



a) $P(\text{strawberry}) =$

$$\begin{aligned}
 &P(\text{strawberry} \cap A) + P(\text{strawberry} \cap B) \\
 &= P(A)P(\text{strawberry}|A) + P(B)P(\text{strawberry}|B) \\
 &= (0.75)(0.5) + (0.25)(0.6) \\
 &= \underline{0.525}
 \end{aligned}$$

b) $P(\text{choc}) = P(\text{choc} \cap A) + P(\text{choc} \cap B)$

$$\begin{aligned}
 &= P(A)P(\text{choc}|A) + P(B)P(\text{choc}|B) \\
 &= 0.75(0) + 0.25(0.4) \\
 &= \underline{0.1}
 \end{aligned}$$

c. i) $P(A | \text{strawberry}) = \frac{P(\text{strawberry} | A) P(A)}{P(\text{strawberry})}$

$$= \frac{(0.5)(0.75)}{(0.525)} = \underline{\frac{5}{7}}$$

ii) $P(\text{next} = \text{grass} | \text{strawberry}) =$

$$\frac{P(\text{next} = \text{grass} \cap \text{strawberry})}{P(\text{strawberry})} =$$

$$\frac{P(\text{next} = \text{grass}) P(\text{strawberry} | A) P(A)}{P(\text{strawberry})} =$$

$$= \left(\frac{5}{9} \cdot 0.5 \cdot 0.75 \right) (0.525)^{-1} = \underline{\frac{25}{63}}$$

2. a)

OUTCOME	PROBABILITY
$C_1=1, H=0$	$1/5$
$C_1=1, H=1$	0
$C_2=1, H=0$	$3/20$
$C_2=1, H=1$	$1/20$
$C_3=1, H=0$	$1/10$
$C_3=1, H=1$	$1/10$
$C_4=1, H=0$	$1/20$
$C_4=1, H=1$	$3/20$
$C_5=1, H=0$	0
$C_5=1, H=1$	$1/5$

$$P(H) = 1/2$$

b)

$$P(C_1|H) = P(C_1, H=1) P(H)^{-1}$$

$$= (0) (1/2)^{-1} = \boxed{0}$$

$$P(C_2|H) = P(C_2, H=1) P(H)^{-1}$$

$$= (1/20) (1/2)^{-1} = \boxed{1/10}$$

$$P(C_3|H) = P(C_3, H=1) P(H)^{-1}$$

$$= (1/10) (1/2)^{-1} = \boxed{1/5}$$

$$P(C_4|H) = P(C_4, H=1) P(H)^{-1}$$

$$= (3/20) (1/2)^{-1} = \boxed{3/10}$$

$$P(C_5|H) = P(C_5, H=1) P(H)^{-1}$$

$$= (1/5) (1/2)^{-1} = \boxed{2/5}$$

c)

$$P(H_2|H_1) = P(C_5|H_1) P(H_2|C_5) + P(C_4|H_1) P(H_2|C_4)$$

$$+ P(C_3|H_1) P(H_2|C_3) + P(C_2|H_1) P(H_2|C_2)$$

$$+ P(C_1|H_1) P(H_2|C_1)$$

$$= (2/5)(1/5) + (3/10)(3/20) + (1/5)(1/10) + (1/10)(1/20)$$

$$+ 0$$

$$= \frac{16}{200} + \frac{9}{200} + \frac{4}{200} + \frac{1}{200} = \frac{30}{200} = \boxed{\frac{3}{20}}$$

$$2d) P(C_i | B_4) = \frac{P(B_4 | C_i) P(C_i)}{P(B_4)}$$

$$\begin{aligned} P(B_4) &= P(T_1) P(T_2 | T_1) P(T_3 | T_2, T_1) P(H_4 | T_3, T_2, T_1) \\ &= (0.5) \left(\frac{3}{20}\right) \left(\frac{1}{4}\right) \left(\frac{1}{5}\right) \\ &= \frac{3}{160} \end{aligned}$$

$$P(C_i | B_4) = \frac{P(T_1 | C_i) P(T_2 | C_i) P(T_3 | C_i) P(H_4 | C_i) P(C_i)}{P(B_4)}$$

$$P(C_1 | B_4) = (1)(1)(1)(0)\left(\frac{1}{5}\right)\left(\frac{1}{16}\right)^{-1} = 0$$

$$P(C_2 | B_4) = \left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{5}\right)\left(\frac{1}{16}\right)^{-1} = \frac{27}{80}$$

$$P(C_3 | B_4) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{5}\right)\left(\frac{1}{16}\right)^{-1} = \frac{1}{5}$$

$$P(C_4 | B_4) = \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\left(\frac{1}{5}\right)\left(\frac{1}{16}\right)^{-1} = \frac{3}{80}$$

$$P(C_5 | B_4) = (0)(0)(0)(1)\left(\frac{1}{5}\right)\left(\frac{1}{16}\right)^{-1} = 0$$

3. a) There are 4 genres, 3 pub. dates, and 2 cover types. This means there are $4 \cdot 3 \cdot 2 = 24$ rows in the probability table.

b) i. $P_{s1}(G, D, C) = P_{s1}(D) P_{s1}(G|D) P_{s1}(C|D)$

ii. $\text{sizeof}(D) = 3$

$\text{sizeof}(D) \cdot \text{sizeof}(G) = 12$

$\text{sizeof}(D) \cdot \text{sizeof}(C) = 6$

total rows = $3 + 12 + 6 = 21$

c) i. $P_{s2}(GDC) = P_{s2}(C)(P_{s2}(G = \text{Music}, 1800) + P_{s2}(\text{Romance}, 2000) + P_{s2}(1900)P_{s2}(\text{History OR Philosophy}))$

ii. $\text{sizeof}(C) = 2$

$\text{sizeof}(\text{Music}, 1800) = 1$

$\text{sizeof}(\text{Rom}, 2000) = 1$

$\text{sizeof}(\text{History OR Philosophy}, 1900) = 2$

total rows = $2 + 1 + 1 + 1 = 5$