

Name: \_\_\_\_\_

Student ID: \_\_\_\_\_

All code that was added to complete this assignment was written by me.

signed: \_\_\_\_\_

A handwritten signature in black ink, appearing to be 'Z. L. Paul', written over a horizontal line.

1 a) My model of  $\Phi$  is as follows:

|       |
|-------|
| $B_1$ |
| $B_2$ |
| $B_3$ |

In this scenario, in English, we would describe  $B_2$  as under  $B_1$ ,  $B_3$  as under  $B_2$ , and  $B_3$  as under  $B_1$ . In other words, the English description of under is transitive.

However, this model and the predicate symbol  $\text{under}(y, x)$  do not satisfy the English def'n:

$$\text{Under}(B_3, B_1) \leftrightarrow (B_1 \text{ is immediately above } B_3) \vee (B_1 \text{ is not above any blocks} \wedge B_1 = B_3)$$

$B_1$  is obviously above  $B_2$  and  $B_3$  by definition and is not equal to  $B_3$  obviously. We can also see  $B_2$  is immediately above  $B_3$ , so  $B_1$  is obviously not. We can substitute false for these statements:

$$\text{Under}(B_3, B_1) \leftrightarrow (\text{false}) \vee (\text{false} \wedge \text{false})$$

$$\text{Under}(B_3, B_1) \leftrightarrow \text{false}$$

Therefore by definition  $B_3$  is not under  $B_1$ , which conflicts with the English definition.

$$2. \quad \forall x ((\forall y (P_1(x, y) \rightarrow P_2(y) \vee P_3(y))) \rightarrow ((\forall y (P_1(x, y) \rightarrow P_2(y))) \vee (\forall y (P_1(x, y) \rightarrow P_3(y))))))$$

First we simplify the implication symbols highlighted in green.

$$\forall x (\neg (\neg \forall y (\neg P_1(x, y) \vee P_2(y) \vee P_3(y))) \vee ((\forall y (\neg P_1(x, y) \vee P_2(y))) \vee (\forall y (\neg P_1(x, y) \vee P_3(y))))))$$

We can then coalesce the second and third terms disjoined together.

$$\forall x (\neg (\neg \forall y (\neg P_1(x, y) \vee P_2(y) \vee P_3(y))) \vee (\forall y (\neg P_1(x, y) \vee P_2(y) \vee \neg P_1(x, y) \vee P_3(y))))$$

We can then simplify with the identity  $x \vee x = x$

$$\forall x (\neg (\neg \forall y (\neg P_1(x, y) \vee P_2(y) \vee P_3(y))) \vee (\forall y (\neg P_1(x, y) \vee P_2(y) \vee P_3(y))))$$

And then we substitute  $P_4(x, y)$  for  $\neg P_1(x, y) \vee P_2(y) \vee P_3(y)$

$$\forall x (\neg (\neg \forall y P_4(x, y)) \vee (\forall y P_4(x, y)))$$

We can therefore see this is a tautology as

$$\forall x \forall y \neg P_4 \vee P_4 \text{ is always true. QED.}$$



3. a)

$$\bullet \exists x ((\text{Murder}(x, M, L, T) \wedge (\forall y (\text{Murder}(y, M, L, T) \rightarrow (y = x))))))$$

$$\bullet \forall x (A(x, L, T) \rightarrow O(x))$$

$$\bullet A(P, L, T)$$

$$\bullet \exists x, \exists y (Mo(x) \wedge Mo(y) \wedge (x \neq y))$$

$$\bullet \text{Murder}(x, M, L, T) \leftrightarrow (Mo(x) \wedge O(x))$$

$$b) \text{Murder}(P, M, L, T) \leftrightarrow (Mo(P) \wedge O(P))$$

$$A(P, L, T) \rightarrow O(P)$$

$$\boxed{Mo(P)?}$$

However, even though Mrs. P had an opportunity to murder Mr. M, we do not know from the given information that Mrs. P had a motive to kill Mr. M. Therefore, we cannot conclude that Mrs. P murdered Mr. M.