6. Wealth Inequality

Adv. Macro: Heterogenous Agent Models

Nicolai Waldstrøm

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Introduction

Wealth inequality

- **Goal for today:** Better understand wealth inequality through the lens of heterogeneous agent general equilibrium models
- **Central economic questions:**
  1. Why are some people rich while others are poor?
  2. To what extent can governments affect inequality?
     - What does the baseline Aiyagari model predict in terms of wealth inequality?
     - How can we augment the baseline model to obtain a closer match of reality?
  3. What explains the rise in wealth inequality in recent decades?
- To answer these questions, we need to better understand why people save, and how this translates into wealth inequality
- **Plan for today:**
  1. Study the predictions of a baseline Bewley-Huggett-Aiyagari model
  2. Consider various model extensions that help match the data
  3. Given such a model, what can we say about optimal wealth taxation?

Wealth inequality in the data

Earnings and wealth inequality

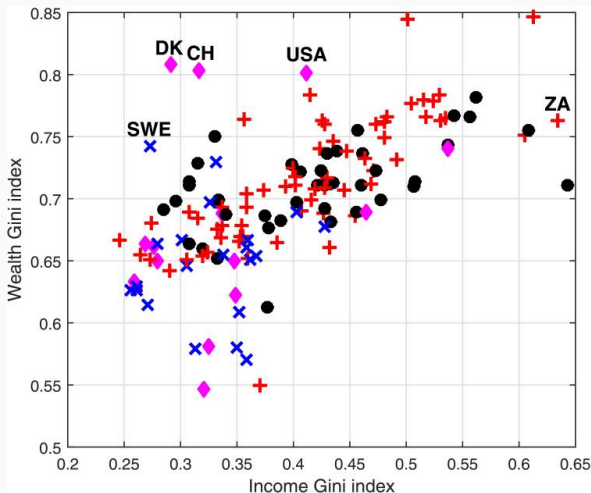- US data on distribution of income and wealth (SCF, 1989)

	Top 1%	Top 5%	Top 20%	Top 40%	Percent at zero or negative
Wealth	29	53	80	93	6
Earnings	6	19	48	72	8

- Wealth more concentrated than earnings
- Skewed distributions with thick upper tails

Wealth more concentrated than earnings

Not only in the US, but also Denmark and almost all other countries

Aiyagari Model

- Infinitely lived agents with preferences

$$\max_{\{c_t\}_{t=0}^{\infty}} E \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

- Budget constraint and borrowing constraint

$$a_t = y_t + (1+r)a_{t-1} - c_t, \quad a_t \geq \underline{a}$$

- Idiosyncratic earnings risk:

$$\ln y_t = \rho \ln y_{t-1} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}\left(0, \sigma_\epsilon^2\right)$$

- As usual, calibrate parameters in earnings process $\left(\rho, \sigma_\epsilon^2\right)$ based on estimates from panel data on earnings, i.e. Floden and Linde (2001)

Aiyagari Model - wealth inequality fit

	Wealth Gini	Wealth in top (%)		
		1%	5 %	20 %
U.S. data, 1989 SCF				
	.78	29	53	80
Aiyagari Baseline				
	.38	3.2	12.2	41.0
Aiyagari higher variability				
	.41	4.0	15.6	44.6

Top wealth inequality

- What about top wealth inequality?
  - Think top 0.01% or 0.001% (Bezos, Musk, Gates etc.)
- The probability of having wealth a above threshold X described as Pareto dist, $P(a > X) \sim x^{-\alpha}$
  - In logs, $\ln P(a > X) \sim -\alpha \ln x$, so linear in wealth with α describing the »thickness of the tail«
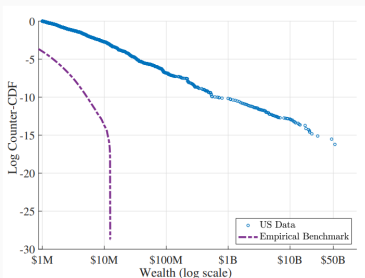
Key mechanism

- Precautionary savings behavior. People save to self-insure against earnings risk

- Once buffer stock savings is reached, people start dissaving (Carroll 1997)

- In the model: The saving rate of the high wealth households is low or even negative
  - Contrasts with much empirical evidence (Dynan Skinner and Zeldes, 2004 and De Nardi, French and Jones, 2010)
  - Will discuss this in next lecture

- Note also: Only driver of wealth inequality is earnings risk
  - Income inequality in data typically lower than *wealth* inequality
  - In reality multiple drivers such as entrepreneurship, preferences, bequests, return heterogeneity

Explaining wealth inequality

Explanations

- Standard Aiyagari model: Income inequality
- Preference heterogeneity
    - Krussel and Smith (1998), Alan, Browning, and Ejenaes (2016), Druedahl and Jorgensen (2015)
- Bequests
    - Kotlikoff and Summers (1981), Modigliani (1988), Gale and Scholz (1994), Di Nardi (2004)
- Entrepreneurship
    - Cagetti and De Nardi (2006), Di Nardi et al. (2007), Guvenen et al. (2023)
- Return heterogeneity
    - Hubmer, Krusell, Smith (2021), Ozkan et al. (2023), Guvenen et al. (2023)

Bequests

$$\max_{\{c_t\}_{t=0}^{T}} E \sum_{t=0}^{T} \beta^t \left(s_t \frac{c_t^{1-\sigma}}{1-\sigma} + (1-s_t)\,\phi\,(a_{t-1}) \right)$$

$$c_t + a_t = y_t + (1+r)a_{t-1} + b_t, \quad a_t \geq \underline{a}$$

1. Bequests and human capital transmission across generations (*warm glow*)

Explaining wealth inequality

$$\max_{\{c_t\}_{t=0}^{T}} E \sum_{t=0}^{T} \beta_i{}^t s_t \frac{c_t^{1-\sigma_i}}{1-\sigma_i}$$

$$c_t + a_t = y_t + (1+r)a_{t-1}, \quad a_t \geq \underline{a}$$

1.
2. Heterogeneous preferences

$$\max_{\{c_t\}_{t=0}^{T}} E \sum_{t=0}^{T} \beta^t s_t \frac{c_t^{1-\sigma}}{1-\sigma}$$

$$c_t + a_t = [l_e f(\theta_t, k_{t-1}) + (1 - l_e) y_t] + (1 + r)(a_{t-1} - k_{t-1}), \quad a_t \geq \underline{a}$$

1.

2.

3. Entrepreneurship.

Explaining wealth inequality

$$\max_{\{c_t\}_{t=0}^{T}} E \sum_{t=0}^{T} \beta^t s_t \frac{c_t^{1-\sigma}}{1-\sigma}$$

$$c_t + a_t = y_t + \left(1 + r_t^i\right) a_{t-1}, \quad a_t \geq \underline{a}$$

1.

2.

3.

4. Idiosyncratic rates of return

Hubmer, Krussel and Smith (2021)

Explaining wealth inequality

- Hubmer, Krussel and Smith (2021): *Sources of US wealth inequality: Past, present, and future*
  - Model which matches key features of US wealth inequality in 1967
  - Can we account for changes in wealth inequality going forward from 1967 based on observables?
    - I.e. changes in income inequality, taxes, asset returns

Model

- Household problem features non-linear tax schedules, heterogeneous returns and β-het.

$$V_t(a_{t-1}, p_t, \beta_t) = \max_{a_{t+1} \geq 0} \{u(c_t) + \beta_t \mathbb{E}[V_{t+1}(a_t, p_{t+1}, \beta_{t+1})|p_t, \beta_t]\}$$

$$c_t + a_t = y_t - \tau_t(y_t) + (1 - \tilde{\tau}_t)\tilde{y}_t + T_t$$

$$y_t = (\underline{r_t} + r_t^X(a_{t-1}))a_{t-1} + w_t l_t(p_t)$$

$$\tilde{y}_t = \sigma_t^X(a_{t-1})\eta_t a_{t-1}$$

- Mean excess return $r_t^X(a_{t-1})$
  - How does mean returns vary with wealth?
- Standard deviation of excess returns: $\sigma_t^X(a_{t-1})$
  - How does return uncertainty vary with wealth?
- Example: If rich HHs primarily invest in stocks, poorer HHs in bonds. Would expect both $r_t^X(a_{t-1}), \sigma_t^X(a_{t-1})$ to be increasing in a_{t-1}

Facts

- Fagereng, Guiso, Malacrino, Pistaferri (2020) find that rates of returns are:
    - Heterogeneous across households (over 200 basis points between 10th and 90th percentile of the distribution of returns)
    - Also heterogenous within asset classes
        - So return differences cannot be explained only by poorer HHs holding banket deposits and rich HHs investing in stocks
    - Persistent
    - Correlated with household wealth and across generations

Equilibrium: capital market clearing

- Need to find two equil. objects $(K_t, \underline{r}_t)$ for capital market clearing:
    1. aggregate capital (as usual)

    $$K_t = \int a_t d\Gamma(a_t)$$

    2. aggregate capital income (redundant if $r_t^x(\cdot) = 0$)

    $$(MPK(K_t) - \delta)K_t = \int \left(\underline{r}_t + r_t^x(a_t) \right) a_t d\Gamma(a_t)$$

- Plus goods market clearing, but redundant given other 2

Calibration strategy summary

1. Calibrate earnings process, tax rates, return process, social safety net to observables

2. Choose randomness in discount factor β residually so as to replicate the wealth distribution in the initial steady state (1967)

3. Then feed in exogenous changes in tax rates, earnings inequality, etc. between 1967 and 2015 to understand the role of these different factors

Return heterogeneity

- Overall return given asset holdings a_{t-1} equals

$$\underline{r}_t + r_t^X(a_{t-1}) + \sigma^X(a_{t-1})\eta_t$$

- $\underline{r}_t$ is endogenous
- $r_t^X(\cdot)$ and $\sigma^X(\cdot)$ are exogenous excess return schedules (mean and st.dev.), taken from the data
- η_t is an i.i.d. standard normal shock
- Reduced form portfolio choice

Calibration: return process

$$r_t^X(a_t) = \sum_{c \in C} w_c(a_t) \left(\bar{r}_{c,t} + \tilde{r}_c^X(a_t) \right)$$

$$\sigma^X(a_t)^2 = \sum_{c \in C} \left(w_c(a_t) \tilde{\sigma}_c^X(a_t) \right)^2$$

- Asset classes C: risk-free, public equity, private equity, housing
- $\bar{r}_{c,t}$: aggregate return on asset class c (U.S. data), time-varying
- Fixed over time, based on Swedish administrative data from Bach, Calvet, Sodini (2016):
    - $w_c(\cdot)$: portfolio weights
    - $\tilde{r}_c^X(\cdot)$: within asset class return heterogeneity
    - $\tilde{\sigma}_c^X(\cdot)$: asset c idiosyncratic return standard deviation

Excess return schedule details

- Aggregate Excess Returns in 1967 steady state:
  - public equity 0.067 (U.S., Kartashova 2014)
  - private equity 0.129 (U.S., Kartashova 2014)
  - housing 0.037 (incl. imputed rent; Jorda, et al, 2017)

- and cross-sectional data from Bach, Calvet, Sodini (2019) implies

	P0-P40	P40-P50	P50-P60	P60-P70	P70-P80	P80-P90	P90-P95	P95-P97.5	P97.5-P99	P99-P99.5	P99.5-P99.9	P99.9-P99.99	Top 0.01%
fixed portfolio weights													
risk-free	0.722	0.412	0.248	0.182	0.156	0.134	0.115	0.102	0.090	0.079	0.071	0.051	0.029
housing	0.162	0.394	0.580	0.662	0.678	0.674	0.658	0.626	0.572	0.482	0.363	0.253	0.155
public equity	0.113	0.189	0.165	0.147	0.153	0.170	0.189	0.207	0.219	0.232	0.230	0.185	0.179
private equity	0.002	0.005	0.007	0.009	0.013	0.021	0.038	0.065	0.118	0.207	0.336	0.511	0.637
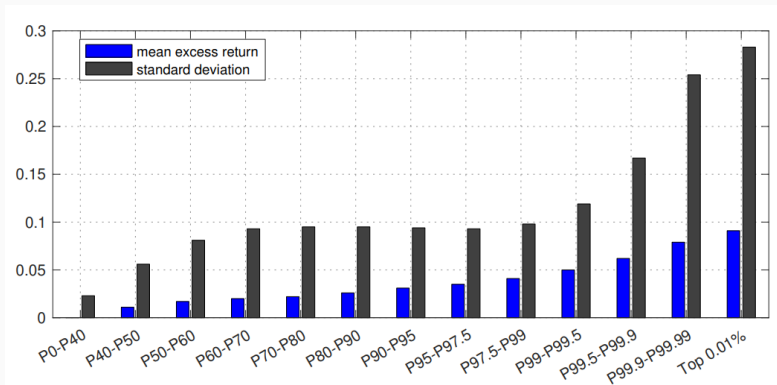
Data sources: Bach, Calvet, Sodini (2019); Kartashova (2014); Jorda, Knoll, Kuvshinov, Schularick, Taylor (2019); Case-Shiller.

Hubmer, Krussel and Smith (2021)

Results

Results, I: Steady state (1967)

- Steady state fit (with and without β-het)

	Top 10%	Top 1%	Top 0.1%	Top 0.01%
Data	70.8%	27.8%	9.4%	3.1%
Single-β Model	66.6%	23.7%	11.2%	7.2%
Benchmark Model	73.8%	27.4%	8.4%	3.2%
	Bottom 50%	Fraction $a < 0$		
Data	4.0%	8.0%		
Single-β Model	3.5%	7.3%		
Benchmark Model	3.0%	6.6%		

Results, I: steady state (1967)

#		top 10%	top 1%	top 0.1%	top 0.01%	Gini
1	β-heterogeneity	8.8%	7.7%	3.8%	2.0%	0.050
2	earnings heterogeneity	−27.5%	−17.8%	−9.5%	−6.4%	−0.173
3	persistent	−5.0%	−7.5%	−4.2%	−2.9%	0.009
4	transitory	−11.6%	−4.3%	−1.7%	−0.9%	−0.109
5	tax progressivity	−21.3%	−61.8%	−71.2%	−67.1%	−0.148
6	return heterogeneity	29.5%	18.4%	6.6%	2.8%	0.192
7	mean differences	25.8%	16.7%	6.0%	2.6%	0.174
8	return risk	0.7%	2.2%	3.3%	2.5%	0.004

- How to read: Shutting of β-het reduces top 10% wealth share by 8.8%
- Model matches wealth distribution well on its entire domain
  - return heterogeneity is key ingredient
  - wealth concentration is mitigated by progressive taxation and labor income risk

Next step: transition

The authors feed in four different factors that have changed during the past 50 years

- Decrease in tax progressivity

- Increase in labor income risk

- Increase in income going to the top

- Changing return premia to different asset classes

Observed change 1: Decrease in tax progressivity

- Federal effective tax rates (Piketty & Saez 2007): income, payroll, corporate and estate taxes
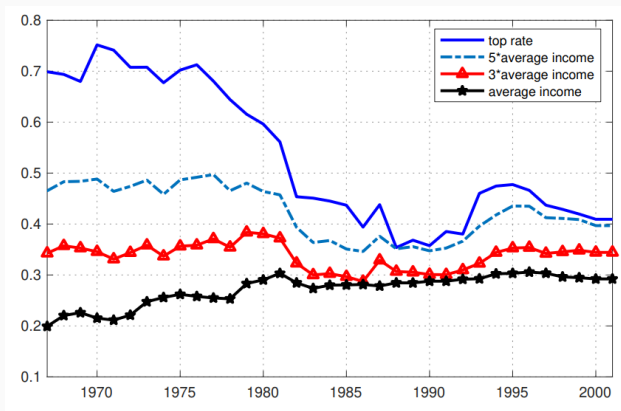
Observed change 2: Increase in labor income risk

- Estimates for variance of persistent and temporary components 1967-2000 (Heathcote, Storesletten & Violante 2010)

cross-sectional standard deviations
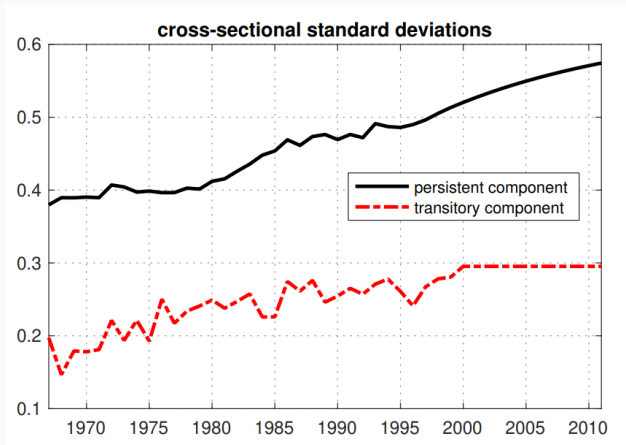
Observed change 3: Increase in top labor income shares

- Adjust standard AR(1) in idiosyncratic productivity by imposing a Pareto tail for the top 10% earners: calibrated tail coefficient decreases from 2.8 to 1.9 (updated Piketty & Saez 2003 series)
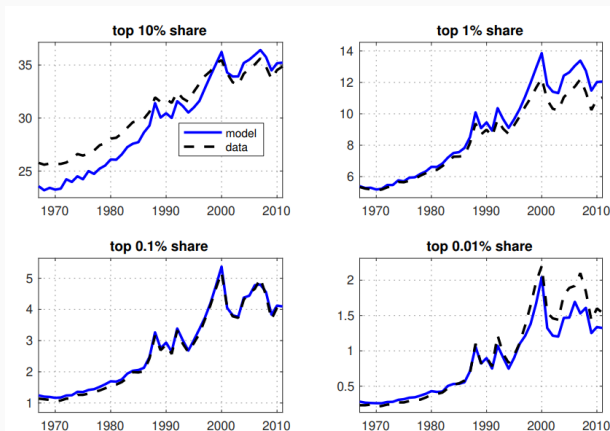
Observed change 4: return premia

- Feed in (smoothed) time series of aggregate U.S. asset premia (Kartashova 2014, Case-Shiller index)
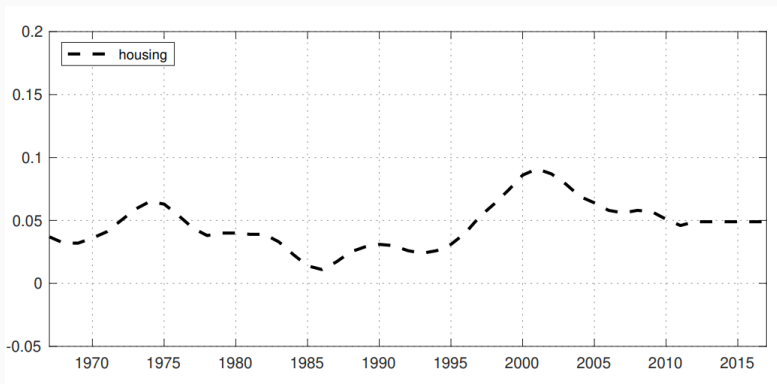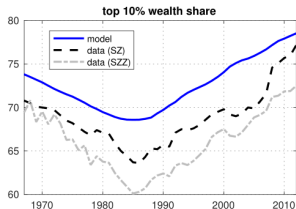
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Observed change 4: return premia
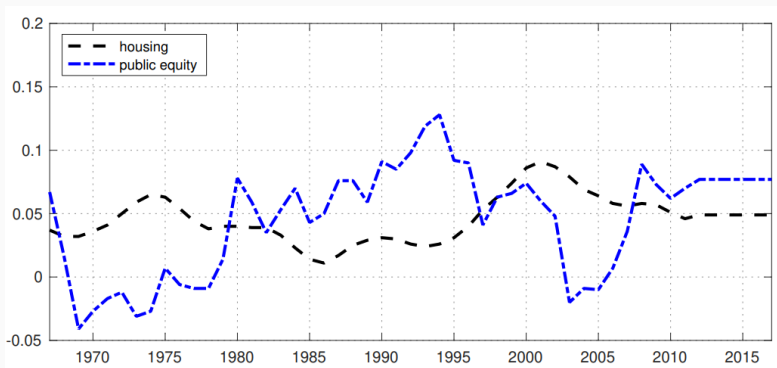
- feed in (smoothed) time series of aggregate U.S. asset premia
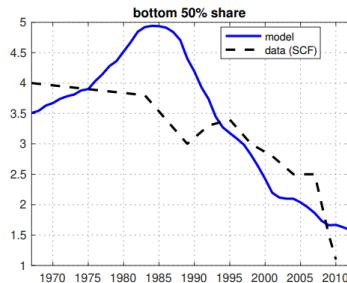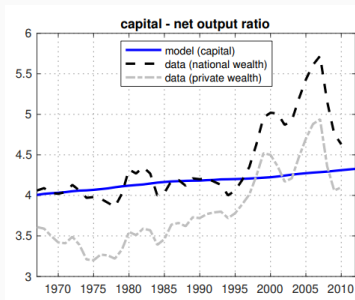 (Kartashova 2014, Case-Shiller index)
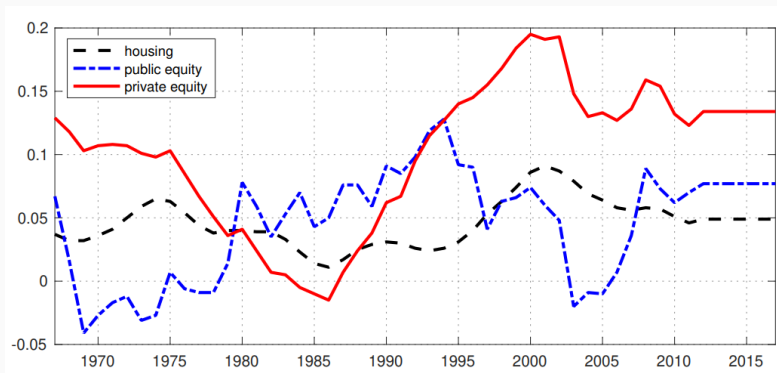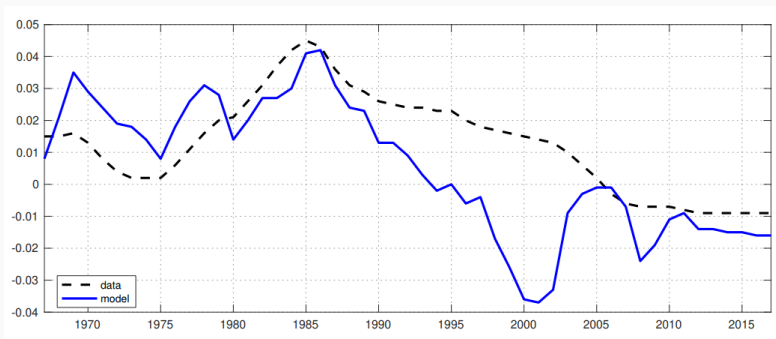
Results, II: historical evolution
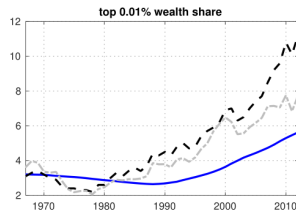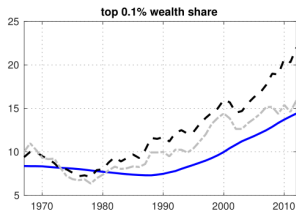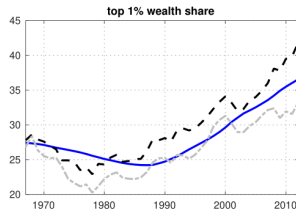
Data sources: dashed black lines refer to Saez & Zucman (2016); dash-dotted gray lines refer to Smith et al. (2020).

- Return premia are matched in model by construction
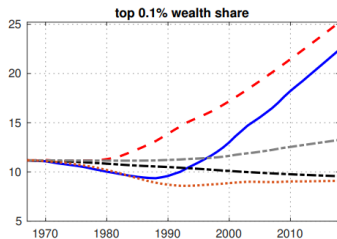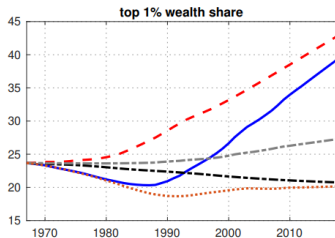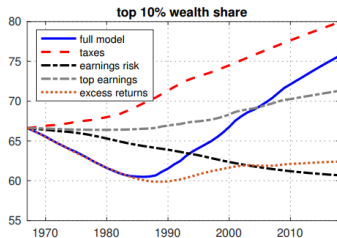- Risk-free rate r is endogenous: comparable level and decline

Decomposition of transitional dynamics
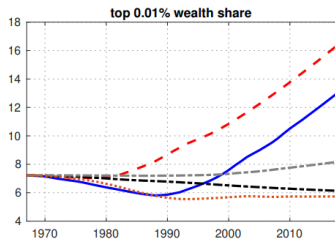
Decomposition of transitional dynamics

- Overall increase in wealth inequality (more than) fully explained by declining tax progressivity
  - Primarily due to direct effect on resource distribution and not due to changing savings behavior

- Time-varying return premia account for U-shape in wealth inequality

- Subtle role of increasing earnings dispersion
  - Thickening Pareto tail in labor income contributes slightly positively to wealth inequality
  - Increase in overall earnings risk *decreases* wealth inequality because precautionary savings motive is stronger for poorer HHs

Summary

- **Hubmer, Krussel and Smith (2021)**
- HANC with:
    - Income risk
    - Return heterogeneity
    - β-heterogeneity
    - Tax system
- Main finding:
    - Return heterogeneity key in matching initial (1967) wealth inequality
    - Can roughly explain evolution in US wealth inequality with observable changes in tax systems

Ozkan et al. (2024)

- Ozkan et al. (2024) takes a *lifecycle perspective*:
    - Why do some people become wealther than others?
    - Use detailed Norwegian admin data
- Evaluate contribution from
    1. Inheritances (bequests) H_{it}
    2. Return heterogeneity r_{it}
    3. Saving rate heterogeneity s_{it}
    4. Labor earnings L_{it}
    5. Initial wealth a_{i0}
- Using budget constraint:

$$a_{it} = a_{it-1} + (L_{it} + H_{it} + r_{it}a_{it-1}) \times s_{it}$$

Results from Ozkan et al. (2024)

- Left panel: Decomposition of wealth for top 0.1%
- Right panel: »Poorest« HHs *within* top 0.1% *(New Money)*

FIGURE 1 – DETERMINANTS OF THE TOP 0.1% WEALTH ACCUMULATION

(A) Top 0.1%

(B) New Money
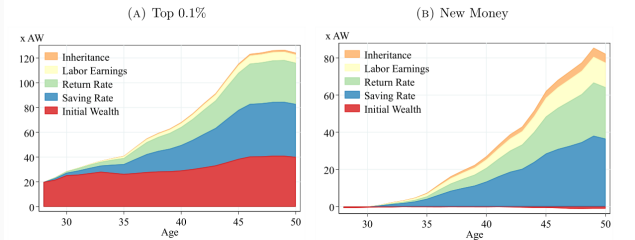
Application to Wealth Taxation

Wealth taxation I

- Spend a lot of time understanding *what* drives wealth inequality
- We will now see an application where the specific source of inequality matters
- **Wealth taxation**
    - Why would we want to tax wealth?

    - Why not?

Wealth taxation II

- Guvenen et al. (2023): *Use It or Lose It: Efficiency and Redistributional Effects of Wealth Taxation*
- Study optimal taxation in two tax systems:
    - Wealth tax: a_i
    - Capital income tax : $r \times a_i$
- Note: Without return heterogeneity two tax system are equivalent
    - After tax wealth /w CI tax : $a_i + (1 - \tau_k) ra_i$
    - After tax wealth /w wealth tax : $(1 - \tau_a) a_i + ra_i$
- Social planner can implement same allocation using these two different instruments by setting $\tau_a = r\tau_k$

Taxation with return heterogeneity

- What if returns differ across agents, r_i?
  - No equivalence between tax systems
- With capital income taxation:
  - A highly productive agent (high r_i) will be taxed more than less productive agents (low r_i)
    - Tax burden falls proportionally more on productive agents $\Rightarrow$distortionary
- With wealth taxation:
  - All agents with same wealth pay same tax regardless of return r_i
  - Shifts tax base towards unproductive agents
- Note: We say HHs with high r_i are more **productive**
  - Think in terms of *entrepreneurial* models
  - High productivity HHs have better technology (i.e. are better entrepreneurs) and can make their wealth growth faster (high r_i)

Model

- HH problem:

$$\max_{\{c_t\}_{t=0}^{T}} E \sum_{t=0}^{T} \beta^t \left(s_t \frac{c_t^{1-\sigma}}{1-\sigma} + (1-s_t) \phi(a_t) \right)$$

$$a_t + c_t = \mathcal{W}(a_{t-1}, z_{t-1}) + w_t(e_t) \ell_t, \quad a_t \geq \underline{a}$$

$$\mathcal{W}(a_{t-1}, z_t) = \begin{cases} a_{t-1} + (\pi(a_{t-1}, z_t) + r a_{t-1})(1-\tau_k) & \text{if CI tax} \\ a_{t-1}(1-\tau_a) + (\pi(a_{t-1}, z_t) + r a_{t-1}) & \text{if wealth tax} \end{cases}$$

- Entrepreneurial abilitiy z follow markov chain with values $z = [0, z_L, z_H]'$ and transition matrix Π_z
  - HHs with $z = 0$ are normal workers
  - HHs with $z = z_L$ are »unproductive« entrepreneurs
  - HHs with $z = z_H$ are »productive« entrepreneurs
- Entrepreneurial profit $\pi(a_{t-1}, z_{t-1})$ given by:

$$\pi(a_{t-1}, z_t) = \max_{k_t < \kappa a_{t-1}} \{ p_t z_t k_t - (r+\delta) k_t \}$$

Empirical fit

- Calibrate model to US. Model reproduces wealth inequality in the
  data, also for the extremely rich
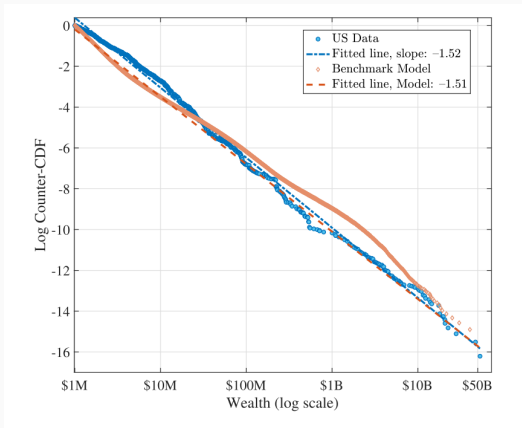
Results

- Exercise: Replace capital income tax $\tau_k = 25\%$ with wealth tax $\tau_a > 0$ in a government revenue-neutral way (requires $\tau_a = 1.2\%$)

TABLE V

TAX REFORM: CHANGE IN MACRO VARIABLES FROM CURRENT U.S. BENCHMARK

	Quantities (% change)						Prices (change)			
	K	Q	TFP_Q	L	Y	C	$\overline{w}$	$\overline{w}$ (net)	$\Delta r^{\dagger}$	$\Delta r^{\dagger}$ (net)
RN reform	16.4	22.6	5.3	1.2	9.2	9.5	8.0	8.0	0.21	−0.36
BB reform	9.2	16.0	6.2	1.2	6.9	7.7	5.6	5.6	0.67	−0.38

- Capital, productivity output, consumption, wages increases
    - Efficency gain from shifting tax base away from productive agents
- Also generates large welfare gain (around 7% consumption equivalent gains)

Results - optimal taxation

- Now find tax rates that maximize aggregate welfare
  - Wealth taxation (OWT) vs. capital income taxation (OKIT)
- Results:

OPTIMAL TAXATION: TAX RATES AND AVERAGE WELFARE EFFECTS

	Benchmark U.S. economy	RN reform	OWT	OWT L-INEQ	OWT-X	WTE-X	OKIT
		(1)	(2)	(3)	(4)	(5)	(6)
Tax rates							
τ_k	25.0	—	—	—	—	—	−13.6
τ_a	—	1.19	3.03	2.54	$3.80^{\dagger}$	3.30	—
τ_ℓ	22.4	22.4	15.4	18.1	14.4	17.7	31.2
ΔWelfare							
$\overline{CE}_1$	—	6.8	9.0	6.0	9.1	8.4	4.2
$\overline{CE}_2$	—	7.2	8.7	5.2	8.8	8.6	5.1

- Wealth taxation: Positive taxation $\tau_a = 3.03\%$, large welfare gain of 9%
- Capital income taxation: *Subsidy* $\tau_K = -13.6\%$ and smaller welfare gain of 4.2%

Summary

- Guvenen et al. (2023) study optimal wealth taxation
- Source of wealth inequality matters for optimal taxation
- If driven by return heterogeneity **wealth tax** strongly preffered to **capital income tax**
  - Why? It distorts investment decisions of high productivity HHs less than a capital income tax

Exercise

Standard HANC model with return heterogeneity

- HH problem:

$$v_t(, e_{it} r_{it}^x, , a_{it-1}) = \max_{c_t} u(c_t) + \beta \underline{v}_{t+1}(e_{it+1}, r_{it+1}^x, a_{it})$$

s.t.

$$a_{it} = (1 + r_t + r_{it}^x)a_{it-1} + w_t e_{it} - c_{it}$$

$$\log e_{it+1} = \rho_e \log e_{it} + \psi_{it+1}^e, \quad \psi_{it+1}^e \sim \mathcal{N}\left(0, \sigma_e^2\right)$$

$$r_{it+1}^x = \overline{r}^x + \rho_z r_{it}^x + \psi_{it+1}^{r^x}, \quad \psi_{it+1}^{r^x} \sim \mathcal{N}\left(0, \sigma_{r^x}^2\right)$$

$$a_{it} \geq 0$$

- **Q1**: Solve the PE HA model with return heterogeneity
- **Q2**: Calibrate the HANC model such that average returns are 4%
- **Q3**: Calibrate a standard HA model without return heterogeneity. Compare the wealth distributions obtained in the two models.

Summary

Summary and next week

- **Today:** Various explanations of wealth inequality
  1. Preferences
  2. Bequests
  3. Returns
- **Next week:** Fall break $\Rightarrow$ No lecture
  - Week after: Secular stagnation
- **Midterm evaluation:** Fill out questionnaire
- **Homework exercise**: Solve model with return heterogeneity
  - See Github repo