CSE384 QUIZ 1

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2. $\forall x((\forall y(P_1(x,y) \rightarrow P_2(y) \vee P_3(y)))$

$\longrightarrow ((\forall y(P_1(x,y) \rightarrow P_2(y))) \vee (\forall y(P_1(x,y) \rightarrow P_3(y)))))$

First we simplify the implication symbols highlighted in green.

$\forall x(\neg(\forall y(\neg P_1(x,y) \vee P_2(y) \vee P_3(y)))$

$\vee ((\forall y(\neg P_1(x,y) \vee P_2(y))) \vee (\forall y(\neg P_1(x,y) \vee P_3(y)))))$

We can then coalesce the second and third terms disjuncted together.

$\forall x(\neg(\forall y(\neg P_1(x,y) \vee P_2(y) \vee P_3(y)))$

$\vee(\forall y(\neg P_1(x,y) \vee P_2(y) \vee \neg P_1(x,y) \vee P_3(y))))$

We can then simplify with the identity $x \vee x = x$

$\forall x(\neg(\forall y(\neg P_1(x,y) \vee P_2(y) \vee P_3(y)))$

$\vee(\forall y(\neg P_1(x,y) \vee P_2(y) \vee P_3(y))))$

And then we substitute $P_4(x,y)$ for $\neg P_1(x,y) \vee P_2(y) \vee P_3(y)$

$\forall x(\neg(\forall y\ P_4(x,y)) \vee (\forall y\ P_4(x,y)))$

We can therefore see this is a tautology as $\forall x \forall y \ \neg P_4 \vee P_4$ is always true. QED.