ECON C103: Problem Set #1

Due on January 30, 2026 at 11:59pm

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Problem 1

A group of people met and some of them shook each-other's hands. Prove that the number of people who shook others' hands an odd number of times is, in fact, even.

Solution

Problem 2

Let X be a nonempty set, and let 2^X denote the set of all subsets of X. Show that there is no onto (aka surjective) function $f : X \mapsto 2^X$. (The function f is surjective if for every set $A \in 2^X$ there is an $x \in X$ such that $f(x) = A$.)

Solution

Problem 3

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a twice-differentiable function. Show that $\frac{\partial^2}{\partial x \partial y} f \geq 0$ if and only if for every $x, x', y, y' \in \mathbb{R}$:

$$x' > x \text{ and } y' > y \Rightarrow f(x', y') - f(x, y') \geq f(x', y) - f(x, y).$$

Solution

Problem 4

A pair $G = (V, E)$ is a *directed graph* if V is a set and $E \subset V \times V$. The graph G has a cycle if there exists an integer $n \geq 1$ and elements $v_1, \ldots, v_n \in V$ such that $(v_1, v_2), (v_2, v_3), \ldots, (v_{n-1}, v_n), (v_n, v_1) \in E$. Show that if V is a non-empty finite set and for all $v \in V$ there is $v' \in V$ such that $(v, v') \in E$, then G has a cycle.

Solution

Problem 5

Consider an $n \times n$ chessboard (the figure below shows an example with $n = 8$).
Show that if n is a power of 2 it is possible to cover the board with L-shaped trominoes (consisting of three squares each, shown in blue) so that all but one of the squares on the board is covered.

Solution