	0.873 (11)	[44]		

Electron-nucleus scattering experiments are the primary technique used to measure moments of nuclear charge and current densities that are relevant to atomic physics[37], and some appropriate values of these quantities are tabulated in Table (2). An exception is the measurement of the deuteron's quadrupole moment ($Q = 0.282(19)$ fm^2) obtained by scattering polarized deuterons from a high-Z nuclear target at low energy[38]. This result is consistent with the molecular determination ($Q = 0.2860(15)$ fm^2)[39, 40], but its error is an order of magnitude larger. Although there is no reason to believe that the (tensor) electric polarizability of the deuteron[45] plays a significant role in the H-D (molecular) quadrupole-hyperfine splitting that was used to determine Q, that correction was not included in the analysis. It was included in the analysis of the nuclear measurement.

I highly recommend the recent review of electron-nucleus scattering by Ingo Sick[37], which contains values of the charge and magnetic radii of light nuclei. That review not only lists the best and most recent values of quantities of interest, but discusses reliability and technical details for those who are interested. One result from that review is listed in Table (2) and is important for the discussion below. The errors of the tritium (^{3}H) radii are nearly an order of magnitude larger than those of deuterium. Of all the light nuclei tritium is the most poorly known experimentally, although the charge radius can now be calculated with reasonable accuracy.

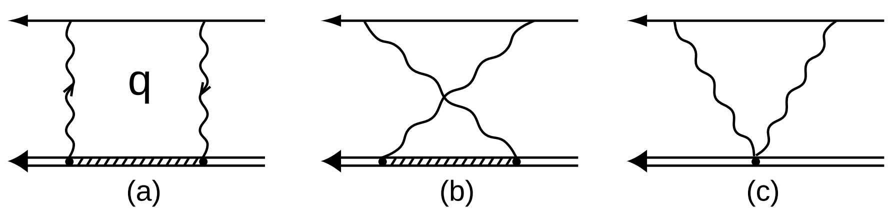

Fig. 5. The direct two-photon process is shown in (a), the crossed-photon process in (b), and "seagull" contributions in (c). The seagulls reflect non-nucleonic processes and terms necessary for gauge invariance. In these graphs the double lines represent a nucleus, the single lines an electron, the wiggly lines a (virtual) photon. The shading represents the set of nuclear excited states. The loop momentum is q, and integrating over this momentum sets the scales of the nuclear part of the process

In addition to moments of the nuclear charge and current densities, various components and moments of the nuclear Compton amplitude can play a significant role. Mechanisms that contribute to the polarizabilities are shown in Fig. (5). The direct (sequential) exchange of photons and the crossed-photon process are shown in (a) and (b), while the "seagull" process is shown in (c). The latter mechanism is required by gauge invariance in any model of hadrons with structure. The exchange of pions between nucleons generates such terms, for example[46]. Because these are loop diagrams, they involve an integral over all momenta (q), and this sets the nuclear scales of the problem. The nuclear size scale (R), the electron mass (m_e), and the average virtual-excitation energy ($\bar{\omega}_N$, appropriate to the shaded part of the line in (a) and (b) that indicates excited nuclear states) determine the generalized polarizabilities[47]. The process is dominated by the usual electric and magnetic polarizabilities and their logarithmic modifications[48].

Specific examples are the (scalar) electric polarizability, α_E, and the nuclear spin-dependent polarizability ($\sim \mathbf{S}$). The latter interacts with the electron spin to produce a contribution to the electron-nucleus hyperfine splitting. There exists a recent calculation of the latter for deuterium[49]. Values of the nuclear electric polarizability for light nuclei obtained from calculations or experiments are listed in Table (3). There are either calculations or measurements of α_E for ^{2}H[48, 50, 51], ^{3}H[52] and ^{3}He[52, 53, 54, 55], and ^{4}He[24, 56]. With the exception of the ^{3}He experimental results, there is reasonable consistency. The decreasing size of the polarizabilities for heavier nuclei is caused by their increased binding. The α-particle has more than 10 times the binding energy of the deuteron, and its polarizability is an order of magnitude smaller.

Table 3. Values of the electric polarizability of light nuclei, both theoretical and experimental, where the latter have been determined by nuclear experiments. No uncertainties were given for the ^{3}H, ^{3}He, and ^{4}He calculations in [52, 53], but they are likely to be smaller than about 10%. The ^{4}He result was used in analyses of muonic He[24, 56]

Nucleus	$\alpha_\mathrm{E}^\mathrm{calc}$ (fm^3)	ref.	$\alpha_\mathrm{E}^\mathrm{exp}$ (fm^3)	ref.
^{2}H	0.6328 (17)	[48]	0.61 (4)	[50]
			0.70 (5)	[51]
^{3}H	0.139	[52]	–	
^{3}He	0.145	[53]	0.250 (40)	[54]
			0.130 (13)	[55]
^{4}He	0.076	[53]	0.072 (4)	[24]

The physics of hyperfine splittings is in general rather different from the physics that contributes to the Lamb shift. It should therefore be no surprise that the nuclear physics that contributes to hyperfine splittings is also quite different; it is also more complicated than its Lamb-shift counterpart. The dominant nuclear physics that we discussed previously was the physics of the nuclear charge density, in the form of moments of the static charge density (i.e., radii) and electric dipole moments that contribute to polarizabilities.

The primary nuclear mechanism in hyperfine splittings is the magnetic interaction caused by the nuclear magnetization density. This density is not as well understood as the charge density. The primary reason is that the same mesons whose exchange binds nuclei together also contribute to the nuclear currents if they carry a charge. The pions that we discussed earlier generate a very important component of that current[46]. The reason for the dichotomy between nuclear charges and currents can be understood by imagining that charged-meson exchange between nucleons is instantaneous. In this limit we know that the transmitted charge is always on a nucleon. In other words only nucleon degrees of freedom matter, which is the normal situation for the charge density. The power counting that we discussed earlier states that corrections to the nuclear charge operator are small ($\sim$ 1%), and include a type that vanishes for instantaneous meson exchanges. That is not the case for the current, however, since any flow of charge (even from a virtual meson) produces a current that is not simply related to nucleon degrees of freedom, and that current can couple to photons. These meson-exchange currents (often denoted "MEC") can be as large as those generated by the usual nuclear convection current. Various tricks can be used to eliminate part of our ignorance, but the nuclear current density is less well understood than the nuclear charge density. Atomic hyperfine splittings provide us with an excellent opportunity to learn about nuclear currents in a very different setting.

Although most of the hyperfine experiments in light atoms were performed decades ago, there has recently been renewed theoretical interest, and the accuracy of the QED calculations is sufficient to extract nuclear information[57]. The differences between the QED calculations and the experimental results can be interpreted as nuclear corrections, and those are significant, as indicated in Table (4). The S-state results in this table (presented as a ratio) have been taken from Table (1) of the recent work of Ivanov and Karshenboim[57].

Table 4. Difference between hyperfine experiments and QED hyperfine calculations for the nth S-state of light hydrogenic atoms times n^3, expressed as parts per million of the Fermi energy. This difference is interpreted as nuclear contributions to the hyperfine splitting[57]. A negative entry indicates that the theoretical prediction without nuclear corrections is too large

	$n^3(E_{\rm hfs}^{\rm exp} - E_{\rm hfs}^{\rm QED})/E_{\rm F}$ (ppm)			
State	H	^{2}H	^{3}H	^{3}He$^+$
1S	-33	138	-38	222
2S	-33	137	–	221

Hyperfine structure is generated by short-range interactions. The dominant Fermi contribution ($E_{\rm F}$) arises from a δ-function, and that produces a dependence on the square of the electron's nth S-state wave function at the origin, $|\phi_n(0)|^2$, which is proportional to $1/{\rm n}^3$. Most nuclear effects have the same dependence ($\sim 1/n^3$), which has been removed from the results in Table (4). The 1S and 2S results are seen to be consistent at this level of accuracy, with 1S experimental results typically being much more accurate.

More calculations of the nuclear contributions to hyperfine splittings in light atoms are badly needed if we are to use this information to learn about the currents in light nuclei. These contributions come in the form of Zemach moments[58] (ground-state quantities) and spin-dependent polarizabilities (discussed above). There exists a considerable literature on the latter subject dating back 50 years. The recent work of Mil'shtein and Khriplovich[49] has pointed out a serious defect in that older work. Although the leading-order terms are essentially non-relativistic in origin (for the nucleons in a nucleus), the sub-leading-order terms are not, and require relativity (for the nucleons) in order to obtain a correct result. This is not terribly surprising, since the same physics that enters that polarizability also enters the Gerasimov-Drell-Hearn sum rule[59], which requires relativity at the nucleon level[60], and is a topic of considerable current interest in nuclear physics[61]. The calculations of [49] suggest that deuterium at least can be understood using fairly simple nuclear models. This needs to be checked using more sophisticated models. We note that the Zemach correction[58] adds to the ratio in Table (4), improving the agreement between experiment and theory for H and ^{3}H. The large positive value of that ratio for deuterium suggests a large polarizability correction, which is confirmed by [49].

6 The Proton Size

One recurring problem in the hydrogen Lamb shift is the appropriate value of the mean-square radius of the proton, $\langle r^2 \rangle_{\rm p}$, to use in calculations. Some older determinations[62] disagree strongly with more recent ones[63]. As shown in (3), the slope of the charge form factor (with respect to $\mathbf{q}^2$) at $\mathbf{q}^2 = 0$ determines that quantity. The form factor is measured by scattering electrons from the proton at various energies and scattering angles.

There are (at least) four problems associated with analyzing the charge-form-factor data to obtain the proton size. The first is that the counting rates in such an experiment are proportional to the flux of electrons times the number of protons in the target seen by each electron. That product must be measured. In other words the measured form factor for low $\mathbf{q}^2$ is $(a - b\frac{\mathbf{q}^2}{6} + \cdots)$, where $b/a = \langle r^2 \rangle_{\rm p}$. The measured normalization a (not exactly equal to 1) clearly influences the value and error of $\langle r^2 \rangle_{\rm p}$. Most analyses unfortunately don't take the normalization fully into account, and [64] estimates that a proper treatment of the normalization of available data could increase $\langle r^2 \rangle_{\rm p}^{1/2}$ by about 0.015 fm and increase its error, as well. In an atom, of course, the normalization is precisely computable.

Another source of error is neglecting higher-order corrections in α (i.e., Coulomb corrections). and [33] demonstrates that this increases $\langle r^2 \rangle_{\rm p}^{1/2}$ by about 0.010 fm. A similar problem in analyzing deuterium data was resolved in [36]. Another difficulty that existed in the past was a lack of high-quality low-$\mathbf{q}^2$ data. The final problem is that one must use a sufficiently flexible fitting function to represent $F(\mathbf{q})$, or the errors in the radius will be unrealistically low. All of the older analyses had one or more of these flaws.

Most of the recent analyses[63, 33, 34] are compatible if the appropriate corrections are made. An analysis by Rosenfelder[33] contains all of the appropriate ingredients, and he obtains $\langle r^2\rangle_{\rm p}^{1/2} = 0.880(15)$ fm. There is a PSI experiment now underway to measure the Lamb shift in muonic hydrogen, which would produce the definitive result for $\langle r^2\rangle_{\rm p}$ [65, 66]. One expects the results of that experiment to be compatible with Rosenfelder's result. Extraction of the proton radius[35] from the electronic Lamb shift is now somewhat uncertain because of controversy involving the two-loop diagrams. These diagrams are significantly less important in muonic hydrogen, where the relative roles of the vacuum polarization and radiative processes are reversed.

7 What Atomic Physics Can Do for Nuclear Physics

The single most valuable gift by atomic physics to the nuclear physics community would be the accurate determination of the proton mean-square radius: $\langle r^2\rangle_{\rm p}$. This quantity is important to nuclear theorists who wish to compare their nuclear wave function calculations with measured mean-square radii. In order for an external source of electric field (such as a passing electron) to probe a nucleus, it is first necessary to "grab" the proton's intrinsic charge distribution, which then maps out the mean-square radius of the proton probability distribution in the wave function: $\langle r^2\rangle_{\rm wfn}$. Thus the measured mean-square radius of a nucleus, $\langle r^2\rangle$, has the following components:

$$\langle r^2\rangle = \langle r^2\rangle_{\rm wfn} + \langle r^2\rangle_{\rm p} + \frac{N}{Z}\langle r^2\rangle_{\rm n} + \frac{1}{Z}\langle r^2\rangle_{\ldots}\,, \tag{6}$$

where the intrinsic contribution of the N neutrons has been included as well as that of the Z protons, and $\langle r^2\rangle_{\ldots}$ is the contribution of everything else, including the very interesting (to nuclear physicists) contributions from strong-interaction mechanisms and relativity in the nuclear charge density[67]. Because the neutron looks very much like a positively charged core surrounded by a negatively charged cloud, its mean-square radius has the opposite sign to that of the proton, whose core is surrounded by a positively charged cloud. It should be clear from (6) that $\langle r^2\rangle_{\rm p}$ (which is much larger than $\langle r^2\rangle_{\rm n}$) is an important part of the overall mean-square radius. Its present uncertainty degrades our ability to test the wave functions of light nuclei.

The next most important measurements are isotope shifts in light atoms or ions. Since isotope shifts measure differences in frequencies for fixed nuclear charge Z, the effect of the protons' intrinsic size cancels in the difference. This is particularly important given the current lack of a precise value for the proton's radius. The neutrons' effect is relatively small and can be rather easily eliminated, and thus one is directly comparing differences in wave functions, or of small contributions from $\langle r^2\rangle_{\ldots}$. Isotope shifts are therefore especially "theorist-friendly" measurements, since they are closest to measuring what nuclear theorists actually calculate.

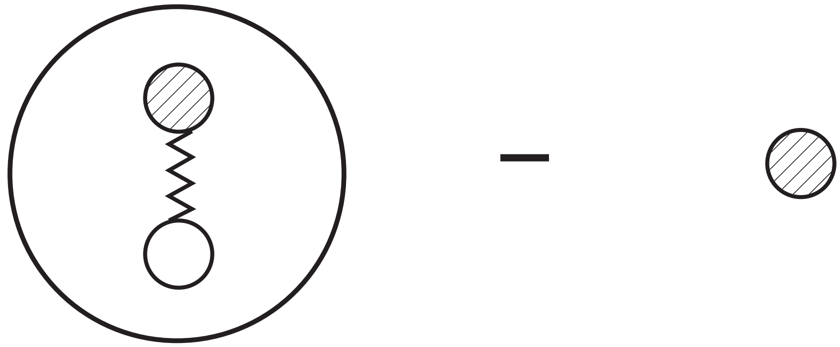

Fig. 6. Cartoon of the ^{2}H - H isotope shift, illustrating how the effect of the finite size of the proton (shaded small circle) in deuterium is cancelled in the measurement. The finite size of the neutron (open small circle) and the electromagnetic interaction mediated by the strong-interaction (binding) mechanism (indicated by the jagged line between the nucleons) do affect the deuteron's charge radius (see text)

Precise isotope-shift measurements have been performed for ^{4}He - ^{3}He[41] and for ^{2}H - ^{1}H (D-H)[5]. A measurement of ^{6}He - ^{4}He is being undertaken[68] at ANL. Gordon Drake has written about and strongly advocated such measurements in the Li isotopes[69]. These are all highly desirable measurements. Because the ^{3}H (tritium) charge radius currently has large errors, in my opinion the single most valuable measurement to be undertaken for nuclear physics purposes would be the tritium-hydrogen (^{3}H - ^{1}H) isotope shift. An extensive series of calculations using first-generation nuclear forces found $\langle r^2 \rangle^{1/2}_{\rm wfn}$ for tritium to be 1.582(8) fm, where the "error" is a subjective estimate[70]. This number could likely be improved by using second-generation nuclear forces, although it will never be as accurate as the corresponding deuteron value, which we discuss next.

The D-H isotope shift in the 2S-1S transition reported by the Garching group[5] was

$$\Delta\nu = 670\ \ 994\ \ 334.64(15)\ \text{kHz}\,. \tag{7}$$

$$\uparrow \qquad \uparrow \quad \uparrow \quad \uparrow$$

Most of this effect is due to the different masses of the two isotopes (and begins in the first significant figure, indicated by an arrow). The precision is nevertheless sufficiently high that the mean-square-radius effect in the sixth significant figure (second arrow) is much larger than the error. The electric polarizability of the deuteron influences the eighth significant figure, while the deuteron's magnetic susceptibility contributes to the tenth significant figure. It becomes difficult to trust the interpretation of the nuclear physics at about the 1 kHz level, so improving this measurement probably wouldn't lead to an improved understanding of the nuclear physics.

Analyzing this isotope shift and interpreting the residue (after applying all QED corrections) in terms of the deuteron's radius leads to the results[71] in Table (5). The very small binding energy of the deuteron produces a long wave function tail outside the nuclear potential (interpretable as a proton cloud around the nuclear center of mass), which in turn leads to an easy and very accurate calculation of the mean-square radius of the (square of the) wave function. Subtracting this theoretical radius from the experimental deuteron radius (corrected for the neutron's size) determines the effect of $\langle r^2 \rangle_{...}$ on the radius. Although this difference is quite small, it is nevertheless significant and half the size of the error in the corresponding electron-scattering measurement (see Table (2)). The high-precision analysis in Table (5) of the content of the deuteron's charge radius would have been impossible without the precision of the atomic D-H isotope-shift measurement. This measurement has given nuclear physics unique insight into small mechanisms that are at present poorly understood[72].

Table 5. Theoretical and experimental deuteron radii for pointlike nucleons. The deuteron wave function radius corresponding to second-generation nuclear potentials and the experimental point-nucleon charge radius of the deuteron (i.e., with the neutron charge radius removed) are shown in the first two columns, followed by the difference of experimental and theoretical results. The difference of the experimental radius with and without the neutron's size is given last for comparison purposes[43]

$\langle r^2 \rangle^{1/2}_{\rm wfn}$ (fm)	$_{\rm exp}\langle r^2 \rangle^{1/2}_{\rm pt}$ (fm)	difference (fm)	$\Delta \langle r^2 \rangle^{1/2}_{\rm n}$ (fm)
1.9687(18)	1.9753(10)	0.0066(21)	−0.0291(7)

8 Summary and Conclusions

Nuclear forces and nuclear calculations in light nuclei are under control in a way never before attained. This progress has been possible because of the great increase in computing power in recent years. Many of the nuclear quantities that contribute to atomic measurements have been calculated or measured to a reasonable level of accuracy, a level that is improving with time. Isotope shifts are valuable contributions to nuclear physics knowledge, and are especially useful to theorists who are interested in testing the quality of their wave functions for light nuclei. In special cases such as deuterium these measurements provide the only insight into the size of small contributions to the electromagnetic interaction that are generated by the underlying strong-interaction mechanisms. In my opinion the tritium-hydrogen isotope shift would be the most useful measurement of that type. One especially hopes that the ongoing PSI experiment is successful in measuring the proton size via the Lamb shift in muonic hydrogen.

9 Acknowledgements

The work of J. L. Friar was performed under the auspices of the United States Department of Energy. The author would like to thank Savely Karshenboim for his interest in nuclear aspects of precise atomic measurements and for giving me the opportunity to discuss this problem.

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