

# **CS 170: Homework #1**

Due on January 26, 2026 at 3:10pm

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## 1 Study Group

List the names and SIDs of the members in your study group. If you have no collaborators, explicitly write "none."

**none**

## 2 Course Policies

Please read through the course policies below:

1. The exams will be at the following times. We do not plan on offering alternate exams, so please mark your calendars:
  - (a) **Midterm 1:** Thursday, 2/26/2026, 7:00 PM - 9:00 PM
  - (b) **Midterm 2:** Thursday, 4/9/2026, 8:00 PM - 10:00 PM
  - (c) **Final:** Friday, 5/15/2026, 7:00 PM - 10:00 PM
2. Homework is due Mondays at 10:00pm, with a late deadline at 11:59pm to protect against technical issues. There is no penalty for submitting before the late deadline, but absolutely no submissions will be accepted after 11:59pm.
3. Your lowest homework will be dropped at the end of the semester. By filling out the mid-semester survey and end-of-semester course evaluation, you have the opportunity for two additional drops.
4. Each homework will have one question for you to attempt on your own, labeled "solo question." You will get detailed feedback on your answer to help calibrate you on your learning. On the remaining questions you may collaborate with your fellow students and with AI tools, but you must always list your collaborators and write up the solutions in your own words. (Details in the policies linked below.)
5. The primary source of communication for CS170 will be through Edstem.
6. **Syllabus and Policies:** <https://cs170.org/policies/>
7. **Homework Guidelines:** <https://cs170.org/resources/homework-guidelines/>
8. **Regrade Etiquette:** <https://cs170.org/resources/regrade-etiquette/>
9. **Forum Etiquette:** <https://cs170.org/resources/ed-etiquette/>

Copy and sign the following sentence on your homework submission.

"I have read and understood the course syllabus and policies."

**I have read and understood the course syllabus and policies.**

### 3 Asymptotic Complexity Comparisons

Order the following functions so that for all  $i, j$ , if  $f_i$  comes before  $f_j$  in the order then  $f_i = O(f_j)$ . Do not justify your answers.

- $f_1(n) = 3^n$
- $f_2(n) = n^{\frac{1}{3}}$
- $f_3(n) = 12$
- $f_4(n) = 2^{\log_2 n}$
- $f_5(n) = \sqrt{n}$
- $f_6(n) = 2^n$
- $f_7(n) = \log_2 n$
- $f_8(n) = 2^{\sqrt{n}}$
- $f_9(n) = n^3$
- $f_{10}(n) = \log_3 n$
- $f_{11}(n) = \log^2 n$

As an answer you may just write the functions as a list, e.g.  $f_8, f_9, f_1, \dots$

*f<sub>3</sub>, f<sub>10</sub>, f<sub>7</sub>, f<sub>11</sub>, f<sub>2</sub>, f<sub>5</sub>, f<sub>4</sub>, f<sub>9</sub>, f<sub>8</sub>, f<sub>6</sub>, f<sub>1</sub>*

## 4 Counting Steps (Solo Question)

You can climb a ladder with  $n$  rungs by climbing either 1 rung or 2 rungs in each step. How many distinct ways are there to climb to the top?

- (a) Give a simple recursive algorithm to compute the answer.
- (b) Prove correctness using induction.

### Solution

- (a) Below is a simple recursive algorithm to compute the answer.

```
1: function STEPS( $n$ )
2:   if  $n = 0$  then
3:     return 1
4:   else if  $n < 0$  then
5:     return 0
6:   else
7:     return STEPS( $n - 1$ ) + STEPS( $n - 2$ )
8:   end if
9: end function
```

- (b) *Proof.* We prove by induction on  $n$  that the algorithm correctly computes the number of distinct ways to climb  $n$  rungs.

#### Base cases:

- When  $n = 0$ : There is exactly 1 way to climb 0 rungs (do nothing), and STEPS(0) returns 1. ✓
- When  $n < 0$ : There are 0 ways to climb a negative number of rungs, and STEPS( $n$ ) returns 0. ✓

**Inductive hypothesis:** Assume that for all  $0 \leq m \leq k$ , STEPS( $m$ ) correctly computes the number of ways to climb  $m$  rungs.

**Inductive step:** For  $n = k + 1$ , the algorithm returns STEPS( $k$ ) + STEPS( $k - 1$ ). By the inductive hypothesis, STEPS( $k$ ) correctly counts ways to reach rung  $k$  (from which we take 1 more step), and STEPS( $k - 1$ ) correctly counts ways to reach rung  $k - 1$  (from which we take 2 more steps). Their sum counts all distinct ways to reach rung  $k + 1$ . □

## 5 Counting Steps Efficiently

- (a) Analyze the running time of your algorithm from the previous question. Is it bounded by a polynomial in  $n$ ? Assume for simplicity that two integers can be added in one timestep.
- (b) Give an efficient algorithm to compute the answer, and show the running time is bounded by  $O(n)$  (polynomial in  $n$  is also acceptable).
- (c) (Extra credit) Give an even more efficient algorithm to compute the answer, and analyze its running time, which should be sub-polynomial, again assuming each integer arithmetic operation takes 1 step.  
*Hint: can you compute the answer by multiplying  $2 \times 2$  matrices?*

Is this running time a fair assessment of how the algorithm will perform in practice?

### Solution

- (a) The recurrence equation for the algorithm is  $T(n) = T(n-1) + T(n-2) + O(n)$ . So it is not bounded in polynomial time because the tree grows approximately exponentially in size with each increase in  $n$ .
- (b) Below is an efficient algorithm to compute the answer.

```

1: function STEPS( $n$ )
2:   rungs[0]  $\leftarrow$  1
3:   rungs[1]  $\leftarrow$  1
4:   for  $i \leftarrow 2$  to  $n$  do
5:     rungs[ $i$ ]  $\leftarrow$  rungs[ $i-1$ ] + rungs[ $i-2$ ]
6:   end for
7:   return rungs[ $n$ ]
8: end function

```

Let  $C(n)$  denote the number of iterations of the for-loop in  $\text{Steps}(n)$ . For  $n \leq 1$ , the loop does not execute, so  $C(n) = 0$ . For  $n \geq 2$ , the loop runs for  $i = 2, 3, \dots, n$ , hence  $C(n) = n - 1$ . Each iteration performs one addition and a constant number of assignments, so costs  $O(1)$  time. The initialization of  $\text{rungs}[0]$  and  $\text{rungs}[1]$  also costs  $O(1)$ . Therefore, the total running time is  $O(1) + C(n) \cdot O(1) = O(n)$ . The space usage is  $O(n)$  to store  $\text{rungs}[0..n]$ .

- (c) Below is an even more efficient algorithm to compute the answer:

```

1: function STEPS( $n$ )
2:    $A \leftarrow \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ 
3:    $A \leftarrow \text{MATRIXPOWER}(A, n + 2)$ 
4:   return  $A[1, 1]$ 
5: end function

```

The above algorithm produces the correct answer on an arbitrary input of  $n \geq 1$ . The function leverages matrix multiplication of a  $2 \times 2$  matrix,  $A$ , by computing  $A^{n+2}$  and returning the bottom-right entry of the result. This works because  $A$  encodes the base cases of the function and computes an update in multiplication

$$\begin{bmatrix} \text{STEPS}(n+1) & \text{STEPS}(n) \\ \text{STEPS}(n) & \text{STEPS}(n-1) \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \text{STEPS}(n+2) & \text{STEPS}(n+1) \\ \text{STEPS}(n+1) & \text{STEPS}(n) \end{bmatrix}$$

and on an arbitrary input of  $n \geq 1$ , computing  $A^{n+2}$  will result in a  $2 \times 2$  matrix with  $\text{STEPS}(n)$  in its bottom-right corner, which is returned.