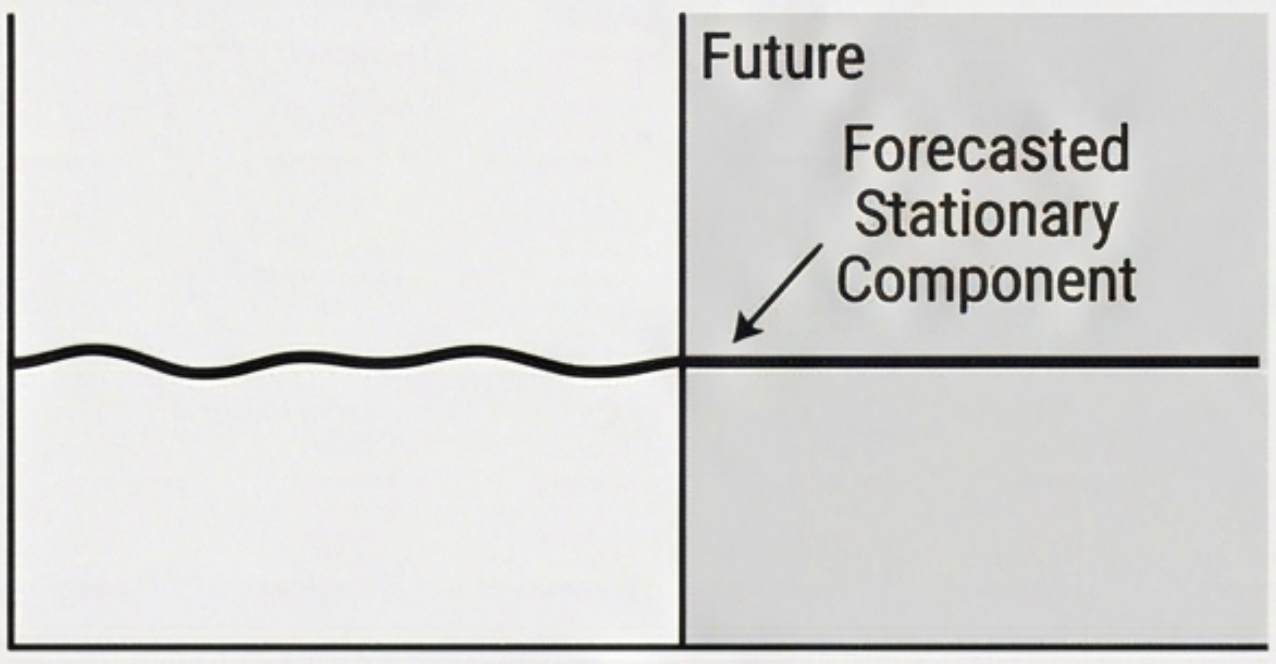
RESEASONALIZATION & RETRENDING FOR TIME SERIES FORECASTING

Reconstructing the Final Forecast by Re-introducing Isolated Pattern Components

MODULE 1: THE INPUTS & GOAL

(Starting from a Stationary Base)

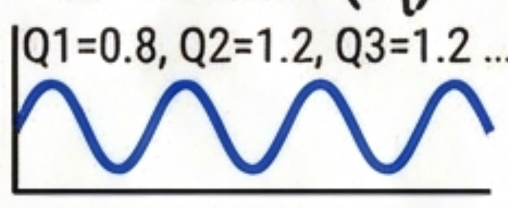
BASE FORECAST (F_{base}) on STATIONARY DATA



RECONSTRUCTION PROCESS

Goal: Transform Base Forecast (F_{base}) back to Original Scale (Y_{hat})

RECALL SEASONAL FACTORS (S_t)



RECALL ESTIMATED TREND (T_t)

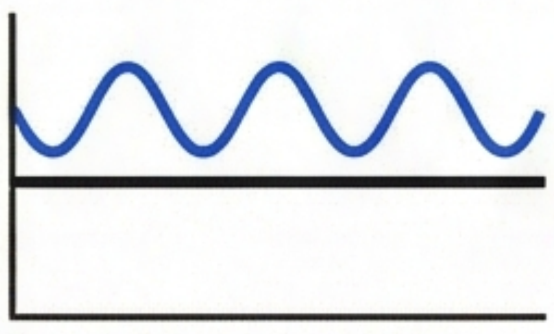
MODULE 2: STEP 1 - RESEASONALIZATION

(Reintroducing the Seasonal Pattern)

PROCESS: REAPPLY SEASONAL FACTORS

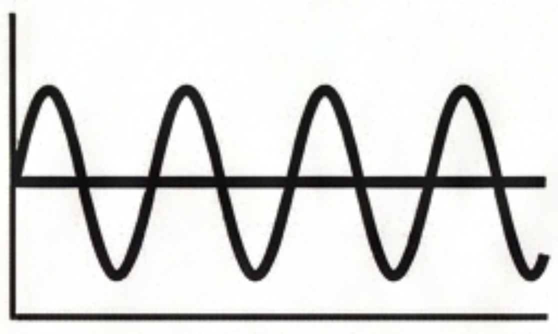
ADDITIVE MODEL
(If Y = T+S+I)

$F_s = F_{base} + S_t$

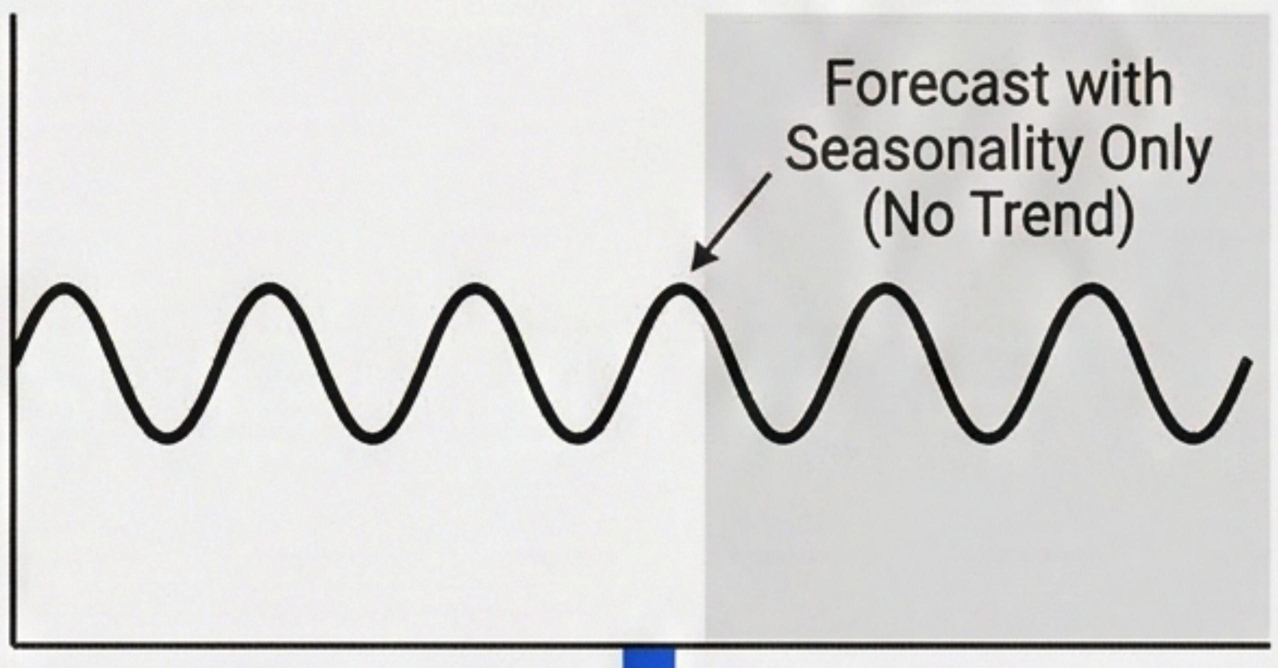


MULTIPLICATIVE MODEL
(If Y = T*S*I)

$F_s = F_{base} * S_t$



RESEASONALIZED FORECAST (F_s)



Flow to Retrending

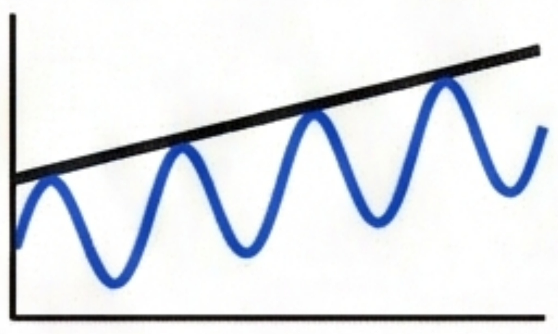
MODULE 3: STEP 2 - RETRENDING

(Reintroducing the Trend)

PROCESS: REAPPLY ESTIMATED TREND

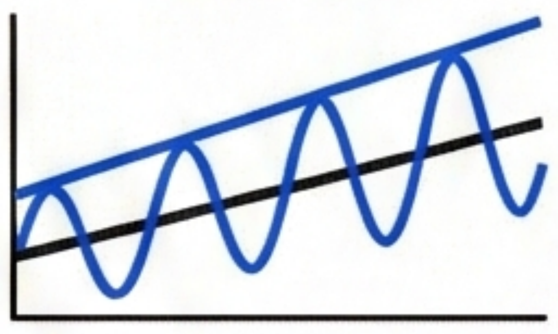
ADDITIVE MODEL
(If Y = T+S+I)

$F_{final} = F_s + T_t$

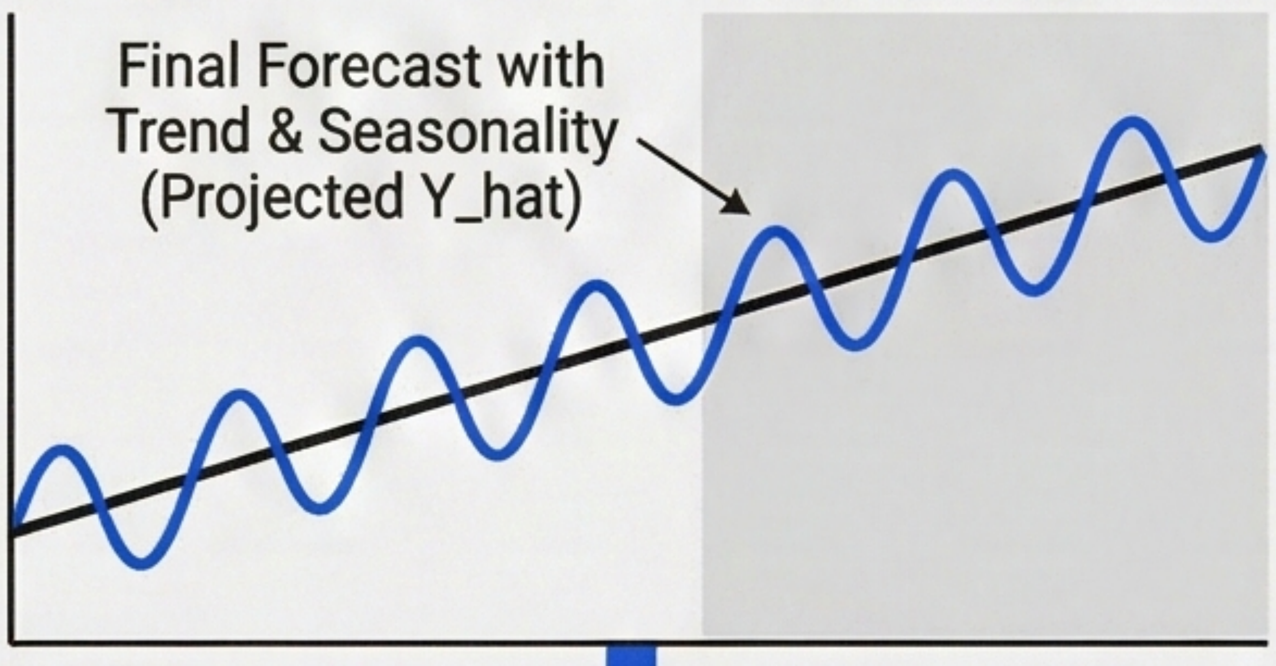


MULTIPLICATIVE MODEL
(If Y = T*S*I)

$F_{final} = F_s * T_t$



FINAL FORECAST (F_{final}) / RETRENDED



Flow to Final Validation

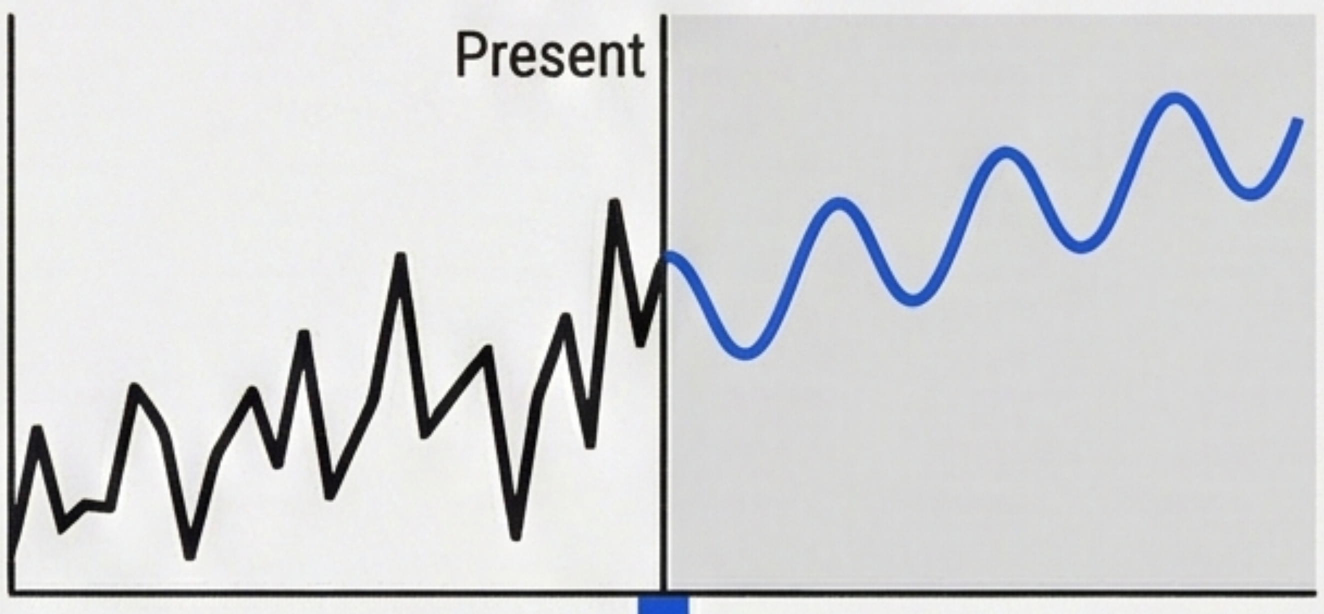
MODULE 4: THE FINAL OUTPUT & VALIDATION

(Completed Forecast)

FINAL OUTPUT:
COMPLETED FORECAST (Y_hat_t)

The fully reconstructed forecast, including projected trend, seasonality, and irregular components.

FINAL FORECAST vs. ACTUALS



VALIDATION & MONITORING

Compare Y_hat with future actuals.
Calculate Error Metrics (e.g., MAPE, RMSE).

ALERT: Continuously monitor forecast accuracy. Significant deviations may require re-estimating S_t and T_t components.

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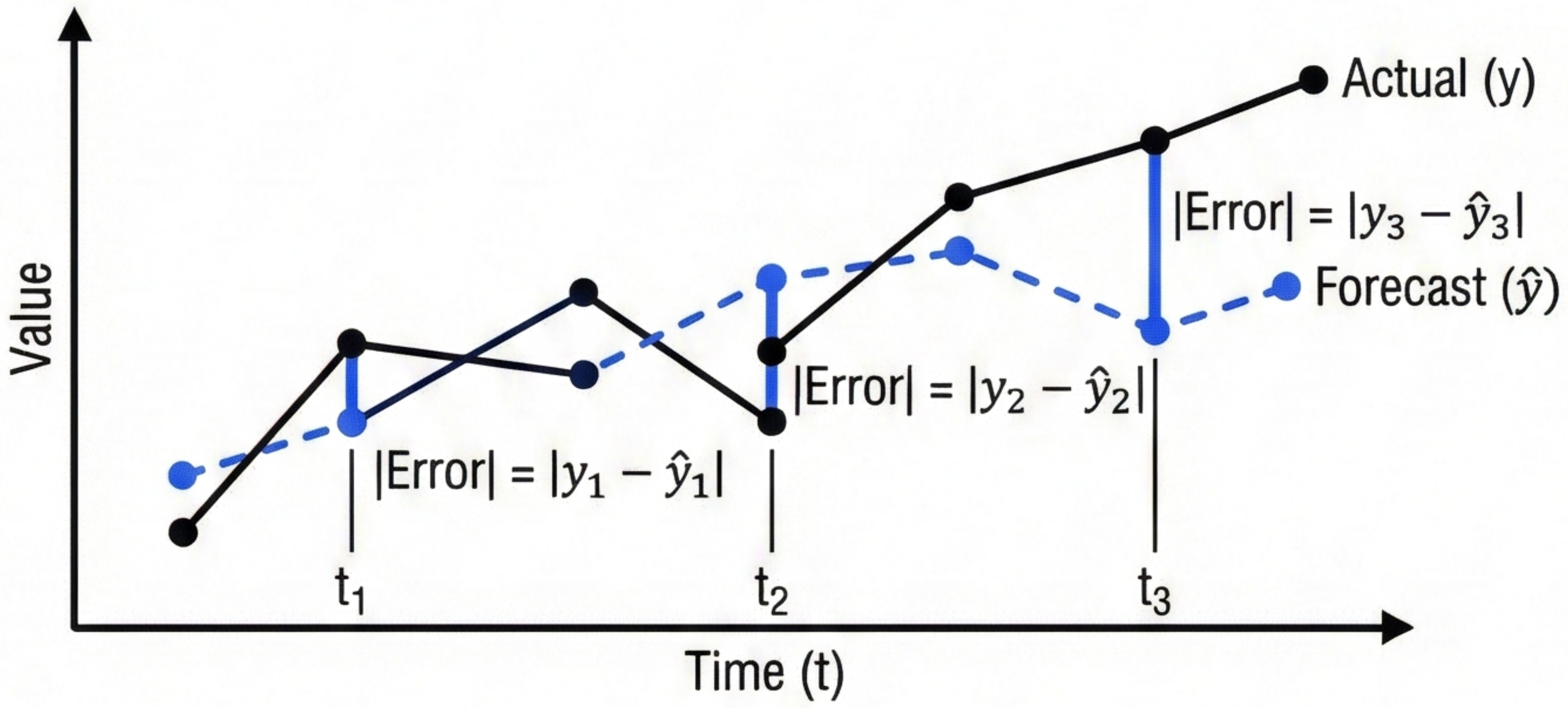
MEAN ABSOLUTE ERROR (MAE) IN TIME SERIES FORECASTING

A metric measuring the average magnitude of errors in a set of predictions, without considering their direction. It is the average of the absolute differences between forecast and actual values.

1. VISUALIZING THE CONCEPT: ACTUAL vs. FORECAST



2. THE FORMULA: CALCULATION STEPS

$$\text{MAE} = \frac{1}{n} \sum |y_t - \hat{y}_t|$$

Average over n observations

Summation

Absolute Error at time t (Magnitude)

Step 1: Calculate Error $(y_t - \hat{y}_t)$ for each point.
Step 2: Take Absolute Value $|y_t - \hat{y}_t|$.
Step 3: Sum all Absolute Errors (Σ).
Step 4: Divide by number of observations (n) to get the Mean.

3. KEY CHARACTERISTICS & INTERPRETATION

Characteristic	Description
Scale-Dependent	The error is in the same units as the original data (e.g., dollars, units). Not directly comparable across different scales.
Robustness	Less sensitive to large outliers compared to squared error metrics (like MSE), as errors are not squared.
Interpretation	Represents the average forecast error magnitude. Lower values indicate better forecasting accuracy.

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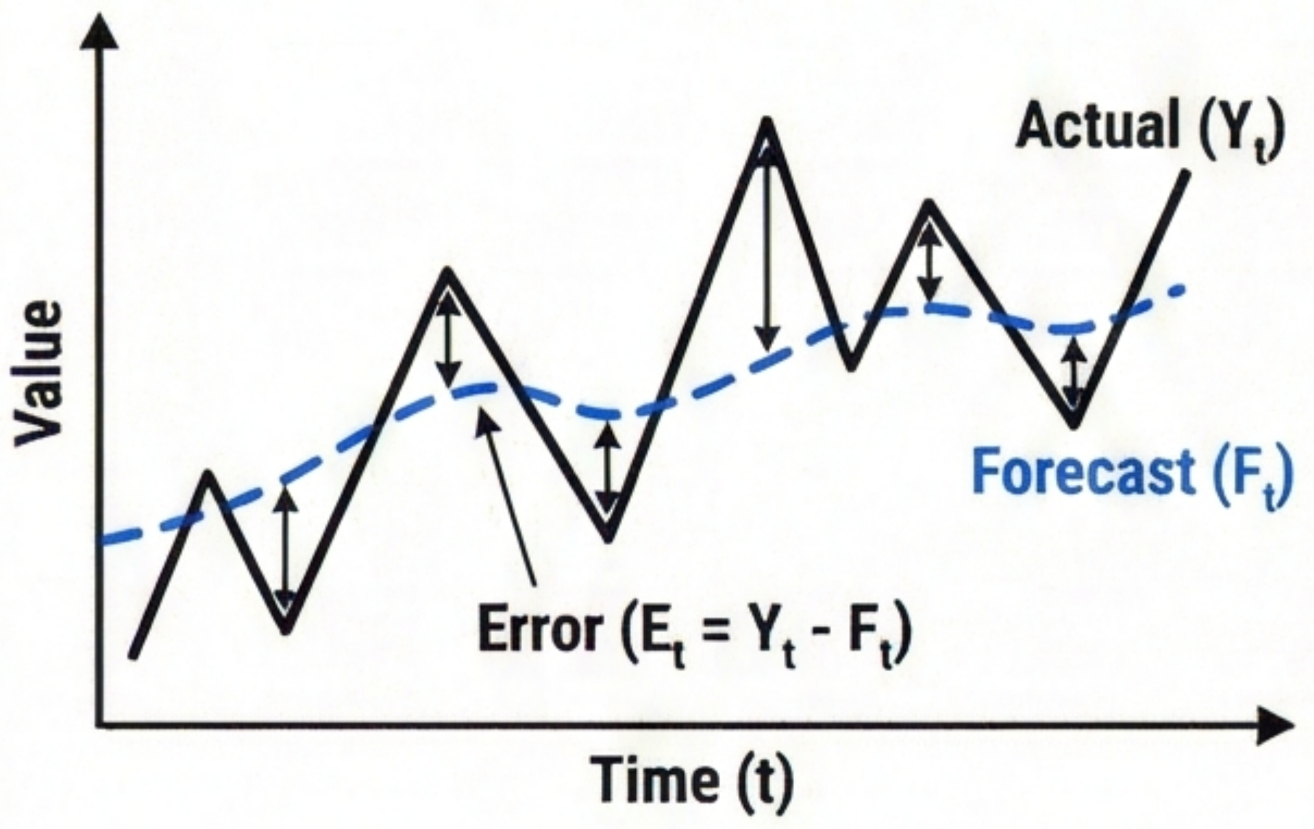
MEAN ABSOLUTE PERCENTAGE ERROR (MAPE) IN TIME SERIES FORECASTING

A Metric for Quantifying Average Relative Forecast Accuracy, Expressed as a Percentage

MODULE 1: CORE CONCEPT & DEFINITION (The Relative Error)

CONCEPT: AVERAGE MAGNITUDE OF RELATIVE ERRORS



MAPE measures the **average percentage difference** between Forecast and Actual values, ignoring direction (+/-). It answers:

"On average, by what percentage is the forecast off?"

MODULE 2: THE FORMULA (Deconstructed Calculation)

THE CALCULATION FORMULA (Step-by-Step)

$$\text{MAPE} = \frac{1}{n}\sum\left|\frac{Y_t - F_t}{Y_t}\right| * 100$$

1. Absolute Percentage Error (APE_t)

$$\left|\frac{Y_t - F_t}{Y_t}\right| * 100$$

Calculate the absolute relative error for each time point 't', expressed as a percentage.

2. Sum of APEs (Σ)

$$\sum(\text{APE}_t)$$

Sum all individual percentage errors over 'n' time points.

3. Average the Sum (1/n)

$$\frac{\sum \text{APE}_t}{n}$$

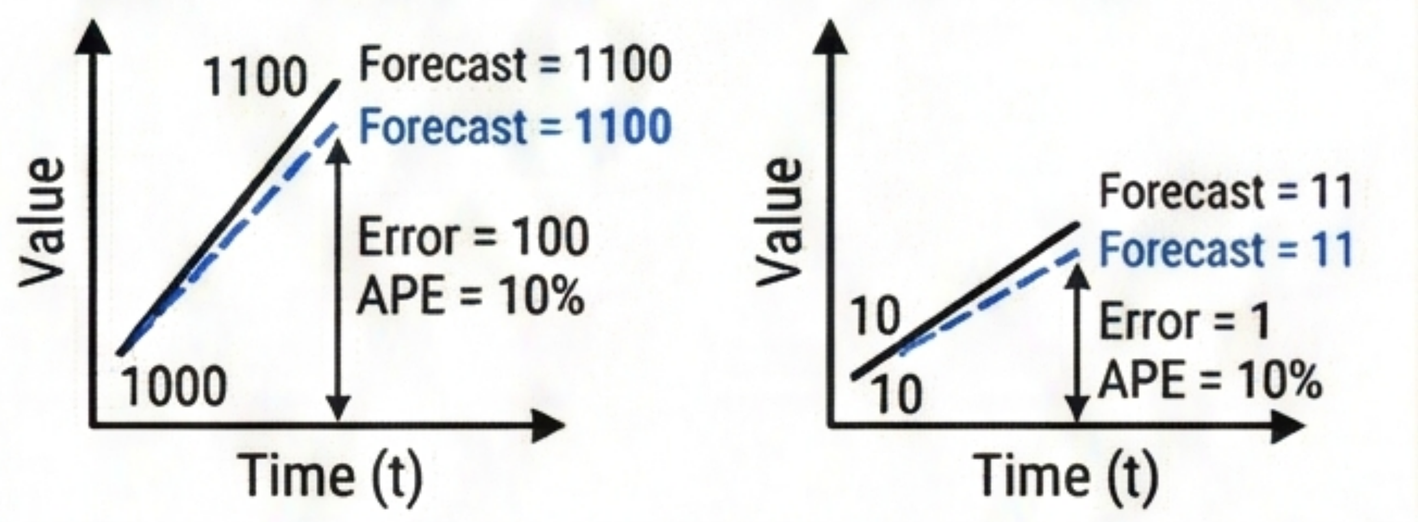
Divide the total sum by the number of observations 'n' to get the mean.

MAPE Value (%)

MODULE 3: KEY VISUAL CHARACTERISTICS (Interpretation & Bias)

VISUALIZING PROPERTIES: SCALE INDEPENDENCE & ASYMMETRY

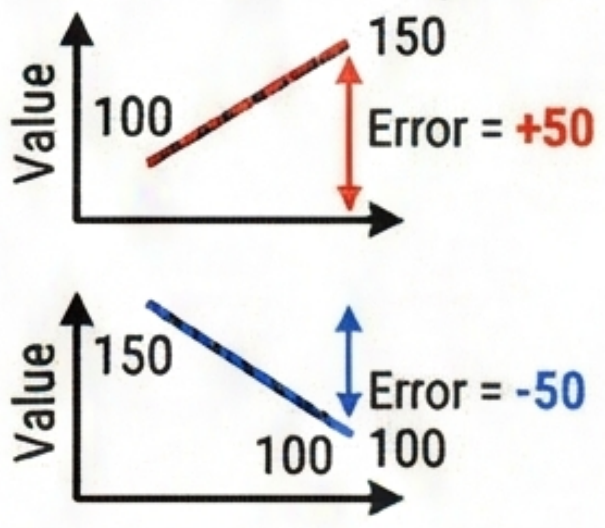
SCALE INDEPENDENCE
(Comparable Across Series)



Different scales, same relative error = Same APE contribution. Useful for comparing performance across different products/markets.

ASYMMETRY
(Penalty for Over-forecasting)

Over-forecast ($F_t > Y_t$)   Under-forecast ($F_t < Y_t$)



$$APE = \left|\frac{100 - 150}{100}\right| * 100 = 50\%$$

$$APE = \left|\frac{150 - 100}{150}\right| * 100 = 33.3\%$$

Vermilion Alert: Equal absolute errors result in higher penalties for over-forecasts. MAPE is biased towards under-forecasting.

MODULE 4: MANAGEMENT IMPLICATION (Limitations & Use Case)

MANAGEMENT INSIGHT: CRITICAL LIMITATION & BEST USE

CRITICAL LIMITATION
(Division by Zero)

$$APE_t = \left|\frac{Y_t - F_t}{0}\right| * 100$$

UNDEFINED / INFINITE ERROR ✗

Cannot be calculated when Actual value (Y_t) is zero. Problematic for intermittent demand data.

BEST USE CASE
(Business Context)

Best suited for comparing forecast accuracy across different datasets or time series with varying scales, *provided* there are no zero values.

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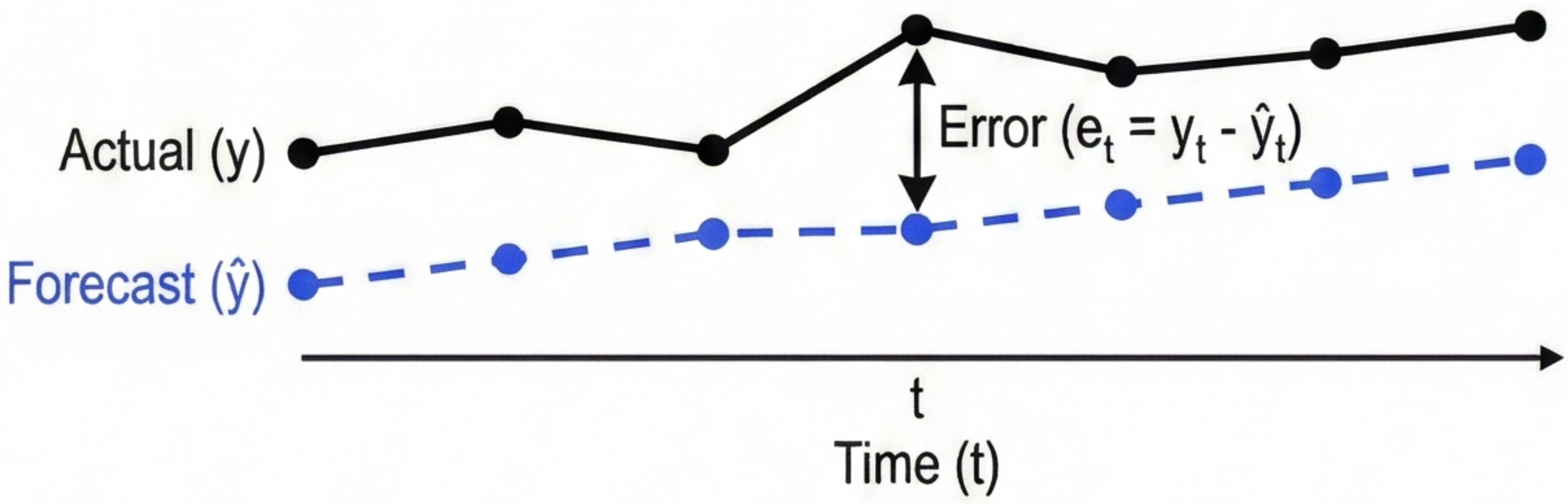
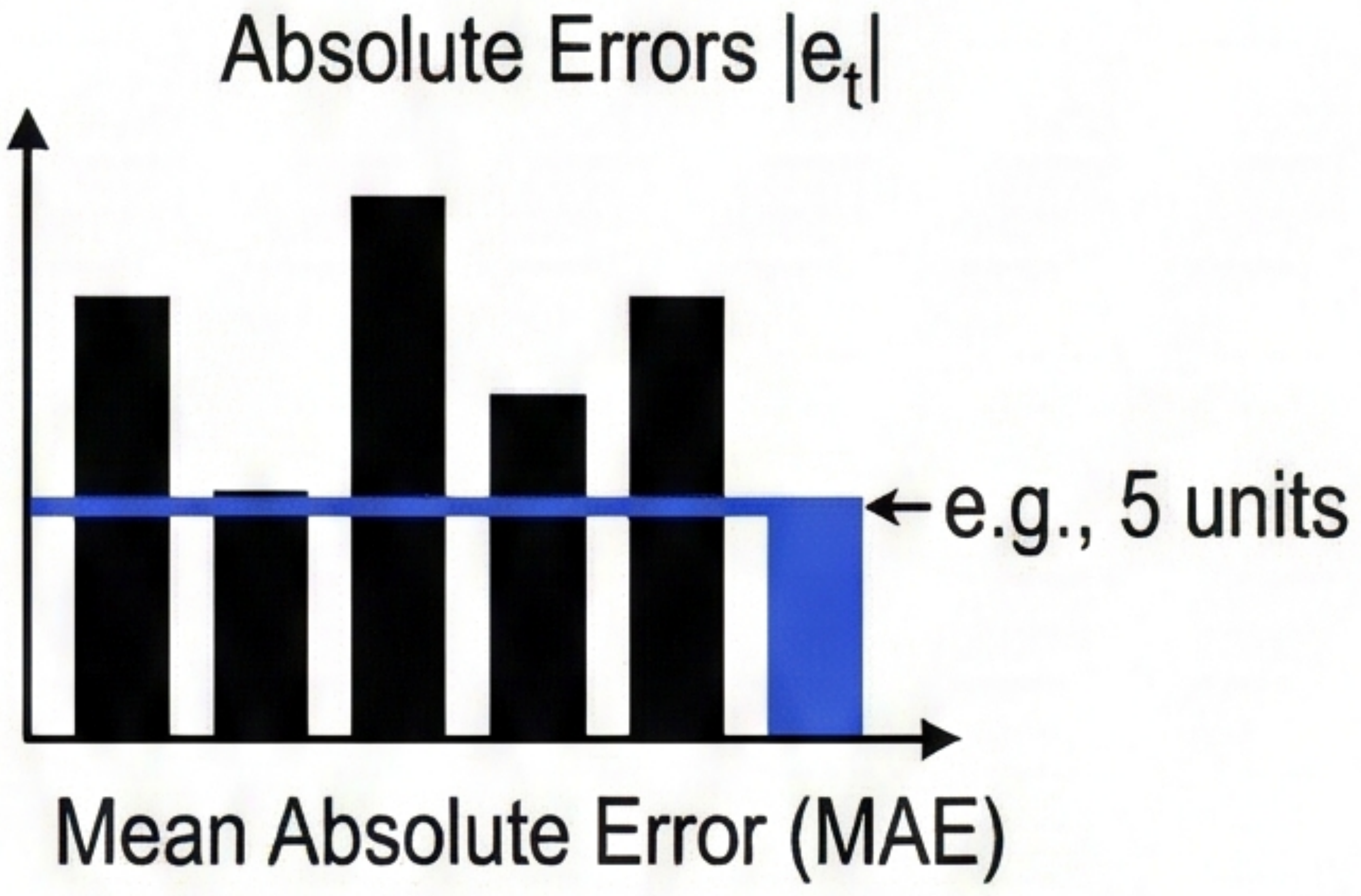
MAE vs. MAPE: Time Series Forecasting Error Metrics

1. Fundamental Concepts: Actual, Forecast, and Error



2. Mean Absolute Error (MAE): Absolute Magnitude

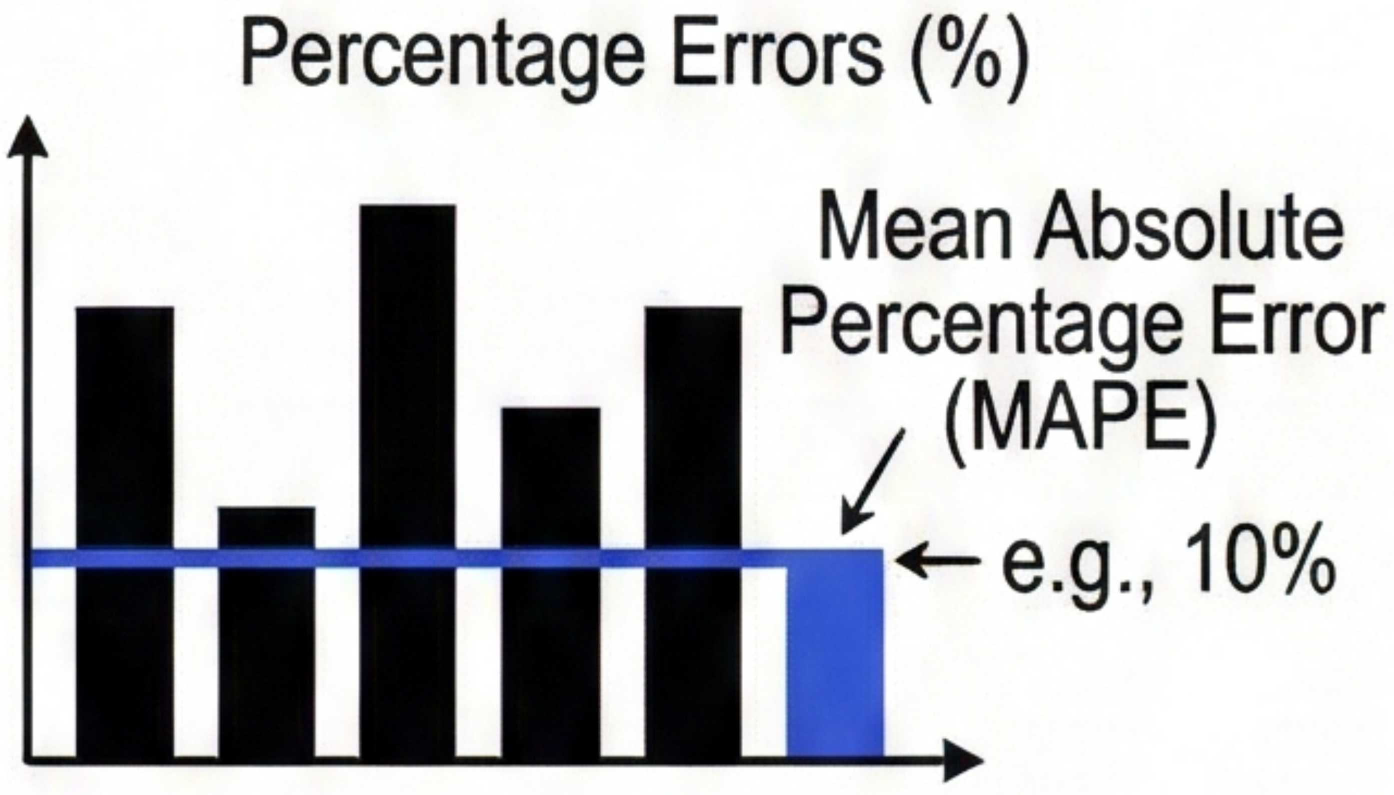
$$\text{MAE} = \underbrace{\frac{1}{n}}_{\text{Average}} \underbrace{\sum_t}_{\text{Sum over all } t} \underbrace{|y_t - \hat{y}_t|}_{\text{Absolute Error at time } t}$$



- **Pros:** Simple interpretation (same units as data). Robust to outliers.
- **Cons:** Scale-dependent (not comparable across different data scales).

3. Mean Absolute Percentage Error (MAPE): Relative Percentage

$$\text{MAPE} = \frac{1}{n} \sum_t \frac{\overbrace{|y_t - \hat{y}_t|}^{\text{Relative Error}}}{\underbrace{|y_t|}_{\text{Actual Value (Denominator)}}} \times 100\%$$



- **Pros:** Scale-independent (comparable across datasets). Percentage is intuitive.
- **Cons:** Undefined when Actual (y) = 0. Biased towards low forecasts. Skewed by small Actuals. !

4. Comparative Summary & Usage Guide

Metric	Key Feature	When to Use
MAE	Absolute Scale (Units)	Prioritizing physical magnitude of error.
MAPE	Relative Percentage (%)	Comparing performance across different scales/series.

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