Macroeconomics: Problem Set #5

Due on November 24, 2024 at 10:00pm

Prof. Barnichon Section 104

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Problem 1

In this problem, we propose to explore two famous episodes when the price level fell substantially. One was in England between 1250 and 1490. The other episode was in the United States between 1879 and 1896.

One commonly argued reason why the price level fell during these periods was that the economies in question were growing and this was putting downward pressure on prices as the supply of money was not growing fast enough to offset the growth in the economy. In this problem, we use an extension of the medieval economy model to formalize this argument.

Consider a version of the medieval economy model in which the desired level of output may change over time. Since it may change over time, we denote it as Y_t^* rather than Y^*. You can think of the reasons why the desired level of output can change over time being either that the size of the population changes over time or that the level of technology changes over time so each person can produce more output per hour worked.

Since the desired level of output can change, the price setting equation becomes:

$$\text{PS:} \qquad \log P_{t+1} - \log P_t = \theta(\log Y_t - \log Y_t^*)$$

Suppose the money market equilibrium condition is the same as before:

$$\text{MME:} \quad \log M_t + \log V = \log P_t + \log Y_t$$

A) Suppose for simplicity that $\log Y_t^*$ is a constant equal to 0. Solve for the steady state level of output and steady state price level when $\log M_t = 0$ and $\log V = 0$.

B) Now suppose that the growth rate of the desired level of ouput rises such that $\Delta \log Y_t^* = 0.02$ but the money supply remains fixed at $\log M_t = 0$ and velocity remains fixed at $\log V = 0$. Solve for the steady state inflation rate (i.e., steady state value of $\Delta \log P_t$) and the steady state output gap (i.e., the steady state value of $\log Y_t - \log Y_t^*$). *Hint*: You should guess that $\Delta \log P_t$ is constant in this steady state (i.e., the change in the log price level is constant). Then, subtract the time $t-1$ version of the PS equation above from the time t version of this equation. The resulting equation should help you solve the problem.

C) Starting from the steady state you calculated in part A), suppose that the growth rate of the desired level of output rises such that $\Delta \log Y_t^* = 0.02$. Solve for the evolution of $\log P_t$ and $\log Y_t$ over the next 20 periods. Assume that $\theta = 0.5$. Again assume that the money supply remains fixed at $\log M_t = 0$ and velocity remains fixed at $\log V = 0$. Graph the result.

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Part A

Suppose for simplicity that $\log Y_t^*$ is a constant equal to 0. Solve for the steady state level of output and steady state price level when $\log M_t = 0$ and $\log V = 0$.

Solution

To solve for steady-state output, I will use the price setting equation where variables no longer change over time and substitue in 0 for $\log Y_t^*$

$$
\begin{aligned}
\log P_{t+1} - \log P_{P_t} &= \theta(\log Y_t - \log Y_t^*) \\
\log P - \log P &= \theta(\log Y - 0) \\
0 &= \theta(\log Y) \\
0 &= \log Y \\
Y &= 1
\end{aligned}
$$

For the steady-state price level, I will use the money market equilibrium condition where again variables don't change over time and substitute in 0 for $\log M_t$, $\log V$ and steady-state output

$$
\begin{aligned}
\log M_t + \log V &= \log P_t + \log Y_t \\
0 + 0 &= \log P + \log Y \\
0 &= \log P + 0 \\
P &= 1
\end{aligned}
$$

Problem 1 continued on next page...

Part B

Now suppose that the growth rate of the desired level of ouput rises such that $\Delta \log Y_t^* = 0.02$ but the money supply remains fixed at $\log M_t = 0$ and velocity remains fixed at $\log V = 0$. Solve for the steady state inflation rate (i.e., steady state value of $\Delta \log P_t$) and the steady state output gap (i.e., the steady state value of $\log Y_t - \log Y_t^*$). *Hint*: You should guess that $\Delta \log P_t$ is constant in this steady state (i.e., the change in the log price level is constant). Then, subtract the time $t-1$ version of the PS equation above from the time t version of this equation. The resulting equation should help you solve the problem.

Solution

I will first find the steady state increase in output by rewriting the left- hand side of the price setting equation for a change in the price level $\Delta \log P_t$. I'll then subtract from it a $t-1$ indexed version of the equation and assume that $\Delta \log P_t$ is constant in steady state.

$$\begin{aligned}
\log P_{t+1} - \log P_t &= \theta(\log Y_t - \log Y_t^*) \\
\Delta \log P_t &= \theta(\log Y_t - \log Y_t^*) \\
\Delta \log P_t - \Delta \log P_{t-1} &= \theta(\log Y_t - \log Y_t^*) - \theta(\log Y_{t-1} - \log Y_{t-1}^*) \\
\Delta \log P - \Delta \log P &= \theta(\log Y_t - \log Y_{t-1} - (\log Y_t^* - \log Y_{t-1}^*)) \\
0 &= \theta(\Delta \log Y_t - \Delta \log Y_t^*) \\
\Delta \log Y_t &= \Delta \log Y_t^*
\end{aligned}$$

To find the steady state value of $\Delta \log P_t$, I will now use the money market equilibrium condition and substitute my value for $\Delta \log Y_t$ into a difference between t and $t-1$ indexed versions of the equation where $\log M_t$ and $\log V$ are both equal to zero (the money supply and velocit remain fixed).

$$\begin{aligned}
\log M_t + \log V &= \log P_t + \log Y_t \\
0 + 0 &= \log P_t + \log Y_t \\
0 &= \log P_t - \log P_{t-1} + \log Y_t - \log Y_{t-1} \\
0 &= \Delta \log P_t + \Delta \log Y_t \\
\Delta \log P_t &= -\Delta \log Y_t \\
\Delta \log P_t &= -\Delta \log Y_t^*
\end{aligned}$$

So the price level is decreasing over time proportionally with the increase in the desired level of output, $\boxed{\Delta \log P_t = -0.02}$. To find the steady state output gap, I will substitute this $\Delta \log P_t$ into the price setting equation.

$$\begin{aligned}
\log P_{t+1} - \log P_t &= \theta(\log Y_t - \log Y_t^*) \\
\Delta \log P_t &= \theta(\log Y_t - \log Y_t^*) \\
\frac{\Delta \log P_t}{\theta} &= \log Y_t - \log Y_t^* \\
-\frac{\Delta \log Y_t^*}{\theta} &= \log Y_t - \log Y_t^*
\end{aligned}$$

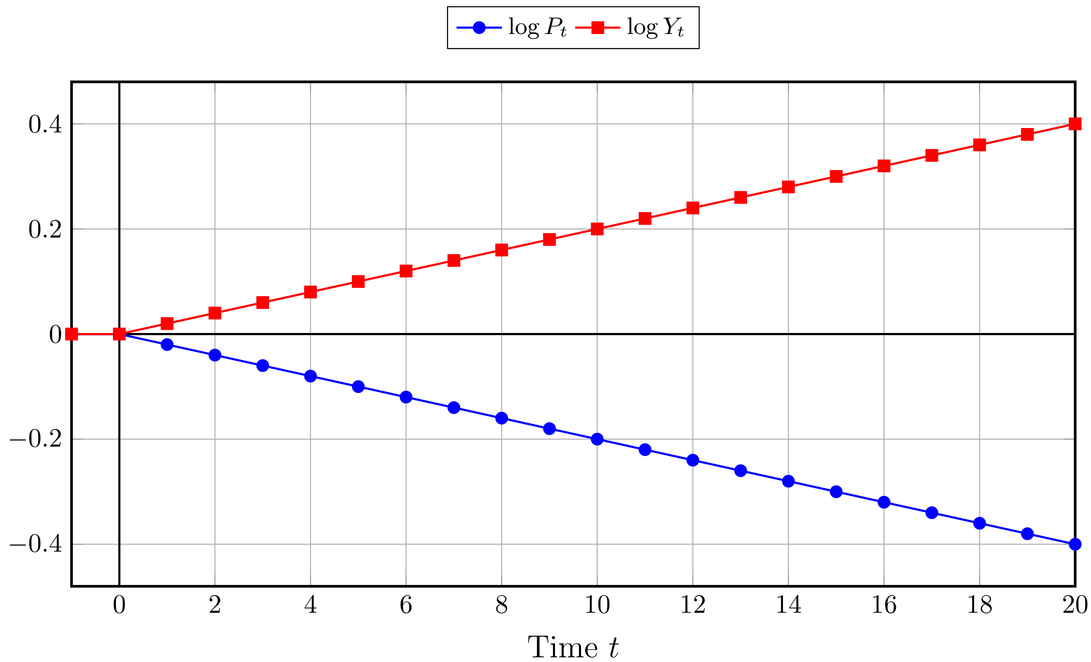
This shows that the steady state output gap is negative and in proportion to the increase in the desired level of output, $\boxed{\log Y_t - \log Y_t^* = -\frac{0.02}{\theta}}$. Output takes time to catch up to the ever changing desired level of output, so the output gap is negative.

Problem 1 continued on next page. . .

Part C

Starting from the steady state you calculated in part A), suppose that the growth rate of the desired level of output rises such that $\Delta \log Y_t^* = 0.02$. Solve for the evolution of $\log P_t$ and $\log Y_t$ over the next 20 periods. Assume that $\theta = 0.5$. Again assume that the money supply remains fixed at $\log M_t = 0$ and velocity remains fixed at $\log V = 0$. Graph the result.

Solution



Problem 2

Indicate whether you think the following statements are true, false or uncertain. Support your answer by giving all necessary reasoning and calculations:

A) The discovery of large gold and silver mines in the Americas in the 16^{th} century greatly increased the purchasing power of people in Europe.

False. The influx of gold and silver into a Europe where these metals served as money did not broadly increase the purchasing power of people. Purchasing power is the amount of goods and services one can buy with money. If only the money supply increases, all other things being equal, this will simply cause an increase in prices, as producers adjust prices to excess demand stimulated by an increase in the money supply.

B) According to the quantity theory of money, "inflation is always and everywhere a monetary phenomenon."

True. According to the quantity theory of money, the price level is directly proportional to the money supply, and that there is a causality between changes in the money supply and changes in the price level. The above statement corresponds with this idea, since it claims that inflation (rising of prices), results from an increase in the money supply.

Problem 3

One view of the cause of the Great Depression is that it is primarily a massive negative money supply shock associated with France (and the U.S.) hoarding gold and exchanging sterling for gold. This question asks you to analyze the implications of such a shock for output, price, and interest rates through the lens of our business cycle model in its IS-LM-PS incarnation.

Consider the following version of our business cycle model:

LM curve: $\Delta \log M_t - \pi_t = -\phi i_t + \phi i_{t-1} + \widetilde{Y}_t - \widetilde{Y}_{t-1} + \Delta \log v_t$

IS curve: $\widetilde{Y}_t = \overline{a} - \overline{b}(R_t - \overline{r})$

Fisher equation: $R_t = i_t - \pi_t$

Price setting equation: $\pi_t = \theta \widetilde{Y}_{t-1}$

The Fisher equation is written assuming adaptive expectations (i.e., $E_t[\pi_{t+1}] = \pi_t$. Also, the version of the LM curve written above comes from taking a first-difference of the LM curve in class and changing variables as before. Throughout this question, you may assume that $\overline{r} = 0.02$, $\phi = 0.5$, $\overline{a} = 0$, $\overline{b} = 1$, and $\theta = 0.2$.

A) Suppose that the economy starts off in a steady state with $\Delta \log M_t = 0$, $\Delta \log v_t = 0$. Calculate the steady state value of the output gap, inflation, the real interest rate, and the nominal interest rate.

B) Suppose that at time 0, the money supply in this economy falls by 0.2 log points (i.e., $\Delta \log M_0 = -0.2$) and remains at that new lower level going forward (i.e., $\Delta \log M_t = 0$ for $t > 0$). Plot the evolution of output, inflation, the nominal interest rate and the real interest rate over time from period 0 to period 20. (You may want to start your figures at period -5 just to have pre-period in the figures to be able to see the response of the economy better. Also, you may ignore the zero lower bound on nominal interest rates. If you don't know what that is, this is OK. We will cover it in a few weeks.)

C) Describe the evolution of the economy in response to the shock in part B) graphically using the IS-LM diagram.

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Part A

Suppose that the economy starts off in a steady state with $\Delta \log M_t = 0$, $\Delta \log v_t = 0$. Calculate the steady state value of the output gap, inflation, the real interest rate, and the nominal interest rate.

Solution

To find the steady state value of the output gap, I will first need to find the steady state interest rate. To find the steady state real interest rate, I will first need to find the steady state inflation and nominal interest rate using the LM curve when there is no change in the money supply or velocity in steady state.

$$\begin{aligned}
\Delta \log M_t - \pi_t &= -\phi i_t + \phi i_{t-1} + \widetilde{Y}_t - \widetilde{Y}_{t-1} + \Delta \log v_t \\
0 - \pi_t &= -\phi i_t + \phi i_{t-1} + \widetilde{Y}_t - \widetilde{Y}_{t-1} + 0 \\
-\pi &= 0 + 0 \\
\pi &= 0
\end{aligned}$$

So steady state inflation is equal to zero, $\boxed{\pi = 0}$. This makes sense considering thequantity theory of money in a context where there is no steady state change in the money supply. Using the price setting equation, we can find the steady state output gap.

$$\begin{aligned}
\pi_t &= \theta \widetilde{Y}_{t-1} \\
\pi &= \theta \widetilde{Y} \\
0 &= \theta \widetilde{Y} \\
\widetilde{Y} &= 0
\end{aligned}$$

Therefore, the output gap is closed at steady state, $\boxed{\widetilde{Y} = 0}$. Substituting this value for the steady state output gap alongstide the given values for $\overline{r}$, $\overline{a}$, and $\overline{b}$ into the IS curve produces the steady state interest rate.

$$\begin{aligned}
\widetilde{Y}_t &= \overline{a} - \overline{b}(R_t - \overline{r}) \\
\widetilde{Y} &= 0 - (R - 0.02) \\
0 &= R - 0.02 \\
R &= 0.02 \\
R &= \overline{r}
\end{aligned}$$

The steady state interest rate takes on the exact same value as the natural real interest rate, $\boxed{R = 0.02}$. From the Fisher equation, we know then that the steady state nominal interest rate equals the steady state real interest rate since inflation is zero, $\boxed{i = 0.02}$.

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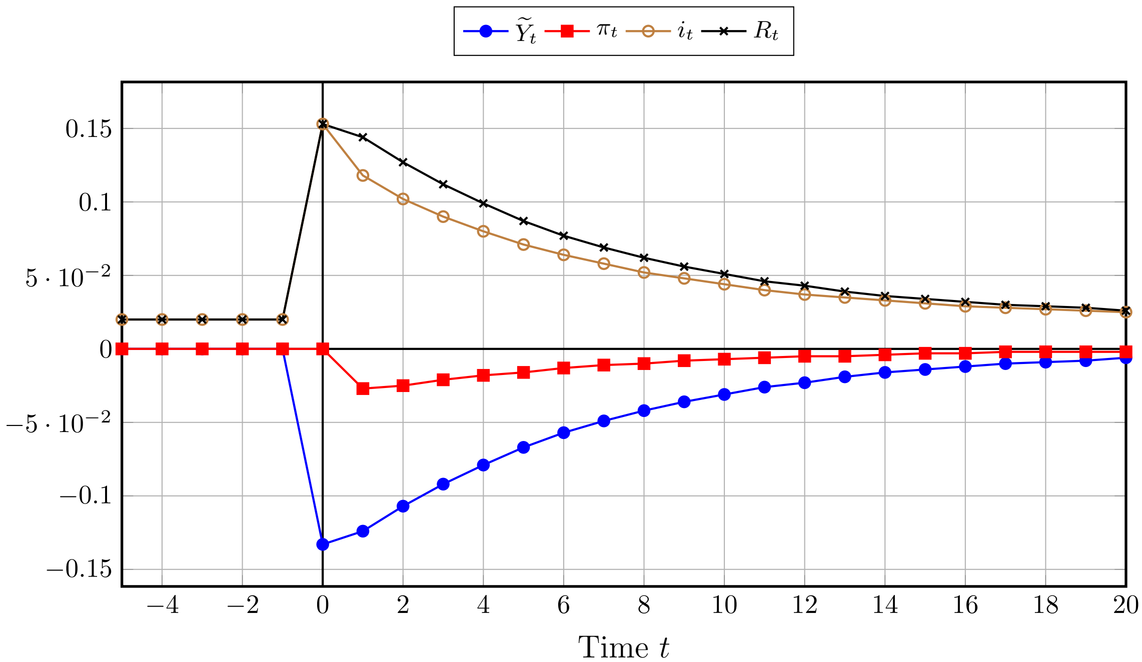
Part B

Suppose that at time 0, the money supply in this economy falls by 0.2 log points (i.e., $\Delta \log M_0 = -0.2$) and remains at that new lower level going forward (i.e., $\Delta \log M_t = 0$ for $t > 0$). Plot the evolution of output, inflation, the nominal interest rate and the real interest rate over time from period 0 to period 20. (You may want to start your figures at period -5 just to have pre-period in the figures to be able to see the response of the economy better. Also, you may ignore the zero lower bound on nominal interest rates. If you don't know what that is, this is OK. We will cover it in a few weeks.)

Solution



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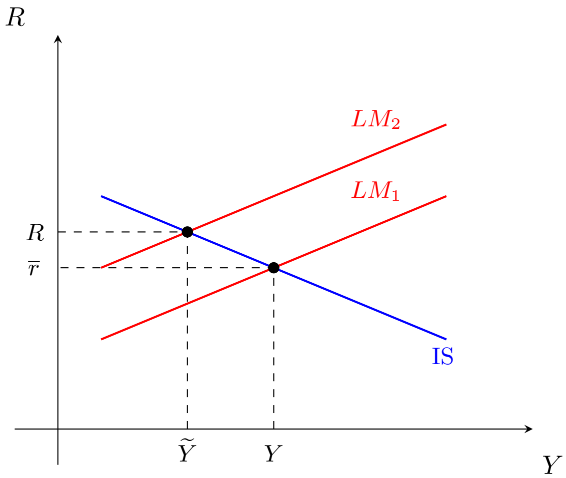
Part C

Describe the evolution of the economy in response to the shock in part B) graphically using the IS-LM diagram.

Solution



In the short run, a sudden drop in the money supply will shift the LM curve to the left, as interest rates will need to adjust to restore equilibrium conditions at all levels of output/income. As this is a short run model, over time the interest rate will return to the natural real interest rate $\overline{r}$ and the output gap will close. This was seen in the previous time series plot as well.