SYMMETRIC MEAN ABSOLUTE PERCENTAGE ERROR (SMAPE) IN TIME SERIES FORECASTING

A Metric for Quantifying Relative Forecast Accuracy, Addressing MAPE's Asymmetry and Zero-Value Issues

MODULE 1: THE PROBLEM (MAPE's Limitations)

PROBLEM: MAPE'S LIMITATIONS (Asymmetry & Zero-Values)

ASYMMETRY BIAS (Penalty for Over-forecasting)

Actual=100
=150
Error= +50
Error=+50

Over-forecast ($F_t > Y_t$)
Forecast= 150
APE = $|\frac{(100-150)}{100}|$*100 = **50%**

Actual=150
=100
Error= -50
Error= 100

Under-forecast ($F_t < Y_t$)
Forecast= 100
APE = $|\frac{(150-100)}{150}|$*100 = **33.3%**

Vermilion Alert: Equal absolute errors result in higher penalties for over-forecasts. MAPE is biased.

DIVISION BY ZERO (Undefined Error)

$$APE_t = \frac{|Y_t - F_t|}{0} * 100$$

UNDEFINED / INFINITE ✗

Cannot be calculated when Actual value (Y_t) is zero. Common in intermittent demand.

Solution: Symmetrize the Denominator

MODULE 2: THE SOLUTION (Core Concept)

SOLUTION: SYMMETRIZING THE DENOMINATOR

MAPE Denominator: |Actual|

$$\boxed{|Y_t|}$$

SMAPE Denominator: Average(|Actual|, |Forecast|)

$$\boxed{|Y_t|} \quad \boxed{|F_t|}$$

$$\boxed{\frac{|Y_t| + |F_t|}{2}}$$

Concept: Instead of dividing only by the Actual value, SMAPE divides by the average of the Absolute Actual and Absolute Forecast values. This creates a symmetric scale and handles zero values.

MODULE 3: THE FORMULA (Deconstructed Calculation)

THE FORMULA (Step-by-Step Calculation)

$$\text{SMAPE} = \frac{1}{n}\sum\left[\frac{|Y_t - F_t|}{\frac{(|Y_t| + |F_t|)}{2}}\right] * 100$$

1. Absolute Error (Numer.)
$|Y_t - F_t|$
Magnitude of difference.

2. Symmetric Average (Denom.)
$\frac{|Y_t| + |F_t|}{2}$
Average scale of Actual & Forecast.

3. Relative Error (Symmetric)
$[\cdots / \cdots]$
Symmetric percentage error for time 't'.

4. Average over Time → SMAPE Value (%)

MODULE 4: KEY CHARACTERISTICS & USE CASE (Interpretation)

CHARACTERISTICS & BEST USE (Management Insight)

KEY CHARACTERISTICS (Symmetry & Range)

- ✔ **Symmetric:** Penalizes over- and under-forecasting equally.
- ✔ **Handles Zero Values:** Defined even when Actual or Forecast is zero (as long as not both).
- ✔ **Range: 0% to 200%.** (Lower is better).

BEST USE CASE (Business Context)

Best suited for comparing forecast accuracy across different datasets or time series, especially when data contains zero or near-zero values and a symmetric penalty is desired (e.g., sparse data, new product launches).

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MAPE vs. SMAPE: FORECAST ACCURACY METRICS

Relative measures comparing Forecast (F) to Actual (A) values over n periods. Focus on percentage error.

1. FORMULA DEFINITION & STRUCTURE

MAPE (Mean Absolute Percentage Error)

$$\text{MAPE} = \frac{1}{n}\sum\left|\frac{A_t - F_t}{A_t}\right| \times 100$$

Actual value in denominator

SMAPE (Symmetric Mean Absolute Percentage Error)

$$\text{SMAPE} = \frac{1}{n}\sum\frac{|(A_t - F_t)|}{\frac{|A_t| + |F_t|}{2}} \times 100$$

Average of Absolute Actual & Forecast in denominator

Key Structural Difference: Denominator Base

2. CRITICAL FAILURE CASE: ACTUAL VALUE (A_t) = 0

MAPE Path ($A_t = 0$)

$A_t = 0$
$F_t = 10$

$$\text{Error} = \left|\frac{0 - 10}{0}\right|$$

UNDEFINED
(Division by Zero)

Critical Flaw:
Cannot handle zero actuals.

SMAPE Path ($A_t = 0$)

$A_t = 0$
$F_t = 10$

$$\text{Error} = \frac{|(0 - 10)|}{\frac{|0| + |10|}{2}}$$

$$\text{Error} = \frac{10}{5} = 2$$

FINITE VALUE
(200%)

Robust: Handles zero actuals.

3. ASYMMETRY & BOUNDS (BEHAVIORAL PROPERTIES)

MAPE: Asymmetric Penalty

Scenario 1:
A=100, F=90
(Under-forecast)

$$\left|\frac{(100-90)}{100}\right| = 10\%$$

Scenario 2:
A=100, F=110
(Over-forecast)

$$\left|\frac{(100-110)}{100}\right| = 10\%$$

Heavier penalty on over- **forecasts** relative to the to the actual value.

Scenario 3:
A=50, F=100
(Over-forecast)

$$\left|\frac{(50-100)}{50}\right| = 100\%$$

Scenario 4:
A=150, F=100
(Under-forecast)

$$\left|\frac{(150-100)}{150}\right| = 33.3\%$$

SMAPE: Symmetric & Bounded

Scenario 1:
A=100,
F=90

$$\frac{|100-90|}{\frac{||100|+|90||}{2}} = \frac{10}{95} \approx 10.5\%$$

Scenario 2:
A=100,
F=110

$$\frac{|100-110|}{\frac{||100|+|110||}{2}} = \frac{10}{105} \approx 9.5\%$$

More symmetric treatment of over- and under-forecasts.

Range: 0% to 200%

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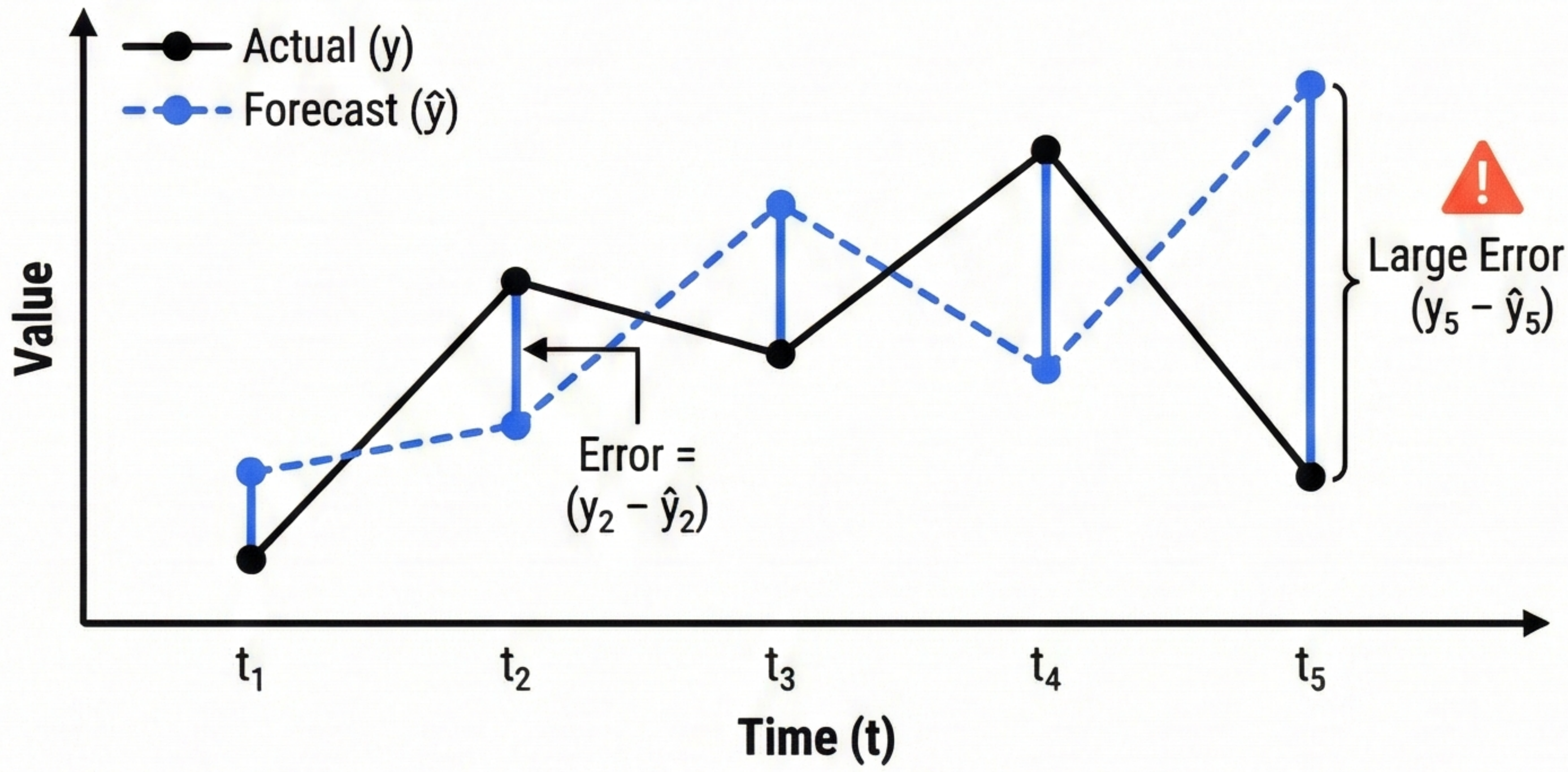
ROOT MEAN SQUARED ERROR (RMSE) IN TIME SERIES FORECASTING

A widely used metric to measure the differences between values **predicted** by a model and the values **actually observed**. It represents the **square root of the average of squared differences** between forecast and actual values.

1. VISUALIZING THE CONCEPT: ACTUAL vs. FORECAST & ERROR



2. THE FORMULA & CALCULATION STEPS

$$\text{RMSE} = \sqrt{\frac{1}{n} \cdot \sum (y_t - \hat{y}_t)^2}$$

Average over n observations

Summation

Squared Error at time t (Magnitude is magnified)

2^2

1. Calculate Error
($y_t - \hat{y}_t$)

→

2. Square the Error
($y_t - \hat{y}_t$)^2
Penalizes large errors significantly

→

$\sum \frac{1}{n}$

3. Mean of Squared Errors (MSE)
MSE = $\frac{1}{n} \cdot \sum (y_t - \hat{y}_t)^2$

→

$\sqrt{\ }$

4. Take Square Root (√)
Returns to original units, gives RMSE

3. KEY CHARACTERISTICS & INTERPRETATION

Characteristic	Description
Scale-Dependent	The error is in the same units as the original data (e.g., dollars, degrees). Not directly comparable across different scales.
Sensitivity to Outliers	Highly sensitive to large errors due to the squaring process. Outliers have a disproportionate impact on the final score (shown by larger errors in section 1).
Interpretation	Represents the typical magnitude of error in the forecast. Lower values indicate better forecasting accuracy.

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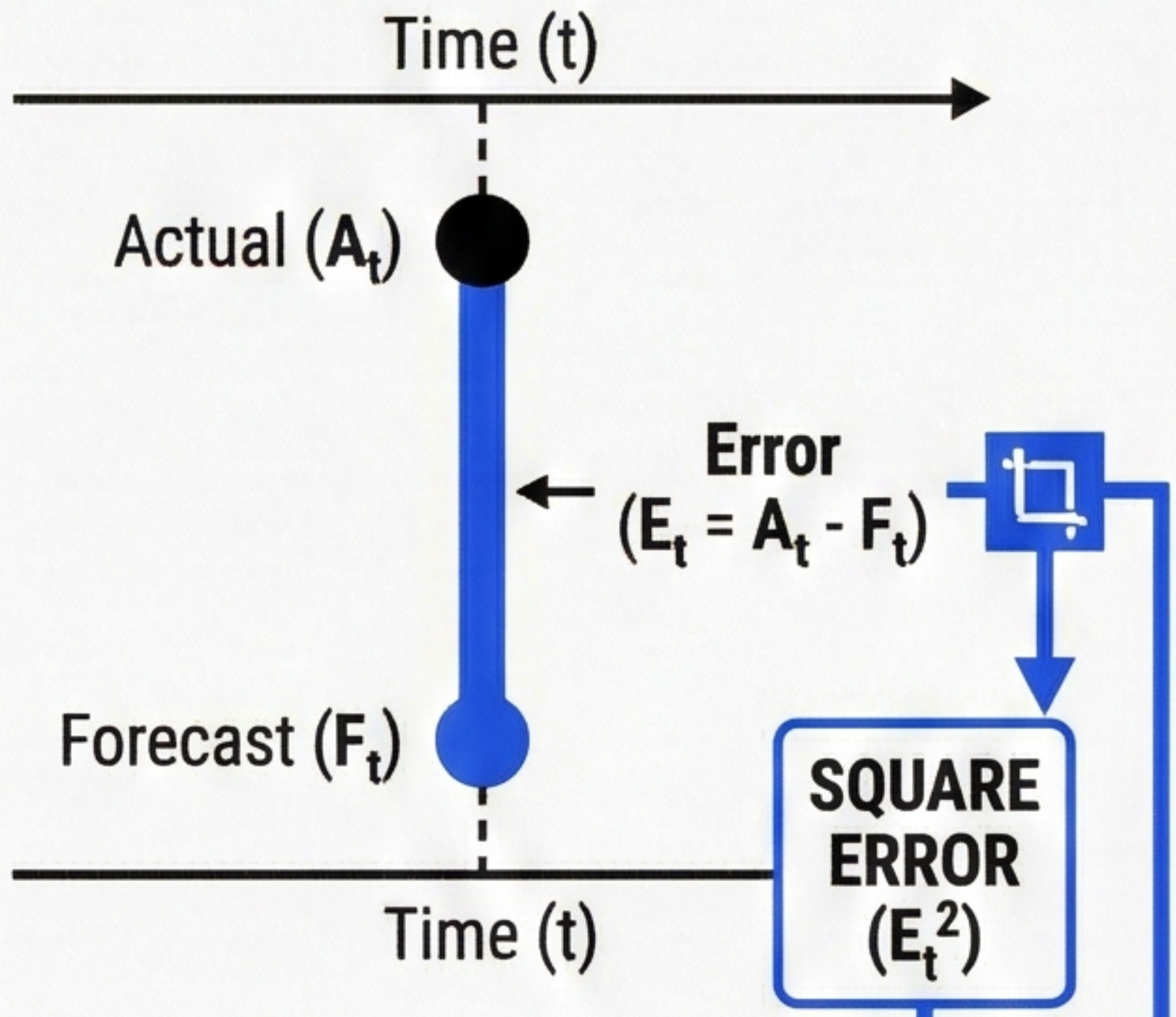
RMSE vs. SMAPE: FORECAST ERROR METRICS COMPARISON

A Comparative Analysis of **Root Mean Squared Error (RMSE)** and Symmetric Mean Absolute Percentage Error (SMAPE) for Time Series Forecasting Decisions

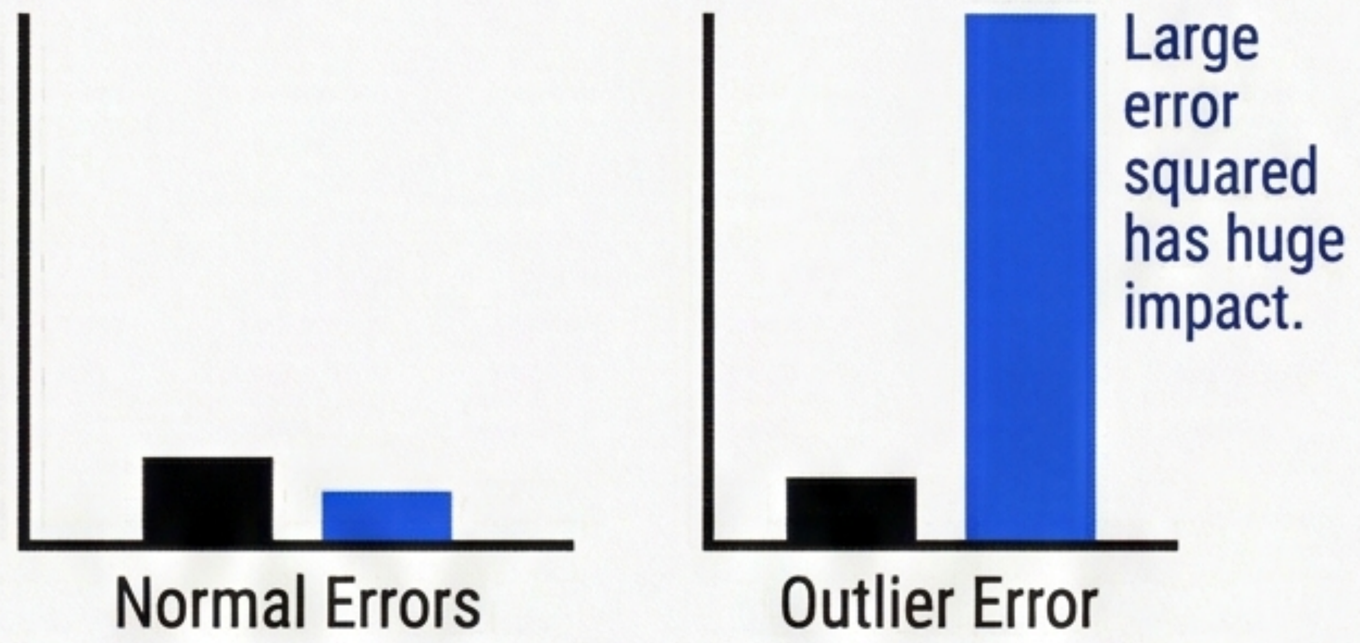
RMSE (ROOT MEAN SQUARED ERROR)



$$\text{RMSE} = \sqrt{\frac{1}{n}\sum (A_t - F_t)^2}$$

Squaring penalizes large errors disproportionately.

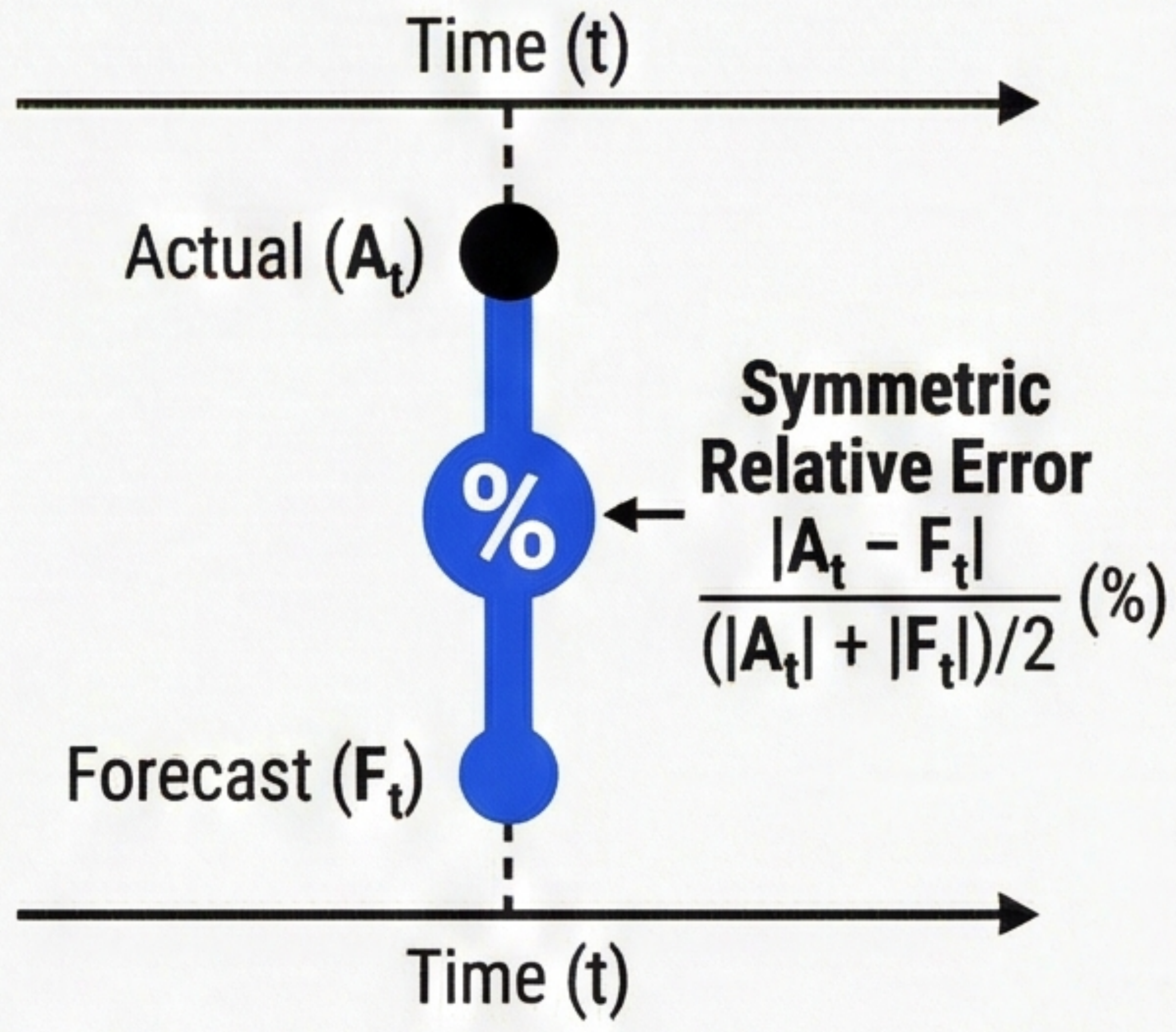
VISUAL PROPERTIES
(Scale-Dependent & Outlier-Sensitive)



CRITICAL LIMITATION: Sensitivity to Outliers
A single large error can significantly inflate RMSE, potentially misleading model selection.

Key Insight: Unit-dependent. Good when large errors are particularly undesirable. Highly sensitive to extreme values.

SMAPE (SYMMETRIC MEAN ABSOLUTE PERCENTAGE ERROR)



Concept: Average of absolute percentage errors with a symmetric denominator to handle near-zero values.

$$\text{SMAPE} = \frac{1}{n}\sum \left[\frac{|A_t - F_t|}{\frac{(|A_t| + |F_t|)}{2}}\right] * 100$$

Symmetric denominator (average of absolute values).

VISUAL PROPERTIES
(Scale-Independent & Symmetric)

CRITICAL LIMITATION: Instability Near Zero
Can be unstable or undefined when both Actual and Forecast are near zero. Range is 0% to 200%.

Key Insight: Unit-independent (%). Good for comparing across different scales. Penalizes over/under-forecasts equally.

COMPARISON SUMMARY & BEST USE CASE

Feature	RMSE	SMAPE
Unit of Measure	Same as Data Units	Percentage (%)
Scale Sensitivity	Scale-Dependent	Scale-Independent
Outlier Sensitivity	High (Squaring effect)	Low (Absolute value)
Symmetry	No (Direction doesn't matter, magnitude does)	Yes (Symmetric denominator)
Best Use Case	When large errors must be minimized.	Comparing across series with different scales.

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