



# 7. Secular Stagnation

Adv. Macro: Heterogenous Agent Models

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# Introduction

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- **Central economic questions:**
  1. How do aging populations affect interest rates and global imbalances?
  2. What will happen going forward?
  3. Should we be concerned about an asset market meltdown?

# Secular Stagnation

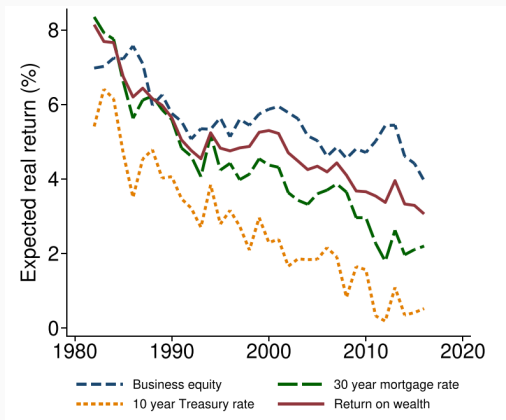
- **Today:** Can we explain *secular stagnation* through the lens of heterogeneous agent models?
  
- **Central economic questions:**
  1. How do aging populations affect interest rates and global imbalances?
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- **Plan for today:** Discuss two possible explanations for observed secular stagnation:
  1. Population aging
  2. Increase in income inequality

# Secular Stagnation

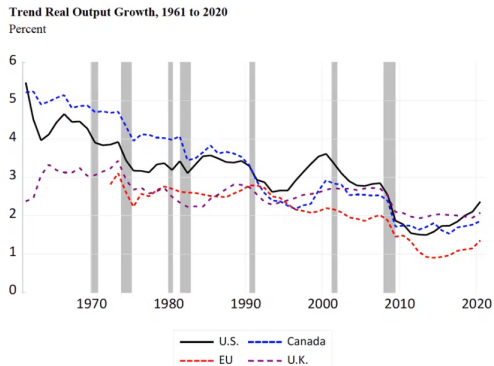
- **Secular stagnation:** A state in which private demand is structurally low
  - Low level of growth in the economy
  - Low interest rates
  - Low level of inflation
- Large literature suggests that advanced economies have been in this state over the past  $\approx 20$  years

# Declining interest rates

Various interest rates from Mian, Sufi, Straub (2021)



# Declining growth



Source: [www.cbo.gov/system/files/2020-12/56891-real%20interest-rates.pdf](https://www.cbo.gov/system/files/2020-12/56891-real%20interest-rates.pdf)

Data sources: Holston et al. (2017) and Federal Reserve Bank of New York, "Measuring the Natural Rate of Interest" (accessed December 10, 2020), [www.newyorkfed.org/research/policy/rstar](https://www.newyorkfed.org/research/policy/rstar).



# Explanations

Large literature on potential explanations

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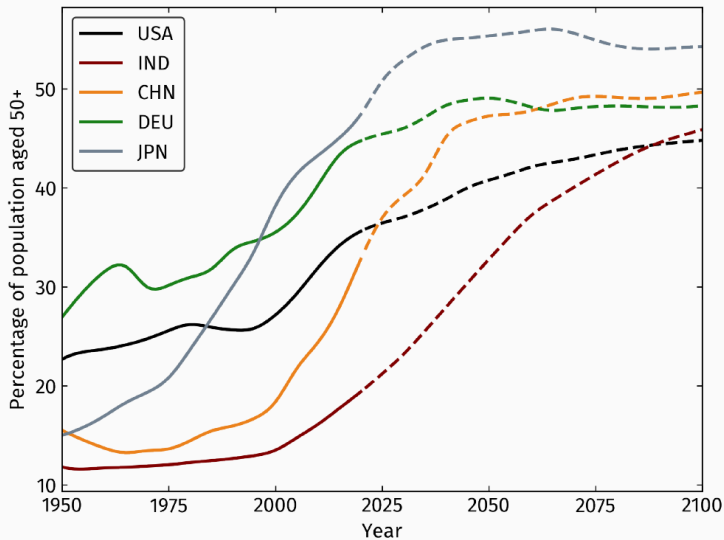
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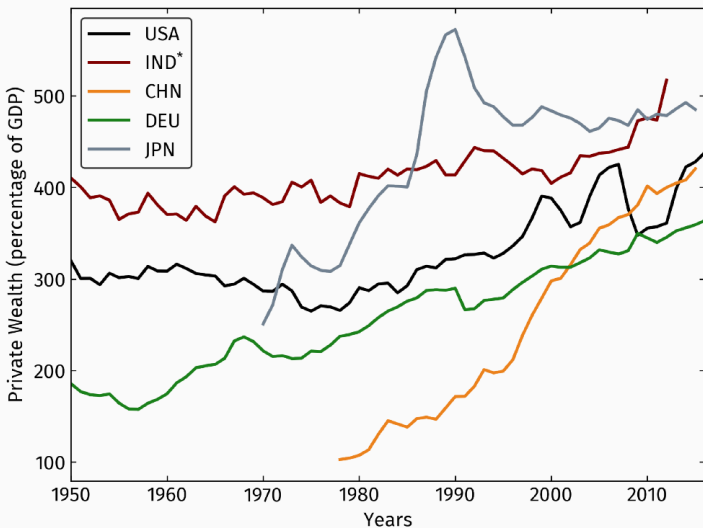
# Demographics

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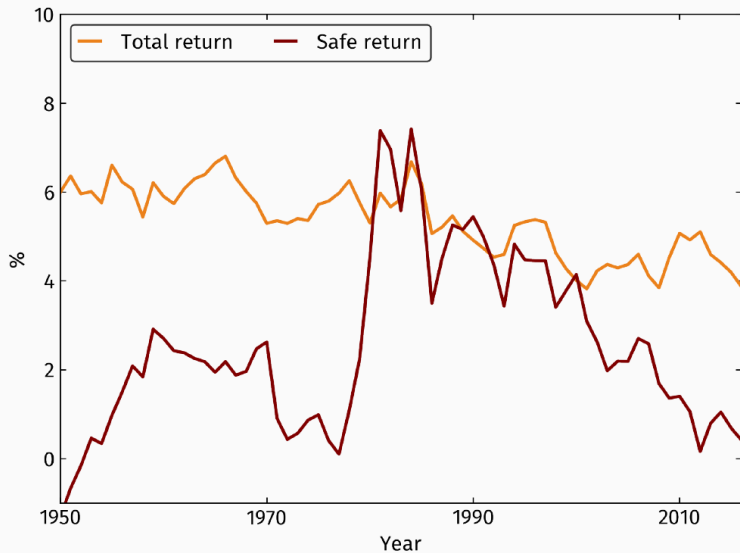
# The world population is aging



## ...wealth-to-GDP ratios are increasing...

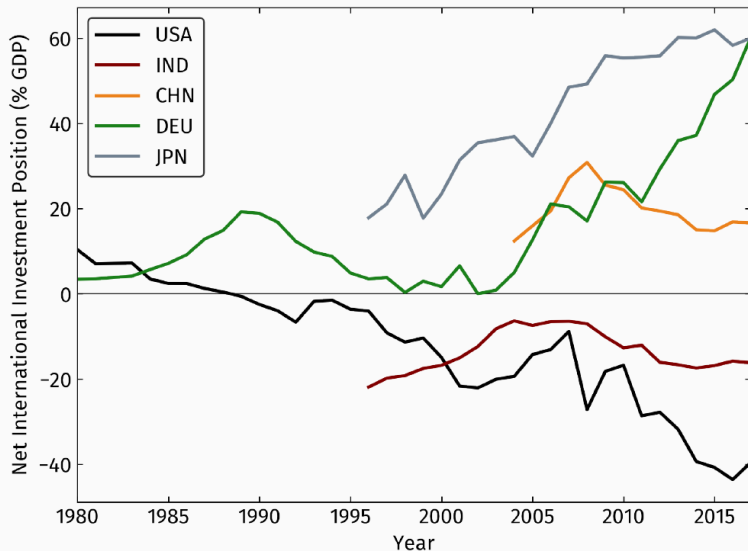


## ...rates of return on wealth are falling...





## ...and “global imbalances” are rising



# How have demographics shaped these trends?

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- Older population saves more, unevenly across countries
- Much less agreement about how much:  $\Delta r$  for 1970-2015 is
  - >  $-100\text{bp}$  in Gagnon-Johannsen-Lopez-Salido 2021
  - <  $-30\text{bp}$  in Eggertsson-Mehrotra-Robbins 2019

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  - “asset market meltdown” hypothesis [Poterba 2001]
    - If large elderly cohort wishes to sell assets to younger, smaller cohort asset prices drop, rates increase
  - “great demographic reversal” hypothesis [Goodhart-Pradhan 2020]
    - Demographic shift may raise rates going forward
    - **(Less)**: Young who consume and produce (i.e. supply labor)
    - **(More)**: Old who only consume
    - $\Rightarrow$  Increase in demand with lower supply: inflationary

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  - Demographic change alters demand for assets in each country
  - This affects world interest rate & financial flows between countries
- Big challenge: how to take this model to the data to discipline the importance of demographics?

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- Second, they use this framework to measure the importance of demographic change
- Admittedly, this approach requires a lot of simplifying assumptions. The authors solve and simulate the full model and show that it gives similar results

# Main results

- The authors reject the “great demographic reversal” hypothesis
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- The authors reject the “great demographic reversal” hypothesis
  - Do not find that aging will decrease savings and increase interest rates
  - Instead, it appears the global savings glut has just begun
- In addition, the authors refute the “asset market meltdown” hypothesis
  - Will dissaving of the old reverse the effects of demographics?
  - Yes, slightly. But it does not cause  $r$  to increase
  - As a result, no asset market meltdown



# Model



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- Government
  - Flow budget constraint

$$G_t + w_t \sum_{j=0}^T N_{jt} \mathbb{E} tr_j + (1 + r_t) B_{t-1} = \tau w_t \sum_{j=0}^T N_{jt} \mathbb{E} l_j + B_t$$

- Balance budget by changing  $G_t$ , not  $\tau_t$  or  $tr_{jt}$ , to keep  $B_t/Y_t$

## Environment: heterogeneous agents

- Problem for heterogeneous agents of cohort  $k$  (age  $j \equiv t - k$ )

$$\max \mathbb{E}_k \left[ \sum_j \beta_j \Phi_j \frac{c_{jt}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \right],$$

$$\text{s.t } c_{jt} + a_{j,t} \leq w_t ((1-\tau)\ell(z_j) + tr(z_j)) + (1+r_t) a_{j-1,t-1}$$

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- $a_{jt}$  : savings

# Equilibrium

Given demographics and policy, in an integrated world equilibrium:

- Individuals optimize
- Firms optimize
- Global asset markets clear

$$\sum_c W_t^c = \sum_c (K_t^c + B_t^c) \quad \forall t$$

where  $W_t^c$  is aggregate household wealth in country  $c$ :

$$W_t^c = \sum_{j=0}^J N_{jt}^c a_{jt}^c$$

Next: consider small country aging alone, with world at steady state  
→  $r$  constant (will adjust later)

# Compositional effects as sufficient statistics

## Proposition 1

The wealth-to-GDP ratio of a small country aging alone with constant rate  $r$  and growth  $\gamma$  follows

$$\frac{W_t}{Y_t} \propto \frac{\sum_j \pi_{jt} a_{j0}}{\sum_j \pi_{jt} h_{j0}}$$

where  $a_{j0} \equiv \mathbb{E}a_{j,0}$  and  $h_{j0} = \mathbb{E}w_0 \ell_{j,0}$  are average initial asset holdings and pretax labor income by age, and  $\pi_{jt} = N_{jt}/N_t$  is the share of the population of age  $j$ .

In a partial equilibrium world (where  $r$  does not adjust to changing demographics) then all changes in  $W/Y$  reflect the changing age composition  $\pi_{jt}$  of the population, given fixed profiles of asset holdings by age ( $a_{j0}$ ) and income by age ( $h_{j0}$ ).

# Compositional effects as sufficient statistics

Based on Proposition 1, we can compute the change in log wealth to GDP ratio as follows:

$$\log\left(\frac{W_t}{Y_t}\right) - \log\left(\frac{W_o}{Y_o}\right) = \log\left(\frac{\sum_j \pi_{jt} a_{jo}}{\sum_j \pi_{jt} h_{j0}}\right) - \log\left(\frac{\sum_j \pi_{j0} a_{jo}}{\sum_j \pi_{j0} h_{j0}}\right) \equiv \Delta_t^{comp}$$

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- Why? Demographics do not affect (normalized) individual decisions
  - **Note:** Comes from SOE **assumption** (constant  $r$ )



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- Later: we'll think about how  $\Delta_t^{comp}$  affects general equilibrium outcomes

## Measuring compositional effects

---

- Calculate  $\Delta_t^{comp}$  for 25 countries:

$$\Delta_t^{comp} \equiv \log \left( \frac{\sum \pi_{jt} a_{j0}}{\sum \pi_{jt} h_{j0}} \right) - \log \left( \frac{\sum \pi_{j0} a_{j0}}{\sum \pi_{j0} h_{j0}} \right)$$

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- Data:
  - $\pi_{jt}$  : projections of age distributions over individuals 2019 UN World Population Prospects
  - $a_{j0}, h_{j0}$  age-wealth and labor income profiles in base year
    - For US: SCF, LIS/CPS, and Sabelhaus-Henriques Volz (2019)
    - $a_{j0}$  includes funded part of DB pensions
    - Household  $\rightarrow$  individual ( $j$ ) by splitting wealth among adults

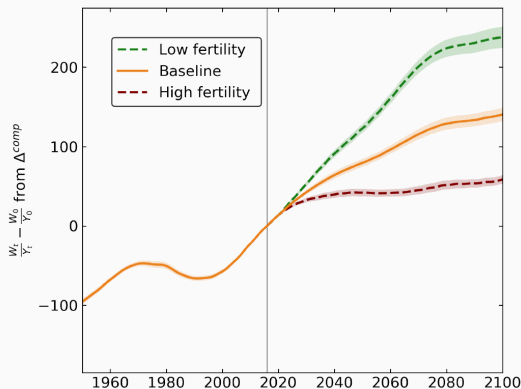
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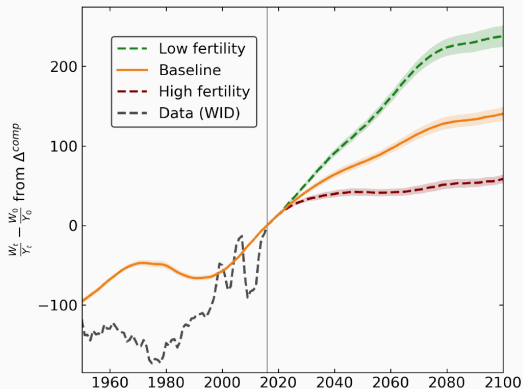
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- Report implied level change  $\frac{W_t}{Y_t} - \frac{W_0}{Y_0} = \frac{W_0}{Y_0} (\exp \{\Delta_t^{comp}\} - 1)$

# $\Delta^{\text{comp}}$ in the United States: 1950-2100

- Composition effect implies that increase in average population age (low fertility scenario) increases  $W/Y$



# $\Delta^{\text{comp}}$ in the United States: 1950-2100



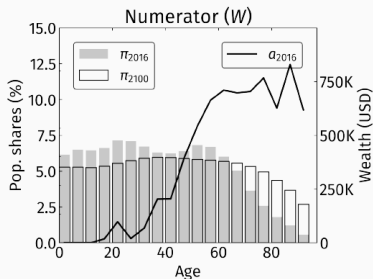
# Where do these large effects come from?

- Does increase in  $W/Y$  come from increase in **agg. wealth  $W$** , or decrease in **agg. income  $Y$** ?



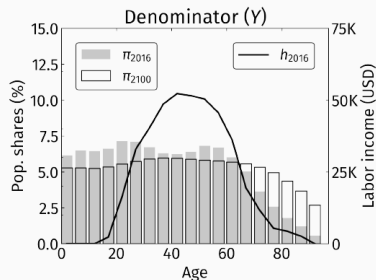
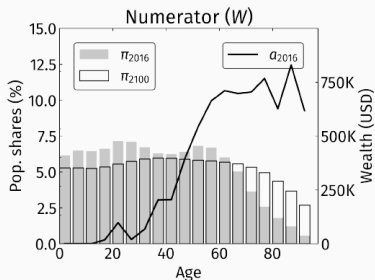
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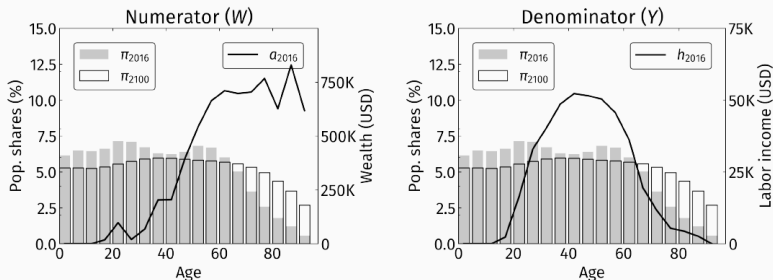


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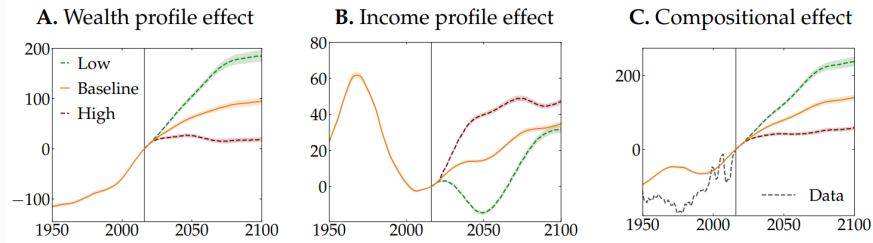


# Where do these large effects come from?



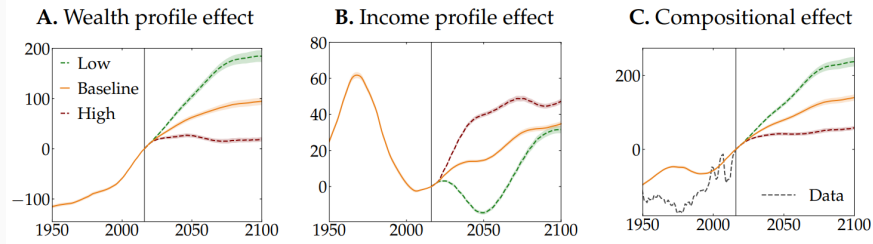
- In paper: separate contribution of numerator (wealth) and denominator (income)
  - Going forward:  $W$  contributes  $\sim 2/3$ ,  $Y$  contributes  $\sim 1/3$
  - Historically demographic dividend pushed  $Y$  up, reversed in 2010

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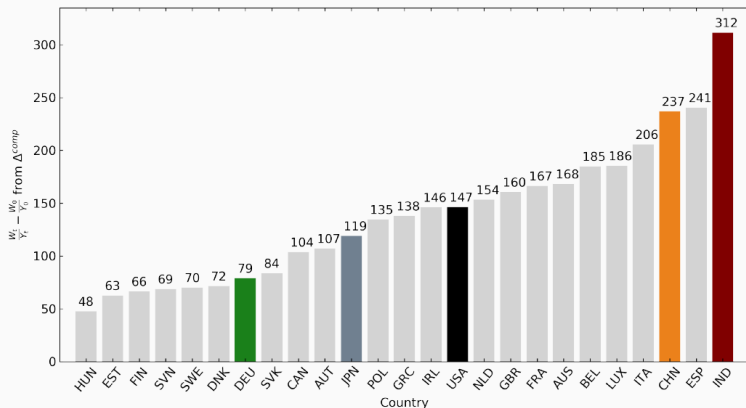
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# Where do these large effects come from?



- Historically (btw. 1970 and 2010) “demographic dividend” pushed up  $Y$ , decreased  $W/Y$  as a larger share of households were at peak working age where labor income is highest
- But this effect has been less pronounced recently, as elderly households earn less

# Across countries, $\Delta^{\text{comp}}$ large and heterogeneous by 2100

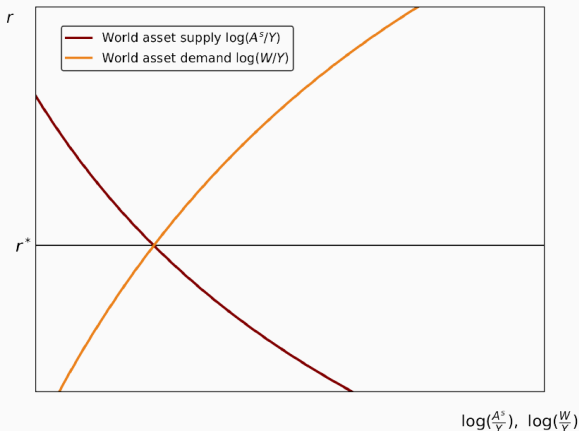


- **Note:** Uses US saving/income age profiles

# General equilibrium

- So far: Change age distribution keeping
  - World interest rate  $r$  fixed
  - Household consumption/saving behavior fixed
- Now: **General equilibrium**
- Changing the age distribution will affect *supply* of wealth (demand for assets)
- Equilibrium  $r$  will depend on supply of assets (gov. bonds + firm capital) as well

# General equilibrium implications

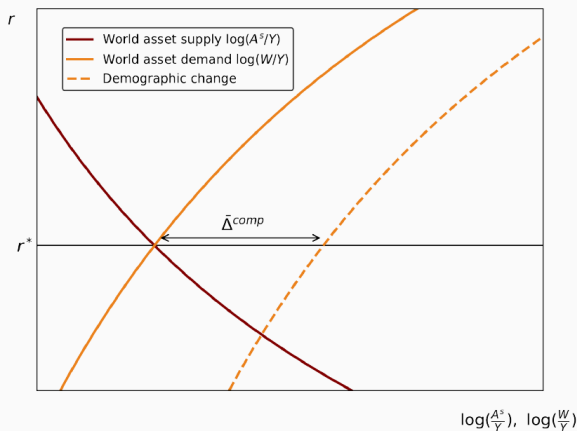


Semielasticity of asset demand  $\bar{\epsilon}_d = \frac{\partial \ln(W/Y)}{\partial r}$  : depends on elasticity of intertemporal substitution  $\sigma$  and observables (HHs)

Semielasticity of asset supply  $\bar{\epsilon}_s = -\frac{\partial \ln((K+B)/Y)}{\partial r}$  : depends on elasticity of



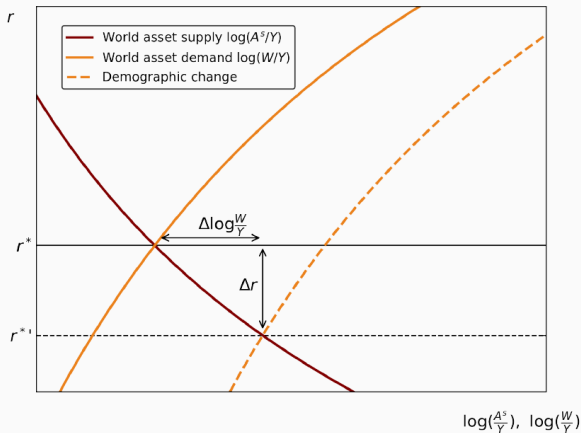
# General equilibrium implications



Asset demand shift of  $\bar{\Delta}^{comp}$  : wealth-weighted average of  $\Delta^{comp, c}$

Large and positive in the data.

# General equilibrium implications



## Proposition 2

If the age profiles of assets and consumption are constant, net foreign assets are zero, and governments maintain constant debt-to-GDP ratios, then the long run change in the rate of return is:

$$\Delta r \approx -\frac{\bar{\Delta}^{\text{comp}}}{\bar{\epsilon}_S + \bar{\epsilon}_d}$$

where  $\bar{\epsilon}_S$  is the average semielasticity of asset supply to  $r$ , and  $\bar{\epsilon}_d$  is the average semielasticity of asset holdings to  $r$ , and  $\bar{\Delta}^{\text{comp}}$  is the average compositional change.

- If asset demand/capital supply is very elastic: Small decline in  $r$ 
  - HHs respond a lot to initial decline in  $r$  by saving less (crowding out direct effect  $\bar{\Delta}^{\text{comp}}$ ), which stabilizes  $r$  in eq.
  - Firms respond by investing a lot in capital thereby driving up  $r$  in eq.

# What determines the asset demand semielasticity?

$$\epsilon^d = \underbrace{\sigma \frac{C}{(1+g)W} \frac{\text{Var } Age_c}{1+r}}_{\equiv \epsilon_{\text{substitution}}^d} + \underbrace{\frac{\mathbb{E}Age_c - \mathbb{E}Age_a}{1+r}}_{\equiv \epsilon_{\text{income}}^d}$$

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- The above can be measured assuming fixed  $Age_a$  and  $Age_c$ 
  - The authors find  $\epsilon_{\text{substitution}}^d = 39.5$ ,  $\epsilon_{\text{income}}^d = -2$ , thus  $\epsilon^d > 0$

# What determines the asset supply semielasticity?

$$\bar{\epsilon}^s = \frac{\eta}{r_0 + \delta} \frac{\bar{K}_0}{\bar{W}_0}$$

- $\eta$  is the elasticity of substitution between capital and labor



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- Based on the above calibration,  $\bar{\epsilon}^s > 0$  for any plausible  $\eta$ .
- Note: Holds for fixed gov. bond supply

# Change in world interest rate

Since  $\bar{\epsilon}_S + \bar{\epsilon}_d > 0$ , then the change in the world interest rate must be negative:

$$\Delta r \approx -\frac{\bar{\Delta}_{\text{comp}}}{\bar{\epsilon}_S + \bar{\epsilon}_d} < 0$$

With different assumptions on the elasticity of intertemporal substitution ( $\sigma$ ) and the elasticity of substitution between capital and labor ( $\eta$ ), this gives:

	$\sigma$		
$\eta$	0.25	0.50	1.00
0.60	-3.24	-1.59	-0.79
1.00	-2.09	-1.25	-0.70
1.25	-1.71	-1.10	-0.65

# Change in capital to income ratio

Proposition 2 gives a similar formula for the change in capital to income:

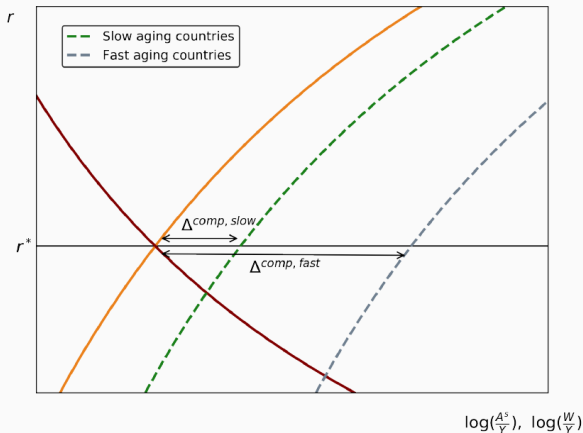
$$\overline{\Delta \log \left( \frac{W}{Y} \right)} \approx \frac{\bar{\epsilon}_S}{\bar{\epsilon}_S + \bar{\epsilon}_d} \bar{\Delta}^{\text{comp}} > 0$$

Again with different assumptions on the IES ( $\sigma$ ) and the elasticity of substitution between capital and labor ( $\eta$ )

$\eta$	$\sigma$		
	0.25	0.50	1.00
0.60	15.6	7.7	3.8
1.00	16.7	10.0	5.6
1.25	17.1	11.1	6.5

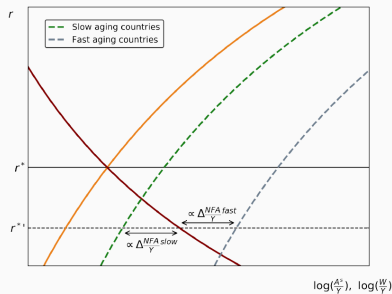
The authors argue that simulations from the general model deliver similar outcomes

# Change in net foreign assets



Country specific  $\Delta^{comp}$  large and heterogeneous in the data

# Change in net foreign assets



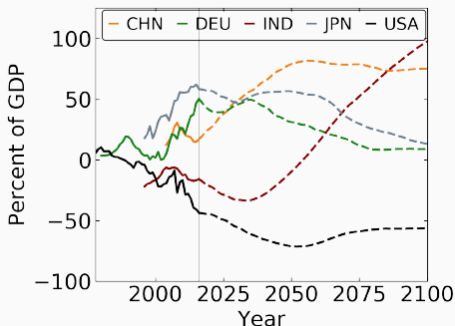
- Countries that age faster will accumulate more wealth, which it supplies to the rest of the world ( $NFA > 0$ )
  - Particularly to countries that age slowly (less wealth accumulation) where domestic firms need to go abroad for investments

$$\Delta \left( \frac{NFA}{Y} \right) \approx \frac{W_0}{Y_0} (\Delta^{\text{comp},c} - \bar{\Delta}^{\text{comp}})$$



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→ Data suggest large global imbalances going forward

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  - No changes in mortality (only birth rates)
  - Demographics have no effect on TFP growth
- To study some of these changes, the authors extend their baseline model → then simulate the transition path



# Results from richer model

- Compositional effect by country from analytical model product of:
  - Combination of demographic changes (exogenous in model) and labor supply/wealth profiles (endogenous)

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- Main finding:  $\Delta^{comp}$  in the richer model is roughly similar to the results from the data

Country	$\Delta^{comp,c}$	
	Model	Data
AUS	30	29
CAN	21	20
CHN	47	45
DEU	21	20
ESP	42	37
FRA	31	30
GBR	27	26
IND	65	56
ITA	34	30
JPN	24	22
NLD	34	33
USA	32	29

# Results from richer model

- GE Effects from the model are also roughly similar

	$\Delta r$	$\overline{\Delta \log \frac{W}{Y}}$	$\bar{\Delta}^{comp}$	$\bar{\Delta}^{soe}$	$\bar{\epsilon}^d$	$\bar{\epsilon}^s$
Sufficient statistic analysis	-1.23	9.9	31.8		17.8	8.0
Preferred model specification	-1.23	10.3	34.1	30.3	17.1	8.0
<i>Alternative model specifications</i>						
+ Constant bequests	-1.18	10.0	34.1	27.0	14.9	8.0
+ Constant mortality	-1.23	10.9	34.1	27.1	13.8	8.0
+ Constant taxes and transfers	-1.33	11.9	34.1	30.1	14.5	8.0
+ Constant retirement age	-1.49	13.4	34.1	34.1	14.6	8.0
+ No income risk	-1.47	13.2	33.9	33.9	13.8	8.0
+ Annuities	-1.33	11.5	34.2	34.2	17.2	8.0
<i>Alternative fiscal rules</i>						
Only lower expenditures	-1.29	11.0	34.1	32.6	17.9	8.0
Only higher taxes	-0.88	6.7	34.1	19.4	14.6	8.0
Only lower benefits	-1.50	12.9	34.1	39.1	18.4	8.0

Notes:  $\Delta r$ ,  $\overline{\Delta \log \frac{W}{Y}}$ ,  $\bar{\Delta}^{comp}$ , and  $\bar{\Delta}^{soe}$  denote the changes in the model simulation between 2016 and 2100, with  $\Delta r$  reported in percentage points and the other three reported in percent ( $100 \cdot \log$ ).

# Conclusion

- How does population aging affect wealth-output ratios, real interest rates, and capital flows?
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  - what matters is the compositional effect  $\Delta^{\text{comp}}$
  - large and heterogeneous in the data
- The approach developed by the authors:
  - Refutes the asset market meltdown hypothesis:  $r$  falls
  - Suggests wealth-to-income ratio will keep rising
  - Larger global imbalances (dispersion of NFAs)

# Income Inequality

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# Secular stagnation and income inequality

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  - See i.e. Mian, Sufi, Straub (2021): *What explains the decline in  $r^*$ ? Rising income inequality versus demographic shifts*
- **Example:** If saving rates increase with income (richer save more) then:
  - Redistribution from poor to rich households *increase* the aggregate supply of savings  $\Rightarrow$  lower rates in GE
  - Suggests link between increasing income inequality and secular stagnation

# Permanent redistribution in the canonical HA model

- Consider standard HA model with permanent income state  $p$  :

$$V(a_{t-1}, z_t, p) = \max_{c_t} \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}[V(a_t, z_{t+1}, p)]$$

subject to

$$c_t + a_t = Ra_{t-1} + z_t p$$

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- Increase in permanent income by 1% increases  $c_t, a_t$  by 1%.
  - Permanent redistribution has *no effect* on aggregates because the dissavings by one group is exactly offset by increase in savings from other groups

# Homothetic household problem I

- Normalize constraints by  $p$  and Bellman by  $p^{1-\sigma}$

$$\frac{V(a_{t-1}, z_t, p)}{p^{1-\sigma}} = \max_{c_t} \frac{\frac{c_t^{1-\sigma}}{1-\sigma}}{p^{1-\sigma}} + \beta \frac{\mathbb{E}[V(a_t, z_{t+1}, p)]}{p^{1-\sigma}}$$

subject to

$$\frac{c_t}{p} + \frac{a_t}{p} = R \frac{a_{t-1}}{p} + z_t \frac{p}{p}$$

$$\frac{a_t}{p} \geq \frac{0}{p}$$



# Homothetic household problem II

- Define  $\tilde{c}_t = \frac{c_t}{p}$ ,  $\tilde{a}_t = \frac{a_t}{p}$ ,  $\tilde{V}_t = \frac{V_t}{p^{1-\sigma}}$  to get:

$$\tilde{V}(a_{t-1}, z_t) = \max_{c_t} \frac{\tilde{c}_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E} [\tilde{V}(a_t, z_{t+1})]$$

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- Then HH problem is independent of  $p$  up to scale!
  - Can always solve above HH problem, and get solution with different  $p$  by simply scaling policy functions  $\tilde{c}_t, \tilde{a}_t$  by  $p$
  - Solution is »scale independent«

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- Two questions
  - What does this elasticity look like empirically?
  - If  $\phi \neq 1$  how can we accommodate this in a HA model?

# Empirical estimate

- Use US panel survey data (PSID)



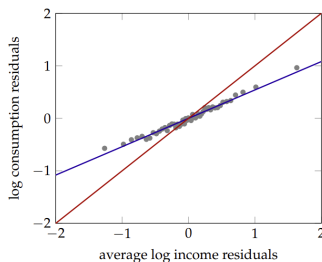
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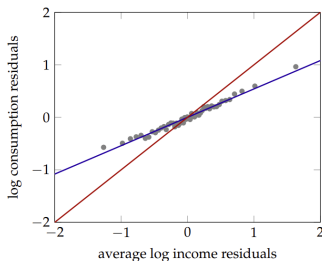
Figure 3: Consumption and average income.



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Figure 3: Consumption and average income.



- More detailed estimates in paper, suggest  $\phi \approx 0.7$

# Non-homothetic HA model

- Extend standard HA model with *taste for wealth* («status«)

$$V(a_{t-1}, z_t, p) = \max_{c_t} \frac{c_t^{1-\sigma}}{1-\sigma} + \phi_a \frac{a_t^{1-\sigma_a}}{1-\sigma_a} + \beta \mathbb{E}[V(a_t, z_{t+1}, p)]$$

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- Note: Still homothetic as long as  $\sigma = \sigma_a$ , but non-homothetic if  $\sigma \neq \sigma_a$ :

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# Non-homothetic HA model

- Extend standard HA model with *taste for wealth* («status«)

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subject to

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- If  $\sigma \neq \sigma_a$  then scaling  $p$  up/down changes relative weight btw  $c, a$

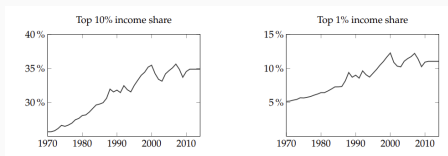
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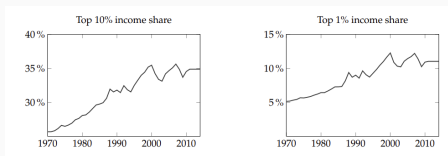
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# Applications

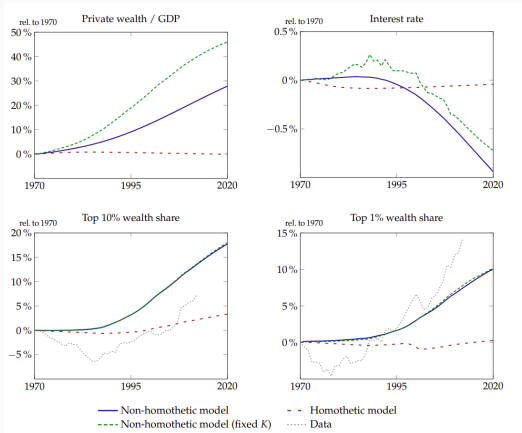
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- Solve for GE (Supply side as in HANC)

# GE implications of rising income inequality

- Compare effects in homothetic and non-homothetic models:



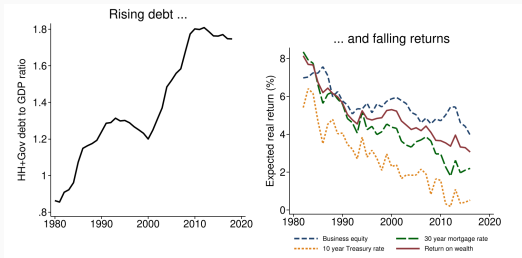
- Note: »Fixed K« is Lucas-tree economy, where wealth grows due to rising asset prices

# Indebted Demand (2021)

- Mian, Sufi, Straub (2021) »Indebted Demand« build on this insight of non-homothetic preferences

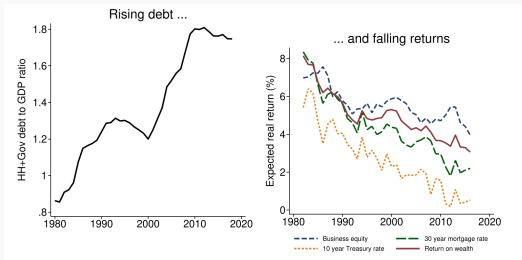
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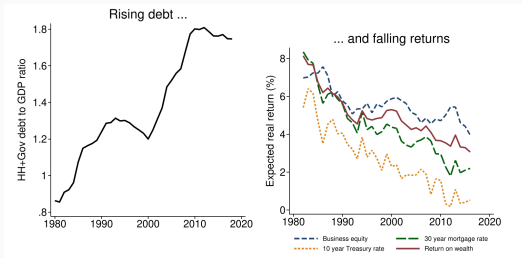
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- Highlights role of increasing household debt and connect to returns:



- Argue that income inequality and less financial regulation leads to more borrowing and lower returns in GE
- **Note:** Would normally expect the opposite
  - If I want to borrow more, I will have to compensate savers by paying **higher** interest rate, not **lower**

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  - **Continuous** time
  - Endowment economy with  $Y = 1$



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- Equilibrium: Debt owed by borrowers equal savings by savers:

$$d_t = a_t^s$$

# Non-homothetic model

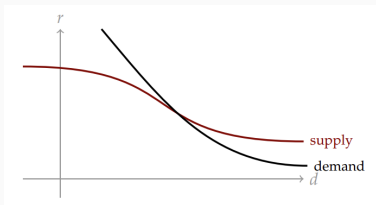
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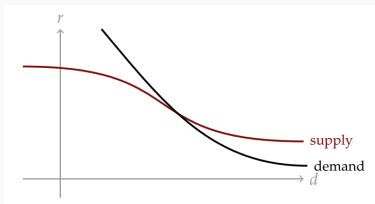
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- Why? Assume rich agent consumes fixed amount  $\bar{c}^s$  and saves everything else: Then  $\bar{c}^s = r_t a_t^s \Leftrightarrow a_t^s = \frac{\bar{c}^s}{r_t}$ 
  - Higher  $r$  will in the long run crowd out  $a_t^s$  in the long run (needed for ss, rules out explosive path)

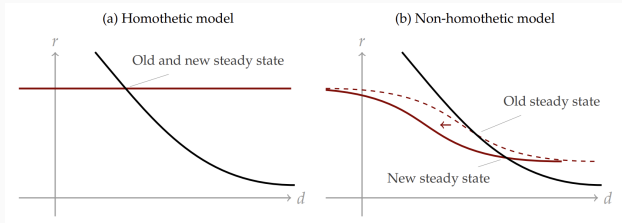
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- Increase in income inequality: Increase in debt and lower interest rates
  - Similar to Straub (2019)
- Increase in financial deregulation (relax borrowing constraint): More borrowing and lower interest rates
  - Note: standard GE argument would imply **higher** rates
  - Lower rates arise due to downward sloping saving schedule for savers

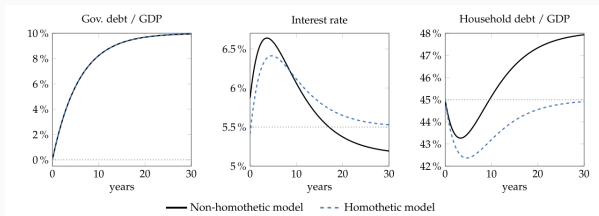
# Results

- Graphically: Increase in income inequality moves supply curve (red) to the left
- Inequality  $\uparrow \Rightarrow$  saving by rich  $\Rightarrow$  lower  $r \Rightarrow$  Incentive debt  $d \uparrow$



# Fiscal Policy

- Temporary government spending shock (deficit financed)
- Short run: Higher  $r$  required for HHs to hold increase in gov. debt
- **Long run** Increase in  $B$  pushes  $r$  down. Rich save more, pushing  $r$  further down and incentivizing holding of debt



# Summary

- Basic premise of Mian, Sufi, Straub (2021): Savers and borrowers have different (permanent) marginal propensities to consume/save
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- Highlight implications for fiscal policy
  - And monetary policy - see paper

# Exercise

- Consider the following PE HA model:

$$V(a_{t-1}, z_t, p) = \max_{c_t} \frac{c_t^{1-\sigma}}{1-\sigma} + \phi_a \frac{a_t^{1-\sigma_a}}{1-\sigma_a} + \beta \mathbb{E}[V(a_t, z_{t+1}, p)]$$

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- Q1:** Derive the Euler equation of the model
- Q2:** Update the EGM algorithm in *household\_problem.py* with the new Euler
- Q3:** Solve the model with 1)  $\phi_a = 0$ , 2)  $\phi_a = 0.1, \sigma = \sigma_a = 1$ , 3)  $\phi_a = 0.1, \sigma = 1, \sigma_a = 0.7$ 
  - Compare the normalized policy function  $c/p$
- Q4:** Conduct an experiment where you permanently redistribute resources across households (change  $p$ ). What are the aggregate effects across the models?

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