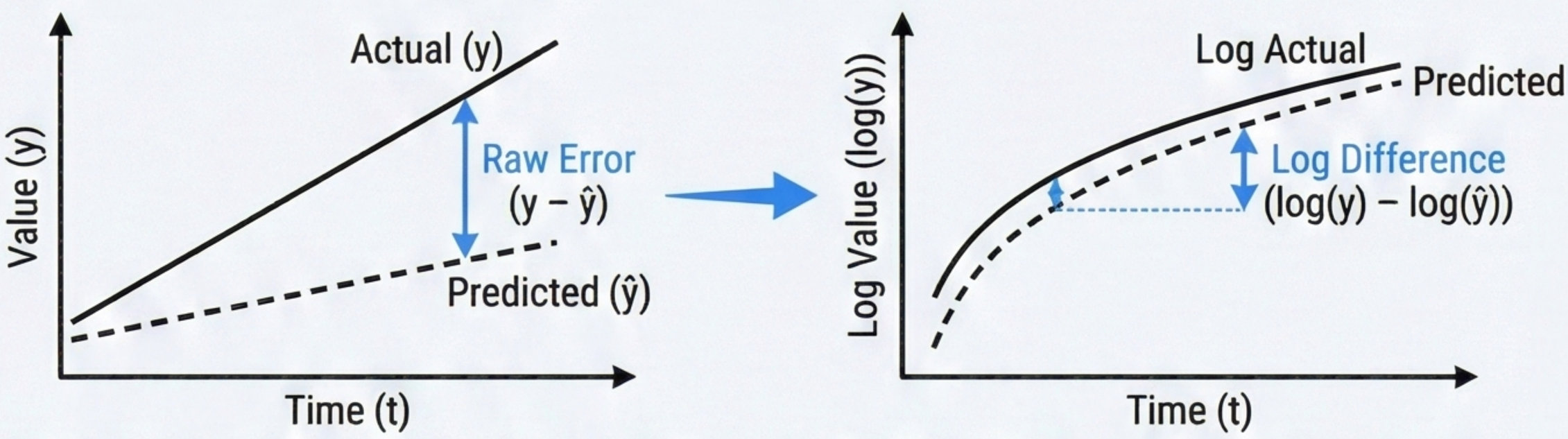
MEAN ABSOLUTE LOG ERROR (MALE) in TIME SERIES FORECASTING

A Metric for Evaluating Forecast Accuracy with Logarithmic Scale Transformation, Emphasizing Relative Errors

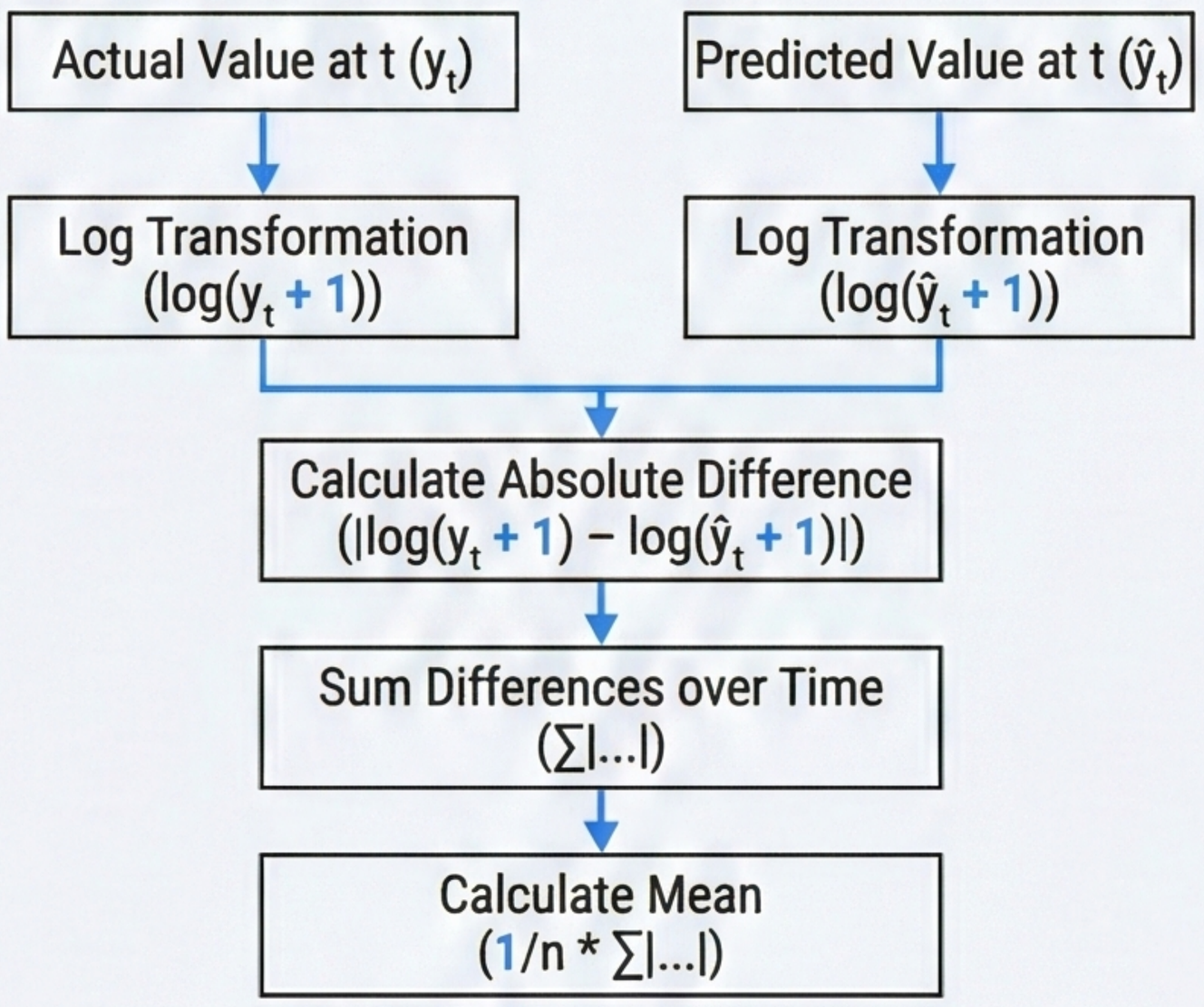
MODULE 1: CORE CONCEPT - LOGARITHMIC TRANSFORMATION & RELATIVE ERROR

1. CORE CONCEPT: LOGARITHMIC DIFFERENCE



MALE measures the average absolute difference between the logarithms of predicted and actual values, emphasizing relative percentage errors over absolute scale.

MODULE 2: CALCULATION FLOW (STEP-BY-STEP)

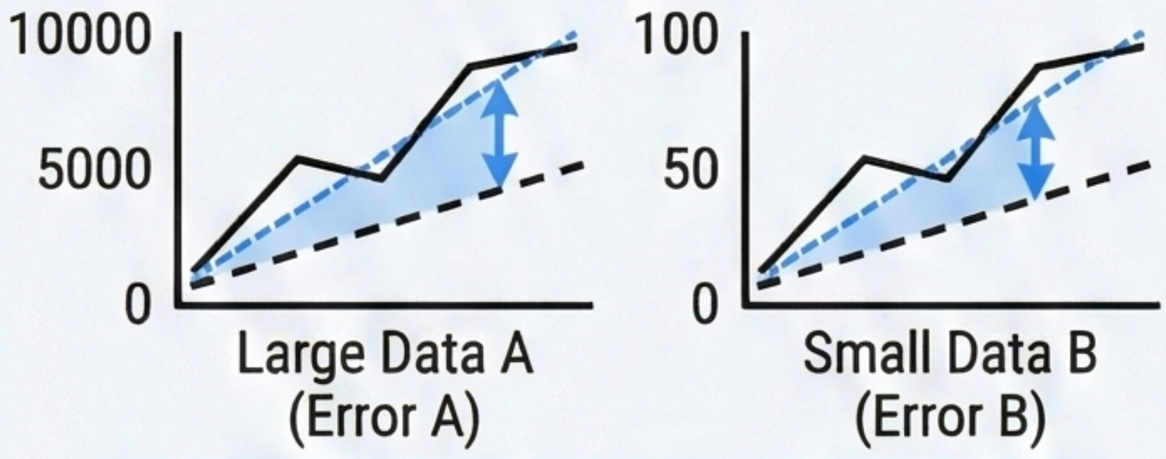


⚠ **Alert:** Requires positive values ($y_t, \hat{y}_t \geq 0$). Adding 1 handles zeros.

$$\text{MALE} = \frac{1}{n} * \sum_{t=1}^{n} | \log(y_t + 1) - \log(\hat{y}_t + 1) |$$

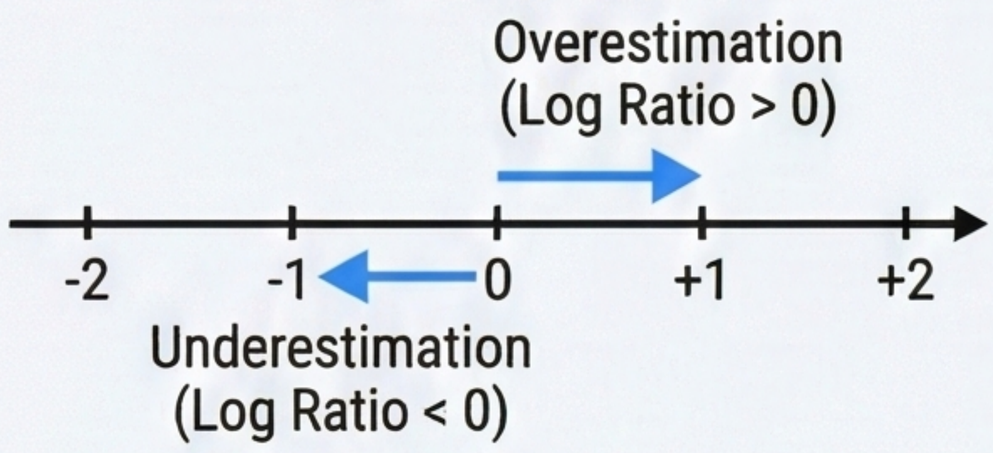
3. KEY CHARACTERISTICS (SCALE INDEPENDENCE & SYMMETRY)

Scale Independence



Similar MALE values for proportionately equal errors across different scales.

Symmetry on Log Scale



Treats over- and under-predictions symmetrically in percentage terms.

4. USE CASES & LIMITATIONS

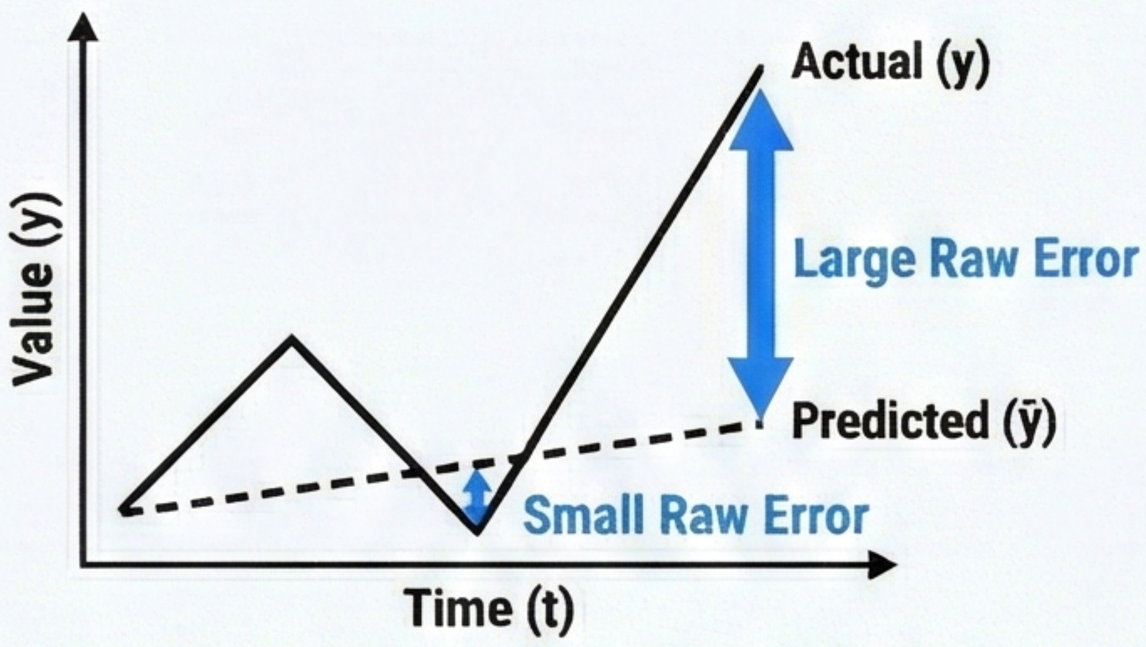
Financial Forecasting: Evaluating stock prices or revenue with large variations.	⚠ **Alert: Zero Values:** Requires adding a constant (e.g., +1) to handle y=0 or ŷ=0.
Demand Planning: Assessing product demand across diverse markets and volumes.	**Interpretation:** Less intuitive for non-technical stakeholders than MAE or RMSE.
Economic Data: Comparing growth rates and indices across different economies.	⚠ **Negative Values:** Not defined for negative data (requires shifting or alternative metric).

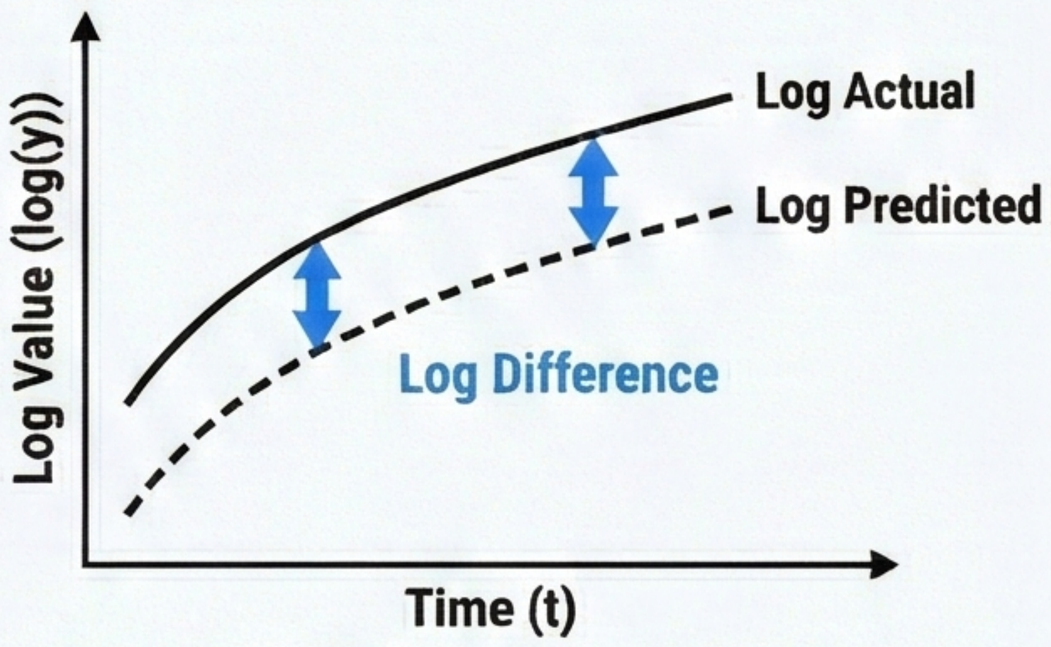
Zbigniew Galar

ROOT MEAN SQUARED LOG ERROR (RMSLE) in TIME SERIES FORECASTING

A Metric for Evaluating Forecast Accuracy with Logarithmic Scale Transformation, Penalizing Relative Errors

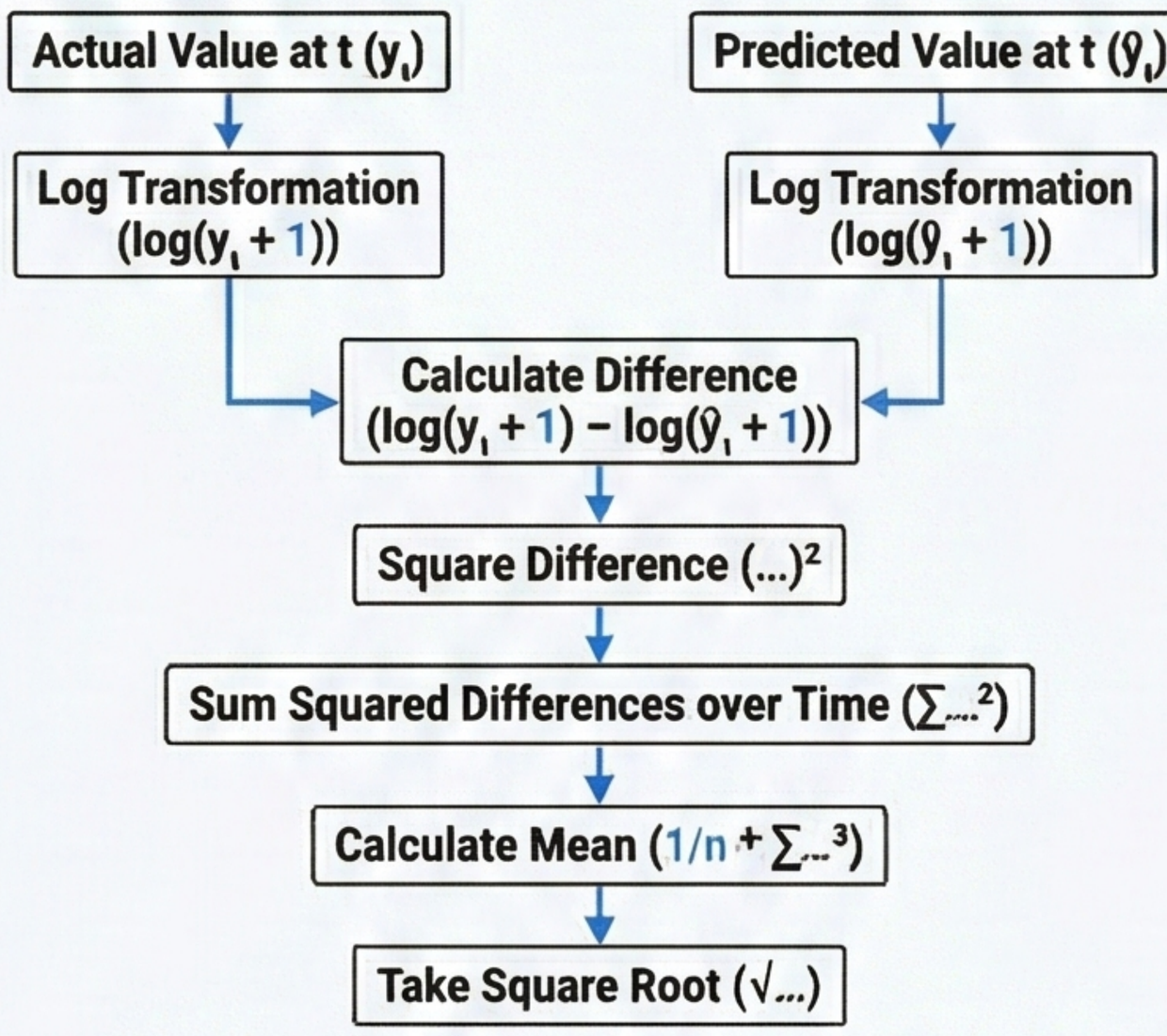
1. CORE CONCEPT: LOGARITHMIC TRANSFORMATION & RELATIVE ERROR





RMSLE measures the root mean squared difference between the logarithms of predicted and actual values, dampening the impact of large errors and focusing on relative differences.

2. CALCULATION FLOW (STEP-BY-STEP)

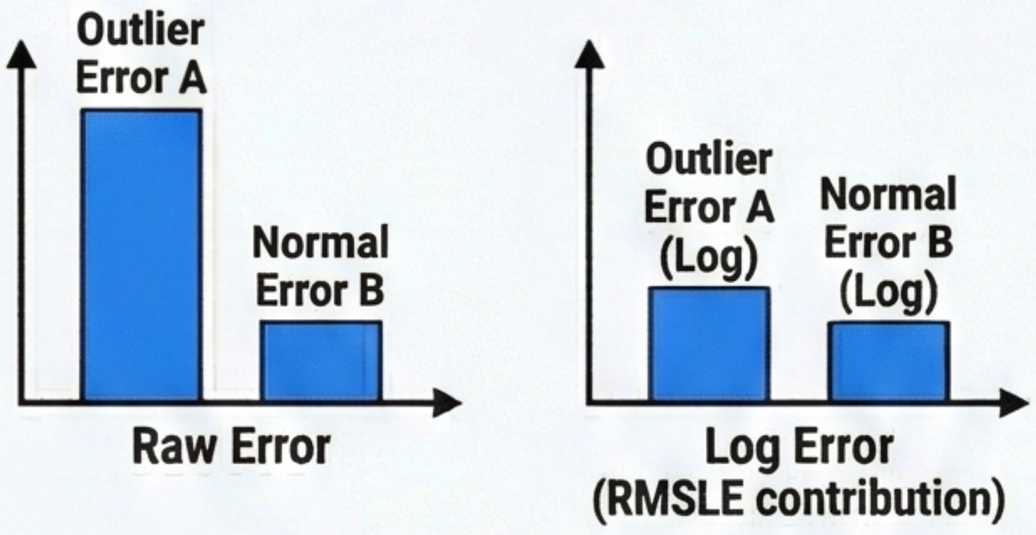


⚠ **Alert:** Requires non-negative values ($y_t, \hat{y}_t \geq 0$). Adding 1 handles zeros.

$$RMSLE = \sqrt{\frac{1}{n} \cdot \sum_{t=1}^{n} (\log(y_t + 1) - \log(\hat{y}_t + 1))^2}$$

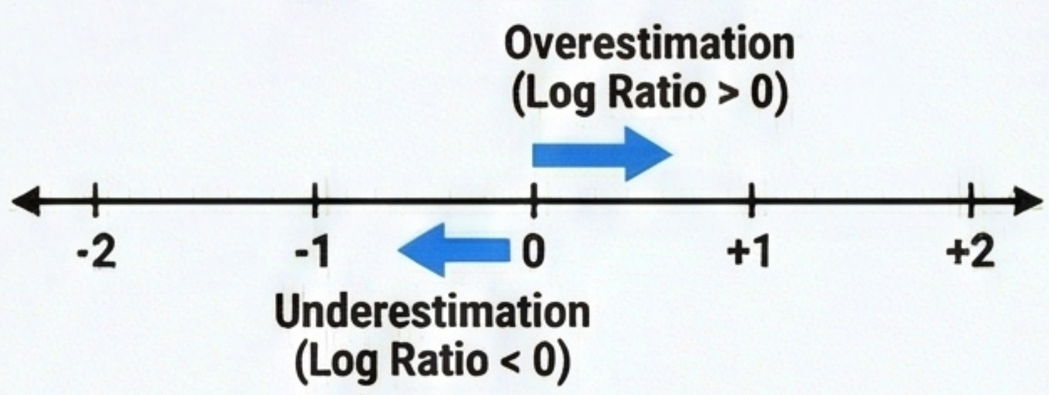
3. KEY CHARACTERISTICS (ROBUSTNESS & ASYMMETRY)

Robustness to Large Values



Log transformation dampens the effect of large values, making RMSLE robust to outliers compared to RMSE.

Asymmetry in Raw Space



Treats over- and under-predictions symmetrically in log space, which translates to asymmetrical treatment in raw space (penalizes under-prediction less than over-prediction for the same absolute error).

4. USE CASES & LIMITATIONS

USE CASES	LIMITATIONS
Demand Forecasting: Evaluating accuracy when products have wide range of sales volumes.	⚠ **Alert: Zero Values:** Requires adding a constant (e.g., +1) to handle y=0 or ŷ=0.
Financial Modeling: Assessing models for variables like stock prices or revenue with exponential growth.	**Interpretation:** Less intuitive for non-technical stakeholders than MAE or RMSE due to log scale.
Count Data: Analyzing errors in forecasting data with a large dynamic range (e.g., web traffic).	⚠ **Negative Values:** Not defined for negative data (requires shifting or alternative metric).

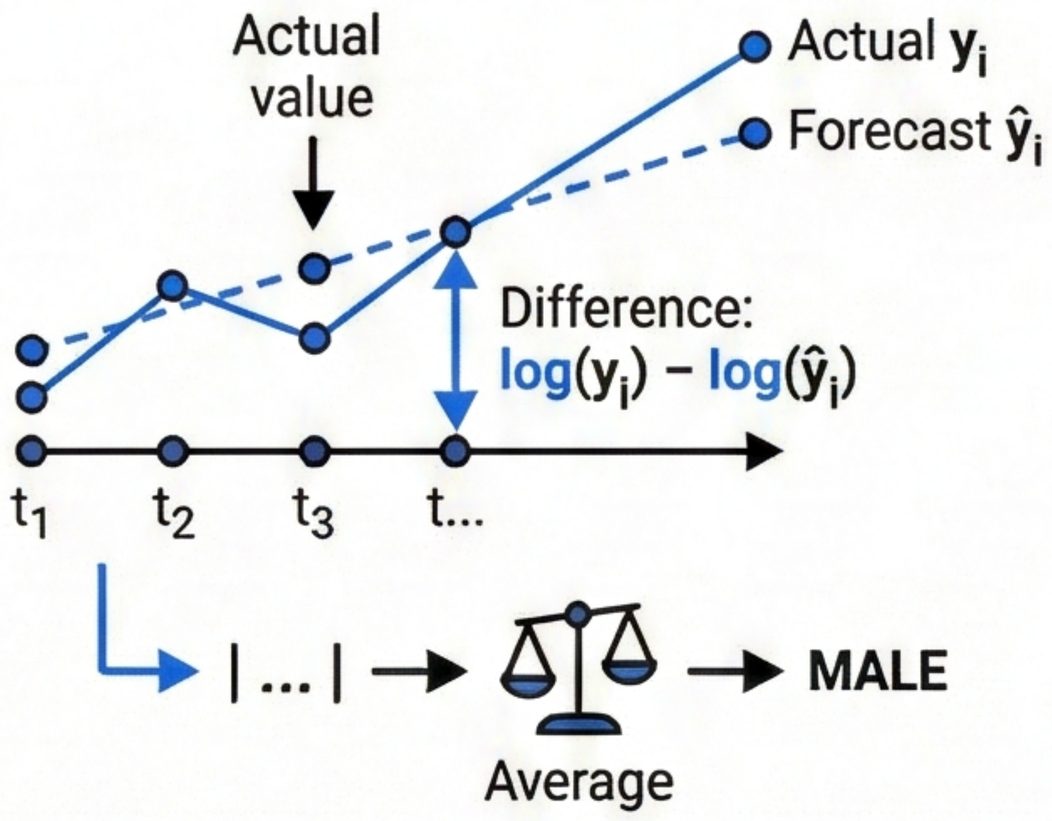
Zbigniew Galar

MALE vs. RMSLE in Monthly Time Series Forecasting

DEFINITIONS & FORMULAS

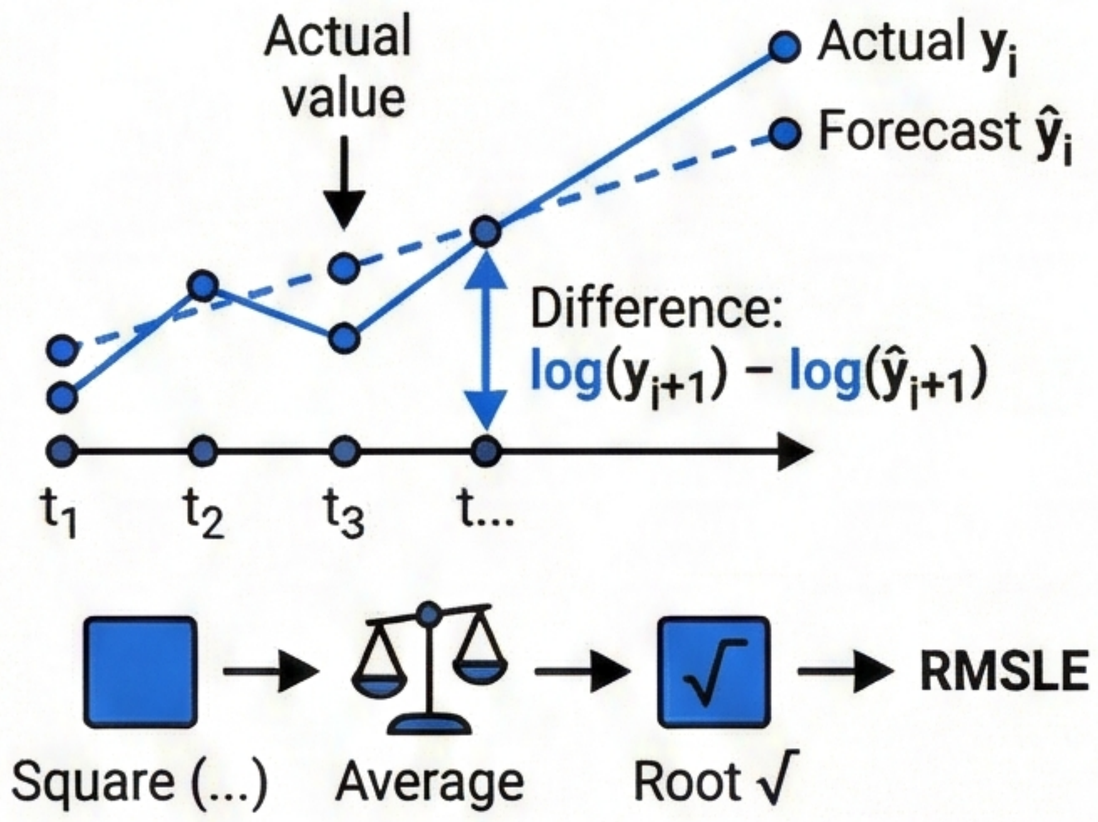
Mean Absolute Logarithmic Error (MALE)

$$\text{MALE} = \frac{1}{N}\sum_{i=1}^{N}|\log(y_i) - \log(\hat{y}_i)|$$



Root Mean Squared Logarithmic Error (RMSLE)

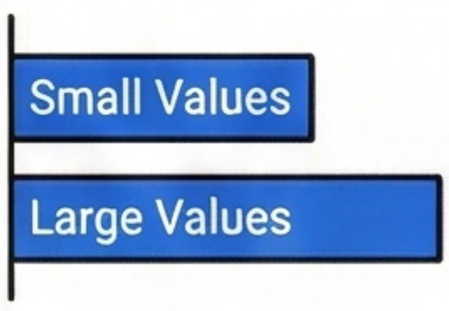
$$\text{RMSLE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\log(y_{i+1}) - \log(\hat{y}_{i+1}))^2}$$



PROPERTIES & SENSITIVITY

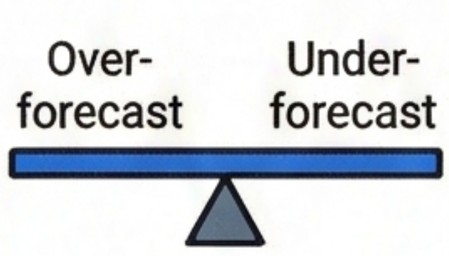
MALE (Symmetric/Absolute)

SCALE



Relative Error is Scale-Independent

FORECAST ERRORS



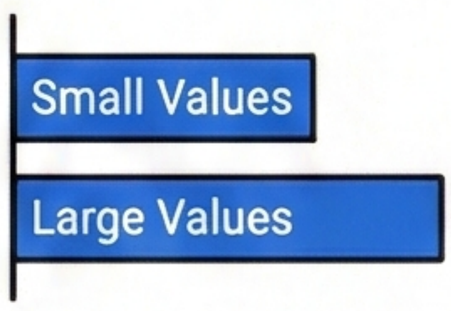
Penalizes Over- and Under-forecasts equally

HANDLING ZEROS

⚠ Undefined for $y_i=0$ or $\hat{y}_i=0$

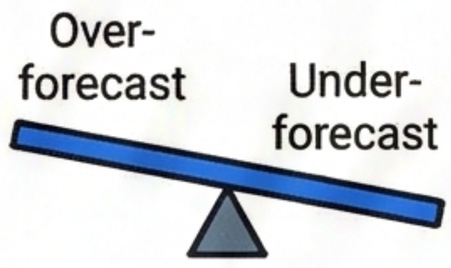
RMSLE (Asymmetric/Squared)

SCALE



Relative Error is Scale-Independent (via log)

FORECAST ERRORS



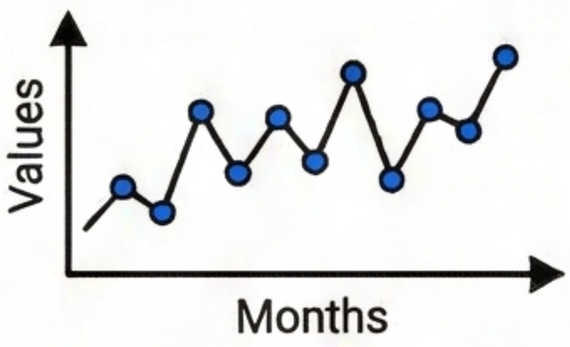
Penalizes Under-forecasts more heavily

HANDLING ZEROS

Uses $\log(x+1)$; Defined for $y_i \geq 0$, $\hat{y}_i \geq 0$

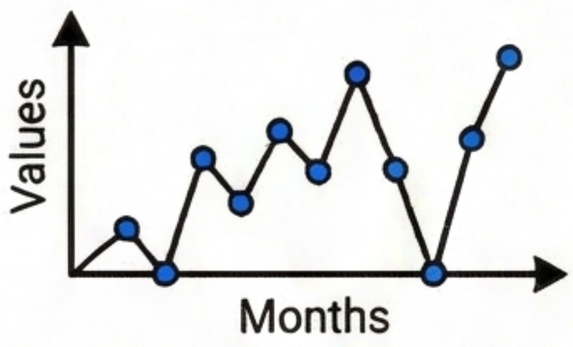
APPLICATION IN MONTHLY TIME SERIES

USE MALE WHEN:



Data is strictly positive. Equal penalty for positive and negative errors is desired. Focus is on **median relative error**.

USE RMSLE WHEN:

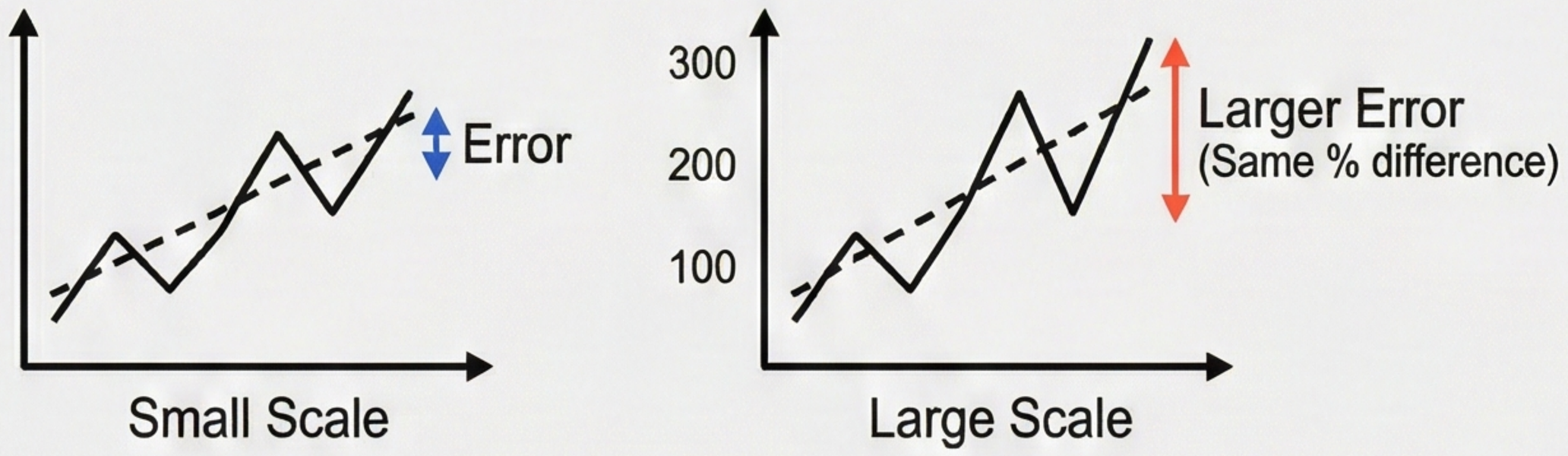


Data may contain zeros. Avoiding under-forecasting is critical (e.g., inventory). Focus is on **mean relative error** and **penalizing large errors**.

Zbigniew Galar

MALE (Mean Absolute Logarithmic Error) in Time Series Forecasting: Natural Logarithm Base

PROBLEM: Scale-Dependent Errors



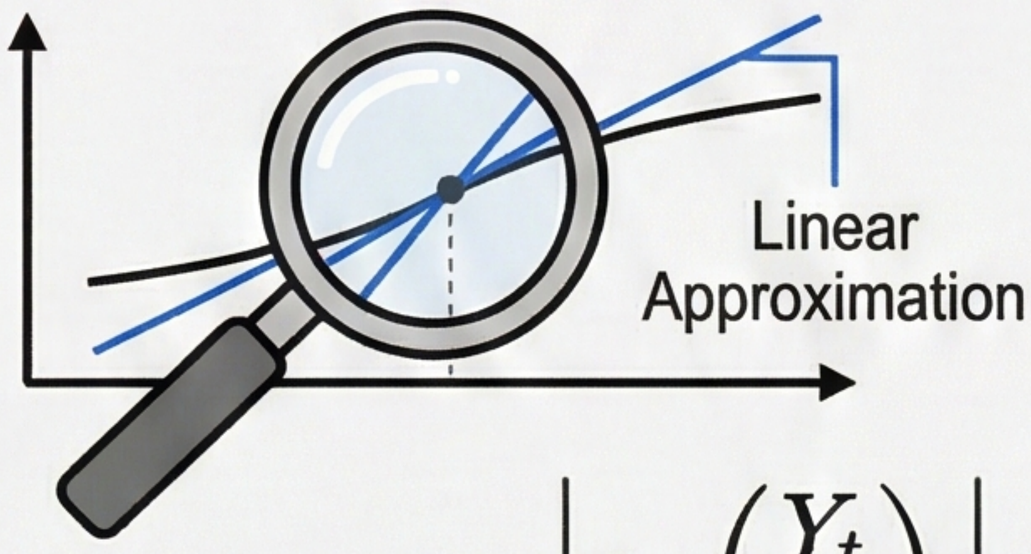
Absolute errors (MAE) are sensitive to data magnitude, favoring smaller scales. Need a scale-independent metric.

SOLUTION: MALE & Natural Logarithm (ln)

$$\text{MALE} = \frac{1}{n}\sum \left| \ln\left(\frac{Y_t}{\hat{Y}_t}\right) \right|$$

- Y_t — Actual Value
- $\hat{Y}_t$ — Forecast Value
- Natural Logarithmic Error (Log Ratio)
- Absolute Log Error

CORE INSIGHT: Small Error Approximation



For small errors ($Y_t \approx \hat{Y}_t$), the log ratio is approximately the relative error.

$$\underbrace{\left| \ln\left(\frac{Y_t}{\hat{Y}_t}\right) \right|}_{\text{Absolute Natural Log Error}} \approx \underbrace{\left| \frac{Y_t - \hat{Y}_t}{\hat{Y}_t} \right|}_{\text{Absolute Relative Error}}$$

Therefore, **MALE** is roughly the Mean Absolute Percentage Error (MAPE) relative to the forecast, providing a clean interpretation for small deviations.

COMPARISON: Metric Behavior

Metric	Scale Sensitivity	Small Error Interpretation
MAE (Mean Absolute Error)	High	Absolute deviation
MAPE (Mean Absolute Percentage Error)	Low	Percentage deviation
MALE (ln Base)	Low	Approx. Relative Error

Note: Using base 10 introduces an extra factor (1/ln(10)), making the approximation less direct.

Zbigniew Galar