



# 6. Wealth Inequality

Adv. Macro: Heterogenous Agent Models

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# Introduction

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- To answer these questions, we need to better understand why people save, and how this translates into wealth inequality
- **Plan for today:**
  1. Study the predictions of a baseline Bewley-Huggett-Aiyagari model
  2. Consider various model extensions that help match the data
  3. Given such a model, what can we say about optimal wealth taxation?

## **Wealth inequality in the data**

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# Earnings and wealth inequality

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Wealth	29	53	80	93	6
Earnings	6	19	48	72	8

# Earnings and wealth inequality

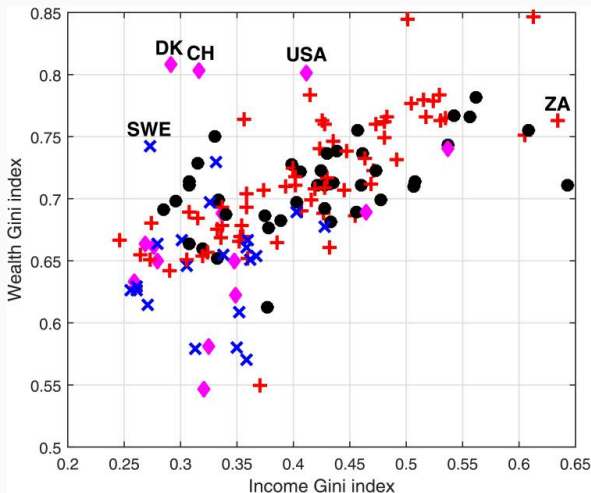
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- Wealth more concentrated than earnings
- Skewed distributions with thick upper tails

# Wealth more concentrated than earnings

Not only in the US, but also Denmark and almost all other countries



# Aiyagari Model

- Infinitely lived agents with preferences

$$\max_{\{c_t\}_{t=0}^{\infty}} E \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

- Budget constraint and borrowing constraint

$$a_t = y_t + (1+r)a_{t-1} - c_t, \quad a_t \geq \underline{a}$$

- Idiosyncratic earnings risk:

$$\ln y_t = \rho \ln y_{t-1} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma_{\epsilon}^2)$$

- As usual, calibrate parameters in earnings process  $(\rho, \sigma_{\epsilon}^2)$  based on estimates from panel data on earnings, i.e. Floden and Linde (2001)

## Aiyagari Model - wealth inequality fit

	Wealth Gini	Wealth in top (%)		
		1%	5 %	20 %
U.S. data, 1989 SCF	.78	29	53	80
Aiyagari Baseline	.38	3.2	12.2	41.0
Aiyagari higher variability	.41	4.0	15.6	44.6

# Top wealth inequality

- What about top wealth inequality?
  - Think top 0.01% or 0.001% (Bezos, Musk, Gates etc.)

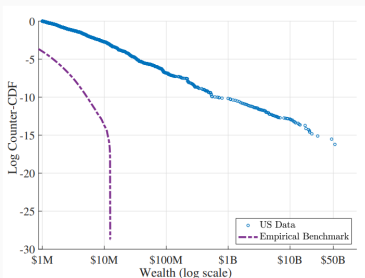
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- The probability of having wealth  $a$  above threshold  $X$  described as Pareto dist,  $P(a > X) \sim x^{-\alpha}$ 
  - In logs,  $\ln P(a > X) \sim -\alpha \ln x$ , so linear in wealth with  $\alpha$  describing the »thickness of the tail«



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  - Will discuss this in next lecture
- Note also: Only driver of wealth inequality is earnings risk
  - Income inequality in data typically lower than *wealth* inequality
  - In reality multiple drivers such as entrepreneurship, preferences, bequests, return heterogeneity

## **Explaining wealth inequality**

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- Return heterogeneity
  - Hubmer, Krusell, Smith (2021), Ozkan et al. (2023), Guvenen et al. (2023)

# Bequests

$$\max_{\{c_t\}_{t=0}^T} E \sum_{t=0}^T \beta^t \left( s_t \frac{c_t^{1-\sigma}}{1-\sigma} + (1-s_t) \phi(a_{t-1}) \right)$$
$$c_t + a_t = y_t + (1+r)a_{t-1} + b_t, \quad a_t \geq \underline{a}$$

1. Bequests and human capital transmission across generations (*warm glow*)

# Explaining wealth inequality

$$\max_{\{c_t\}_{t=0}^T} E \sum_{t=0}^T \beta_i^t s_t \frac{c_t^{1-\sigma_i}}{1-\sigma_i}$$

$$c_t + a_t = y_t + (1+r)a_{t-1}, \quad a_t \geq \underline{a}$$

- 1.
2. Heterogeneous preferences

# Explaining wealth inequality

$$\max_{\{c_t\}_{t=0}^T} E \sum_{t=0}^T \beta^t s_t \frac{c_t^{1-\sigma}}{1-\sigma}$$

$$c_t + a_t = [l_e f(\theta_t, k_{t-1}) + (1 - l_e) y_t] + (1 + r)(a_{t-1} - k_{t-1}), \quad a_t \geq \underline{a}$$

- 1.
- 2.
3. Entrepreneurship.

# Explaining wealth inequality

$$\max_{\{c_t\}_{t=0}^T} E \sum_{t=0}^T \beta^t s_t \frac{c_t^{1-\sigma}}{1-\sigma}$$

$$c_t + a_t = y_t + (1 + r_t^i) a_{t-1}, \quad a_t \geq \underline{a}$$

- 1.
- 2.
- 3.
4. Idiosyncratic rates of return

**Hubmer, Krussel and Smith  
(2021)**

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# Explaining wealth inequality

- Hubmer, Krussel and Smith (2021): *Sources of US wealth inequality: Past, present, and future*
  - Model which matches key features of US wealth inequality in 1967
  - Can we account for changes in wealth inequality going forward from 1967 based on observables?
    - I.e. changes in income inequality, taxes, asset returns

# Model

- Household problem features non-linear tax schedules, heterogeneous returns and  $\beta$ -het.

$$V_t(a_{t-1}, p_t, \beta_t) = \max_{a_{t+1} \geq 0} \{u(c_t) + \beta_t \mathbb{E}[V_{t+1}(a_t, p_{t+1}, \beta_{t+1}) | p_t, \beta_t]\}$$

$$c_t + a_t = y_t - \tau_t(y_t) + (1 - \tilde{\tau}_t)\tilde{y}_t + T_t$$

$$y_t = (\underline{r}_t + r_t^X(a_{t-1}))a_{t-1} + w_t l_t(p_t)$$

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- Example: If rich HHs primarily invest in stocks, poorer HHs in bonds. Would expect both  $r_t^X(a_{t-1}), \sigma_t^X(a_{t-1})$  to be increasing in  $a_{t-1}$

- Fagereng, Guiso, Malacrino, Pistaferri (2020) find that rates of returns are:
  - Heterogeneous across households (over 200 basis points between 10th and 90th percentile of the distribution of returns)
  - Also heterogeneous within asset classes
    - So return differences cannot be explained only by poorer HHs holding bank deposits and rich HHs investing in stocks
  - Persistent
  - Correlated with household wealth and across generations

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- Plus goods market clearing, but redundant given other 2

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1. Calibrate earnings process, tax rates, return process, social safety net to observables
2. Choose randomness in discount factor  $\beta$  residually so as to replicate the wealth distribution in the initial steady state (1967)
3. Then feed in exogenous changes in tax rates, earnings inequality, etc. between 1967 and 2015 to understand the role of these different factors

# Return heterogeneity

- Overall return given asset holdings  $a_{t-1}$  equals

$$\underline{r}_t + r_t^X(a_{t-1}) + \sigma^X(a_{t-1})\eta_t$$

- $\underline{r}_t$  is endogenous
- $r_t^X(\cdot)$  and  $\sigma^X(\cdot)$  are exogenous excess return schedules (mean and st.dev.), taken from the data
- $\eta_t$  is an i.i.d. standard normal shock
- Reduced form portfolio choice

# Calibration: return process

$$r_t^X(a_t) = \sum_{c \in C} w_c(a_t) (\bar{r}_{c,t} + \tilde{r}_c^X(a_t))$$
$$\sigma^X(a_t)^2 = \sum_{c \in C} (w_c(a_t) \tilde{\sigma}_c^X(a_t))^2$$

- Asset classes  $C$ : risk-free, public equity, private equity, housing
- $\bar{r}_{c,t}$ : aggregate return on asset class  $c$  (U.S. data), **time-varying**
- Fixed over time, based on Swedish administrative data from Bach, Calvet, Sodini (2016):
  - $w_c(\cdot)$ : portfolio weights
  - $\tilde{r}_c^X(\cdot)$ : within asset class return heterogeneity
  - $\tilde{\sigma}_c^X(\cdot)$ : asset  $c$  idiosyncratic return standard deviation

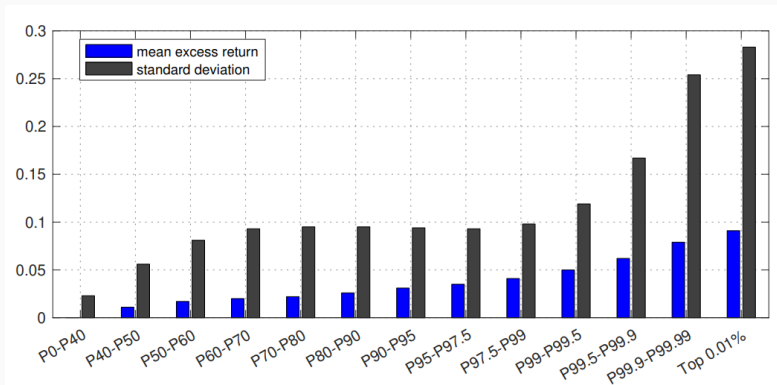
# Excess return schedule details

- Aggregate Excess Returns in 1967 steady state:
  - public equity 0.067 (U.S., Kartashova 2014)
  - private equity 0.129 (U.S., Kartashova 2014)
  - housing 0.037 (incl. imputed rent; Jorda, et al, 2017)
- and cross-sectional data from Bach, Calvet, Sodini (2019) implies

	P0-P40	P40-P50	P50-P60	P60-P70	P70-P80	P80-P90	P90-P95	P95-P97.5	P97.5-P99	P99-P99.5	P99.5-P99.9	P99.9-P99.99	Top 0.01%
fixed portfolio weights													
risk-free	0.722	0.412	0.248	0.182	0.156	0.134	0.115	0.102	0.090	0.079	0.071	0.051	0.029
housing	0.162	0.394	0.580	0.662	0.678	0.674	0.658	0.626	0.572	0.482	0.363	0.253	0.155
public equity	0.113	0.189	0.165	0.147	0.153	0.170	0.189	0.207	0.219	0.232	0.230	0.185	0.179
private equity	0.002	0.005	0.007	0.009	0.013	0.021	0.038	0.065	0.118	0.207	0.336	0.511	0.637



# Schedule of excess returns



Data sources: Bach, Calvet, Sodini (2019); Kartashova (2014); Jorda, Knoll, Kuvshinov, Schularick, Taylor (2019); Case-Shiller.

# Hubmer, Krussel and Smith (2021)

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## Results

# Results, I: Steady state (1967)

- Steady state fit (with and without  $\beta$ -het)

	Top 10%	Top 1%	Top 0.1%	Top 0.01%
Data	70.8%	27.8%	9.4%	3.1%
Single- $\beta$ Model	66.6%	23.7%	11.2%	7.2%
Benchmark Model	73.8%	27.4%	8.4%	3.2%
	Bottom 50%	Fraction $a < 0$		
Data	4.0%	8.0%		
Single- $\beta$ Model	3.5%	7.3%		
Benchmark Model	3.0%	6.6%		

# Results, I: steady state (1967)

#		top 10%	top 1%	top 0.1%	top 0.01%	Gini
1	$\beta$ -heterogeneity	8.8%	7.7%	3.8%	2.0%	0.050
2	earnings heterogeneity	-27.5%	-17.8%	-9.5%	-6.4%	-0.173
3	persistent	-5.0%	-7.5%	-4.2%	-2.9%	0.009
4	transitory	-11.6%	-4.3%	-1.7%	-0.9%	-0.109
5	tax progressivity	-21.3%	-61.8%	-71.2%	-67.1%	-0.148
6	return heterogeneity	29.5%	18.4%	6.6%	2.8%	0.192
7	mean differences	25.8%	16.7%	6.0%	2.6%	0.174
8	return risk	0.7%	2.2%	3.3%	2.5%	0.004

- How to read: Shutting of  $\beta$ -het reduces top 10% wealth share by 8.8%
- Model matches wealth distribution well on its entire domain
  - return heterogeneity is key ingredient
  - wealth concentration is mitigated by progressive taxation and labor income risk

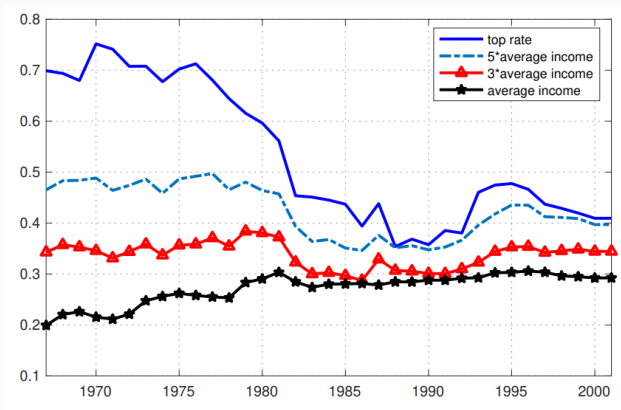
## Next step: transition

The authors feed in four different factors that have changed during the past 50 years

- Decrease in tax progressivity
- Increase in labor income risk
- Increase in income going to the top
- Changing return premia to different asset classes

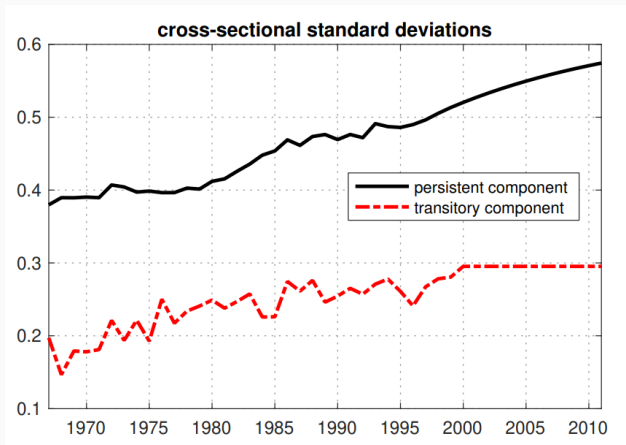
# Observed change 1: Decrease in tax progressivity

- Federal effective tax rates (Piketty & Saez 2007): income, payroll, corporate and estate taxes



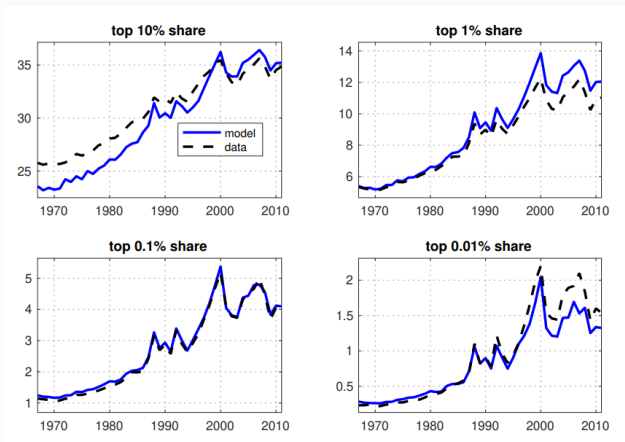
## Observed change 2: Increase in labor income risk

- Estimates for variance of persistent and temporary components 1967-2000 (Heathcote, Storesletten & Violante 2010)



# Observed change 3: Increase in top labor income shares

- Adjust standard AR(1) in idiosyncratic productivity by imposing a Pareto tail for the top 10% earners: calibrated tail coefficient decreases from 2.8 to 1.9 (updated Piketty & Saez 2003 series)



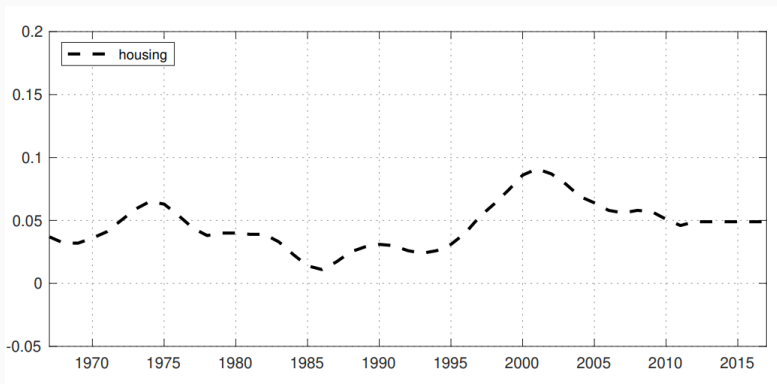


## Observed change 4: return premia

- Feed in (smoothed) time series of aggregate U.S. asset premia (Kartashova 2014, Case-Shiller index)

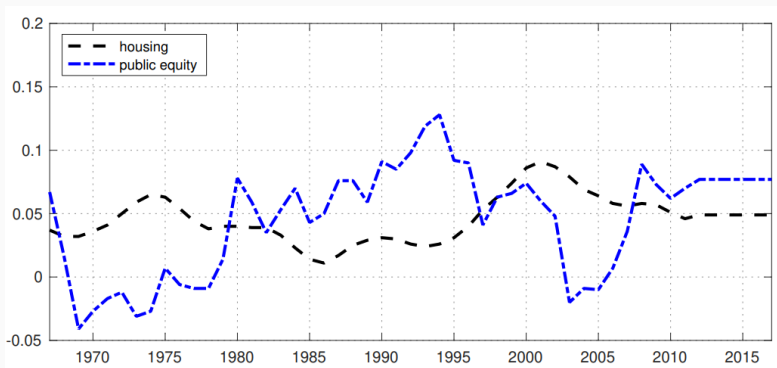
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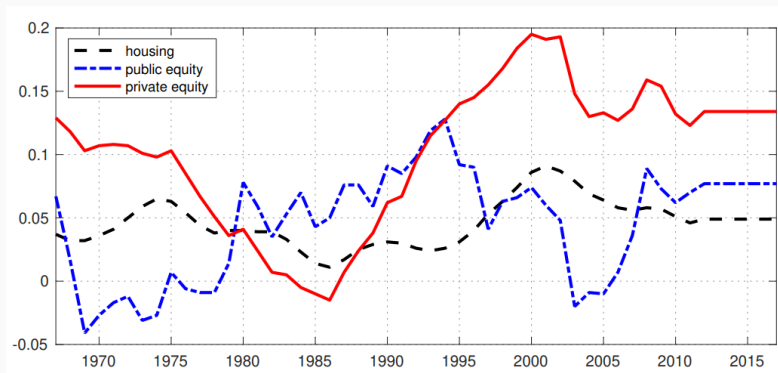
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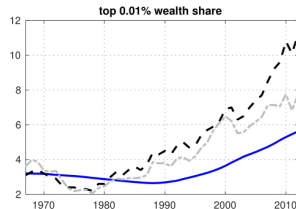
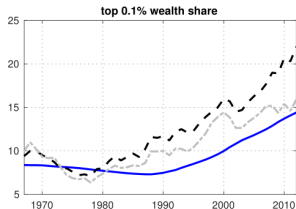
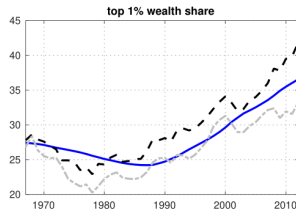
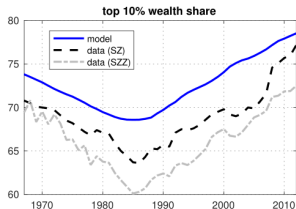


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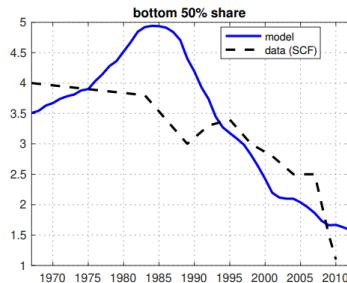
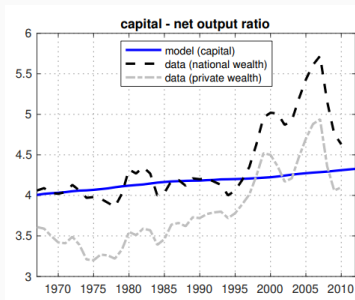


# Results, II: historical evolution



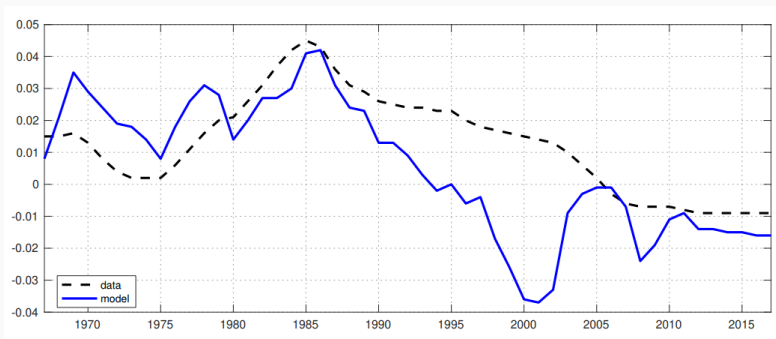
Data sources: dashed black lines refer to Saez & Zucman (2016); dash-dotted gray lines refer to Smith et al. (2020).

# Results: Capital-output ratio and bottom 50 %

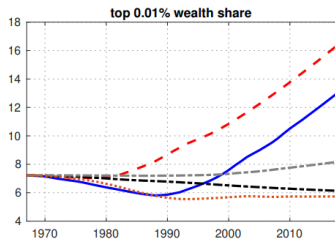
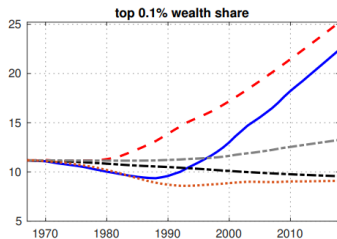
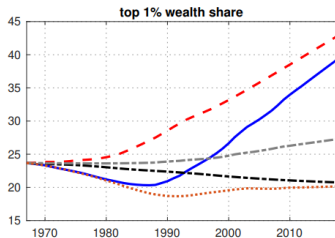
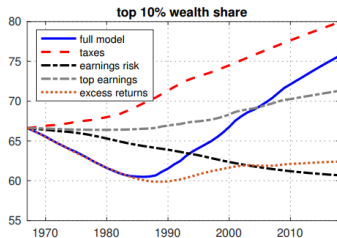


# Results: Risk-free rate

- Return premia are matched in model by construction
- Risk-free rate  $r$  is endogenous: comparable level and decline



# Decomposition of transitional dynamics





# Decomposition of transitional dynamics

- Overall increase in wealth inequality (more than) fully explained by declining tax progressivity
  - Primarily due to direct effect on resource distribution and not due to changing savings behavior
- Time-varying return premia account for U-shape in wealth inequality
- Subtle role of increasing earnings dispersion
  - Thickening Pareto tail in labor income contributes slightly positively to wealth inequality
  - Increase in overall earnings risk *decreases* wealth inequality because precautionary savings motive is stronger for poorer HHs

# Summary

- **Hubmer, Krussel and Smith (2021)**
- HANC with:
  - Income risk
  - Return heterogeneity
  - $\beta$ -heterogeneity
  - Tax system
- Main finding:
  - Return heterogeneity key in matching initial (1967) wealth inequality
  - Can roughly explain evolution in US wealth inequality with observable changes in tax systems

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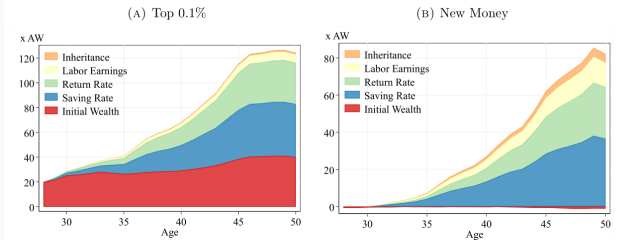
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- Using budget constraint:

$$a_{it} = a_{it-1} + (L_{it} + H_{it} + r_{it}a_{it-1}) \times s_{it}$$

# Results from Ozkan et al. (2024)

- Left panel: Decomposition of wealth for top 0.1%
- Right panel: »Poorest« HHs *within* top 0.1% (*New Money*)

FIGURE 1 – DETERMINANTS OF THE TOP 0.1% WEALTH ACCUMULATION



# **Application to Wealth Taxation**

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# Wealth taxation I

- Spend a lot of time understanding *what* drives wealth inequality
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    - Efficiency concern 2: In the Ramsey model aggregate K is generally below the golden rule level

# Wealth taxation II

- Guvenen et al. (2023): *Use It or Lose It: Efficiency and Redistributive Effects of Wealth Taxation*
- Study optimal taxation in two tax systems:
  - Wealth tax:  $a_i$
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  - After tax wealth /w CI tax :  $a_i + (1 - \tau_k) ra_i$
  - After tax wealth /w wealth tax :  $(1 - \tau_a) a_i + ra_i$



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- Social planner can implement same allocation using these two different instruments by setting  $\tau_a = r \tau_k$

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  - Shifts tax base towards unproductive agents
- Note: We say HHs with high  $r_i$  are more **productive**
  - Think in terms of *entrepreneurial* models
  - High productivity HHs have better technology (i.e. are better entrepreneurs) and can make their wealth grow faster (high  $r_i$ )

# Model

- HH problem:

$$\max_{\{c_t\}_{t=0}^T} E \sum_{t=0}^T \beta^t \left( s_t \frac{c_t^{1-\sigma}}{1-\sigma} + (1-s_t) \phi(a_t) \right)$$

$$a_t + c_t = \mathcal{W}(a_{t-1}, z_{t-1}) + w_t(e_t) \ell_t, \quad a_t \geq \underline{a}$$

$$\mathcal{W}(a_{t-1}, z_t) = \begin{cases} a_{t-1} + (\pi(a_{t-1}, z_t) + r a_{t-1})(1 - \tau_k) & \text{if CI tax} \\ a_{t-1}(1 - \tau_a) + (\pi(a_{t-1}, z_t) + r a_{t-1}) & \text{if wealth tax} \end{cases}$$

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- Entrepreneurial ability  $z$  follow markov chain with values  $z = [0, z_L, z_H]'$  and transition matrix  $\Pi_z$ 
  - HHs with  $z = 0$  are normal workers
  - HHs with  $z = z_L$  are »unproductive« entrepreneurs
  - HHs with  $z = z_H$  are »productive« entrepreneurs

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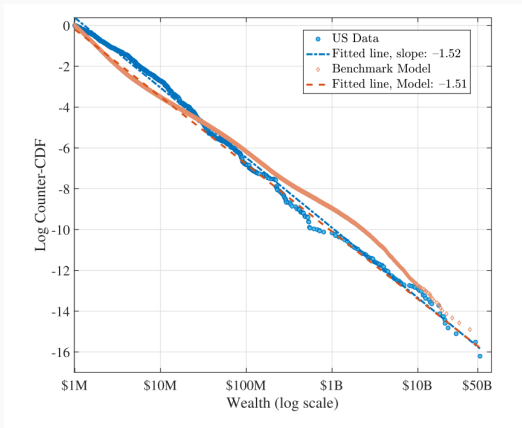
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- Entrepreneurial profit  $\pi(a_{t-1}, z_{t-1})$  given by:

$$\pi(a_{t-1}, z_t) = \max_{k_t \leq \kappa a_{t-1}} \{p_t z_t k_t - (r + \delta) k_t\}$$



# Empirical fit

- Calibrate model to US. Model reproduces wealth inequality in the data, also for the extremely rich



# Results

- Exercise: Replace capital income tax  $\tau_k = 25\%$  with wealth tax  $\tau_a > 0$  in a government revenue-neutral way (requires  $\tau_a = 1.2\%$ )

TABLE V										
TAX REFORM: CHANGE IN MACRO VARIABLES FROM CURRENT U.S. BENCHMARK										
	Quantities (% change)						Prices (change)			
	$K$	$Q$	$TFP_Q$	$L$	$Y$	$C$	$\bar{w}$	$\bar{w}$ (net)	$\Delta r^\dagger$	$\Delta r^\dagger$ (net)
RN reform	16.4	22.6	5.3	1.2	9.2	9.5	8.0	8.0	0.21	-0.36
BB reform	9.2	16.0	6.2	1.2	6.9	7.7	5.6	5.6	0.67	-0.38

- Capital, productivity output, consumption, wages increases
  - Efficiency gain from shifting tax base away from productive agents
- Also generates large welfare gain (around 7% consumption equivalent gains)

# Results - optimal taxation

- Now find tax rates that maximize aggregate welfare
  - Wealth taxation (OWT) vs. capital income taxation (OKIT)
- Results:

OPTIMAL TAXATION: TAX RATES AND AVERAGE WELFARE EFFECTS							
	Benchmark U.S. economy	RN reform	OWT	OWT L-INEQ	OWT-X	WTE-X	OKIT
		(1)	(2)	(3)	(4)	(5)	(6)
Tax rates							
$\tau_k$	25.0	—	—	—	—	—	-13.6
$\tau_a$	—	1.19	3.03	2.54	3.80 <sup>†</sup>	3.30	—
$\tau_\ell$	22.4	22.4	15.4	18.1	14.4	17.7	31.2
$\Delta$ Welfare							
$\overline{CE}_1$	—	6.8	9.0	6.0	9.1	8.4	4.2
$\overline{CE}_2$	—	7.2	8.7	5.2	8.8	8.6	5.1

- Wealth taxation: Positive taxation  $\tau_a = 3.03\%$ , large welfare gain of 9%
- Capital income taxation: *Subsidy*  $\tau_K = -13.6\%$  and smaller welfare gain of 4.2%

# Summary

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- Guvenen et al. (2023) study optimal wealth taxation
- Source of wealth inequality matters for optimal taxation
- If driven by return heterogeneity **wealth tax** strongly preferred to **capital income tax**
  - Why? It distorts investment decisions of high productivity HHs less than a capital income tax

## Exercise

---

# Standard HANC model with return heterogeneity

- HH problem:

$$v_t(e_{it} r_{it}^x, a_{it-1}) = \max_{c_t} u(c_t) + \beta v_{t+1}(e_{it+1}, r_{it+1}^x, a_{it})$$

s.t.

$$a_{it} = (1 + r_t + r_{it}^x) a_{it-1} + w_t e_{it} - c_{it}$$

$$\log e_{it+1} = \rho_e \log e_{it} + \psi_{it+1}^e, \quad \psi_{it+1}^e \sim \mathcal{N}(0, \sigma_e^2)$$

$$r_{it+1}^x = \bar{r}^x + \rho_z r_{it}^x + \psi_{it+1}^{r^x}, \quad \psi_{it+1}^{r^x} \sim \mathcal{N}(0, \sigma_{r^x}^2)$$

$$a_{it} \geq 0$$

- **Q1:** Solve the PE HA model with return heterogeneity
- **Q2:** Calibrate the HANC model such that average returns are 4%
- **Q3:** Calibrate a standard HA model without return heterogeneity.  
Compare the wealth distributions obtained in the two models.



# Summary

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# Summary and next week

- **Today:** Various explanations of wealth inequality
  1. Preferences
  2. Bequests
  3. Returns
- **Next week:** Fall break  $\Rightarrow$  No lecture
  - Week after: Secular stagnation
- **Midterm evaluation:** Fill out questionnaire
- **Homework exercise:** Solve model with return heterogeneity
  - See Github repo