Computer Science 384
St. George Campus

Monday, March 07, 2022
University of Toronto

Take-Home Quiz #1: Knowledge Representation and Reasoning
Due: March 14, 2022 by 11:59 PM

Late Policy: You may use grace days to hand in this Take-Home Quiz after the due date, without penalty. (Recall you were given 5 grace days at the beginning of term.) Quizzes may not be handed in late otherwise. You do ***NOT*** need to notify us about using your grace days.

What to hand in on paper: Nothing.

What to submit electronically: **Submit written answers in a file called** `answers.pdf` **as well as** `acknowledgment_form.pdf` using MarkUs. Assignments must be completed ***individually***, and you must submit a single document. Handwritten submissions are acceptable as long as they are written ***neatly*** and ***legibly*** (typed submissions are preferred but **not** required).

How to submit: You will submit your assignment using MarkUs. Your login to MarkUs is your teach.cs username and password. It is your responsibility to include all necessary files in your submission. You can submit a new version of any file at any time, following the above noted restrictions on late submissions. For the purposes of determining the number of grace days required for a late submission, the submission time is considered to be the time of your latest submission. More detailed instructions for using MarkUs are available at: https://markus.teach.cs.toronto.edu/2022-01.

Clarifications: Important corrections (hopefully few or none) and clarifications to the assignment will be posted on Quercus.

Questions: Questions about the assignment should be posted to Piazza.

1. **(13 points)** Consider the blocks world domain (NBW), and suppose the vocabulary of the domain, $\mathscr{L}_{NBW}$ includes the following symbols:

   Predicate Symbols:

   - $above(x,y)$ iff x is above y, either directly or indirectly.
   - $under(y,x)$ iff ((x is the **unique block immediately above** block y) or (x is not above any blocks and $y = x$)).
   - $clear(x)$ iff no blocks are above x.
   - $ontable(x)$ iff x is not above any blocks.

   Let Φ be a set containing the following sentences:

   - $\forall x(\neg above(x,x))$
   - $\forall x\forall y\forall z((above(x,y) \wedge above(x,z) \wedge \neg(y=z)) \rightarrow (above(z,y) \vee above(y,z)))$
   - $\forall x\forall y\forall z((above(x,y) \wedge above(y,z)) \rightarrow above(x,z))$

   (a) **(5 points)** Construct a model of Φ with size three (i.e., that includes 3 blocks) which does not satisfy the English description of *under*.

   (b) **(8 points)** Modify Φ, without modifying the vocabulary, so that the models of the resulting set of sentences would be those structures that satisfy the English definition of *under*, *clear* and *ontable*.

2. **(10 points)** A **tautology** is a formula that is true in every possible structure. The sentence $(\forall x)(P(x) \vee \neg P(x))$ is an example of a tautology.
   Determine whether or not the following sentence is a tautology. **Justify** your answer.

$$\forall x\Big[\Big(\forall y\big(P_1(x,y) \rightarrow (P_2(y) \vee P_3(y))\big)\Big) \rightarrow \Big(\big(\forall y(P_1(x,y) \rightarrow P_2(y))\big) \vee \big(\forall y(P_1(x,y) \rightarrow P_3(y))\big)\Big)\Big]$$

3. **(17 points)** Consider the following vocabulary:
   $Murder(x, y, z, u)$: x murdered y at place z at time u
   $At(x, y, z)$: x was at place y at time z
   $O(x)$: x had an opportunity
   $M(x)$: x had a motive.
   Constant Symbols: M stands for Mr M.; P for Mrs P.; L for library; and T for 12 p.m.

   (a) **(7 points)**
   Translate the sentences below to first-order logic using the above-mentioned vocabulary.

   - Somebody murdered Mr M. in the library at 12 p.m., and there is exactly one person who is the murderer of Mr M.
   - Anybody who was in the library at 12 p.m. had an opportunity to murder Mr M.
   - Mrs P. was in the library at 12 p.m.
   - More than one person had a motive to murder Mr M.
   - A person is a murderer of Mr M. if only if they had a motive to murder Mr M. and an opportunity to do so.

   (b) **(10 points)**
   Do the sentences from part (a) logically entail that Mrs P. murdered Mr M.?
   Formally justify your answer.

4. **NOTE:** This question is for exercise purposes only and will ***NOT*** be marked.
   Do ***NOT*** include your solution to this exercise in your submission.

   Given predicates P, Q, R and variables x, y, z; convert the following sentences to clausal form: (Recall that the 8 steps are: eliminate implication, move negation inward, standardize variables, skolemize, convert to prenex, distribute $\wedge$ over $\vee$, flatten conjunctions and disjunctions, convert to clauses.) **Clearly indicate any Skolem functions or constants used in the conversion**. Note each formula might give rise to more than one clause so **number each clause generated**.

   (a) $\forall x.R(x) \rightarrow \exists y.P(x,y)$

   (b) $\exists x, \forall y.(Q(x,y) \wedge Q(y,x)) \vee \neg R(y)$

   (c) $\forall x, \forall w.\ \neg P(w,x) \rightarrow \exists z.(R(z) \wedge Q(x,z))$

   (d) $\exists y.R(y) \wedge \forall x.Q(y,x)$

5. **NOTE:** This question is for exercise purposes only and will ***NOT*** be marked.
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   For each of the pairs below, give the most general unifier (MGU) or state why no unifier exists. If a unifier exists, **provide the expression** that results from the unification. In all of the expressions that follow, `Move` is a 3-ary predicate name, functions and constants are lower case and variables are upper case. Remember that it can be helpful to line up the commas to figure out what needs to be unified.

   (a) $Move(f(X), f(h(Y)), g(h(Y)))$ and $Move(Z, f(h(a)), g(X))$

   (b) $Move(X, h(Y), f(X))$ and $Move(f(Y), h(b), f(g(a)))$

   (c) $Move(g(f(Y)), X, f(Z))$ and $Move(g(X), f(b), f(g(h(c))))$

HAVE FUN and GOOD LUCK!