Macroeconomics: Problem Set #4

Due on November 12, 2024 at 1:00pm

Prof. Barnichon Section 104

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Problem 1

Consider the model of the medieval economy discussed in class:

$$\log M_t + \log V = \log P_t + \log Y_t$$

$$\log P_{t+1} - \log P_t = \theta(\log Y_t - \log Y^*)$$

A) Suppose $V = 1$ and $Y^* = 1$. Calculate the steady state of this economy when $\log M_t = 1$.

B) Suppose that time is measured in years and the economy is in the steady calculate in part A) at time $t = -1$. Suppose that at time $t = 0$ Vikings bring back a boatload of gold coins that raises the money supply to $\log M_0 = 3$. Suppose that the money supply remains constant at this level for the next 20 years. Suppose that $\theta = 0.25$. Trace out the dynamics of the logarithm of the price level and the logarithm of output over these 20 years using the two equations from part A). Plot the resulting "time series" for logarithm of output, the logarithm of the price level, and the logarithm of the money supply from $t = -1$ to $t = 20$ (i.e. plot each variable as a function of time).

C) Now suppose that $\theta = 0.5$. Trace out the dynamics in this case. Again plot the results as in the previous part.

D) Comment on the transiton dynamics and the difference between the two cases. Relate to the concept of the half-life (one paragraph).

Problem 1 continued on next page. . .

Part A

Suppose $V = 1$ and $Y^* = 1$. Calculate the steady state of this economy when $\log M_t = 1$.

Solution

In the long run the economy will reach a steady state where the variables do not change across periods. In this case $P_{t+1} = P_t = P$, and we can subtsitute in our values for V, Y^*, and $\log M_t$ into the given equations.

$$\begin{aligned}
\log P_{t+1} - \log P_t &= \theta(\log Y_t - \log Y^*) \\
0 &= \theta(\log Y - \log(1)) \\
0 &= \log Y \\
Y &= 1
\end{aligned}$$

$$\begin{aligned}
\log M_t + \log V &= \log P_t + \log Y_t \\
1 + \log(1) &= \log P + \log Y \\
1 &= \log P + \log Y \\
\log P &= 1
\end{aligned}$$

From the above we can see that the steady state value of Y is 1, the same value as the desired level of output Y^*. The log of the price level is also 1, which is the same value as the log of the money supply. This suggests that there is a long-run monetary neutrality, in that prices and money supply change proportionally, but leave output unaffected.

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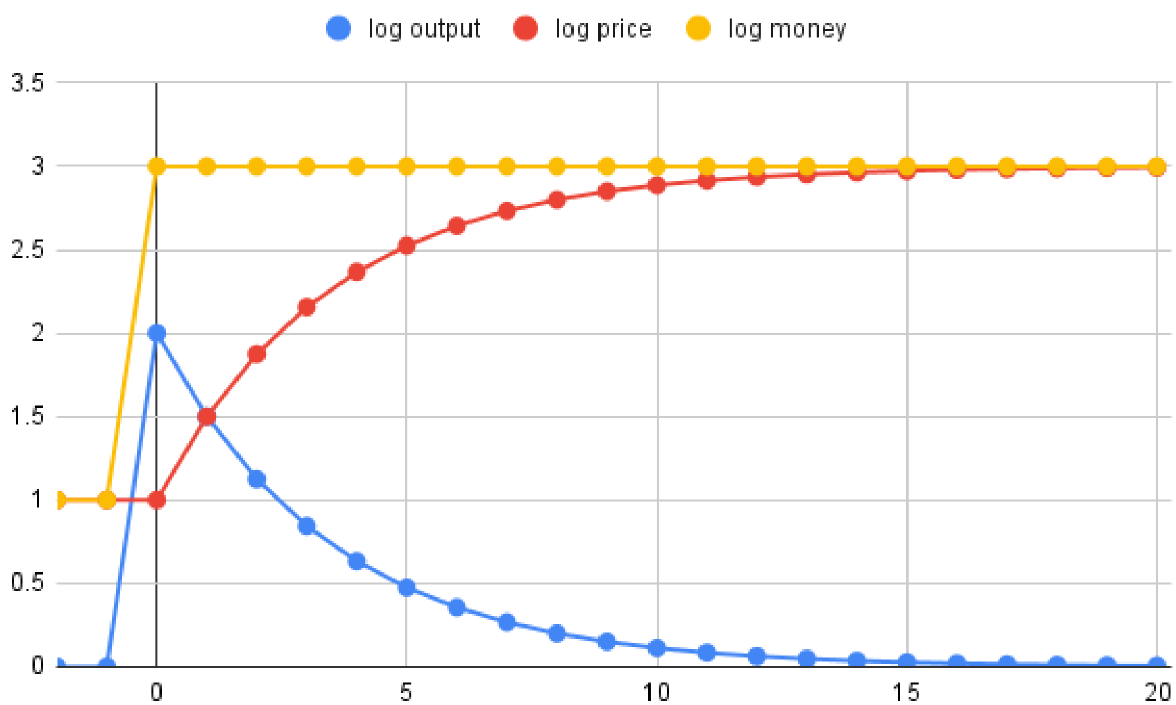
Part B

Suppose that time is measured in years and the economy is in the steady calculate in part A) at time $t = -1$. Suppose that at time $t = 0$ Vikings bring back a boatload of gold coins that raises the money supply to $\log M_0 = 3$. Suppose that the money supply remains constant at this level for the next 20 years. Suppose that $\theta = 0.25$. Trace out the dynamics of the logarithm of the price level and the logarithm of output over these 20 years using the two equations from part A). Plot the resulting "time series" for logarithm of output, the logarithm of the price level, and the logarithm of the money supply from $t = -1$ to $t = 20$ (i.e. plot each variable as a function of time).

Solution



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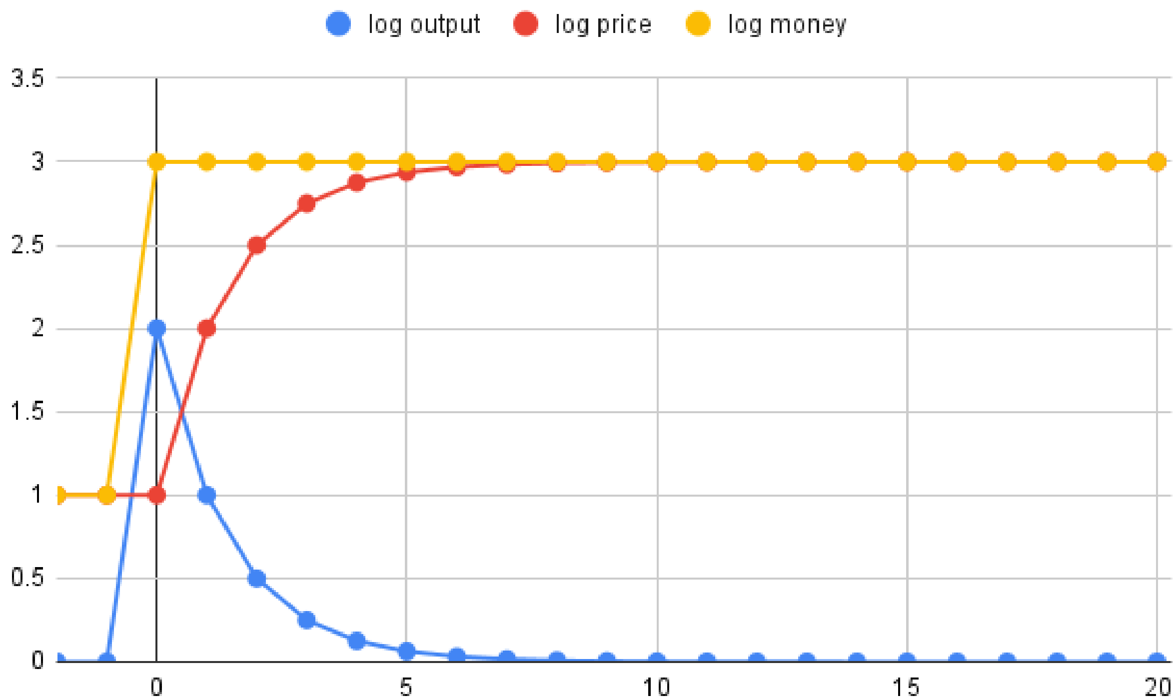
Part C

Now suppose that $\theta = 0.5$. Trace out the dynamics in this case. Again plot the results as in the previous part.

Solution



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Part D

Comment on the transiton dynamics and the difference between the two cases. Relate to the concept of the half-life (one paragraph).

Solution

Steady state logarithms of output and price are reached quicker in the second case where the speed of the price adjustment, θ, is equal to 0.5, whereas in the first case it is 0.25. The half-life is the time required for, in this case, the logarithms of output and price to reduce to half their value. In the second case, the half-life is then naturally shorter. This makes sense considering that, if prices take less time to adjust, suppliers will adjust prices quicker to catch up with inflated demand, thereby reducing output quicker as well.

Problem 2

Consider again the model of the medieval economy discussed in class:

$$\log M_t + \log V = \log P_t + \log Y_t$$

$$\log P_{t+1} - \log P_t = \theta(\log Y_t - \log Y^*)$$

We would like to rewrite the model in terms of inflation and the "output gap"—i.e., the gap between actual output and steady state ouput. We denote inflation by π_t. Recall that inflation is defined as $(P_{t+1} - P_t)/P_t$. For inflation rates close to zero the following approximation is quite accurate $\pi_t \approx \log(1+\pi_t) = \log(P_t/P_{t-1})$. Please make use of this as needed in solving the problem. We denote the output gap by $\tilde{Y}_t = \log Y_t - \log Y^*$.

A) Using this notation, show that the medieval economy model can be rewritten as:

$$\text{AD:} \quad \Delta \log M_t = \pi_t + \widetilde{Y}_t - \widetilde{Y}_{t-1}$$

$$\text{SRAS:} \quad \pi_t = \theta \widetilde{Y}_{t-1}$$

where AD stands for "aggregate demand" and SRAS stands for "short-run aggregate supply." These labels will be explained in class.

In the simplest version of the medieval economy, the money supply is exogenously given by, say, the number of gold coins in the economy. Suppose now that the government starts issuing paper money and can thus change the supply of the money at will. For simplicity, suppose the economy is in a steady state with zero inflation at time 0. In other words, $\pi_0 = 0$ and $\widetilde{Y}_0 = 0$. Also, assume that $\theta = 0.25$.

B) By an unfortunate turn of events, a crazy person takes over as the chairman of the central bank in our medieval economy with paper money. Suppose this person decides to raise the money supply by one log unit every odd period and reduce it by one log unit every even period. In other words $\Delta \log M_t = 1$ in odd periods and $\Delta \log M_t = -1$ in even periods. Solve for the dynamics of the output gap and inflation for 20 periods in this case.

C) The dynamics that come out of part (b) may strike you as strange. Do, you think this is actually what would happen if such a crazy person were to take over at the central bank? Remember that the logic of the model is that the reason why output deviates from desired output is that producers are surprised by changes in the stock of money. We motivated this assumption by the notion that in the medieval economy without paper money the money supply rarely changed and any changes were true surprises. Remembering that the producers are trying to achieve a zero output gap, explain how their behavior may eventually differ from the behavior embodied in the SRAS equation above.

Now suppose that the crazy central banker is thrown out of office (this is not the answer I am looking for in part C)) and the money supply doesn't change for a while so that economy again settles down to the steady state with zero inflation. At that point, a new central banker is hired and (s)he decides to start steadily increasing the money supply. More specifically, suppose the money supply grows at a constant rate $\Delta \log M_t = \Delta \log M$.

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D) Assuming that the behavior of the people in the economy is well described by the AD and SRAS equations above, what is the steady state value of the output gap and inflation for this economy as a function of $\Delta \log M$?

E) Again assuming that the behavior of the people in the economy is well described by the AD and SRAS equations above, plot the relationship between steady state inflation and the steady state output gap for different constant values of money growth with inflation on the vertical axis and the output gap on the horizontal axis. We refer to this relationship as the "long run aggregate supply" (LRAS) relationship in the medieval model.

F) The following quote is attributed to Abraham Lincoln: "You can fool some of the people all of the time, and all of the people some of the time, but you cannot fool all of the people all of the time." Discuss the realism of the price setting behavior that underlies the SRAS curve in light of the LRAS curve derived above and Lincoln's quote.

Part A

Show that the medieval economy model can be rewritten as:

$$\begin{aligned} &\text{AD:} \quad \Delta \log M_t = \pi_t + \widetilde{Y}_t - \widetilde{Y}_{t-1} \\ &\text{SRAS:} \quad \pi_t = \theta \widetilde{Y}_{t-1} \end{aligned}$$

where AD stands for "aggregate demand" and SRAS stands for "short-run aggregate supply." These labels will be explained in class.

Solution

First, I will derive the equation for aggregate demand using the first equation by taking a difference in the logarithm of the money supply between time t and $t-1$

$$\begin{aligned} \log M_t + \log V &= \log P_t + \log Y_t \\ \log M_t &= \log P_t + \log Y_t - \log V \\ \log M_t - \log M_{t-1} &= \log P_t - \log P_{t-1} + \log Y_t - \log Y_{t-1} \\ \Delta \log M_t &= \log \frac{P_t}{P_{t-1}} + \log Y_t - \log Y^* - (\log Y_{t-1} - \log Y^*) \\ \Delta \log M_t &= \pi_t + \widetilde{Y}_t - \widetilde{Y}_{t-1} \end{aligned}$$

For short-run aggregate supply, I will reindex the second equation given in the specification and rewrite for inflation as a function of the output gap

$$\begin{aligned} \log P_{t+1} - \log P_t &= \theta(\log Y_t - \log Y^*) \\ \log P_t - \log P_{t-1} &= \theta(\log Y_{t-1} - \log Y^*) \\ \log \frac{P_t}{P_{t-1}} &= \theta(\widetilde{Y}_{t-1}) \\ \pi_t &= \theta \widetilde{Y}_{t-1} \end{aligned}$$

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Part B

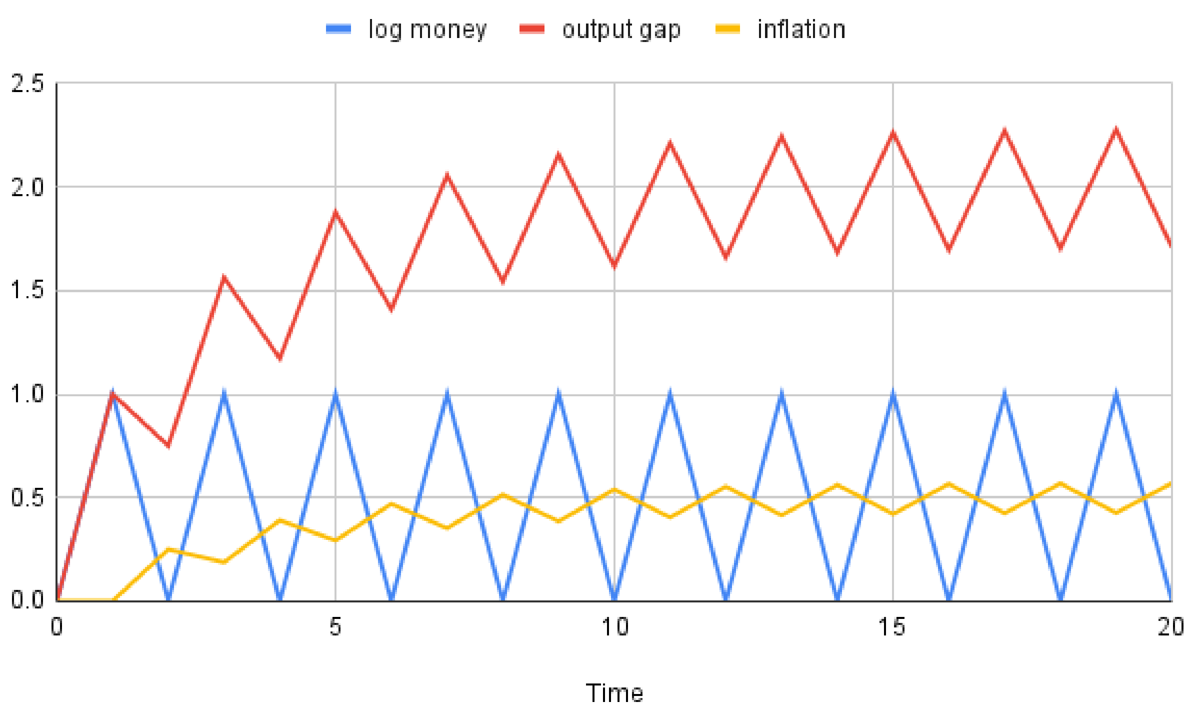
By an unfortunate turn of events, a crazy person takes over as the chairman of the central bank in our medieval economy with paper money. Suppose this person decides to raise the money supply by one log unit every odd period and reduce it by one log unit every even period. In other words $\Delta \log M_t = 1$ in odd periods and $\Delta \log M_t = -1$ in even periods. Solve for the dynamics of the output gap and inflation for 20 periods in this case.

Solution

To solve for the output gap and inflation across these 20 time periods, I will rewrite the aggregate demand equation in terms of the output gap and then plot the log money supply, the output gap, and inflation over time

$$\Delta \log M_t = \pi_t + \widetilde{Y}_t - \widetilde{Y}_{t-1}$$
$$\widetilde{Y}_t = \Delta \log M_t - \pi_t + \widetilde{Y}_{t-1}$$

Given that the log money supply, the output gap, and inflation all start at zero, below is a plot of their dynamics over 20 time periods when θ is equal to 0.25



Part C

The dynamics that come out of part (b) may strike you as strange. Do, you think this is actually what would happen if such a crazy person were to take over at the central bank? Remember that the logic of the model is that the reason why output deviates from desired output is that producers are surprised by changes in the stock of money. We motivated this assumption by the notion that in the medieval economy without paper money the money supply rarely changed and any changes were true surprises. Remembering that the producers are trying to achieve a zero output gap, explain how their behavior may eventually differ from the behavior embodied in the SRAS equation above.

Solution

If the changes in the monetary supply and inflation were as predictable as seen in the plot, producers will probably not suddenly continuously be shocked such that their output gap jumps up and down forever. Instead, they will adjust their expectations accordingly to minimize sudden deviations from their preferred output.

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Part D

Now suppose that the crazy central banker is thrown out of office (this is not the answer I am looking for in part C)) and the money supply doesn't change for a while so that economy again settles down to the steady state with zero inflation. At that point, a new central banker is hired and (s)he decides to start steadily increasing the money supply. More specifically, suppose the money supply grows at a constant rate $\Delta \log M_t = \Delta \log M$.

Assuming that the behavior of the people in the economy is well described by the AD and SRAS equations above, what is the steady state value of the output gap and inflation for this economy as a function of $\Delta \log M$?

Solution

In a steady state, time-dependent variables don't change across time periods, so $\widetilde{Y}_t = \widetilde{Y}_{t-1}$, and if we know that the money supply grows at a constant rate, we can substitute int our aggregate demand and short-run aggregate supply equations

$$\begin{aligned}
\Delta \log M_t &= \pi_t + \widetilde{Y}_t - \widetilde{Y}_{t-1} \\
\Delta \log M &= \pi + \widetilde{Y} - \widetilde{Y} \\
\Delta \log M &= \pi
\end{aligned}$$

And now using our short-run aggregate supply equation to solve for the output gap, substituting in $\Delta \log M$ in for inflation π

$$\begin{aligned}
\pi_t &= \theta \widetilde{Y}_{t-1} \\
\pi &= \theta \widetilde{Y} \\
\log M &= \theta \widetilde{Y} \\
\frac{\log M}{\theta} &= \widetilde{Y}
\end{aligned}$$

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Part E

Again assuming that the behavior of the people in the economy is well described by the AD and SRAS equations above, plot the relationship between steady state inflation and the steady state output gap for different constant values of money growth with inflation on the vertical axis and the output gap on the horizontal axis. We refer to this relationship as the "long run aggregate supply" (LRAS) relationship in the medieval model.

Solution

Below is a plot of steady state inflation over steady state output gap, relating the two across many different growths of the money supply. The plot is linear, with a slope of $d\pi/d\widetilde{Y} = \theta$

Problem 2 continued on next page. . .

Part F

The following quote is attributed to Abraham Lincoln: "You can fool some of the people all of the time, and all of the people some of the time, but you cannot fool all of the people all of the time." Discuss the realism of the price setting behavior that underlies the SRAS curve in light of the LRAS curve derived above and Lincoln's quote.

Solution

The quote in the context of our discussion highlights the short-run nature of increased output due to monetary policy. Although with some constant inflation it is possible to maintain an output gap, in the long run, steady state output is not determined by the money supply in an economy. And if prices were able to adjust instantly to the money supply, it would be impossible to maintain elevated output by tweaking the money supply.