Macroeconomics: Problem Set #6

Due on December 8, 2024 at 10:00pm

Prof. Barnichon Section 104

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Problem 1

In this question, we explore whether government budget deficits are inflationary. In the medieval economy, we assumed that prices adjusted to a change in the money supply with a delay. To simplify things in this question, we will instead assume that prices are "perfectly flexible" — i.e., respond immediately one-for-one to any change in the money supply. In other words, we assume that $M_t = P_t$, where M_t is the nominal value of money in the economy in year t — i.e., the quantity of dollar bills outstanding — and P_t is the price level in year t. Notice that this is a simplifed version of $M_t\overline{V} = P_tY_t$, where $\overline{V} = 1$, and $Y_t = 1$.

Let's denote the <u>real value</u> of government spending on goods and services in year t by G_t and the real value tax revenue in year t by T_t. Suppose the government can issue one-year debt. Let's denote by B_t the real value of such debt issued by the government in year t to be paid off with interest in year $t+1$. Finally, let's denote the net real interest rate that the government has ot pay on its debt between year $t-1$ and year t by R_{t-1}.

The consolidated government budget constraint is then given by

$$G_t + (1 + R_{t-1})B_{t-1} = T_t + B_t + \frac{M_t - M_{t-1}}{P_t}$$

The way to interpret this equation is that the terms on the left hand side are all the things the government spends money on in year t: It purchases goods and services, and it pays off the debt it issued in year $t-1$; The terms on the right hand side are all the sources of inflow of money into the government coffers in year t: tax revenue, income from the sale of new debt, and new money issued by the central bank. The equation is written in real terms, which is why the change in the quantity of money must be divided by the price level to get how much "stuff" the new money can buy in year t.

Suppose that in year $t = 0$ the government starts off with no debt to pay off — i.e., $B_{-1} = 0$. Suppose also that the real interest rate that the government must pay on its debt is constant at 5%, i.e., $R_t = 0.05$ for all t. Furthermore, suppose the amount of money in the economy in year $t = -1$ is M_{-1}.

Suppose that the institutional setup in this economy is that there is an "independent" central bank that chooses M_t in each year and Congress chooses G_t and T_t. Jointly, these choices will then determine how much debt will need to be issued by the government in year t.

A) Suppose the central bank is determined not to allow any inflation. How should it set the money supply to achieve this?

B) Suppose that dysfunctional politics in this country implies that Congress refuses to collect enough taxes to pay for the amount of spending it decides to do. (Alternatively, one could say that it decides to spend more than the amount of taxes it decides to collect. Specifically, suppose $G_t = 0.25$, while $T_t = 0.15$. Given the monetary policy from part A), use the government budget constraint to calculate how government debt evolves over the next 30 years. It is convenient to do this in Excel.

C) Now suppose that the bond market in this country puts an upper limit on the amount that it is willing to lend to the government. Suppose this upper limit is equal to 1 — i.e., 100% of GDP. This means that government debt is constrained to be less than or equal to 1, i.e., $B_t \leq 1$. In which year will this constraint bind?

Imagine that we reach the year in which the constraint in part C) starts binding. Some adjustment is needed to balance the government's budget. But what type of adjustment? We will now see that it depends on which institution is stronger, the central bank or Congress. Let's consider each case in turn.

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D) Suppose the central bank is compeletely independent and that its commitment to no inflation is sufficiently strong that Congress cannot force the central bank to abandon its policy. Describe the adjustment Congress is forced to make in this case once the constraint binds. In particular, calculate the primary surplus, $T_t - G_t$, that Congress will need to set in the period that the constraint binds and in subsequent periods (up to period 30).

E) Suppose instead the Congress can push the central bank around and get it to change its monetary policy once the constraint binds. What will inflation be after the government's budget hits the constraint under the assumption that Congress doesn't change the primary surplus but rather forces the central bank to "monetize" its deficit spending after that point? More specifically, calculate the inflation rate in the year the constraint binds and in subsequent years up to year 30.

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Part A

Suppose the central bank is determined not to allow any inflation. How should it set the money supply to achieve this?

Solution

Since prices are perfectly flexible in this problem, to ensure that there is no inflation, the central bank should not change the money supply over time, i.e., $\boxed{M_{t+1} = M_t}$. Since $M_t = P_t$, this implies that prices won't change over time either, $P_{t+1} = P_t$, and hence no inflation.

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Part B

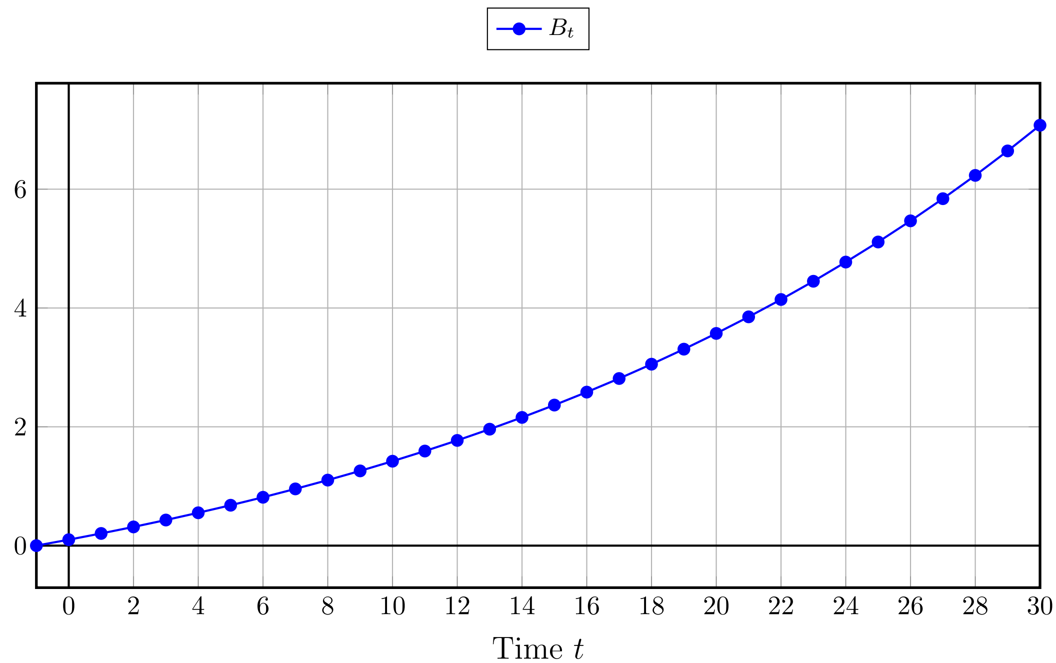
Suppose that dysfunctional politics in this country implies that Congress refuses to collect enough taxes to pay for the amount of spending it decides to do. (Alternatively, one could say that it decides to spend more than the amount of taxes it decides to collect. Specifically, suppose $G_t = 0.25$, while $T_t = 0.15$. Given the monetary policy from part A), use the government budget constraint to calculate how government debt evolves over the next 30 years.

Solution

If the money supply does not change across time, the government budget constraint becomes the following

$$G_t + (1 + R_{t-1})B_{t-1} = T_t + B_t + \frac{M_t - M_{t-1}}{P_t}$$
$$G_t + (1 + R_{t-1})B_{t-1} = T_t + B_t + \frac{0}{P_t}$$
$$G_t + (1 + R_{t-1})B_{t-1} - T_t = B_t$$

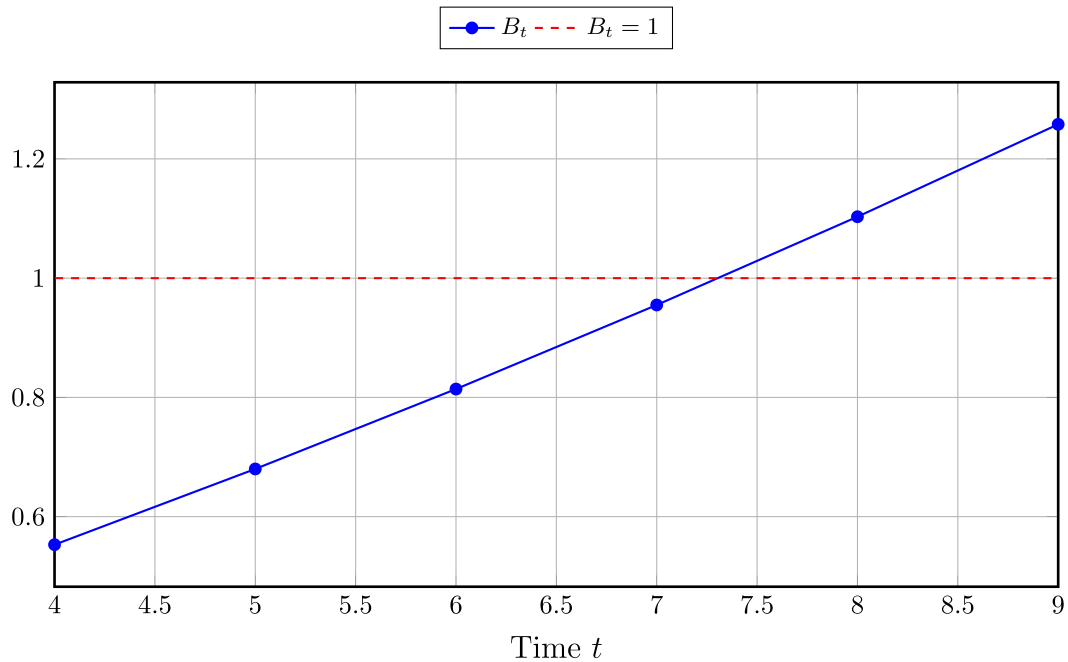
So the government debt evolves according to $B_t = G_t + (1 + R_{t-1})B_{t-1} - T_t$ over the next 30 years, which is plotted below starting at $t = -1$ where $G_t = 0.25$, $R_t = 0.05$, and $T_t = 0.15$



Part C

Now suppose that the bond market in this country puts an upper limit on the amount that it is willing to lend to the government. Suppose this upper limit is equal to 1 — i.e., 100% of GDP. This means that government debt is constrained to be less than or equal to 1, i.e., $B_t \leq 1$. In which year will this constraint bind?

Solution



I've replotted above an enhanced version of the graph from the last part with a horizontal line at the $B_t = 1$ level, denoting the upper limit for the bond market. From the plot, it appears the upper limit is crossed after $\boxed{\text{year 7}}$ going into year 8.

Part D

Suppose the central bank is compeletely independent and that its commitment to no inflation is sufficiently strong that Congress cannot force the central bank to abandon its policy. Describe the adjustment Congress is forced to make in this case once the constraint binds. In particular, calculate the primary surplus, $T_t - G_t$, that Congress will need to set in the period that the constraint binds and in subsequent periods (up to period 30).

Solution

If Congress cannot force the central bank to abandon its policy, the government will have to run a surplus such that $B_t = B_{t-1}$. I'll solve for the value of $T_t - G_t$ that satisfies this constraint

$$\begin{aligned}
G_t + (1 + R_{t-1})B_{t-1} - T_t &= B_t \\
G_t - T_t &= B_t - (1 + R_{t-1})B_{t-1} \\
G_t - T_t &= B_t - (1 + R_{t-1})B_t \\
G_t - T_t &= B_t(1 - (1 + R_{t-1})) \\
G_t - T_t &= -B_t R_{t-1} \\
T_t - G_t &= B_t R_{t-1}
\end{aligned}$$

The primary surplus required going forward is the product of the debt and the interest rate for each period, $\boxed{T_t - G_t = B_t R_{t-1}}$.

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Part E

Suppose instead the Congress can push the central bank around and get it to change its monetary policy once the constraint binds. What will inflation be after the government's budget hits the constraint under the assumption that Congress doesn't change the primary surplus but rather forces the central bank to "monetize" its deficit spending after that point? More specifically, calculate the inflation rate in the year the constraint binds and in subsequent years up to year 30.

Solution

If the central bank is forced to monitize the debt, that means that the money supply needs to change in a way so that $(M_t - M_{t-1})/P_t$ balances the government budget constraint so that $B_t = B_{t-1}$. Since prices are perfectly flexible in this problem, $M_t = P_t$, we can rewrite this real money supply change as inflation since $\pi_t = (P_t - P_{t-1})/P_t$

$$
\begin{aligned}
G_t + (1 + R_{t-1})B_{t-1} &= T_t + B_t + \frac{M_t - M_{t-1}}{P_t} \\
G_t + (1 + R_{t-1})B_t &= T_t + B_t + \frac{P_t - P_{t-1}}{P_t} \\
G_t + (1 + R_{t-1})B_t - B_t - T_t &= \pi_t \\
G_t + B_t((1 + R_{t-1}) - 1) - T_t &= \pi_t \\
G_t + B_t R_{t-1} - T_t &= \pi_t
\end{aligned}
$$

Now substituing in values for G_t, R_t, and T_t

$$
\begin{aligned}
G_t + B_t R_{t-1} - T_t &= \pi_t \\
0.25 + 0.05B_t - 0.15 &= \pi_t \\
0.10 + 0.05B_t &= \pi_t
\end{aligned}
$$

So inflation will equal $\boxed{\pi_t = 0.10 + 0.05B_t}$.

Problem 2

Consider the business cycle model with the monetary policy rule we saw in class (see also chapter 13 of the Jones textbook):

Monetary policy rule: $R_t - \overline{r} = \overline{m}(\pi_t - \overline{\pi})$

IS curve: $\widetilde{Y}_t = \overline{a}_t - \overline{b}(R_t - \overline{r})$

Phillips $\pi_t = \pi_{t-1} + \overline{v}\widetilde{Y}_t + \overline{o}_t$

Here we allow the aggregate demand and cost-push shocks to vary with time (so, they have time subscripts).

A) Combine the monetary policy rule and the IS curve to derive an "aggregate demand" equation for this model.

B) This leaves two equations and two unknown endogenous variables (output gap and inflation). Solve these two equations, i.e., use these two equations to express the endogenous variables as functions of only past endogenous variables, exogenous variables, and parameters.

C) Use the result from part B) and the original monetary policy rule to express the real interest rate as a function of only past endogenous variables, exogenous variables, and parameters.

Suppose $\overline{r} = 3\%$, $\overline{\pi} = 2\%$, $\overline{b} = 1/3$, $\overline{m} = 1$, $\overline{v} = 1$. Suppose furthermore that $\pi_{-1} = 2\%$. Suppose $\overline{a}_t$ and $\overline{o}_t$ are zero at all times unless otherwise noted.

D) Calculate the values of inflation, the output gap, and real interest rate in period 0.

E) Now suppose the economy is hit by a positive aggregate demand shock that lasts for 3 periods. More specifically, suppose $\overline{a}_1 = \overline{a}_2 = \overline{a}_3 = 2\%$. After period 3, the aggregate demand shock returns to 0. Using the three equation model, calculate the evolution of inflation, the output gap, and the real interest rate form period 1 to period 10. Plot the resulting time series for these three variables. You may find it useful to do this in Excel.

F) Starting form the point calculated for period 0 in part D), suppose the economy is instead hit by a positive aggregate supply shock that lasts for 3 periods. More specifically, suppose $\overline{o}_1 = \overline{o}_2 = \overline{o}_3 = 2\%$. After period 3, the aggregate supply shock returns to 0. Using the three equation model, calculate the evolution of inflation, the output gap, and the real interest rate from period 1 to period 10. Plot the time series for these three variables.

G) Comment on the difference between the response of the economy to an aggregate demand schock and an aggregate supply shock.

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Part A

Combine the monetary policy rule and the IS curve to derive an "aggregate demand" equation for this model.

Solution

To find the aggregate demand equation for this model I will substitute in the monetary policy rule equation for R_t in the IS curve equation to obtain an equation for the output gap in terms of inflation

$$\widetilde{Y}_t = \overline{a}_t - \overline{b}(R_t - \overline{r})$$
$$\widetilde{Y}_t = \overline{a}_t - \overline{b}(\overline{m}(\pi_t - \overline{\pi}) + \overline{r} - \overline{r})$$
$$\widetilde{Y}_t = \overline{a}_t - \overline{b}\overline{m}(\pi_t - \overline{\pi})$$

This curve describes the aggregate demand mechanism. When inflation increases, the central bank raises real rates according to the monetary policy rule, resulting in a decline in consumption and investment, thereby decreasing output, and the output gap.

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Part B

This leaves two equations and two unknown endogenous variables (output gap and inflation). Solve these two equations, i.e., use these two equations to express the endogenous variables as functions of only past endogenous variables, exogenous variables, and parameters.

Solution

To solve for the endogenous variable of inflation, I will substitute in the equation I derived for the aggregate demand into the Phillips curve equation.

$$\begin{aligned}
\pi_t &= \pi_{t-1} + \overline{v}\widetilde{Y}_t + \overline{o}_t \\
\pi_t &= \pi_{t-1} + \overline{v}(\overline{a}_t - \overline{b}\overline{m}(\pi_t - \overline{\pi})) + \overline{o}_t \\
\pi_t &= \pi_{t-1} + \overline{v}\overline{a}_t - \overline{v}\overline{b}\overline{m}\pi_t + \overline{v}\overline{b}\overline{m}\overline{\pi} + \overline{o}_t \\
\pi_t + \overline{v}\overline{b}\overline{m}\pi_t &= \pi_{t-1} + \overline{v}\overline{a}_t + \overline{v}\overline{b}\overline{m}\overline{\pi} + \overline{o}_t \\
\pi_t(1 + \overline{v}\overline{b}\overline{m}) &= \pi_{t-1} + \overline{v}\overline{a}_t + \overline{v}\overline{b}\overline{m}\overline{\pi} + \overline{o}_t \\
\pi_t &= \frac{\pi_{t-1} + \overline{v}\overline{a}_t + \overline{v}\overline{b}\overline{m}\overline{\pi} + \overline{o}_t}{1 + \overline{v}\overline{b}\overline{m}}
\end{aligned}$$

Now to solve for the output gap I can substitute in this equation for the inflation into the aggregate demand equation I found earlier

$$\begin{aligned}
\widetilde{Y}_t &= \overline{a}_t - \overline{b}\overline{m}(\pi_t - \overline{\pi}) \\
\widetilde{Y}_t &= \overline{a}_t - \overline{b}\overline{m}(\frac{\pi_{t-1} + \overline{v}\overline{a}_t + \overline{v}\overline{b}\overline{m}\overline{\pi} + \overline{o}_t}{1 + \overline{v}\overline{b}\overline{m}} - \overline{\pi})
\end{aligned}$$

Part C

Use the result from part B) and the original monetary policy rule to express the real interest rate as a function of only past endogenous variables, exogenous variables, and parameters.

Solution

To solve for the real interest rate as a function of only past endogenous variables, exogenous variables, and parameters, I will again substitute in the equation I found for inflation but this time into the monetary policy rule equation

$$R_t - \overline{r} = \overline{m}(\pi_t - \overline{\pi})$$
$$R_t - \overline{r} = \overline{m}(\frac{\pi_{t-1} + \overline{v}\overline{a}_t + \overline{v}\overline{b}\overline{m}\overline{\pi} + \overline{o}_t}{1 + \overline{v}\overline{b}\overline{m}} - \overline{\pi})$$
$$R_t = \overline{r} + \overline{m}(\frac{\pi_{t-1} + \overline{v}\overline{a}_t + \overline{v}\overline{b}\overline{m}\overline{\pi} + \overline{o}_t}{1 + \overline{v}\overline{b}\overline{m}} - \overline{\pi})$$

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Part D

Suppose $\overline{r} = 3\%$, $\overline{\pi} = 2\%$, $\overline{b} = 1/3$, $\overline{m} = 1$, $\overline{v} = 1$. Suppose furthermore that $\pi_{-1} = 2\%$. Suppose $\overline{a}_t$ and $\overline{o}_t$ are zero at all times unless otherwise noted.

Calculate the values of inflation, the output gap, and real interest rate in period 0.

Solution

First to solve for inflation at time $t = 0$, I will substitute in the given values into the equation I found for inflation in part B)

$$\pi_t = \frac{\pi_{t-1} + \overline{v}\overline{a}_t + \overline{v}\overline{b}\overline{m}\overline{\pi} + \overline{o}_t}{1 + \overline{v}\overline{b}\overline{m}}$$
$$\pi_0 = \frac{\pi_{-1} + \overline{v}\overline{a}_0 + \overline{v}\overline{b}\overline{m}\overline{\pi} + \overline{o}_0}{1 + \overline{v}\overline{b}\overline{m}}$$
$$\pi_0 = \frac{2\% + 1 \cdot 0 + 1 \cdot \frac{1}{3} \cdot 1 \cdot 2\% + 0}{1 + 1 \cdot \frac{1}{3} \cdot 1}$$
$$\pi_0 = (2\% + \frac{2}{3}\%) \cdot \frac{3}{4}$$
$$\pi_0 = \frac{3}{2}\% + \frac{1}{2}\% = 2\%$$

So inflation in period zero equals $\boxed{\pi_0 = 2\%}$. To find the output gap, I will now substitute in this value for π_0 into the aggregate demand equation

$$\widetilde{Y}_t = \overline{a}_t - \overline{b}\overline{m}(\pi_t - \overline{\pi})$$
$$\widetilde{Y}_0 = \overline{a}_0 - \overline{b}\overline{m}(\pi_0 - \overline{\pi})$$
$$\widetilde{Y}_0 = 0 - 1/3 \cdot 1 \cdot (2\% - 2\%)$$
$$\widetilde{Y}_0 = 0\%$$

So output is at potential at time period 0, i.e., $\boxed{\widetilde{Y}_0 = 0\%}$. To find the real interest rate at time period 0, I will again substitue in the value I found for π_0 but now into the equation for the monetary policy rule

$$R_t - \overline{r} = \overline{m}(\pi_t - \overline{\pi})$$
$$R_0 = \overline{r} + \overline{m}(\pi_0 - \overline{\pi})$$
$$R_0 = 3\% + 1 \cdot (2\% - 2\%)$$
$$R_0 = 3\%$$

So the real interest rate in period 0 equals the natural rate $\boxed{R_0 = 3\%}$.

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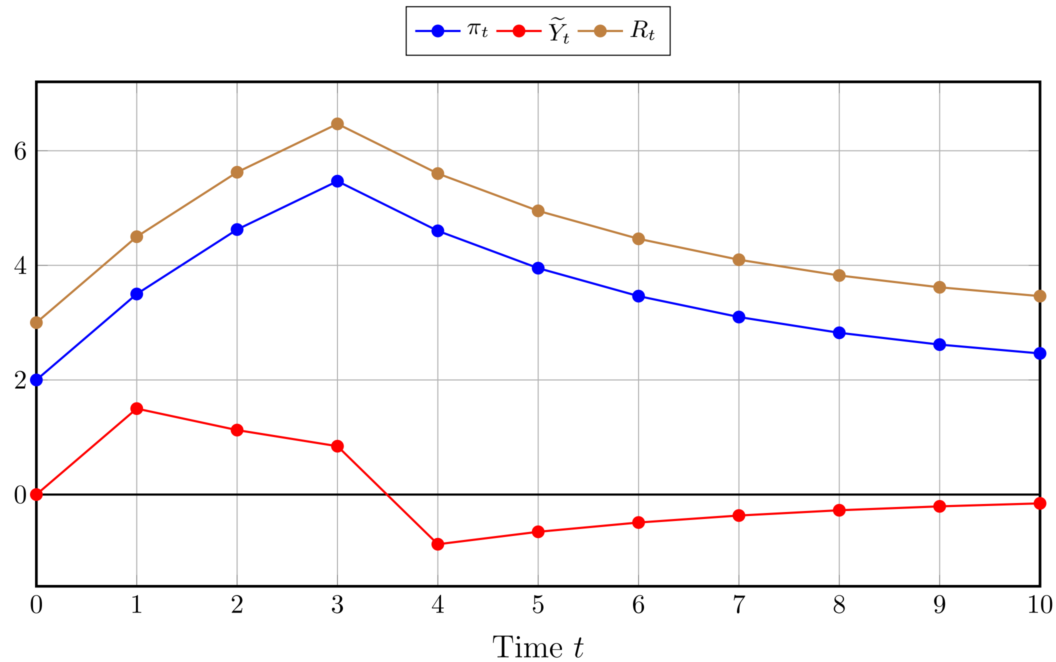
Part E

Now suppose the economy is hit by a positive aggregate demand shock that lasts for 3 periods. More specifically, suppose $\overline{a}_1 = \overline{a}_2 = \overline{a}_3 = 2\%$. After period 3, the aggregate demand shock returns to 0. Using the three equation model, calculate the evolution of inflation, the output gap, and the real interest rate form period 1 to period 10. Plot the resulting time series for these three variables.

Solution



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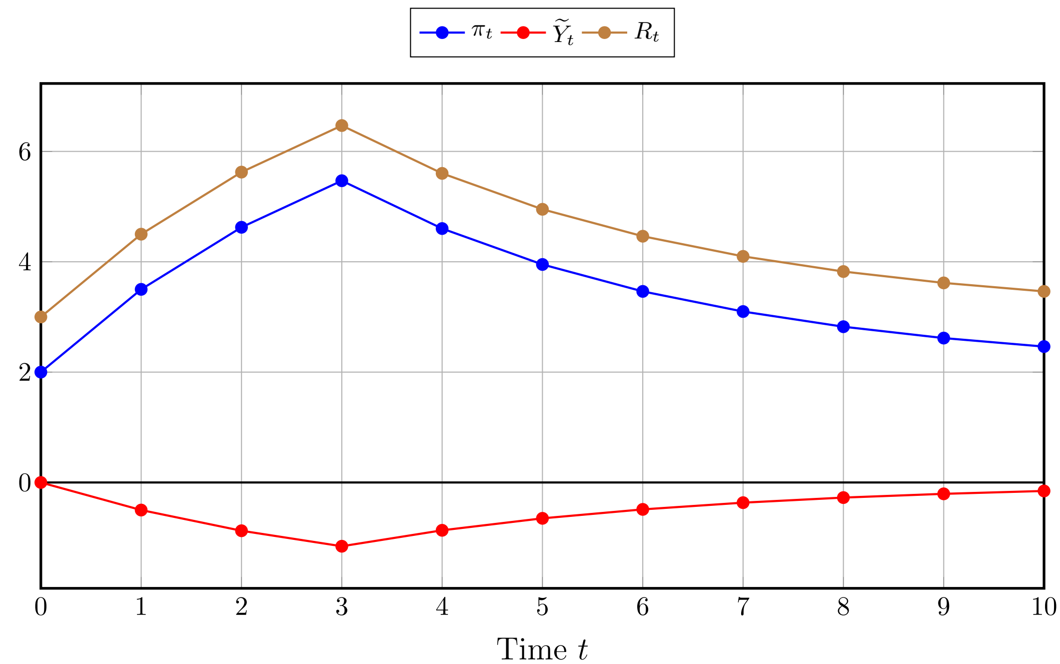
Part F

Starting form the point calculated for period 0 in part D), suppose the economy is instead hit by a positive aggregate supply shock that lasts for 3 periods. More specifically, suppose $\bar{o}_1 = \bar{o}_2 = \bar{o}_3 = 2\%$. After period 3, the aggregate supply shock returns to 0. Using the three equation model, calculate the evolution of inflation, the output gap, and the real interest rate from period 1 to period 10. Plot the time series for these three variables.

Solution



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Part G

Comment on the difference between the response of the economy to an aggregate demand schock and an aggregate supply shock.

Solution

The economy's response to the aggregate demand shock is initially a positive, yet decreasing output gap for the duration of the aggregate demand shock. Inflation and interest rates increase duing this period in response to the increased demand. However, after the demand shock ends, the output gap becomes negative as the elevated demand can no longer lift output above potential amidst high rates. Over time, inflation, the output gap, and real rates all return to their steady state levels.

The economy's response to the aggregate supply shock is similar in terms of inflation and real rates, however, the effect on the output gap is not the same. The output gap is negative and increasing in magnitude for the duration of the aggregate supply shock. All variables eventually return to their steady state levels over time after the supply shock ends.